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Vortex infected plasmons and phonons

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Abstract An experimentally suggested addition (for energy balance in high power laser induced plasmas for fusion reaction) to the usual complexities of a physical plasma, is a possibility of existence of particles carrying vorticity in addition to their having mass and charge. These particles of mass, charge and vorticity (PMCVs) may also be called vortex infected plasmons and vortex infected phonons

The plasma is regarded as a mixture of several species of fluids, each of which is specified by a per particle mass, charge and vorticity and population per unit volume A crude theory, is suggested for application and has been used to investigate small amplitude transverse waves, for understanding the dynamics of PMCVs. Only low frequency transverse waves are excited. Birefringence occurs in which one component is a decay wave and other is dispersed through the plasma volume.

Keywords Vorticity, plasmons, phonons

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1. Introduction

Experiments on laser induced plasmas for fusion reaction show that the sum of the estimated loss of wave energy through collisional absorption and electromagnetic radiation is somewhat less than the actual amount of energy loss [1]. This extra loss of wave energy is acquired by the plasma in some form which is not yet known clearly. One guess is that energy is transferred to some negative ions to convert these into vortices, the formation of which requires rotational energy. These vortices essentially are particles having mass, charge and vorticity (PMCVs). In other words, these quasi particles are examples of vortex infected positively charged and negatively charged plasmons and phonons. Of these the positively charged plasmons do not exist in nature, because positive charges having electronic mass do not exist in nature. The PMCVs have therefore, their birth, life and death in laser excited plasmas for fusion progresses. Here are elementary excitations of vortex spectrum of charges. These wave functions would probably change by movements over distances less than an atomic spacing. When quantum of energies is small, long wave lengths or long distances are not necessary for their manifestation.

Since the plasma is a many body (particles of atomic and electronic size) interaction of long range, vortices in plasmas should have a basis in the classical particle dynamics. Existence of these quasiparticles, however short their life span may be confirmed experimentally. When the vorticity is destroyed by nonlinearities, the affected plasma constituent joins the company of vortex free plasmons and phonons.

Laser fields are about 10^5 volts/cm, or more. At a laser focal spot the field intensity is about 10^{15} watts or more. So, oscillating electric and magnetic fields are about 9×10^{18} volts/cm. The radiation pressure is about 3×10^5 atmosphere and the field quiver velocity is that of 16 KeV electrons. These fields are of the order of quantum mechanical forces binding charges inside molecules. They dissociate the orbital

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charges from bondage in atoms and molecules. The very high pressure of laser light generates forces so strong that mechanical phenomena overtake optical effects. Mechanical and optical effects are therefore mixed up in laser effected materials. Also, the sharpness of incidence of a laser on matter can induce the rotation of the vorticity. So, induction of spin in some charges is not an impossibility in these interactions. As the fluid expands in space at high speed from the superdense region, small pockets of energy of circulation vorticity are formed in it.

Assumption for this study of the formation and action of the PMCVs are : (i) The plasma containing the PMCVs can be studied classically; (ii) The macroscopically neutral plasma is a mixture of several species of fluids; (iii) The parameters of the n-th species of the plasma are : per particle mass m_n, charge q_n and vorticity ξ_n^0, per unit volume population N_n and force of vorticity $(v_n \times \xi_n^0)/c$ which is in addition to other usual forces (Lorentz force, gravitational force, viscous force). Here v_n is the average velocity and ξ_n^0 is the parameter defining the vorticity; (iv) The grains of PMCVs are of size small enough for validity of the fluid mixture theory. So, their dimensions would lie between atomic dimensions and Debye shielding radius.

Earlier Das *et al* [2] investigated small amplitude transverse waves of circular polarization in an unmagnetised, cold collision free mixture of fluids of vortex free positive ions, vortex free electrons and vortex infected negatives ions. Low frequency waves are found to be excited by perturbation. Some of these waves are dispersive waves and some are decay waves. So this type of a three fluid mixture can filter out waves of some frequency and prevent waves of some other frequencies to pass through the plasma. This is also birefringence of circularly polarized waves of low frequency, where the right circularly polarized (RCP) part is dispersive wave and the left circularly polarized (LCP) part is a decay wave.

Vortex sources, in the available theory (Section 2), are two dimensional vortex distributions in continuous media. Vortices in these distributions are formed by the energy of streams of fluids by surface tension. The energy required to form the vortices is small in macroscopic laboratory dimensions. Internal motions do not change the density; however, these can excite the vortices.

2. Existing theory of vortices in plasmas

Vortex sources are two dimensional vortex distributions in continuous media. These are formed by the energy of streams of fluids by surface tension. The energy required to form the vortices is small in macroscopic laboratory dimensions. Internal motions do not change the density. These can, however, excite the vortices. The two dimensional vorticity is specified by stream function $\psi(x,y,z)$ given by [3] $v = \xi \times \nabla\psi(x,y,z)$ where $\xi = \nabla \times v$, pockets are formed in it of circulation (vorticity) of charges taking energy of the supplied wave field for birth and life.

A flowing fluid does not directly form vortices. Vortices form and travel in the background of the following fluid when resistance beyond some critical value exists. To have resistance, the energy flow and momentum must go into heat, that is, internal excitation, when the fluid velocity is very high, because of neutral fluid is possible at low velocities without resistance. For plane fluid flows, say in the XY-plane, the viscosity arises from a transfer of momentum in the Z-direction, like the viscosity of a gas in the kinetic theory. The heat generated can set in motion local convective motion of circulation in the Benard cells which is a vortex motion. This can happen in some regions if not in the whole volume of the fluid. This motion for vorticity is not possible in solids. So, viscous force of collision, acting normal to the boundary of an expanding dense material core, promotes vorticity generation.

PMCVs must scatter PMCVs, other phonons and plasmons. These scatterings would depend on mass per particle, vorticity and sign of the changes of the colliding particles. The number of PMCVs present depends on temperature. The mean free path for collision and the viscosity for collisions between these vortices would also depend on temperature. Parameters of collisions between phonons and plasmons, between phonons and phonons and between plasmons and plasmons are known. Knowledge of apporaches for these estimations would help to estimate the collision cross sections for collisions between the particles of mass, charge and vorticity (PMCVs) and PMCVs, PMCVs and phonons, and between the PMCVs and plasmons. Collision cross sections for collisions involving PMCVs are to be determined in an adhock manner because their estimation from first principles is not known.

The vorticity force $(v_l \times \xi_l^0)$, (where $\xi_l^0 (= \nabla \times v_l)$ is the perturbation vorticity), follows from one of the two terms of the vector expansion of the nonlinear substantial derivative term $(v \cdot \nabla)v$ of the expression for the acceleration vector. So, the force of vorticity is similar to the force of static magnetic fields on a charged particle. The other force term of it is the vorticity destroyed term $\nabla(v^2/2)$, because it acts parallel to the direction of the wave path. So, it destroys the vorticity. The nonlinear centrifugal force of rotation which is $(\Omega \times r) \times \Omega [= (v^2/2r_c)\hat{n}$, where r_c is the radius of curvature, Ω is the angular velocity and $\hat{n}$ is the unit vector parallel to the principal normal at the field point $P(r)$], does not contribute to the formation and dynamics of vorticity. Actually, both the centrifugal force and the force $\nabla(v^2/2)$ are destructive for vorticity of particles of a plasma. So the characteristic time of survival of the PMCVs is of the order of the time period of the nonlinearly induced second harmonic. Further, the coriolies force, like the vorticity force, vanishes if there is no motion in the rotating frame of co-ordinates used. So, the identification is not total of vorticity of our model with rotation. Only the coriolis force evidently, is identified with the force of vorticity in this theory. Existence of these quasi particles, however, short their life span may be, should be experimentally studied. When the vorticity is destroyed by nonlinearities, the affected plasma constituent joins the company of vortex free plasmons and phonons.

The formula for contribution to the viscosity from collision between PMCV and other PMCVs may be taken [4]

$$\eta = \frac{1}{15} t_\nu N_\nu \left(\frac{p_0^2}{\mu} \right) \quad (1)$$

where t_ν is the mean time between collisions, and N_ν represents the number of PMCVs, μ is the effective mass of a PMCV and p_0 is its momentum. The factor $t_\nu N_\nu$ is independent of temperature and η become independent of temperature. The contributions of phonons and plasmons to viscosity are determined by their scattering by PMCVs and other phonons and plasmons.

If P is the sum of energies lost in different forms in a laser excited plasma, then we can write

$$P = P_r + P_c + P_{\text{rot}}, \quad (2)$$

where P_r is the loss in the form of electromagnetic radiation, P_c is the collisional loss of energy from the field to the plasma, and P_{rot} is the loss of power to generate vorticity (rotation).

Electrons do not couple to the rotational degree of freedom because their angular momentum at a given impact parameter are very small. The angular momentum of the ions is also not sufficiently large for that of vorticity.

Since PMCVs are affected by collision, it is likely that vortex energy per unit volume in the plasma can be estimated from the difference $(\boldsymbol{E} \cdot \boldsymbol{j}) - (\boldsymbol{E} \cdot \boldsymbol{j}_0)$ where $\boldsymbol{j}_0$ is the electric current induced by vortex-free plasma in presence of the applied wave field.

The vorticity evaluation equation

$$\frac{\partial \xi}{\partial t} = \nabla \times (\boldsymbol{v} \times \xi) + \nabla \times (\boldsymbol{H} \cdot \nabla) \left(\frac{\mu \boldsymbol{H}}{4\pi\rho} \right) + \nu \nabla^2 \xi \quad (3)$$

obtains by taking curl of all the terms of the momentum transfer equation for incompressible MHD media :

$$\frac{\partial \boldsymbol{v}}{\partial t} - (\boldsymbol{v} \times \xi) = -\nabla \left| \frac{p}{\rho} + \frac{\mu |\boldsymbol{H}|^2}{8\pi\rho} + \frac{1}{2} |\boldsymbol{v}|^2 \right. + (\boldsymbol{H} \cdot \nabla) \left(\frac{\mu \boldsymbol{H}}{4\pi\rho} \right) + \nu \nabla^2 \boldsymbol{v} \quad (4)$$

where ν is the kinematic viscosity, ρ is the density of the fluid, p is the pressure and H is magnetic moment field. Hence, vorticity is created and not conserved by magnetic field; it is conceived as a field vector like $\boldsymbol{B}$. It cannot have a velocity for motion. Like $\boldsymbol{B}$ lines and tubes, vorticity lines and tubes are developed. Vorticity filaments are useful quantities.

In the existing theory of plasma vortices, the product of the cross section σ and angular velocity Ω on a vortex filament is defined as its strength β. It is a constant on the filament. When the forces acting in a medium have a single valued potential and the density is a function of pressure, the circulation in any closed circuit moving with the fluid is constant for all time. These vortices have no mass, charge, momentum and rate of change of momentum. So, the particle concepts are not conspicuous in the theory.

Solutions of the field equations for two dimensional plasma vortices are compared with the field equations for the electric monopoles, and electric and magnetic dipoles and other multipoles. Concepts of monopole vortices of both sign, dipoles and multipolar vortices of both signs have been developed and used. An initial distribution of many vortices of both polarities evolve into a pair of large counter rotating vortices. This is a coalescence or condensation of all vortices of some polarity. The counter rotating vortices of different polarity interact through a mutual perturbation of orbits. The monopolar and decays into two monopolar vortices can reconstruct their initial shapes after experiencing a strong distortion due to head-on collisions and overtaking collisions. Vortex induced transport is possible. Particles trapped in a vortex are large in size than radius of an electron. So anomalous diffusion is explained [3].

Vortex solutions follow from the curl of all terms of the nonlinear Navier-Stokes (N–S) equation

$$\left[\frac{\partial}{\partial t} + (\boldsymbol{v} \cdot \nabla) \right] \boldsymbol{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \boldsymbol{v} \quad (5)$$

in compressible fluids. The definition of vorticity [$\xi = (\nabla \times \boldsymbol{v})$] shows that a spatial region exists where there would be one or more closed stream lines. In incompressible media the viscous force in the N–S equation is $\nu\nabla^2 v\,[=\nu\nabla\times(\nabla\times v)]$. The rate of work done by this force on the medium per unit volume at a point $P(x, y, z)$ is

$$v\cdot\frac{\partial v}{\partial t}+v\cdot(v\cdot\nabla)v=-\left(v\cdot\nabla\frac{p}{\rho}\right)+\nu\left(v\cdot\nabla^2 v\right). \quad (6)$$

Integration over a volume V and surface Σ of the fluid reduces the last term to

$$\nu\iiint_V\left[v\cdot(\nabla\times v)\right]dV=-\nu\iiint_V(\nabla\times v)^2\,dV \quad (7)$$

which is the heat dissipation effect, where we have used the vector relation

$$\nabla\cdot\left[v\times(\nabla\times v)\right]=(\nabla\times v)^2-v\cdot\nabla\times(\nabla\times v) \quad (8)$$

before applying Stoke's theorem for vanishing of an integral. Influence of the force $\nu\nabla^2 v$ is assumed to be negligible in the short time of existence of the PMCVs in our theory.

3. Basic equations of the model

For convenience we use the subscript n_i on the variables of the n-th species of positive charges, and subscript n_e on the n-th species of negative charges. The full set of the basic equations of our model in CGS Gaussian units is

$$\frac{\partial v_{n_i}}{\partial t}-\left(v_{n_i}\times\xi^0_{n_i}\right)=\frac{q_{n_i}}{M_{n_i}}E+\frac{q_{n_i}}{M_{n_i}c}\left(v_{n_i}\times H^0\right)+\nabla\left(v^2_{n_i}/2\right)$$
$$-\sum_{n_e=1}^{N_e}\nu^{n_e}_{n_i}\left(v_{n_i}-v_{n_e}\right)-\sum_{n_j=1}^{N_i}\nu^{n_j}_{n_i}\left(v_{n_i}-v_{n_j}\right), \quad (9)$$

$$\frac{\partial v_{n_e}}{\partial t}-\left(v_{n_e}\times\xi^0_{n_e}\right)=\frac{q_{n_e}}{M_{n_e}}E+\frac{q_{n_e}}{M_{n_e}c}\left(v_{n_e}\times H^0\right)+\nabla\left(v^2_{n_e}/2\right)$$
$$-\sum_{n_i=1}^{N_e}\nu^{n_e}_{n_i}\left(v_{n_e}-v_{n_i}\right)-\sum_{n_j=1}^{N_i}\nu^{n_j}_{n_e}\left(v_{n_e}-v_{n_j}\right), \quad (10)$$

$$j=\sum_{n_i=1}^{N_i}\left(q_{n_i}N_{n_i}v_{n_i}\right)+\sum_{n_e=1}^{N_e}\left(q_{n_e}N_{n_e}v_{n_e}\right), \quad (11)$$

$$\rho^c=\sum_{n_i=1}^{N_i}\left(q_{n_i}N_{n_i}\right)+\sum_{n_e=1}^{N_e}\left(q_{n_e}N_{n_e}\right);$$

$$\rho^m=\sum_{n_i=1}^{N_i}\left(M_{n_i}N_{n_i}\right)+\sum_{n_e=1}^{N_e}\left(M_{n_e}N_{n_e}\right), \quad (12)$$

$$\nabla\times H=\frac{1}{c}\frac{\partial E}{\partial t}+\frac{4\pi}{c}j, \quad (13)$$

$$\nabla\times E=-\frac{1}{c}\frac{\partial H}{\partial t}, \quad (14)$$

$$\nabla\cdot E=4\pi\rho^c;\ \nabla\cdot H=0, \quad (15)$$

$$\frac{\partial N_{n_i}}{\partial t}+\nabla\cdot\left(v_{n_i}N_{n_i}\right)=0;\ \frac{\partial N_{n_e}}{\partial t}+\nabla\cdot\left(v_{n_e}N_{n_e}\right)=0. \quad (16)$$

Here, the vorticity per particle of the n-the species of ions is $\xi^0_{n_i}$, that of electrons is $\xi^0_{n_e}$; ρ^c is the space charge separation per unit volume, ρ^m is the average mass density. q_{n_e} is the n-th species of negative charge per particle, and q_{n_i} is the same for the positive charges. For transverse waves $\rho^c = 0$, $N_{n_i} = N_{n_e} = N^0_n$ and $\nabla\cdot E = 0$. These vorticity parameters and the force of vorticity are new things in physics. The eqs. (9–16) are solved for waves of very small amplitude, in the linearized approximation for studying the behaviour of plasma of vortex infected plasmons and phonons (VIPPs).

We assume that the dependence of the field variable on space and time is ~exp $i(kz-\omega t)$, where 'ω' and 'k' are the frequency and wave number of the wave.

In collision free plasma in which both electrons and ions are mobile in presence of transverse waves these equations give

$$\omega v_{n_{e_x}}+i\Omega_{n_e}v_{n_{e_y}}=-\frac{ie}{M_{n_e}}E_x,$$
$$-i\Omega_{n_e}v_{n_{e_x}}+\omega v_{n_{e_y}}=-\frac{ie}{M_{n_e}}E_y, \quad (17)$$

$$\omega v_{n_{i_x}}-i\Omega_{n_i}v_{n_{e_y}}=-\frac{ie}{M_{n_i}}E_x,$$
$$i\Omega_{n_i}v_{n_{i_x}}+\omega v_{n_{i_y}}=-\frac{ie}{M_{n_i}}E_y, \quad (18)$$

$$\left(k^2c^2-\omega^2\right)E=4\pi i\omega j, \quad (19)$$

where

$$\Omega_{n_e}=\frac{q_{n_e}H^0}{M_{n_e}c}+\xi^0_{n_e}=\Omega^0_{n_e}+\xi^0_{n_e},\ \Omega^0_{n_e}=\frac{q_{n_e}H^0}{M_{n_e}c}, \quad (20)$$

$$\Omega_{n_i}=\frac{q_{n_i}H^0}{M_{n_i}c}+\xi^0_{n_i}=\Omega^0_{n_i}+\xi^0_{n_i},\ \Omega^0_{n_i}=\frac{q_{n_i}H^0}{M_{n_i}c}. \quad (21)$$

Relation (11) for the electric current of convection in the linearized approximation becomes

$$j=e\sum_n N^0_n\left(v_{n_i}-v_{n_e}\right). \quad (22)$$

Hence (19) reads

$$\left(k^2c^2-\omega^2\right)E=4\pi ie\omega\sum_n N^0_n\left(v_{n_i}-v_{n_e}\right). \quad (23)$$

Eqs. (17) and (18) give

$$v_{n_{e_x}} = -\frac{ie}{M_{n_e}} \frac{\left(\omega E_x - i\Omega_{n_e} E_y\right)}{\omega^2 - \Omega_{n_e}^2},$$

$$v_{n_{e_y}} = -\frac{ie}{M_{n_e}} \frac{\left(\omega E_y - i\Omega_{n_e} E_x\right)}{\omega^2 - \Omega_{n_e}^2}, \tag{24}$$

$$v_{n_{i_x}} = -\frac{ie}{M_{n_i}} \frac{\left(\omega E_x + i\Omega_{n_i} E_y\right)}{\omega^2 - \Omega_{n_i}^2},$$

$$v_{n_{i_y}} = -\frac{ie}{M_{n_i}} \frac{\left(\omega E_y - i\Omega_{n_i} E_x\right)}{\omega^2 - \Omega_{n_i}^2}. \tag{25}$$

Here, we have supposed $\omega^2 \neq \Omega_{n_e}^2$, $\omega^2 \neq \Omega_{n_i}^2$. Using (24)–(25) in (23) we obtain

$$(k^2c^2 - \omega^2)E_x = -\sum_n \omega \left[\omega_{n_i}^2 \frac{\left(\omega E_x + i\Omega_{n_i} E_y\right)}{\omega^2 - \Omega_{n_i}^2} + \omega_{n_e}^2 \frac{\left(\omega E_x - i\Omega_{n_e} E_y\right)}{\omega^2 - \Omega_{n_e}^2} \right], \tag{26}$$

$$(k^2c^2 - \omega^2)E_y = -\sum_n \omega \left[\omega_{n_i}^2 \frac{\left(\omega E_y - i\Omega_{n_i} E_x\right)}{\omega^2 - \Omega_{n_i}^2} + \omega_{n_e}^2 \frac{\left(\omega E_y + i\Omega_{n_e} E_x\right)}{\omega^2 - \Omega_{n_e}^2} \right], \tag{27}$$

where $\omega_{n_e}^2 = 4\pi N_n^0 e^2/M_{n_e}$, $\omega_{n_i}^2 = 4\pi N_n^0 e^2/M_{n_i}$. These two equations can be written as

$$AE_x = iBE_y, \; AE_y = -iBE_x, \tag{28}$$

where
$$A = k^2c^2 - \omega^2 + \sum_n \left(\frac{\omega^2\omega_{n_i}^2}{\omega^2 - \Omega_{n_i}^2} + \frac{\omega^2\omega_{n_e}^2}{\omega^2 - \Omega_{n_e}^2} \right), \tag{29}$$

$$B = \sum_n \omega \left[\frac{\omega_{n_i}^2 \Omega_{n_i}}{\omega^2 - \Omega_{n_i}^2} - \frac{\omega_{n_e}^2 \Omega_{n_e}}{\omega^2 - \Omega_{n_e}^2} \right]. \tag{30}$$

Relation (28) shows that

$$|E_x|^2 = |E_y|^2 \tag{31}$$

and
$$A^2 = B^2. \tag{32}$$

Eq. (31) is the condition for circularly polarized waves. So, this model indicates the excitation of circularly polarized waves in the plasma of VIPPs. For these waves, let the electric field be

$$\boldsymbol{E} = (a\cos\theta, \lambda a \sin\theta, 0), \tag{33}$$

where a is the field amplitude, $\theta = kz - \omega t$, the helicity $\lambda = 1$ for the LCP waves and $\lambda = -1$ for the RCP waves. Then, in place of (32), explicitly, the dispersion relation is

$$k_{\pm}^2c^2 - \omega^2 + \sum_n \left(\frac{\omega^2\omega_{n_i}^2}{\omega^2 - \Omega_{n_i}^2} + \frac{\omega^2\omega_{n_e}^2}{\omega^2 - \Omega_{n_i}^2} \right) = \pm \sum_n \omega \left[\frac{\omega_{n_i}^2 \Omega_{n_i}}{\omega^2 - \Omega_{n_i}^2} - \frac{\omega_{n_e}^2 \Omega_{n_e}}{\omega^2 - \Omega_{n_e}^2} \right], \tag{34}$$

where $\omega_{n_e}^2 = 4\pi N_n^0 e^2/M_{n_e}$, $\omega_{n_i}^2 = 4\pi N_n^0 e^2/M_{n_i}$. With the help of (20) and (21), these become

$$k_{\pm}^2c^2 - \omega^2 + \sum_n \left[\frac{\omega^2\omega_{n_i}^2}{\omega^2 - \left(\Omega_{n_i}^0 + \xi_{n_i}^0\right)^2} + \frac{\omega^2\omega_{n_e}^2}{\omega^2 - \left(\Omega_{n_e}^0 + \xi_{n_e}^0\right)^2} \right] = \pm \sum_n \omega \left[\frac{\omega_{n_i}^2 \left(\Omega^{n_i} + \xi_{n_i}^0\right)}{\omega^2 - \left(\Omega_{n_i}^0 + \xi_{n_i}^0\right)^2} - \frac{\omega_{n_e}^2 \left(\Omega^{n_e} + \xi_{n_e}^0\right)}{\omega^2 - \left(\Omega_{n_e}^0 + \xi_{n_e}^0\right)^2} \right]. \tag{35}$$

Eq. (35) simplifies to

$$k_{\pm}^2c^2 = \omega^2 - \omega \sum_n \frac{\omega_{n_i}^2 \left(\omega \pm \xi_{n_e}^0\right) + \omega_{n_e}^2 \left(\omega \pm \xi_{n_i}^0\right)}{\left[\omega \pm \left(\Omega_{n_i}^0 + \xi_{n_i}^0\right)\right]\left[\omega \mp \left(\Omega_{n_e}^0 + \xi_{n_e}^0\right)\right]}. \tag{36}$$

One new case of low frequency waves arises when one of the two quantities 'x' and 'y' say 'x' is greater than 'y' but less than the middle term of expansion of $(x \pm y)^2$ dominates, that is $(x \pm y)^2 \approx \pm 2xy$ where presently $x = \Omega_{n_e}^0$ or $\Omega_{n_i}^0$ and $y = \xi_{n_e}^0$ or $\xi_{n_i}^0$. In this case, eq. (35) reduces to

$$k_{\pm}^2c^2 \approx \mp \frac{\omega}{2} \sum_n \left(\frac{\omega_{n_i}^2}{\xi_{n_i}^0} + \frac{\omega_{n_e}^2}{\xi_{n_e}^0} \right). \tag{37}$$

So, one of these two waves is a decay wave and the other is dispersive. These waves have no cut-off frequency.

For the waves having Alfvén wave frequencies $\omega < \Omega_{n_i}^0$, we obtain from (37),

$$k_{\pm}^2 = \pm \sum_n \frac{\left(\dfrac{\omega\xi_{n_e}^0}{C_{Ae}^2} + \dfrac{\omega\xi_{n_i}^0}{C_{Ai}^2} \right)}{\left(1 + \dfrac{\xi_{n_i}^0}{\Omega_{n_i}^0}\right)\left(1 - \dfrac{\xi_{n_e}^0}{\Omega_{n_e}^0}\right)}, \tag{38}$$

where

$$C_{Ae}^2 = \frac{H^{0^2}}{4\pi\rho^{n_e}}, \; C_{Ai}^2 = \frac{H^{0^2}}{4\pi\rho^{n_i}}, \; \rho^{n_e} = M_{n_e} N_n^0, \; \rho^{n_i} = M_{n_i} N_n^0.$$

In this case, the LCP wave is dispersive but the RCP wave is a decay wave.

The DC magnetic moment density χ has been evaluated for the general case using the relation [5]

$$\chi = \frac{1}{2c}\sum(\zeta_s \times j_s) \tag{39}$$

for which the dispersion relation is (35). ζ_s is the wave induced displacement, j_s is the current density and subscript 's' represents the species of the plasma. As for finding the Poynting flux $c(\boldsymbol{E}\times\boldsymbol{H})/4\pi$ and the density of rate of work done $(\boldsymbol{E}\cdot\boldsymbol{J})$, which like χ are second order effects, only the real expressions of the applied wave field, given in (33) has to be used. Thus we obtain

$$\chi_x = 0,\ \chi_y = 0,$$

$$\chi_z = -\frac{\lambda a^2}{2c\omega}\sum_n\left[\frac{N_{n_i} q_{n_i}^3}{M_{n_i}^2\left[\omega-\lambda\left(\Omega_{n_i}^0+\xi_{n_i}^0\right)\right]^2} + \frac{N_{n_e} q_{n_e}^3}{M_{n_e}^2\left[\omega-\lambda\left(\Omega_{n_e}^0+\xi_{n_e}^0\right)\right]^2}\right]. \tag{40}$$

Evidently, the DC moment field of the LCP waves and the RCP waves are in opposite directions and have non-identical magnitudes.

4. Circularly polarized waves affecting a collisionally damped two component plasma

Writing $E_\pm = E_x \pm iE_y$ *etc.*, for the LCP waves, we find that

$$(k_+^2c^2-\omega^2)E_+ = 4\pi i Ne\omega\left(v_{i_+}-v_{e_+}\right), \tag{41}$$

$$\left(\omega+\Omega_e^0-\xi_e^0+i\nu_e\right)v_{e_+} - i\nu_e v_{i_+} + \frac{ie}{m}E_+ = 0, \tag{42}$$

$$\left(\omega-\Omega_i^0-\xi_i^0+i\nu_i\right)v_{i_+} - i\nu_i v_{e_+} - \frac{ie}{M}E_+ = 0, \tag{43}$$

where ν_i is the collision frequency of an ion with an electron, $i=\sqrt{-1}$ when it appears as a factor, otherwise it is used as a superscript or subscript and ν_e is the collision frequency of an electron with an ion. For the RCP waves we similarly have

$$(k_-^2c^2-\omega^2)E_- = 4\pi i Ne\omega\left(v_{i_-}-v_{e_-}\right), \tag{44}$$

$$\left(\omega+\Omega_e^0-\xi_e^0+i\nu_e\right)v_{e_-} - i\nu_e v_{i_-} + \frac{ie}{m}E_- = 0, \tag{45}$$

$$\left(\omega-\Omega_i^0-\xi_i^0+i\nu_i\right)v_{i_-} - i\nu_i v_{e_-} - \frac{ie}{M}E_- = 0. \tag{46}$$

Solving the linear simultaneous eqs. (42) and (43) for v_{e_+} and v_{i_+} and (45) and (46) for v_{e_-} and v_{i_-} in terms of $E_\pm$, we obtain

$$v_{e_\pm} = E_\pm\frac{-\frac{e}{M}\nu_e-\frac{ie}{m}\left[\omega\mp\left(\Omega_i^0+\xi_i^0\right)+i\nu_i\right]}{\left[\omega\pm\left(\Omega_e^0-\xi_e^0\right)+i\nu_e\right]\left[\omega\mp\left(\Omega_i^0+\xi_i^0\right)+i\nu_i\right]+\nu_e\nu_i}, \tag{47}$$

$$v_{i_\pm} = E_\pm\frac{\frac{e}{m}\nu_i+\frac{ie}{M}\left[\omega\pm\left(\Omega_e^0-\xi_e^0\right)+i\nu_e\right]}{\left[\omega\pm\left(\Omega_e^0-\xi_e^0\right)+i\nu_e\right]\left[\omega\mp\left(\Omega_i^0+\xi_i^0\right)+i\nu_i\right]+\nu_e\nu_i}. \tag{48}$$

The dispersion relations for the LCP and the RCP waves thus become

$$k_\pm^2c^2-\omega^2 = \frac{-\omega^2\left(\omega_e^2-\omega_i^2\right)\pm\left(\xi_e^0\omega_i^2+\xi_i^0\omega_e^2\right)}{\left[\omega\pm\left(\Omega_e^0-\xi_e^0\right)+i\nu_e\right]\left[\omega\mp\left(\Omega_i^0+\xi_i^0\right)+i\nu_i\right]+\nu_e\nu_i}, \tag{49}$$

where $\omega_e^2 = 4\pi N_0e^2/m$, $\omega_i^2 = 4\pi N_0e^2/M$. For $\nu_i = 0$, $\nu_e = 0$, this dispersion relation is identified with that of (36). The imaginary terms in the dispersion relation from collisional loss are necessary for some resonant interactions. For non-resonent interactions we ignore the collision terms.

The expressions for the wave field induced displacement vector ζ(the time derivative of which is the velocity) for each constituent of a plasma is necessary for finding the DC magnetic moment χ per unit volume. We find that

$$\zeta_{e_\pm} = \frac{1}{\omega}E_\pm\frac{-\frac{ie}{M}\nu_e+\frac{e}{m}\left[\omega\mp\left(\Omega_i^0+\xi_i^0\right)+i\nu_i\right]}{\left[\omega\pm\left(\Omega_e^0-\xi_e^0\right)+i\nu_e\right]\left[\omega\mp\left(\Omega_i^0+\xi_i^0\right)+i\nu_i\right]+\nu_e\nu_i} \tag{50}$$

$$\zeta_{i_\pm} = \frac{1}{\omega}E_\pm\frac{\frac{ie}{m}\nu_i-\frac{e}{M}\left[\omega\pm\left(\Omega_e^0-\xi_e^0\right)+i\nu_e\right]}{\left[\omega\pm\left(\Omega_e^0-\xi_e^0\right)+i\nu_e\right]\left[\omega\mp\left(\Omega_i^0+\xi_i^0\right)+i\nu_i\right]+\nu_e\nu_i} \tag{51}$$

The relevant magnetic moment formula following form is

$$\chi_z = \frac{iN_0e}{4c}\left(\zeta_{e_-}v_{e_+}-\zeta_{e_+}v_{e_-}\right)-\frac{iN_0e}{4c}\left(\zeta_{i_-}v_{i_+}-\zeta_{i_+}v_{i_-}\right). \tag{52}$$

As mentioned earlier, only real values of field expressions and all other factors have to be used in this formula. So, in place $\zeta_{e_\pm}$ of (50) and $\zeta_{i_\pm}$ of (51), the expressions $(\zeta_{e_\pm}+\zeta_{e_\pm}^*)/2$ and $(\zeta_{i_\pm}+\zeta_{i_\pm}^*)/2$ have to be used where $\zeta_{e_\pm}^*$ and $\zeta_{i_\pm}^*$ are the complex conjugates of $\zeta_{e_\pm}$ and $\zeta_{i_\pm}$ fo finding the DC magnetic moment.

Further analysis without any simplifying assumptions if the above quantities would be very cumbrous and unimportant for physics. We consider some simple cases now.

Special case : (a)

Waves of Alfvén wave frequencies in collision free plasmas :

For Alfvén wave frequencies, $\omega < \Omega_i^0$, in a collision-free two component plasma having an ambient magnetic field in the direction of wave propagation (the direction of Z-axis), the disperson relation (49) simplifies to

$$k_\pm^2 = \left(\frac{\omega^2}{C_{A_i}^2} + \frac{\omega^2}{C_{A_e}^2}\right) \mp \left(\frac{\omega\xi_e^0}{C_{A_e}^2} + \frac{\omega\xi_i^0}{C_{A_i}^2}\right), \tag{53}$$

where $C_{A_i}^2 = \frac{H^{0 2}}{4\pi\rho^i}$, $C_{A_e}^2 = \frac{H^{0 2}}{4\pi\rho^e}$, $\rho^i = MN_0$, $\rho^e = mN_0$.

Since $\omega < \Omega_e^0$, k_+^2 becomes negative and the LCP waves become purely decay waves and the RCP waves remain dispersive because k_-^2 is positive.

The total non-zero magnetic moment field, evaluated with the help of the formula (39) is

$$\chi_z^0 = \frac{eE^2}{8\pi\omega c}\; \frac{\omega}{m(\Omega_e^0 - \xi_e^0)^2}\; \frac{\omega_i^2}{M(\Omega_i^0 + \xi_i^0)^2} \tag{54}$$

where $\omega_e^2 = 4\pi N_0 e^2/m$, $\omega_i^2 = 4\pi N_0 e^2/M$.

The first harmonic magnetic moment in the plasma is evaluated. The non-zero component of this field is given by

$$\chi_x = \gamma \sin\theta, \quad \chi_y = -\gamma\cos\theta, \quad \chi_z = 0. \tag{55}$$

where $\gamma = \chi_z^0 N_0 e/2mc\omega$. Now using the relation $B_i = \mu_{ij} H_i$ $H_i + 4\pi\chi_i$, the non-zero elements of the magnetic permeability tensor μ_{ij} are evaluated :

$$\mu_{11} = 1 + \frac{4\pi\gamma\omega}{ck}, \quad \mu_{22} = 1 - \frac{4\pi\gamma\omega}{ck} \tag{56}$$

All the other elements of the magnetic permeability tensor μ_{ij} are zero. These non-identical in value of diagonal elements indicate existence of spin wave features in plasmas of VIPPs. This suggests the possibility of investigating magneitc material like behaviour of fully ionized plasma affected by waves and vorticity.

Special case : (b)

Vortex free electrons, vortex free ions and vortex affected negative ions :

This case is relevant to the experimental findings on the loss energy in the excitation of the expanding super dense plasma in the programmes of fusion relation [2]. A cognizable amount of laser energy would be spent for acquiring the vorticity by the few negative ions which exist in the laser induced plasma, because negative ions are the most mobile ones of the heavy particles and because rotation energy acquired will be maximum if the negative ions becomes vorticity affected.

The three component plasma is a specified by the following nonzero parameters and relations

$$q_i = e, \quad q_e = -e, \quad N^i = N_i^e + N_n^e,$$
$$\rho^q = e\left(N^i - N_i^e - N_n^e\right),$$
$$J = e\left(N^i v_i - N_i^e v_i^e - N_n^e v_n^e\right). \tag{57}$$

Since we are here concerned with purely transverse waves we put $\rho^c = 0$ and hence have $N^i = N_i^e + N_n^e$. So the existence of populations of charged vortex elements, in our model, does not violate the macroscopic charge neutralization condition in the prefield state and for transverse waves in the perturbed state. The relevant guiding equations of motion are

$$\dot{v}_i = \frac{e}{M}E, \quad \dot{v}_i^e = -\frac{e}{m}E, \quad \dot{v}_n^e = -\frac{e}{M}E + \left(v_n^e \times \xi_e^0\right). \tag{58}$$

These with the help of Maxwell equations, give the dispersion relation

$$k_\pm^2 c^2 - \omega^2 - \omega_{pe}^2 \quad \frac{\omega_{pv}^2\omega}{(\omega \mp \xi_e^0)} \tag{59}$$

for transverse waves of infinitesimally small amplitude where $\omega_{pe}^2 = 4\pi N^e e^2/m$, $\omega_{pv}^2 = 4\pi N^n e^2/M$. This relation also follows as a special case of the general dispersion relation (35). For waves of low frequencies, $\omega < |\xi_e^0|$; so from (59), we obtain

$$k^2c^2 = -\omega_{pe}^2 + \lambda\omega_{pe}^2\frac{\omega}{\xi_e^0} = \omega_{pe}^2\left(-1 + \lambda\frac{\omega}{\xi_e^0}\right). \tag{60}$$

This dispersion relation shows that the low frequency LCP waves are decay waves and the RCP waves disperse through the plasma. This is a birefringence and also a noncollisional absorption of waves in plasmas.

5. Mechanisms of local growth of vortex

A fluid in flow does not directly form vortices. Resistance sets in beyond a critical velocity. To have resistance the flow energy and momentum must go into heat, that is internal excitations at the necessary momentum. For flow of a fluid over a plane surface, say the XY-plane, the viscosity arises from a transfer of momentum along OZ.

A state of local circulation is very difficult to have without a high excitation energy. A liquid cannot rotate as a rigid body unlike a solid, when the excitation energy is low.

However, a part of the liquid, unlike that in a solid, when the energy is low, can turn independently of the whole body. Any motion of the body is compounded of motions of the tiny parts. But, to set any small part into a rotational state requires a high energy because the moment of inertia is small. When only a limited energy is available, nearly all the parts must be frozen out in their ground state, so that every where the local angular momentum is zero. It takes energy to create circulation, and this circulation may be distributed uniformly throughout the fluid. The liquid is therefore assumed to be made up of quasi independent units of nearly atomic dimensions; some of these can even acquire vorticity from circulation.

Local development and growth of vorticity is possible in fluids in motion [4,6,7]. The slowing down of a fluid in motion at velocity v by the amount δv, such that the entire fluid changes its velocity, is possible when the velocity remains irrotational. Then the mechanical momentum is

$$\delta p = M\delta v\,, \quad \delta\varepsilon = M\,v\,\delta v = v\;\delta p, \tag{61}$$

where M is the fluid mass, and $\delta\varepsilon$ is the energy loss from the value ε. When the loss is converted into heat, this $\delta\varepsilon$ is transformed into internal excitations of rotations [7]. Resistance to motion sets in even at low velocities, when small part of the fluid stop or slow down through the entire fluid does not necessarily decrease at once. This fluid flow becomes rotational, and possesses some local circulations. Therefore, states of fluid motion are possible for which $\nabla\times v = 2\Omega \neq 0$ where Ω is the angular velocity vector. This relation is valid where there is circulation.

6. Conclusion

Critical assessment is required of the propriety, capacity, applications and extension of the proposed model for theory of the vortices in particles having mass and charge. A proper dynamics of PMCVs should have a 4-vector formalism of special theory of relativity and Lagrangian and Hamiltonian formalisms as well. A kinetic theory formalism for PMCVs in Vlasov plasma and partially ionized plasma should exist. These cannot be obtained with the help of our model. The assumptions of the expression for froce per unit volume of the vorticity of particles should have a physical basis. The statement that the prefield value of the vorticity is ξ is an adhoc imposition on the field equations, which must be replaced by a more proper theory.

Low frequency, circularly polarized waves propagate through such plasmas containing PMCVs. These waves exhibit birefringence, and at the same time are noncollisionaly absorbed. Hence a plasma of PMCVs acts as a wave filter. Dispersion features of Alfvén wave frequencies are modified in presence of the vorticity.

The magnetic permeability tensor elements, depending upon the wave parameters, are different for LCP and RCP waves. This suggests the possibility of investigating magnetic material like behaviour of plasmas, affected by vorticity and waves [8–10].

The field equations (17)–(25) for this model can be considered for waves of other types. For instance the modification of Appleton-Hartree equation can be investigated, including that of the Hall effect, the Whistler modes, *etc*.

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References

[1] H C Pant *Centre of Advanced Technology (CAT), Indore, India* (Verbal Communication) (1996)

[2] Chandra Das, Rajarshi Bhattacharyya and B Chakraborty *Phys Plasmas* 4 p 1484 (1997)

[3] P K Shukla *Proc Int Conf on Plasma Physics* (eds) A Sen and P K Kaw (Bangalore : Indian Academy of Science) p 297 (1989)

[4] A C Hollies Hallet *Progress in Low Temperature Physics* (ed) C J Gorter (New York : Interscience) p 64 (1955)

[5] B Chakraborty, R Bhattacharya and S N Paul *Indian J. Phys* 75B p 439 (2001)

[6] C J Gorter *Progress in Low Temperature Physics* (ed) C J Gorter (New York : Interscience) p 1 (1953)

[7] F P Feynman *Progress in Low Temperature Physics* (ed) C J Gorter (New York : Interscience) p 64 (1955)

[8] A K Dekker *Solid State Physics* (London : Macmillan) p 452 (1958)

[9] M C Steele and B Vural *Wave Interaction in Solid Plasmas* (New York : McGraw-Hill) p 162 (1969)

[10] A J Chorin *Vorticity and Turbulence* (New York : Springer-Verlag) (1994)