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Wavevector and frequency dependent complex dielectric function and the dynamical structure factor of two-dimensional weakly-coupled quantum and classical hot plasmas

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Abstract Complete expressions for the wavevector and frequency dependent complex dielectric function for two-dimensional quantum and classical weakly-coupled one-component thermal plasma have been obtained and computed in the entire domain of plasma frequency. The dynamical structure factor and other related quantities have been computed to obtain full knowledge about the collective dynamics of the two dimensional plasma. Comparisons have also been made with other models suitably modified for the two dimensional case.

Keywords Surfaces and interfaces, thin films, dielectric response

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1. Introduction

Two-dimensional weakly-coupled hot plasma can be realized experimentally, amongst other solid state methods such as, thin semiconductor interfaces, thin films, inversion layers *etc* , at very low temperatures, around 1 K when the electrons of concentration approximately, $10^5 \leq n \leq 10^{10}$ cm^{-2}, are trapped on the surface of a liquid dielectric like He [1,2] or Ne. The combination of external applied field and the image potential provide the necessary positive neutralizing background and confine electrons to the lowest energy state perpendicular to the surface [3]. The electrons are completely free to move on the surface unrestricted. The electrons in such a plasma, unlike three-dimensional plasma, possess only two degrees of freedom. Such a change, results in dramatic changes in the plasma behaviour. For instance, while the plasma frequency in three dimensions is independent of the wave vector, in two-dimensional plasma, it turns out to be dependent on the wavevector $\boldsymbol{q}$ with a power law, *i.e.*, $\omega_r \propto q^{1/2}$

In a weakly-coupled plasma, the coupling parameter $\Gamma((= e^2(\pi n)^{1/2}/k_BT)$, where n is surface number density of electrons and other symbols have their usual meaning), is equal to or less than one. Here, an electron is assigned a circular area of radius r_s. The change over from classical to quantum behaviour is on the other hand, governed by the value of $n\,\lambda_{\rm th}^2$ (where $\lambda_{\rm th}\left(= h/\sqrt{2mk_BT}\right)$ is the thermal de-Broglie wavelength of electrons carrying mass m at temperature T, h is Planck's constant and k_B is Boltzmann's constant). When $n\,\lambda_{\rm th}^2 \ll 1$, the plasma behaves classically both in its dynamics and statistics, but as $n\,\lambda_{\rm th}^2$ approaches larger values, the statistical behaviour of the system changes to quantum mechanical description and one has to use two particle distribution function [4-6] in its description instead of one particle distribution function. In the present communication, an attempt is made to study such a one-component quantum plasma which has not been hitherto studied, along with the classical plasma for which, the complete frequency ω and wavevector $\boldsymbol{q}$-dependent dielectric function and collective dynamics have not been worked out. It may be noted that when $n\,\lambda_{\rm th}^2 \gg 1$, the plasma is highly degenerate and strongly coupled where quantum Hall effects both integral and fractional, occur. For such a degenerate plasma, wave vector and frequency-dependent dielectric function has already been reported [7].

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2. Mathematical formalism

The complex dielectric function $\varepsilon^{Q}(q,\omega)$ for a two-dimensional, weakly-coupled, one-component quantum plasma, is related to the two-particle quantum distribution function $F_{\pm}(q,\omega)$ [5] as follows :

$$\varepsilon^{Q}(q,\omega) = 1 + \frac{2\pi i}{h}\left[F_{+}(q,\omega) - F_{-}(q,\omega)\right], \quad (1)$$

where $F_{\pm}(q,\omega)$ are given as

$$F_{\pm}(q,\omega) = \frac{1}{2\pi i}\frac{2\pi e^{2}}{q}\int \frac{dp}{\left(\omega - \frac{q\cdot p}{m} + i\varepsilon\right)} \times f\left(p \pm \frac{hq}{2}\right), \quad (2)$$

e and p are the charge and two dimensional linear momentum of the electron respectively. The single particle momentum distribution function $f\left(p+\frac{hq}{2}\right)$ here, corresponds to the Maxwellian and is given by the expression

$$f\left(p+\frac{hq}{2}\right) = A\exp\left(\frac{-\left(p+\frac{hq}{2}\right)^{2}}{2mk_{B}T}\right). \quad (3)$$

A can be evaluated using the sum rule

$$\int_{0}^{\infty} \omega \,\mathrm{Im}\, \varepsilon(q,\omega)\, d\omega = \frac{\pi}{2}\omega_{p}^{2}, \quad (4)$$

where ω_p is the angular plasma frequency of the two-dimensional plasma.

Substituting expression (3) in (2) and solving the two-dimensional integral, $F_{\pm}(q,\omega)$ can be obtained. Using the derived $F_{\pm}(q,\omega)$ in the expression (1), one gets the complete wavevector and frequency dependent complex dielectric function for one-component quantum plasma as

$$\varepsilon^{Q}(q,\omega) = \varepsilon_{1}^{Q}(q,\omega) + i\varepsilon_{2}^{Q}(q,\omega) = 1 + \sqrt{2}\,\frac{\omega_{p}^{2}}{q^{2}v^{2}}\frac{mv}{hq}$$
$$\times\left[e^{-\left(\frac{\omega}{\sqrt{2}qv}+\frac{hq}{2\sqrt{2}mv}\right)^{2}}\left(\frac{\omega}{\sqrt{2}qv}+\frac{hq}{2\sqrt{2}mv}\right)\right.$$
$$\times\left(1+\frac{1}{3}\left(\frac{\omega}{\sqrt{2}qv}+\frac{hq}{2\sqrt{2}mv}\right)^{2}+\cdots\right)$$
$$-e^{-\left(\frac{\omega}{\sqrt{2}qv}-\frac{hq}{2\sqrt{2}mv}\right)^{2}}\left(\frac{\omega}{\sqrt{2}qv}-\frac{hq}{2\sqrt{2}mv}\right)$$
$$\left.\times\left(1+\frac{1}{3}\left(\frac{\omega}{\sqrt{2}qv}-\frac{hq}{2\sqrt{2}mv}\right)^{2}+\cdots\right)\right]$$
$$+ i\sqrt{\frac{\pi}{2}}\frac{\omega^{2}}{q^{2}v^{2}}\frac{mv}{hq}$$
$$\times\left[e^{-\left(\frac{\omega}{\sqrt{2}qv}-\frac{hq}{2\sqrt{2}mv}\right)^{2}} - e^{-\left(\frac{\omega}{\sqrt{2}qv}+\frac{hq}{2\sqrt{2}mv}\right)^{2}}\right], \quad (5)$$

where $v = \left(\sqrt{\frac{k_{B}T}{m}}\right)$ is the thermal velocity.

The complex dielectric function for classical plasma can be obtained from expression (5) when $h = 0$ and is given as :

$$\varepsilon^{cl}(q,\omega) = \varepsilon_{1}^{cl}(q,\omega) + i\varepsilon_{2}^{cl}(q,\omega)$$
$$= 1 + \frac{\omega_{p}^{2}}{q^{2}v^{2}} - \frac{\omega_{p}^{2}}{q^{2}v^{2}}\frac{\omega^{2}}{q^{2}v^{2}}e^{-\omega^{2}/2q^{2}v^{2}}$$
$$\times\left(1+\frac{1}{3}\left(\frac{\omega^{2}}{2q^{2}v^{2}}\right)+\frac{1}{10}\left(\frac{\omega^{2}}{2q^{2}v^{2}}\right)^{2}+\cdots\right)$$
$$+ i\sqrt{\frac{\pi}{2}}\frac{\omega_{p}^{2}}{q^{2}v^{2}}\frac{\omega}{qv}e^{-\omega^{2}/2q^{2}v^{2}}. \quad (6)$$

Expressions (5) and (6) appear to be similar to the corresponding expressions for three-dimensional plasma [5], but here $\omega_{p} = \sqrt{2\pi n e^{2}q/m}$ and q is a two-dimensional vector and therefore, quantities related to these dielectric functions would have to be evaluated keeping this fact in mind [8,9].

The collective modes of the plasma can be obtained from the relation $\varepsilon_1(q,\omega) = 0$. Such an exercise, strictly speaking, is quite involved as $\varepsilon(q,\omega)$ is a polynomial of very high order in ω^2 as can be seen from the analysis of expressions (5) and (6), though it is possible to obtain such modes approximately [10]. Further, one cannot get the detailed knowldege about how well-defined these collective modes are, which one can obtain by studying the related dynamical structure factor $S(q,\omega)$ of the plasma. One can make use of the fluctuation dissipation theorem [5,6,11] to obtain the following expression for $S^{Q}(q,\omega)$:

$$S^{Q}(q,\omega) = \frac{1}{\pi\omega}\frac{q^{2}}{q_{\cdot}^{2}}\frac{\varepsilon_{2}^{Q}(q,\omega)}{\left[\left(\varepsilon_{1}^{Q}(q,\omega)\right)^{2}+\left(\varepsilon_{2}^{Q}(q,\omega)\right)^{2}\right]}, \quad (7)$$

where $q^{2} = \frac{1}{\lambda_{D}^{2}} - \frac{\omega_{p}^{2}}{v^{2}}$, λ_D being the two dimensional Debye screening length. $\varepsilon_{1}^{Q}(q,\omega)$ and $\varepsilon_{2}^{Q}(q,\omega)$ are the real and imaginary parts of the complex dielectric function given in expression (5). The singularity in $S(q,\omega)$ yields the value of the collective modes which corresponds exactly to the value given by $\varepsilon_1(q,\omega) = 0$. $\varepsilon_2(q,\omega)$ gives essentially the damping accompanying the mode. It is also clear from the expression that $S(q,\omega)$ may not be close to delta function in which case, it describes the presence of extremely well defined collective mode. $S(q,\omega)$ can also be experimentally observed by appropriate electron inelastic scattering experiment from the plasma.

The static structure factor $S(q)$ is the zero-th sum rule

$$S(q) = \int_{-\infty}^{\infty} S(q,\omega)\, d\omega, \quad (8)$$

which is also given by the expression

$$S^Q(q) = -\frac{q^2}{q_-^2}\left(\frac{1}{\varepsilon^2(q,0)} - 1\right), \tag{9}$$

$\varepsilon^Q(q, 0)$ is the zero frequency dielectric function which can be obtained from expression (5).

The zero frequency dynamic structure factor $S^Q(q, 0)$ which can be obtained from expression (7), is given as follows :

$$S^Q(q,0) = \frac{1}{\sqrt{2\pi}} \frac{1}{\left(\varepsilon_1^Q(q,0)\right)^2} \frac{1}{qv} e^{-(hq/2\sqrt{2}mv)^2} \tag{10}$$

The corresponding expressions for the classical plasma are :

$$S^{cl}(q,w) = \frac{1}{\pi\omega}\frac{q^2}{q_-^2}\frac{\varepsilon_2^{cl}(q,\omega)}{\left[(\varepsilon_1^{cl}(q,\omega))^2 + (\varepsilon_2^{cl}(q,\omega))^2\right]}, \tag{11}$$

$$S^{cl}(q) = \frac{q^2}{q^2 + q_-^2}, \tag{12}$$

$$S^{cl}(q,0) = \frac{1}{\sqrt{2\pi}} \frac{1}{(1 + q_-^2/q^2)^2} \frac{1}{qv}. \tag{13}$$

In computing $S(q, \omega)$, the value of $S(q)$ should be consistent from expressions (8) and (9) for the quantum case and (8) and (12) for the classical case. Like $S(q, \omega)$, $S(q)$ and $S(q, 0)$ are also experimentally observable [12,13].

3. Results and discussion

In order to compute the dielectric function given by expression (5), we take the surface number density of electrons $n = 7.65 \times 10^4$ cm^{-2} at $T = 1$ K. Computations have been made for $\varepsilon_1(q, \omega)$ and $\varepsilon_2(q, \omega)$ for different values of the quantum parameter, $R(= hq/mv)$. When $h = 0$, $R = 0$ and the expression (5) reduces to the expression (6) of the classical plasma. The computed values $\varepsilon_1(q, \omega)$ have been plotted in Figure 1 for different values of R. When R is 0.0001, the computed values from expression (5) for $\varepsilon_1(q, \omega)$ are very close to the ones calculated using expression (6) of the classical plasma, As R increases, the deviation from the classical case starts showing up as shown in the figure for $R = 1$, 2 and 3. The computed $\varepsilon_2(q, \omega)$ for various values of R are shown in the inset of the figure. For $R = 0.0001$, the computed values of $\varepsilon_2(q, \omega)$ match very closely with the computations of $\varepsilon_2(q, \omega)$ from expression (6). Here too, the deviation from the classical result becomes increasingly different as R increases from 1 to 3 as is evident from the figure. The collective mode frequencies ω_c exist for $R \lesssim 2$. As for $R = 2$, $\varepsilon_1(q, \omega)$ does not show any intersection with $\varepsilon_1(q, \omega) = 0$. Thus, the presence of collective modes is restricted to weakly quantum and classical case. Further, the Landau damping given by $\varepsilon_2(q, \omega)$, keeps on increasing with the increase in the value of R as shown in the inset. It may be mentioned that no analytical expression which covers the entire ω/qv was available even for the classical plasma. But here, due care has to be taken to ensure the convergence of the series.

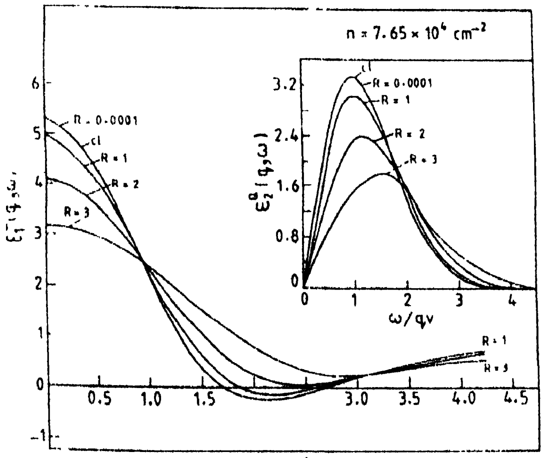

Figure 1. Variation of the computed values of real $\varepsilon_1^Q(q, \omega)$ and imaginary $\varepsilon_2^Q(q, \omega)$ parts of quantum two dimensional plasma dielectric function with angular plasma frequency ω, expressed in units of qv for different values of quantum parameter R when the surface number density of electrons $n = 7.65 \times 10^4$ cm^{-2}, temperature $T = 1$ K and wave vector $q = 1.87 \times 10^2$ cm^{-1}. Cl refers to the classical variation of the dilectric function.

In Figure 2 have been plotted, the computed values of $S(q, \omega)$ using expressions (7) and (11) for the quantum and

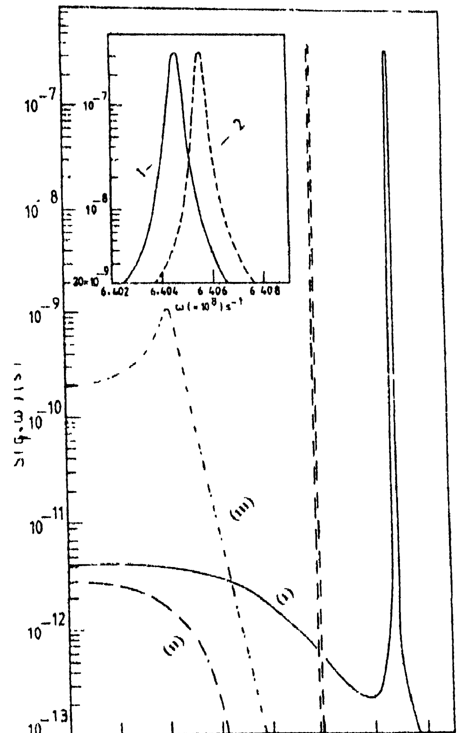

Figure 2. Variation of the dynamical structure factor $S(q, \omega)$ with the angular plasma frequency ω, for different values of the wavevector q and surface number density n at $T = 1$ K for the two dimensional plasma. For $n = 7.65 \times 10^5$ cm^{-2}, (——) curve (i) is shown for $q = 3.0 \times 10^2$ cm^{-1}, and (-●-) curve (ii) for $q = 1.87 \times 10^2$ cm^{-1}. For $n = 7.65 \times 10^4$ cm^{-2} (-●●-) curve (iii) is plotted for $q = 1.87 \times 10^2$ cm^{-1}.
In the inset are shown, the details of the $S^Q(q, \omega)$ and $S^{Cl}(q, \omega)$ by (——) and (- - -) curves respectively, near the peak values of the collective modes for $n = 7.65 \times 10^5$ cm^{-2} at $q = 3.0 \times 10^2$ cm^{-1}

classical plasma respectively when $n = 7.65 \times 10^4$ cm^{-2}. The computed value of $S(q, \omega)$ for $q = 1.87 \times 10^2$ cm^{-1} is shown by curve (iii). As the density is increased to 7.65×10^5 cm^{-2}, the $S(q, \omega)$ turns out to be that given by curve (ii), which shows the presence of much more well defined collective modes at higher energy, in comparison with that given for the lower density. In both these cases, the difference between $S^Q(q, \omega)$ and $S^{cl}(q, \omega)$ is very small as not to be evident in the figure. The values of $S(q)$ computed using the expressions (8) and (9), (8) and (12) for quantum and classical plasma respectively, are consistent with one another. Also, the values of $S(q, 0)$ plotted in Figure 2 are same as those calculated from (10) and (13). In Figure 2 is also shown the $S(q, \omega)$ for $q = 3.0 \times 10^2$ cm^{-1} and $n = 7.65 \times 10^5$ cm^{-2} by line (i) Here too, the collective mode is very well defined. The difference between the classical and quantum collective mode forms have been shown in the inset of the figure by curve 1 for quantum plasma and curve 2 for classical plasma The collective mode in the quantum case occurs at somewhat lower frequency and is more damped as is clear from the figure. The computations have also been done for larger values of R by changing temperature and wavevector, in which case, the difference between classical and quantum plasma increases but the absolute value of $S(q, \omega)$ turns out to be very small. It may by pointed out, that the frequency ω_c at which the collective mode occurs, (in all $S(q, \omega)$) studied, it has been checked that these also satisfy the relation $\varepsilon_1(q, \omega_c) = 0$ numerically.

In Figure 3 have been shown, the computed values of $S(q, \omega)$ given by expression (11) by curve (i) for different values of q when $n = 7.65 \times 10^4$ cm^{-2} and $T = 1$ K. As the value of q is increased from 2.079×10^1 cm^{-1} to 1.87×10^2 cm^{-1}, the collective mode frequency ω_c shifts to higher values as shown in Figure 3 and also plotted in Figure 4(b) by curve (i). The values from an earlier study [10] where only ω_c have been worked out, are close to our results. For the sake of comparison, we have also shown in Figure 4(b) the dispersion relation of the cold plasma by curve (ii) and for two-dimensional Wigner solid by curve (iii). By curve (iv), is shown the dispersion relation followed by neutral Maxwellian electron gas. The damping of the modes given by the full width at half maximum (FWHM) of $S(q, \omega)$, increases with the increase in q and is also plotted in Figure 4(c) by curve (i). The collective modes therefore, become increasingly ill-defined with increase in q. The variation in the value of the peak in the collective modes is also evident from Figure 4(a). $S(q, 0)$ keeps shifting to higher values with q and its variation is plotted in Figure 4(d) by curve (i) which matches with the values computed using expression (13). Finally, the static structure factor $S(q)$, as computed from expression (8), is found to be same as that given by expression (12) and have been plotted by curve (i) in Figure 4(e). Therefore, from this study one gets consistently both the dynamical structure factor and static structure factor and hence, can determine all relevant statistical thermodynamic properties of the plasma.

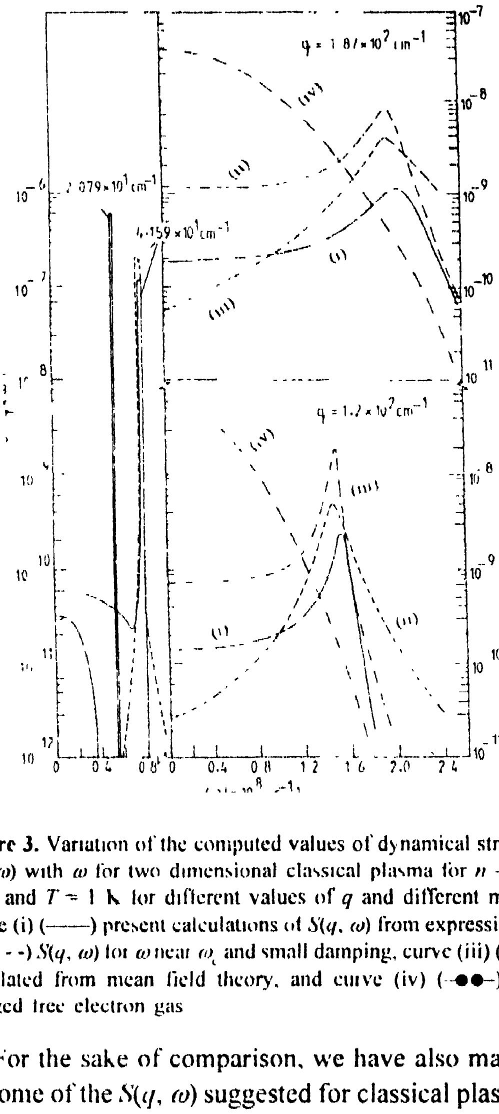

Figure 3. Variation of the computed values of dynamical structure factor $S(q, \omega)$ with ω for two dimensional classical plasma for $n = 7.65 \times 10^4$ cm^{-2} and $T = 1$ K for different values of q and different models Curve (i) (——) present calculations of $S(q, \omega)$ from expression (6), curve (ii) (- - -) $S(q, \omega)$ for ω near ω_c and small damping, curve (iii) (-●-) $S(q, \omega)$ calculated from mean field theory, and curve (iv) (-●●-) $S(q, \omega)$ for charged free electron gas

For the sake of comparison, we have also made studies on some of the $S(q, \omega)$ suggested for classical plasma. When one considers the Coulomb interaction to be zero and appropriately takes account of the two dimensional nature of charged system, one gets an expression for $S(q, \omega)$ [14,15], which has been worked out, computed and plotted in the Figure 3 by curves (iv) for $q = 1.2 \times 10^2$ cm^{-1} and 1.87×10^2 cm^{-1}.

Expression for $S(q, \omega)$ near $\omega = \omega_c$ and small damping [16] has been used to compute $S(q, \omega)$ and is plotted by curve (ii) in Figure 3 and the various characterstics of $S(q, \omega)$ have been plotted in Figure 4 by curve (ii). The computed value of $S(q)$ is not consistent with computation of $S(q)$ from expression (12) as is evident from Figure 4(e). This was expected as strictly speaking, the suggested $S(q, \omega)$ is valid for ω near ω_c. But as postulated [16], the damping does not turn out to be small, particularly for $q \geq 0.8 \times 10^1$ cm^{-1} and is infact, more than that given by full $S(q, \omega)$, as is evident from Figure 4(c). The values of ω_c however, are not very different from actual one excepting for large values of q, as shown in the inset of Figure 4(b) by curve (ii). The values of $S(q, \omega_c)$ become increasingly different from the present values for larger q as can be seen from Figure 4(a).

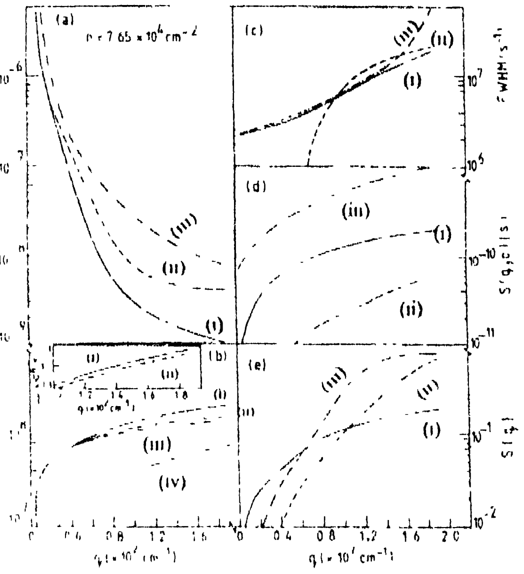

Figure 4. Variation of various characteristic quantities, Peak value of $S(q, \omega)$, *i.e.* $S(q, \omega_c)$ collective mode frequency ω_c, full width at half maximum (FWHM), $S(q, \omega)$ at $\omega \to 0$, *i.e.*, $S(q, 0)$ and static structure factor $S(q)$ with q for two dimensional classical plasma. In Figures (a), (c), (d) and (e), curve (i) ( ) corresponds to the present calculations from expression (6), curve (ii) (- - -) represent the calculations done for $S(q, \omega)$ near $\omega \sim \omega_c$ and curve (iii) (-●) for the expression of mean field theory. In Figure (b), the different curves represent : (i) (— --) calculations done using $S(q, \omega)$ from expression (6), (ii) (- - -) cold plasma, (iii) (-●-) two dimensional Wigner solid, and (iv) (●●) neutral Maxwellian electron gas.

In the inset of the Figure (b), present calculations shown by (·-—) curve (i) have been compared with the calculations done for the expression given in reference 16 shown by (- - -) curve (ii).

Computations have also been made using the mean field expression for $S(q, \omega)$ [14,15], appropriately modified for the two dimensional case and the results have been plotted by curve (iii) for various studied quantities in Figures 3 and 4. As in the earlier study, for low values of q there is a little difference from the present calculations but, as q increases significant differences start appearing. While, the collective mode frequency occurs at almost the same value, the damping is somewhat larger and increases sharply for $q \geq 1.4 \times 10^2$ cm^{-1} as shown in the Figure 4(c). Both, $S(q, 0)$ and $S(q, \omega_c)$, are much larger than that given by present calculations, as is evident from Figures 4(a) and 4(d). Moreover, values of the zeroth sum rule from this form of $S(q, \omega)$ are quite different, as shown in Figure 4(e).

4. Conclusion

One may conclude that it is not only possible to obtain complete analytical expressions for both quantum and classical two-dimensional plasmas, but also to compute these for the entire range of frequency, unlike earlier studies. Further, it turns out that as the degree of quantum nature of classical plasma is increased by either increasing the surface number density of electrons or decreasing the temperature, the collective mode frequency changes and plasma becomes more damped. There is no collective mode for quantum parameter $R > 2$. The dynamical structure factor which yields the complete dynamics of the collective modes, have been computed using the real and imaginary parts of the dielectric function. Explicit calculations show that though the value of collective mode frequency ω_c from some earlier approximate studies are close to the exact values from the present work, the details like the damping, the zero frequency dynamical structure factor *etc.* are quite different besides the detailed spectrum of the dynamical structure factor. The results of the present sutdy can be checked by performing appropriate electron inelastic scattering experiment from such two-dimensional plasmas.

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