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Effect of an aligned magnetic field on the growth of Rayleigh-Taylor instability in a viscous and conducting fluid

T Sankarappa*, S Basavaraja and R D Mathad
Department of Physics, Gulbarga University, Gulbarga-585 106, Karnataka, India
E-mail : sankarappa@rediffmail.com

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Abstract The influence of an aligned magnetic field on the Rayleigh-Taylor instability in a thin layer of finite electrically conducting and highly viscous fluid has been investigated using the linear theory in the creeping flow limit. It is for the first time that the effect of an aligned field on the Rayleigh-Taylor instability in a layer of finite electrically conducting and highly viscous fluid is studied. The growth rate of the instability is found to be controlled by the ratio of surface tension to the stress gradient, thickness of the fluid layer and the Chandrashekhar number which is a function of magnetic field. It is also noted that an aligned magnetic field prevent the growth of finger instabilities which otherwise exist for thicker films

Keywords : Rayleigh-Taylor instability, finite conducting fluid, aligned magnetic field

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1. Introduction

The Rayleigh-Taylor (RT) instability is an instability of interface between two fluids of different densities, which occurs when the light fluid is pushing the heavy fluid [1]. The study of RT instabilities have been a subject of considerable interest due to their applications in the areas of inertial fusion target design, astrophysics, plasma physics and physics of fluids [2–6]. The instabilities are usually considered in a viscous creeping flow situations. The mathematical models of the instabilities are widely used in the areas of failure of metallic glasses [7,8], grain boundary failure in metals [7], non-cross linked polymers [9] and cracking of cross linked polymers [10]. Brown [11] theoretically investigated RT instability in a finite thickness layer of viscous fluid making creeping flow approximation. The RT like instabilities in the presence of magnetic field arise in many astrophysical and geophysical situations, however, a particular area of interest is the inertial fusion where the application of magnetic field is used to control the growth of the instabilities. Davalos-Orozco [12] studied the effect of horizontal magnetic field on RT instability of a two fluid layer for an unstable density stratification and concluded that in the case of an adverse density stratification, the horizontal magnetic field gives stability and propagate the perturbation as traveling waves. Sharma and Chhajlani [13] and Chandrasekhar [14] have carried out the instability studies on ideal plasma in the presence of magnetic field. The aim of this Brief Communication is to examine the stability of the interface of a fluid layer of finite thickness. The particular concerns are the effects of the thickness of the layer and the strength of the magnetic field on the growth rates of instability.

2. Theory

We shall consider a situation where an electrically finite conducting lighter fluid (layer of finite thickness) accelerates towards a heavier fluid causing the interface between the fluids to be unstable (Figure 1). The other interface of the lighter fluid is assumed to be bounded by a rigid material. The fluid is considered to be highly viscous, so the inertial terms can be ignored. The flow is taken to be steady and two dimensional. The relevant parameters are the fluid viscosity,

*Corresponding Author

μ_f, magnetic permeability, μ, electrical conductivity, σ, fluid layer thickness, h, surface tension, γ, stress gradient, δ, and the applied magnetic field, H_x.

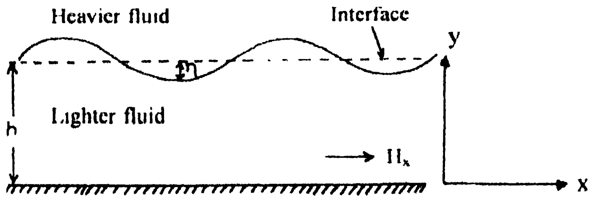

Figure 1. The geometry of the perturbation

The basic system can be defined as

$$u = w = 0,\ H_x = H_x,\ H_y = 0. \tag{1}$$

The basic equations of motion of the system in the absence of body force are given by

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}\left(H_x\frac{\partial H_x}{\partial x}+H_y\frac{\partial H_x}{\partial y}\right)+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\mu}{\rho}\left(H_x\frac{\partial H_y}{\partial x}+H_y\frac{\partial H}{\partial y}\right)+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right), \tag{3}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial y}=0, \tag{4}$$

$$u\frac{\partial H_x}{\partial x}+w\frac{\partial H_x}{\partial y}=H_x\frac{\partial u}{\partial x}+H_y\frac{\partial u}{\partial y}+\nu_m\left(\frac{\partial^2 H_x}{\partial x^2}+\frac{\partial^2 H_x}{\partial y^2}\right), \tag{5}$$

$$u\frac{\partial H_y}{\partial x}+w\frac{\partial H_y}{\partial y}=H_x\frac{\partial w}{\partial x}+H_y\frac{\partial w}{\partial y}+\nu_m\left(\frac{\partial^2 H_y}{\partial x^2}+\frac{\partial^2 H_y}{\partial y^2}\right), \tag{6}$$

$$\frac{\partial H_x}{\partial x}+\frac{\partial H_y}{\partial y}=0, \tag{7}$$

where $P = P_0 + (\mu H^2/2)$ is the total pressure, P_0 is the hydrostatic pressure, (u, w) are the velocity components in x and y directions respectively, $H(H_x, H_y)$ is a time independent magnetic field, ρ is the fluid density, $\nu(= \mu_f/\rho)$ is the kinematic viscosity and $\nu_m(= 1/\mu\sigma)$ is the magnetic viscosity.

We superimpose on the basic state given in eq. (1) a small symmetric disturbance of the form

$$u = u',\ w = w',\ P = P',\ H_{ox} = H_x + h_x,\ H_{oy} = h_y, \tag{8}$$

where the primes indicate the perturbed quantities, H_{ox} and H_{oy} are the total fields (sum of applied and induced fields) and h_x and h_y are the induced magnetic fields in the x and y directions respectively.

Substituting eq. (8) into the eqs. (2–7) and linearizing, we get after omitting the primes for simplicity, the required linearized equations :

$$-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}H_{ox}\frac{\partial h_x}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=0, \tag{9}$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\mu}{\rho}H_{ox}\frac{\partial h_y}{\partial x}+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=0, \tag{10}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial y}=0, \tag{11}$$

$$H_{ox}\frac{\partial u}{\partial x}+\nu_m\left(\frac{\partial^2 h_x}{\partial x^2}+\frac{\partial^2 h_x}{\partial y^2}\right)=0, \tag{12}$$

$$H_{ox}\frac{\partial w}{\partial x}+\nu_m\left(\frac{\partial^2 h_y}{\partial x^2}+\frac{\partial^2 h_y}{\partial y^2}\right)=0, \tag{13}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}=0, \tag{14}$$

The boundary conditions and the interface conditions shown in Figure 1 are

$$w = Dw = 0;\ h_y = Dh_y = 0 \text{ at } y = 0, \tag{15}$$

where D refers to d/dy.

The free surface is at $y = h$ and we shall consider the development of small perturbations in this interface. The actual deviation of the interface from $y = h$ is given by η. The continuity of tangential stress at the interface is given by

$$\frac{\partial w}{\partial x}+\frac{\partial u}{\partial y}=0, \text{ at } y = h. \tag{16}$$

The continuity of normal stress at the interface gives

$$P=-\delta.\eta-\gamma\frac{\partial^2\eta}{\partial x^2}+\mu_f\frac{\partial w}{\partial y}, \text{ at } y = h. \tag{17}$$

The kinematic condition with $y = h - \eta$ as the interface, after the linearization gives

$$w=\frac{\partial\eta}{\partial t}, \text{ at } y = h. \tag{18}$$

The continuity of magnetic tangential stress at $y = h$ is given by

$$\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}=0, \text{ at } y = h. \tag{19}$$

The continuity of normal magnetic stress is given by

$$\frac{\mu H_{ox}^2}{2}=-\gamma\frac{\partial^2\eta}{\partial x^2}+\mu H_{ox}\frac{\partial h_y}{\partial y}, \text{ at } y = h. \tag{20}$$

By eliminating the pressure gradient from the eqs. (9) and (10) and using (11–14) we get

$$\frac{\mu H_{ox}^2}{\rho\nu\nu_m}\frac{\partial w}{\partial x}+\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 u}{\partial y^3}-\frac{\partial^3 w}{\partial x^3}-\frac{\partial^3 w}{\partial y^2\partial x}=0. \tag{21}$$

Assuming the variations of u, w, P and η in the x-direction to be of the form

$$\begin{bmatrix} u \\ w \\ p \\ \eta \end{bmatrix} = \begin{bmatrix} u(y) \\ w(y) \\ p(y) \\ \eta(y) \end{bmatrix} \exp(i\alpha x + nt). \tag{22}$$

where n is the growth rate of the RT instability and α is the wave number.

By using eqs. (9), (11) and (22) in (21) we get

$$D^4 w - 2\alpha^2 D^2 w + \alpha^4 (1 + a_0^2) w = 0, \tag{23}$$

where $a_0^2 = \left(\mu H_{ox}^2 / \rho \nu \nu_m \alpha^2\right)$.

The general solution of eq. (23) can be written as

$$w = \exp(\alpha a_1 y)\left[A \sin(\alpha a_2 y) + B\cos(\alpha a_2 y)\right] + \exp(-\alpha a_1 y)\left[C \sin(\alpha a_2 y) + E\cos(\alpha a_2 Y)\right], \tag{24}$$

where $a_1 = \left[(1 + a_0^2)^{\frac{1}{2}} / 2\right]^{\frac{1}{2}}$ and $a_2 = \left[(a_0^2 - 1)^{\frac{1}{2}} / 2\right]^{\frac{1}{2}}$.

By imposing the boundary conditions given in eqs. (15) and (16) on the solution given in eq. (24), we obtain the constants B, C and E to be

$B = A(1 - R)(a_2/2a_1)$, $C = -AR$ and $E = A(1 - R)(a_2/2a_1)$,

where

$$R = \frac{\exp(M)\left[(a_2^3 + 3a_1^2 a_2 - a_1)\cos(N) + 2a_1(1 + a_1^2)\sin(N)\right] - \exp(-M)\left[(a_2^3 - a_1^2 a_2 - a_2)\cos(N) - 2a_1^3 \sin(N)\right]}{\exp(M)\left[(a_2^3 - a_1^2 a_2 - a_2)\cos(N) + 2a_2^2 a_1 \sin(N)\right] + \exp(-M)\left[(a_2^3 + 3a_1^2 a_2 - a_2)\cos(N) - 2a_1(a_1^2 + 1)\sin(M)\right]}, \tag{25}$$

$M = \alpha a_1 h$ and $N = \alpha a_2 h$.

On substituting $H^2 = (H_{ox} + h_x)^2 + h_y^2$ in the total pressure P and using eq. (8), we get

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0. \tag{26}$$

Making use of eqs. (10), (17), (18), (22) and (26), the dispersion relation is obtained to be

$$n = \frac{(\delta - \alpha^2 \gamma)\alpha^2 w}{\mu_f (2\alpha^2 Dw - D^3 w)} \quad \text{at } y = h. \tag{27}$$

On making this equation dimensionless using the relations $n^* = (n\mu_f)/\sqrt{(\gamma\delta)}$, $\lambda^* = \sqrt{(\gamma/\delta)}$, $\alpha^* = (\alpha\lambda^*)$, $h^* = (h/\lambda^*)$ and $D^* = Dh$ and for simplicity neglecting the asterisks, become

$$n = \frac{(1 - \alpha^2) h^3 \alpha^2 w}{(2h^2 \alpha^2 Dw - D^3 w)}. \tag{28}$$

In the eq. (24), the quantity a_0 in the dimensionless form is given as $a_0 = [Q/(h^2\alpha^2)]^{\frac{1}{2}}$, with Chandrashekhar number, $Q = (\mu H_x^2 h^2)/(\rho \nu \nu_m)$.

3. Discussion and conclusion

The eq. (28) is a function of the dimensionless wave number α, the ratio of surface tension to the stress gradient λ, the film thickness h and the Chandrashekhar number Q which is a function of an applied horizontal magnetic field H. The growth rate n is numerically computed for different values of h and Q and, plotted in Figures 2–5.

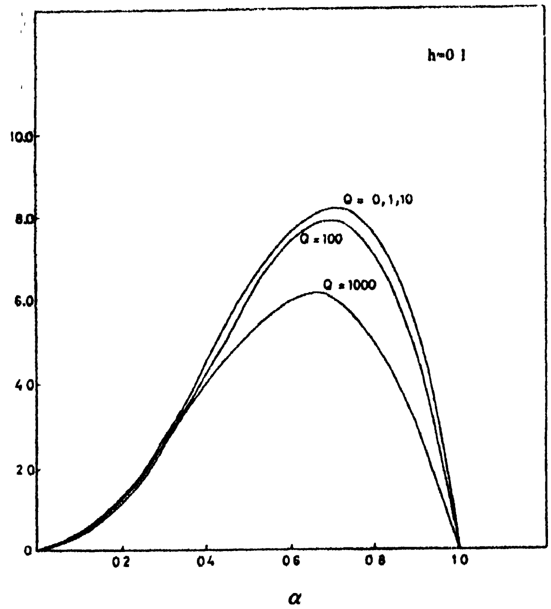

Figure 2. A plot of growth rate n *versus* wave number α at different Chandrashekhar number, Q, for film thickness, $h = 0.1$.

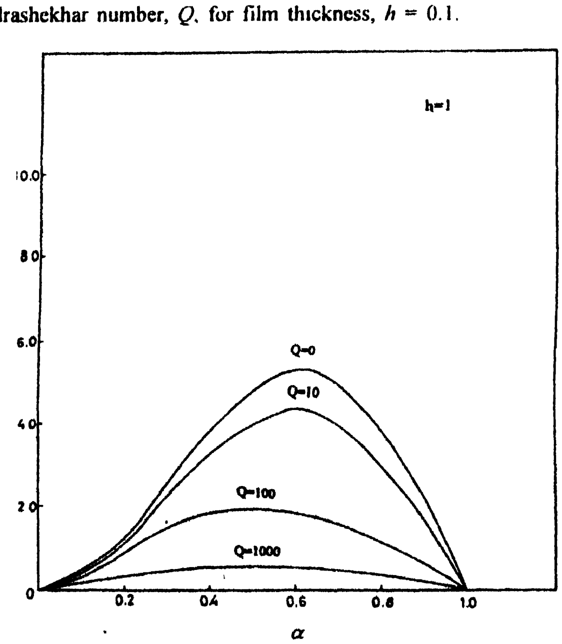

Figure 3. A plot of growth rate n *versus* wave number α at different Chandrashekhar number, Q, for film thickness, $h = 1$.

These plots reveal that the size scale of the RT instabilities of the fluid of finite layer thickness is controlled by λ, h and Q. Further, it can be seen that the increase in the film thickness h, produce a finger type of instability but the effect

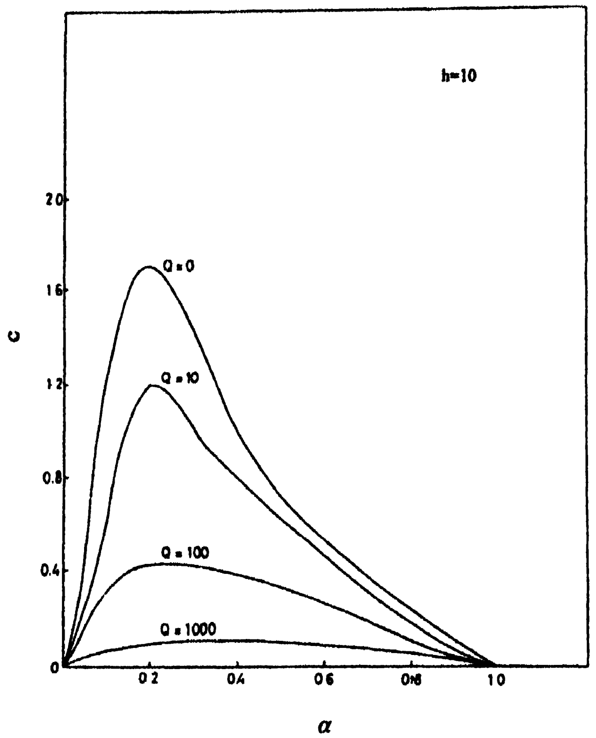

Figure 4. A plot of growth rate n *versus* wave number α at different Chandrashekhar number Q, for film thickness h = 10.

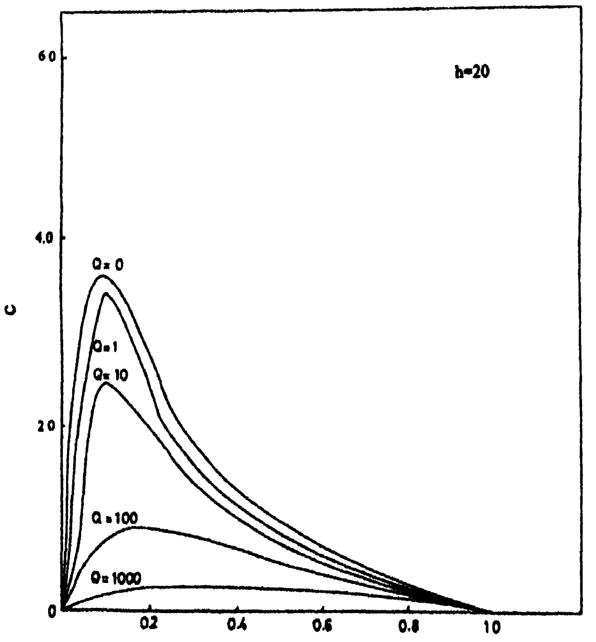

Figure 5. A plot of growth rate n *versus* wave number α at different Chandrashekhar number Q, for film thickness h = 20.

of an aligned magnetic field is to suppress the growth rates When Q is made zero in the dispersion relation (28), the hydrodynamic results of Brown [11] are recovered. This can be easily realized by substituting the value of applied field to be zero in eq. (23). These results may be of great use in inertial control of fusion (ICF). The RT instabilities grow in imploding ICF targets when the heavy fluid is accelerated by a lighter fluid. All the ICF targets undergo such an instability at some time or the other during the implosion The outer ablating surface may be stabilized in the presence of an aligned magnetic field.

It is for the first time that the RT instability in a finite thickness layer of highly viscous, electrically finite conducting fluid has been studied in the presence of an aligned magnetic field under the approximation of creeping flow limit. It is found that the growth rate of the RT instabilities is controlled by the ratio of surface tension to the stress gradient, fluid layer thickness and the Chandrashekhar number. The magnetic field is also found to prevent the finger instabilities which otherwise exist for thick films. The results tend to hydrodynamic domain in the absence of an applied field.

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