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Single photo-ionization of He—a QED approach

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Abstract : Present paper deals with the field theoretic study of single photo-ionization (SPI) cross sections σ_{ph} and σ_c of helium due to photoelectric effect (PEE) and Compton scattering (CS) respectively, by photons of energy lying between 0–12 keV. In this energy range, σ_{ph} gradually decreases with energy while σ_c gradually increases. There is a crossover point around 6.5 keV Near about this crossover point, graphs of σ_{ph} and σ_c are like mirror images. For photon energy below the crossover point, $\sigma_{ph} > \sigma_c$ while above that point $\sigma_c > \sigma_{ph}$ The ratio σ_c/σ_{ph} lies between 0.3 to 1.5 in the energy range under consideration Present results are compared with existing theoretical results

Keywords Single ionization, photoelectric effect, Compton effect, Feynman diagram

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1. Introduction

We have calculated here SPI of helium by both photoelectric effect (PEE) and Compton scattering (CS) using field theory, Feynman diagram and covariant Lorentz gauge [1–4]. Second order Feynman diagram (Figure 1) represents SPI due to PEE, where the high energy photon interacts with the bound-electron cloud and ionizes it leaving only the He^+ ion in 1*s*-state. In the case of CS there are two Feynman diagrams Figure 2a and Figure 2b, corresponding to direct and exchange interactions respectively. Contributions from the direct and exchange diagrams in CS are very small as compared to the contribution from the

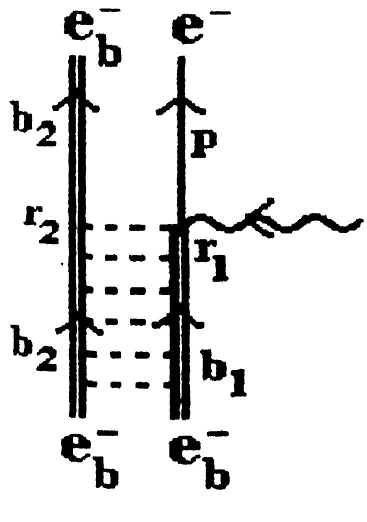

Figure 1. Feynman diagram for photoionization. Double line represents bound electron, single line represents free electron, broken line represent Coulomb photon, wavy line with arrow represents photon.

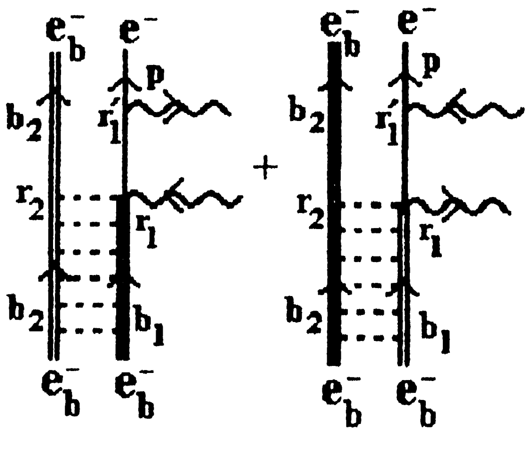

Figure 2. Feynman diagram for Compton scattering, same as in Figure 1. (a) direct scattering, (b) exchange scattering.

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interference of the two diagrams. In our calculations the cross sections due to PEE gradually decreases with energy of the incident photon, while those due to CS increases. There is a crossover point near 6.5 KeV, where cross sections for both the processes are same. It is interesting to note that Compton curve is mirror image of the photoelectric curve about this crossover point. Although the present results on σ_{ph} deviates slightly from the existing theoretical results [2,8], a good agreement is obtained in the case of σ_c [3].

2. Mathematical formalism

2.1. Photo-electric effect :

The reaction for SPI by PEE is

$$He(1s^2) + h\nu \rightarrow He^+(1s) + e. \qquad (1)$$

The amplitude for the process is obtained from the first order Feynman diagram Figure 1 and is given by R_p [2].

$$R_p = \bar{\Psi}_f(r_1, r_2, R)\gamma_\mu A_\mu(r_1, k)\Psi_i(r_1, r_2, R) \qquad (2)$$

where γ_μ is the Dirac matrix and $A_\mu(r_1, k)$ is the wave function for transverse photon at r_1 with momentum k. r_1, r_2 and R are the four-coordinates of the two bound electrons and the nucleus respectively. The transverse potential $A_\mu(r_1, k)$ with photon polarization vectors λ and momentum k is given by

$$A_\mu(r_1,k) = \frac{\lambda_\mu e^{ikr_1}}{\sqrt{2k_0(2\pi)^3}}$$

p is the 4-momentum of the electron and k is the 4-momentum of the photon. The wave function of He atom in the initial state.

$$\Psi_i(r_1,r_2,R) = \phi_{1s^2}(x,y)\frac{1}{(2\pi)^3}\frac{m}{\sqrt{\varepsilon_{b1}\varepsilon_{b2}}} \qquad (3)$$

$$u(b_1,r_1)u(b_2,r_2)\exp(iRL)\exp(ib_1r_1 + ib_2r_2)$$

where $u(b_i, r_i)$ is the Dirac spinor for the bound electron with four momentum b_i at the position $r_i(i = 1, 2)$. ε_{b1} and ε_{b2} are the binding energies of the two bound electrons. L and L' are the 4-momenta of the nucleus in the initial and in the final state respectively. The space coordinates of the electrons relative to the nucleus are $\boldsymbol{x} = \boldsymbol{r}_1 - \boldsymbol{R}$ and $\boldsymbol{y} = \boldsymbol{r}_2 - \boldsymbol{R}$.

$$\Phi_{1s^2}(x,y) = \chi_{1s}(x)\chi_{1s}(y) \qquad (4)$$

Φ_{1s^2} is the ground sate wave function of He atom [4,5]

$$\chi_{1s}(x) = Ae^{-z_1x} + Be^{-z_2x}, A = 0.7349,$$

$$B = 0.799, Z_1 = 1.41, Z_2 = 2.61 \qquad (5)$$

The final wave function of the system is

$$\Psi_f(r_1,r_2,R) = \frac{1}{\sqrt{v(2\pi)^3}}\frac{m}{\sqrt{E_{p-b_2}}}u(p)e^{i(r_1p)}$$

$$F_c(r_1,p)\Phi_{1s}^{He^+}(y)u(b_2)\exp(ir_2b_2 + iRL') \qquad (6)$$

$F_c(r_1, p)$ is the Coulomb distorsion factor of the plane wa for ionized electron in the field of the He^+ ion.

$$F_c(r_1,p) = e^{-\pi\eta/2}\Gamma(1+i\eta)_1F_1\left[-i\eta,1,i(|\bar{p}||\bar{r}_1| - \bar{p}.\bar{r}_1)\right]$$

where $\eta = \dfrac{m_e M_{He^+}/(m_e + M_{He^+})}{|\bar{p}|}$

The square of the probability amplitude for PEE

$$|M_{fi}|^2 = |R_pR_p^*|$$

$$= C'^2\left(c^{\mathcal{A}}(P\ \ k - b_1 + L' - L)\right)^2 I_p^2 T_p T_p^*$$

The corss section for SPI due to PEE becomes

$$\sigma_{ph} = \int\frac{1}{|k|}|M_{fi}|^2\frac{d^3\bar{p}}{(2\pi)^2}\frac{d^3\bar{L}}{(2\pi)^2}$$

2.2. Compton scattering .

The reaction for SPI due to CS

$$He + \gamma = He^+(1s) + \bar{e} + \gamma,$$

is represented by the second order Feynman diagrams Figu 2a and 2b corresponding to the direct and exchange te respectively. The second order S-matrices R_D, R_E for direct and exchange terms are respectively,

$$R_D = \left[\left(J_\mu^e\right)_{r_1}A_\mu(k,r_1)\right]\left[\left(J_\mu^e\right)_{r_1'}A'(k^*,r_1^*)\right],$$

$$R_E = \left[\left(J_\mu^e\right)_{r_1}A_\mu(k,r_1^e)\right]\left[\left(J_\mu^e\right)_{r_1}A(k^*,r_1)\right],$$

where the electron-currents at r_1 and r_1' are respective

$$\left(J_\mu^e\right)_{r_1} = \bar{\Psi}(r_1)\gamma_\mu\Psi(r_1,r_2,R),$$

$$\left(J_\mu^e\right)_{r_1} = \bar{\Psi}(r_1^*,r_2,R)\gamma_\mu\Psi(r_1^*),$$

and matrix element for SPI by CS becomes

$$R_{cs} = R_D + R_E.$$

The amplitude for SPI by the CS process is given by

$$|M_{fi}^c|^2 = R_D^*R_D + R_E^*R_E + R_D^*R_E + R_D^*R_E$$

The interference of the direct and exchange term

$$R_D^*R_E + R_E^*R_D = 2R_D^*R_E$$

The cross section for single ionization by Compton scatteri is finally given by

$$\sigma_c = \int\frac{1}{|k|}|M_{fi}^c|^2\frac{d^3k'}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}.$$

3. Results and discussions

We have calculated Single photoionization cross section PEE for helium from diagram (Figure 1). It is a first or process in which one of the bound electron escapes while the second electron remain bound with the helium ion in 1s-state In Compton scattering the Feynman diagrams are of second order and consist of direct and exchange diagram. Contribution from direct and exchange diagrams (Figure 2) in CS are small compared to the contribution from the interference term between the direct and exchange processes.

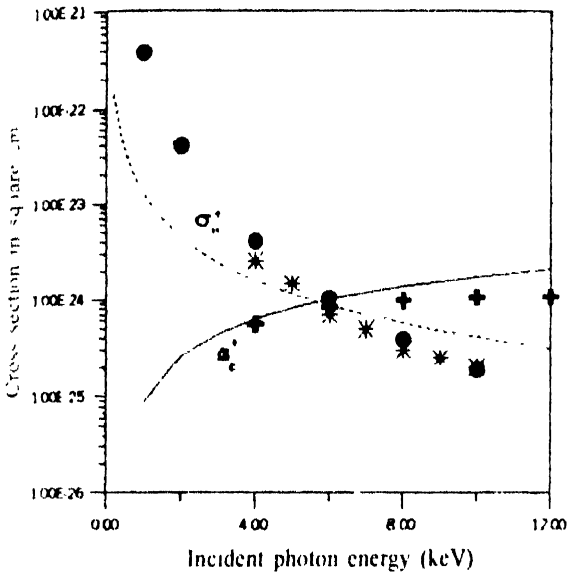

Figure 3. Cross section for single photoionization *versus* incident photon energy Photoionization · Present field theoretic calculation - calculation done by Anderson [8] *, MBPT [6] calculation by Hino •• Compton scattering Present field theoretic calculation - - - , calculation by [7] Hino ·

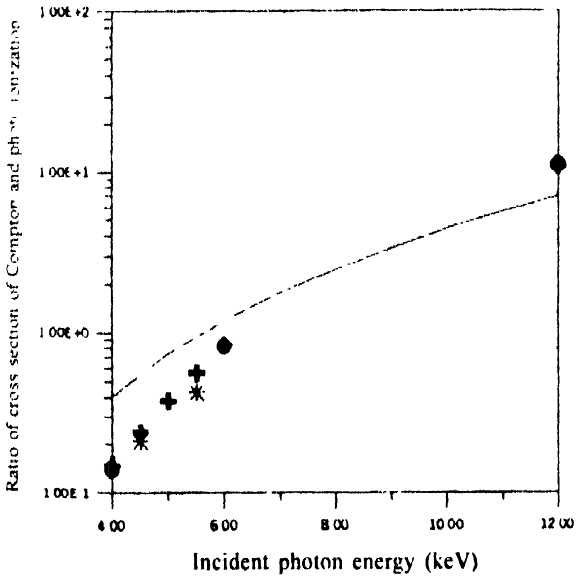

Figure 4. Ratio of Compton scattering cross section and photoionization cross section *versus* incident photon energy Present field theoretic calculation ———, calculation by Viegle + [5], Hino [6] ••, and Morgan [5]]*.

As a first attempt for SPI in the field theoretic approach we have ignored the distortion factor for the time being. In principle it will include very little correction to the cross section. The energy range of photon is 0.2 1.2 keV In the case of photoelectric effect the cross section asymptotically decreases with energy as in the case of Hino [1,2]. Present value for σ_{ph} upto about 5 keV, is slightly below that of the existing theoretical value (Figure 4), while the present result above 5 keV remains slightly greater than the existing result. In the case of σ_c present result almost agree with the existing calculation (Figure 4). It is interesting to note that Compton curve is mirror image of the photoelectric curve about this cross-over point. We hope to improve on our present result by including Coulomb distortion in the final wave function in our subsequent calculations.

4. Conclusion

In conclusion we find it most encouraging that this compat yet simple QED formulation can elegantly explain single photoionization by photoelectric effect and Compton effect. The formulation of this photon-atom collision problem in a gauge-invariant language that is similar to those describing most of the fundamental interactions in nature has its own intrinsic appeal.

Acknowledgement

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References

[1] S Bhattacharyya and S Mitra *Phys Rev* **A60** 2269 (1999)

[2] S Bhattacharyya and S Mitra *Phys Rev* **A62** 032709 1 (2000)

[3] S Bhattacharyya and K Pathak *Indian J Phys* **74B** 1 (2000)

[4] S Bhattacharyya and K Pathak *Acta Phys Polonica* **A98** 335 (2000)

[5] D V Morgan and R J Barlett *Phys Rev* **A59** 4075 (1999)

[6] K Hino, P M Bergstrom and Joseph H Macek *Phys Rev Lett* **72** 1620 (1994)

[7] K Kino and J H McGuire *Phys Rev* **A48** 1271 (1993)

[8] Lans R Andersson and J Burgdorfer *Phys. Rev Lett* **71** 50 (1993)