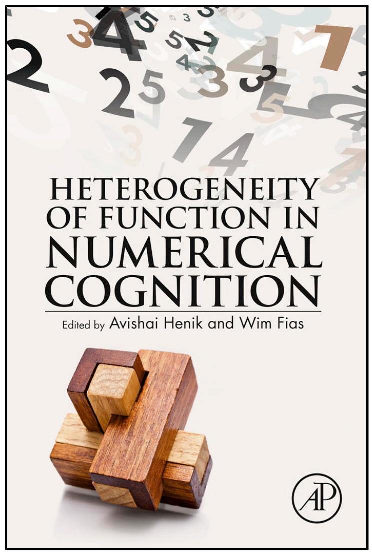

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CHAPTER

1

Numbers and Language: What's New in the Past 25 Years?

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OUTLINE

Heterogeneity of Function in Numerical Cognition
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The relationship between mathematical and verbal performance is not a clear one. On the one hand, both abilities seem to be related; on the other hand, many school systems offer pupils the opportunity to choose between a language-oriented education and a mathematics-oriented education, suggesting the two types of skills diverge.

THE RELATIONSHIP BETWEEN VERBAL AND ARITHMETICAL PERFORMANCE IN THE WISC INTELLIGENCE TEST

One way to assess the relationship between verbal and arithmetical performance is to see how they correlate in intelligence tests. Wechsler (1949), for instance, made a distinction between verbal intelligence and performance intelligence, and he included a test of arithmetic in the verbal scale. In the Wechsler Intelligence Scale for Children (WISC), the test of arithmetic correlated .6 with the verbal scale and .45 with the performance scale (Seashore, Wesman, & Doppelt, 1950). For comparison, the vocabulary subtest (the most typical verbal test in the WISC) correlated .75 with the verbal scale and .55 with the performance scale. The test with the highest correlation to the performance scale was the object assembly test, correlating .35 with the verbal scale and .6 with the performance scale. The arithmetic test was retained as part of the verbal scale when the WISC was revised for the first time (and called the WISC-R).

Factor analyses indicated, however, that a third factor was present in the WISC and the WISC-R. This factor was difficult to interpret but was called "freedom from distractibility." A new test was added to the WISC-III (the symbol search test) to better measure the elusive third factor, but

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this did not succeed very well because a new factor analysis hinted at four factors (Keith & Witta, 1997) as shown in Fig. 1.1. Surprisingly, in this analysis, the arithmetic subtest became the best measure of the factor freedom from distractibility. In addition, this factor had the highest correlation with the overall score (considered to be a measure of general intelligence, or *g*). Both findings suggested that the factor freedom from distractibility was more central to intelligence than its name suggested. Keith and Witta (1997) proposed to rename it as "quantitative reasoning." Unfortunately, only two tests loaded on the factor, which is weak evidence for a factor.

The fourth revision of the WISC included extra tests to better measure the four first-order factors. In particular, it was hypothesized that the

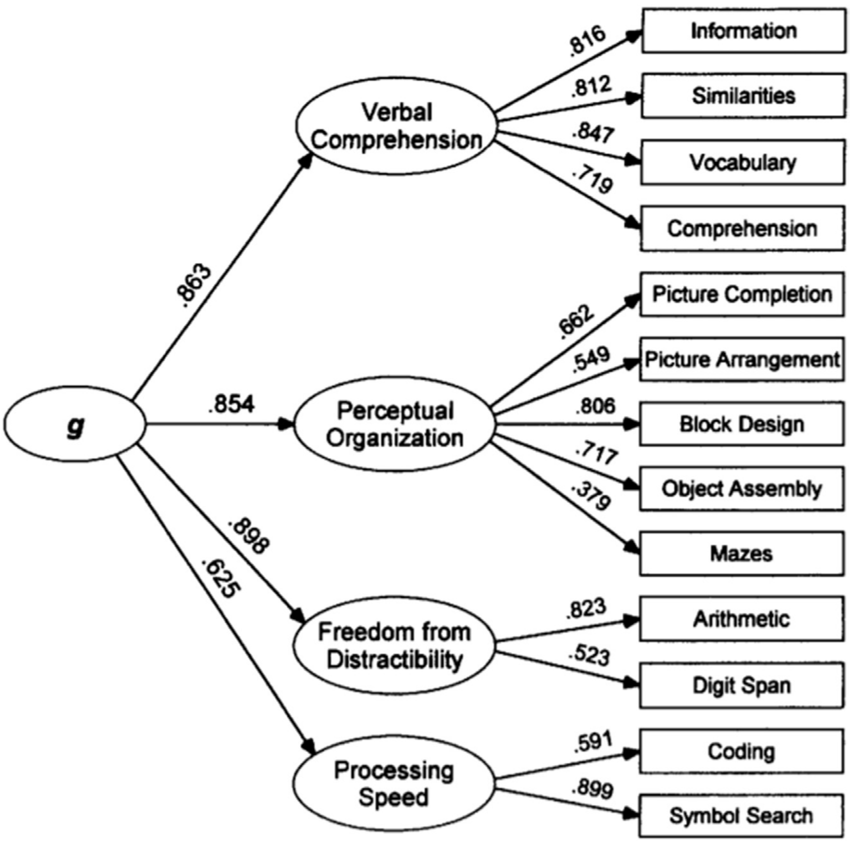

FIGURE 1.1 Outcome of a hierarchical factor analysis of the WISC-III. Subtests are a measure of both general intelligence (*g*) and a first-order variable. Four first-order variables could be discerned. The test of arithmetic loaded on the first-order variable freedom from distractibility, together with the digit span test. *From Keith, T. Z., & Witta, E. L. (1997). Hierarchical and cross-age confirmatory factor analysis of the WISC-III: What does it measure?* School Psychology Quarterly, 12*(2), 89–107.*

freedom from distractibility factor actually could be a working memory factor and new tests were added to better capture it. This seemed to work reasonably well (Keith, Fine, Taub, Reynolds, & Kranzler, 2006), as shown in Fig. 1.2 (although a solution with five first-order factors provided a better fit).

All in all, the analyses of the WISC-III and WISC-IV confirm that arithmetic skills are correlated to language skills (via the high correlations with *g*) and at the same time form two different intelligence factors on which individuals can score high or low (see also Reynolds, Keith, Flanagan, & Alfonso, 2013).

A similar conclusion was reached on the basis of an analysis specifically geared toward mathematical knowledge. Taub, Floyd, Keith, and McGrew (2008) predicted mathematics achievement on the basis of IQ subtests. The mathematics tests consisted of a calculation test (ranging from simple addition facts to calculus) and an applied problems test in which the nature of the problem had to be comprehended, relevant information identified, calculations performed, and solutions stated. Data were available from 5-year-old children to 19-year-old children. Fluid reasoning (similar to working memory) had an effect in all age groups (see also Primi, Ferrão, & Almeida, 2010). For the younger pupils, processing speed also contributed to the performance, whereas for the older participants, crystallized intelligence became more important (arguably to retrieve simple solutions from long-term memory; see also Calderón-Tena & Caterino, 2016).

Interestingly, the broader Cattell–Horn–Carroll (CHC) model, on which the analyses of Figs. 1.1 and 1.2 were based and which is currently seen as the best summary of intelligence research, postulates the existence of a separate first-order factor quantitative knowledge (McGrew, 2009), as shown in Fig. 1.3. Therefore, the expectation is that with the right tests included, numerical knowledge will come out as an individual type of intelligence, although so far attempts have not been successful (e.g., Keith, Low, Reynolds, Patel, & Ridley, 2010).

COGNITIVE PROCESSES INVOLVED IN NUMERICAL COGNITION: THE 1990s

The relationship between verbal and mathematical performance in intelligence tests provides an interesting background but is limited by the type of tests used to assess the various skills. In the psychometric tradition, tests have mainly been proposed via trial and error, starting with the first intelligence test published by Binet and Simon (1907). Tests that correlated with school achievement were retained, others were replaced. In addition, there is a strong force not to change existing arrangements too much, as

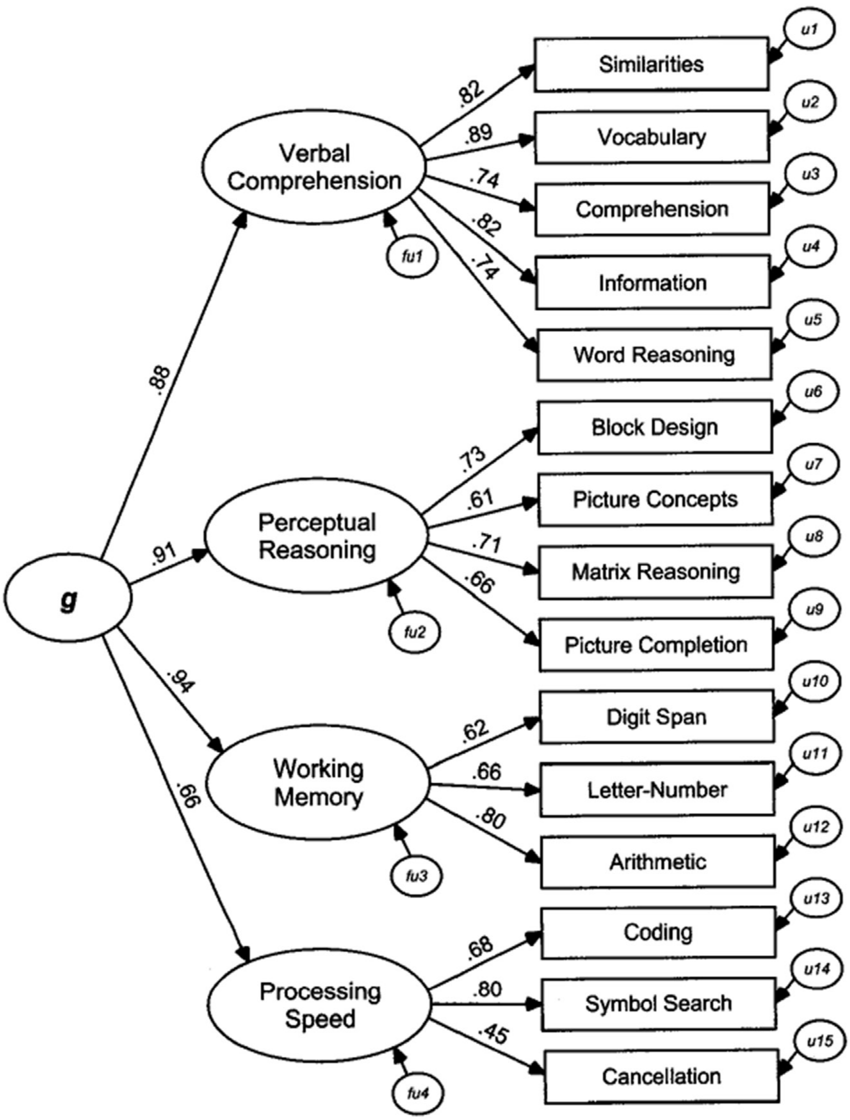

FIGURE 1.2 Outcome of a hierarchical factor analysis of the WISC-IV. Subtests are a measure of both general intelligence (*g*) and a first-order variable. Four first-order variables could be discerned. The test of arithmetic loaded on the first-order variable working memory. *From Keith, T. Z., Fine, J. G., Taub, G. E., Reynolds, M. R., & Kranzler, J. H. (2006). Higher order, multisample, confirmatory factor analysis of the Wechsler Intelligence Scale for Children – fourth edition: What does it measure?* School Psychology Review, 35*(1), 108–127.*

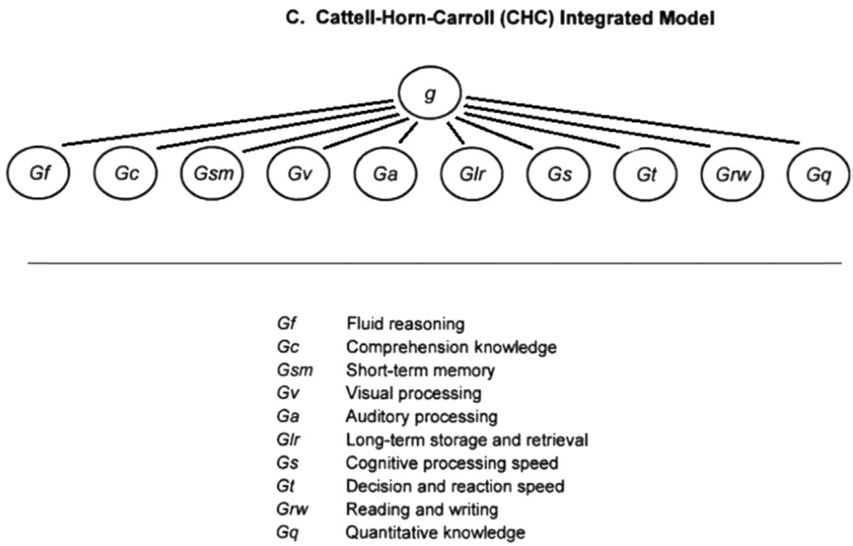

FIGURE 1.3 The Cattell–Horn–Carroll model of intelligence. The model postulates a general intelligence factor and ten first-order factors, of which quantitative knowledge is one. *Based on McGrew, K. S. (2009). CHC theory and the human cognitive abilities project: Standing on the shoulders of the giants of psychometric intelligence research.* Intelligence, *37(1), 1–10. Used with permission from Elsevier.*

practitioners do not like radically new revisions of existing tests. Therefore, the factors emerging from the factor analyses may to some extent be an artifact of the initial choices made in the design of intelligence tests (but see Jewsbury, Bowden, & Strauss, 2016 for an interesting study about the overlap between the processes postulated in cognitive models of executive function and factors emerging from the CHC model of intelligence).

A different approach is to start from an analysis of the cognitive processes involved in number and word processing, independent of whether these processes are related to individual differences in performance. The 1990s were a particularly fruitful decade in this respect, largely because of the publications of Dehaene and the responses they elicited from other researchers.

According to Dehaene (1992; Dehaene, Dehaene-Lambertz, & Cohen, 1998), a tacit hypothesis in cognitive arithmetic was that numerical abilities derived from human linguistic competence. In his own words (Dehaene, 1992, pp. 2–3): "For the lay person, calculation is the numerical activity par excellence. Calculation in turn rests on the ability to read, write, produce or comprehend numerals […]. Therefore number processing, in its fundamental form, seems intuitively linked to the ability to mentally manipulate sequences of words or symbols according to fixed transcoding or calculation rules."

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Based on data from adults, infants, and animals, Dehaene (1992) argued that number processing on the basis of symbols is not the only processing the human brain is capable of. A second pathway makes use of an innate quantity system. The quantity system is based on analog encoding and allows accurate representations for numbers up to 3 or 4 and approximate quantities for larger numbers. Evidence for the involvement of such a quantity-based pathway was found in animals, young children, and in number comparison tasks with adults. When participants are asked to indicate whether a two-digit number is larger or smaller than 65, response times decrease as a function of the logarithm of the distance between the number and 65. Therefore, participants are faster at indicating that 61 is smaller than 65 than that 63 is smaller than 65. Importantly, they are also faster at indicating that 51 is smaller than 65 than that 59 is smaller than 65, a finding that would not be predicted if two-digit numbers were encoded entirely as ordered sequences of two symbols. Both 51 and 59 start with the tens digit 5, which is different from the tens digit of the comparison number 65. Therefore, if the comparison was based on the tens digits alone, no difference would be predicted in deciding that 51 is smaller than 65 than in deciding that 59 is smaller than 65 (as both comparisons would boil down to deciding that 5 is smaller than 6). The metaphor of an analog, compressed *number line* was proposed, with clearer distinctions at the low end than at the high end.

The verbal pathway and the number line pathway were part of the triple-code model Dehaene (1992) proposed for number processing. He argued that the meaning of numbers is encoded in three ways:

1. An auditory-verbal code, similar to the semantic representations of words.
2. A visual-Arabic code, in which numbers are manipulated in Arabic format on a spatially extended representational medium.
3. An analog-magnitude code, in which numerical quantities are represented as inherently variable distributions of activation over a compressed analogical number line.

Dehaene further proposed that the three codes interact with each other and are activated by different types of input, as shown in Fig. 1.4.

WHAT HAVE WE LEARNED ABOUT NUMBERS AND THEIR RELATION TO LANGUAGE SINCE?

Dehaene's (1992) article and the special journal issue, of which it was part, were a catalyst in number processing research. While research before had been scattered, now a sufficiently large group of scholars took up the topic and became organized by arranging symposia and workshops and by publishing special journal issues and edited handbooks (e.g., Campbell,

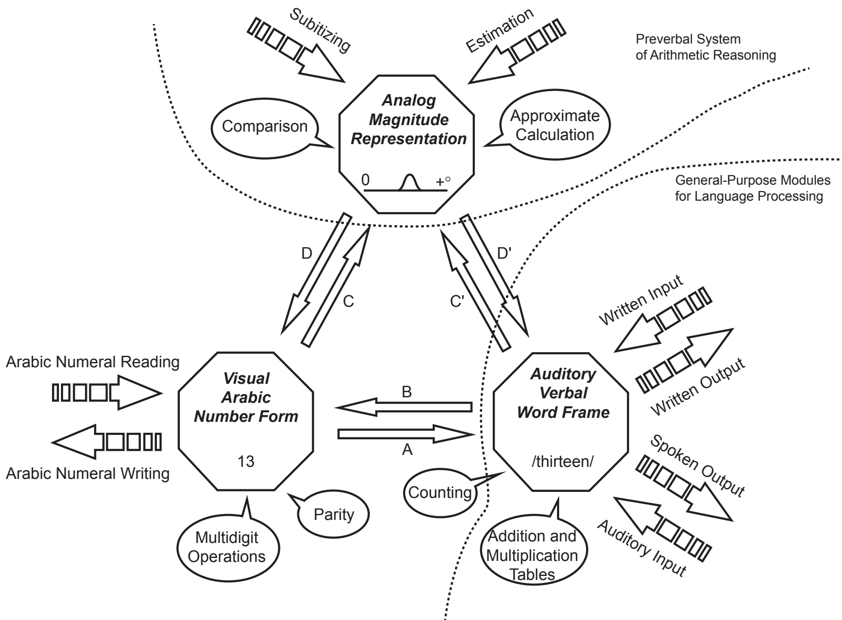

FIGURE 1.4 Dehaene's triple-code model of number processing. The three octagons represent the three codes that together form the meaning of numbers. For each code, the input and output and the main operations involving the code are given. *From Dehaene, S. (1992). Varieties of numerical abilities.* Cognition, 44*(1), 1–42.*

2005; Kadosh & Dowker, 2015). The number of paper submissions to journals grew so substantial that editors started to appoint dedicated action editors for the topic.

Unfortunately, the large number of publications has not (yet?) led to a flurry of "established findings" related to numerical and verbal performance. As it happens, I could find five. Four other topics are still in full debate. I discuss them successively.

THINGS WE HAVE LEARNED I: SMALL NUMBERS ARE EASIER TO PROCESS THAN LARGE NUMBERS

A consistent finding in number processing is that small numbers are easier to process than large numbers. One of the first robust findings was that people can easily discriminate between one, two, three, and sometimes four elements but require increasingly more time to discern five, six, seven,… elements. The fast perception of small numbers of elements is called *subitizing* (Kaufman, Lord, Reese, & Volkmann, 1949; Taves, 1941).

Small numbers are also easier to compare with each other than large numbers: People are faster to indicate that two is smaller than three than that eight is smaller than nine (Moyer & Landauer, 1967).

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Finally, arithmetic operations are easier with small numbers than with large numbers (Knight & Behrens, 1928). The problem 2+3 is easier to solve than 4+5; the same is true for 2×3 versus 4×5. This problem size effect is present when the numbers are presented as Arabic digits and when they are presented as words (Noël, Fias, & Brysbaert, 1997).

THINGS WE HAVE LEARNED II: THE ANALOG-MAGNITUDE SYSTEM ACTIVATES A PART OF THE CORTEX THAT IS NOT INVOLVED IN LANGUAGE PROCESSING

As Fig. 1.5 shows, most brain regions of the left cerebral cortex are involved in language processing. Still, the areas that consistently light up when a task assesses number magnitude processing—the left and right intraparietal sulci—fall outside the zone, as can be seen in Fig. 1.6. Bueti and Walsh (2009) reviewed the literature indicating that this region is active not only in number processing but also in time and space understanding. Van Opstal and Verguts (2013), however, pointed to problems with this view of the intraparietal sulcus as a generalized magnitude system.

For the sake of completeness, it is important to keep in mind that the intraparietal sulci do not work in isolation but form part of larger networks. In particular, interactions with the lateral prefrontal cortex are important (Nieder & Dehaene, 2009).

THINGS WE HAVE LEARNED III: THERE IS A DIRECT ARABIC-VERBAL TRANSLATION ROUTE

An element from the triple-code model that elicited some controversy was whether it was possible to directly translate Arabic numbers into spoken (and written) numbers. An alternative model proposed by McCloskey, Caramazza, and Basili (1985) postulated that all numerical processing required mediation by the semantic system. Some evidence pointed in this direction. Fias, Reynvoet, and Brysbaert (2001), for instance, presented a digit and a number word on the same screen and asked the participants to name the word or the digit. The word and the digit either pointed to the same number (e.g., 6—six) or to different numbers (6—five). Fias et al. observed that the digit was named faster when the two stimuli referred to the same number than when they referred to different numbers. No such interference effect was observed for the naming of number words. In contrast, when the participants had to indicate whether the digit or the word was an odd or an even number, there was equivalent interference for both notations. On the basis of this pattern of results, Fias et al. concluded that digits were processed like pictures and could not be named via a nonsemantic, direct translation route.

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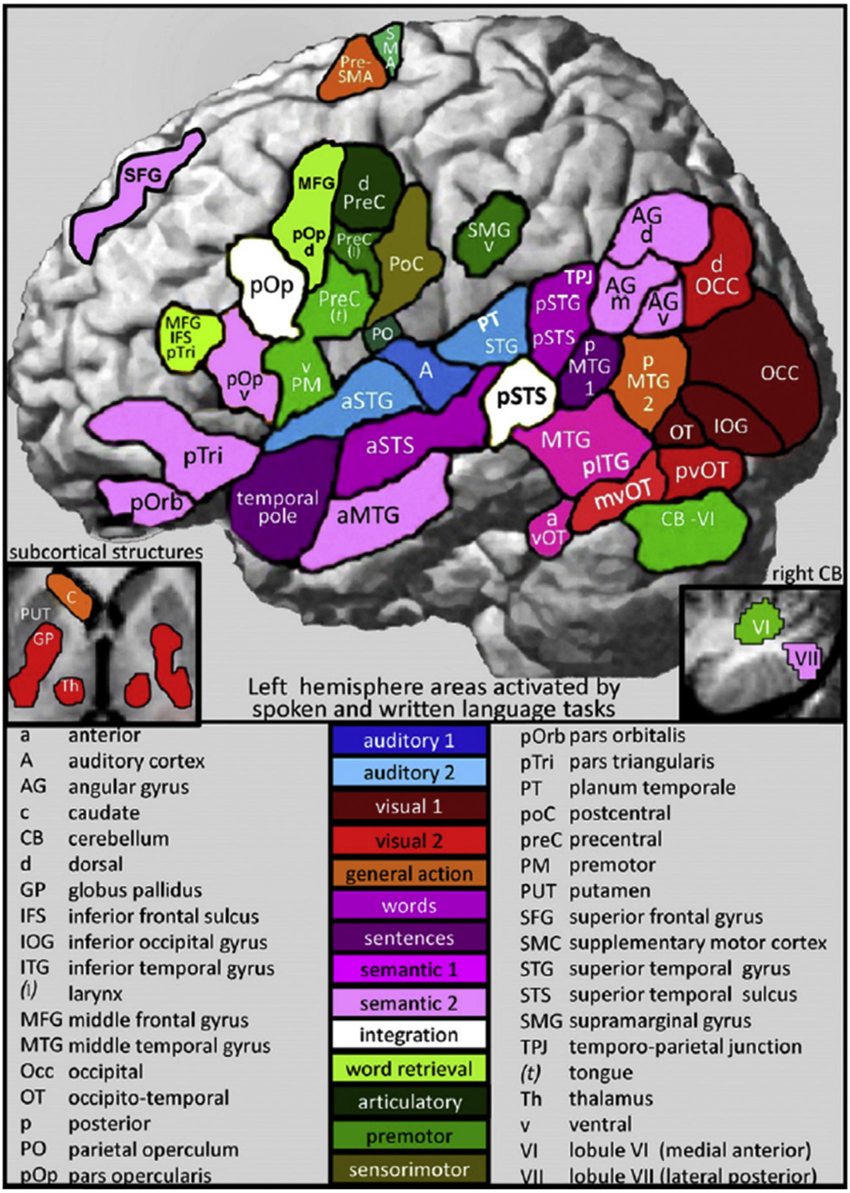

FIGURE 1.5 Brain areas of the left hemisphere active in language processing. *From Price, C. J. (2012). A review and synthesis of the first 20 years of PET and fMRI studies of heard speech, spoken language and reading.* Neuroimage, 62*(2), 816–847.*

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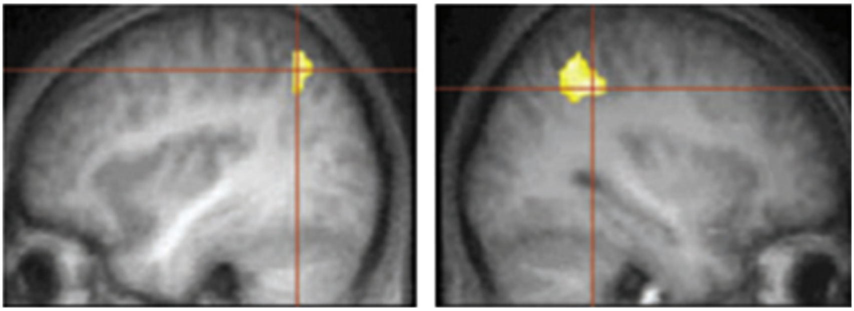

FIGURE 1.6 The intraparietal sulci (left and right) are active whenever number magnitude is addressed in a task. *From Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus.* Neuron, 44*(3), 547–555.*

In recent years, however, several paradigms have shown that nonsemantic naming of Arabic numbers is possible (see also Roelofs, 2006, for evidence based on the interference effect used by Fias et al., 2001). One series of experiments made use of the semantic blocking paradigm (Herrera & Macizo, 2012). In this paradigm, five stimuli are presented over and over again to be named. Two different conditions are distinguished: A blocked condition in which the stimuli come from the same semantic category (e.g., five animals) and a mixed condition in which the stimuli come from different semantic categories (e.g., an animal, a body part, a piece of furniture, a vehicle, and a piece of clothing). The typical finding in this paradigm is that words are named faster in the blocked condition than in the mixed condition but that pictures are named *more slowly* in the blocked condition. The difference in naming cost is explained by assuming that words can be named directly, whereas pictures require semantic mediation to be named. In the blocked picture naming condition, the various concepts and names compete and hinder each other. The semantic blocking paradigm is ideal to test whether digits are named like words or pictures, as the alternative interpretations predict opposite effects. In a series of experiments, Herrera and Macizo (2012) showed that digits are named *faster* in a blocked condition than in a mixed condition, thus resembling words and deviating from pictures.

THINGS WE HAVE LEARNED IV: THERE ARE DIFFERENCES IN PROCESSING ARABIC NUMBERS AND VERBAL NUMBERS

Arabic and verbal numbers are not interchangeable, even not for numbers below 10 (it was traditionally thought that the Arabic notation was particularly efficient for multidigit numbers). An important finding is

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that calculations are more efficient when the problems are given in Arabic notation than in verbal notation. Therefore, "4+2" is solved faster than "four+two," even though naming times of digits and number words are the same (Clark & Campbell, 1991; Noël et al., 1997; see also Megías & Macizo, 2016, for other evidence that digits activate arithmetic information more strongly than words).

The advantage of digits over words is also true when the numbers are presented as part of word problems. Therefore, children are better at solving the visually presented problem "Manuel had 3 marbles and then Pedro gave him 5" than at solving the problem "Manuel had three marbles and then Pedro gave him five" (Orrantia, Múñez, San Romualdo, & Verschaffel, 2015). More in general, magnitude information is activated faster by Arabic numbers than by verbal numbers (Ford & Reynolds, 2016; Kadosh, Henik, & Rubinsten, 2008).

THINGS WE HAVE LEARNED V: INDIVIDUALS WITH DYSLEXIA HAVE POORER ARITHMETIC PERFORMANCE

Despite the differences between Arabic number processing and verbal number processing, people with reading difficulties are likely to experience mathematical deficits as well. For a start, there is a high comorbidity of dyslexia and dyscalculia. In a sample of 2586 primary school children, Landerl and Moll (2010) observed 181 children (7%) with a reading deficit, of whom 23% had an additional arithmetic deficit. Toffalini, Giofrè, and Cornoldi (2017) analyzed the data of 1049 children referred to psychologists for assessment of learning difficulties. Of these children, 308 (29%) had a specific reading difficulty, 147 (14%) a specific spelling problem, 93 (9%) a specific calculation deficit, and 501 (48%) a mixed deficit (not further specified). At present, it is not clear whether the comorbidity of dyslexia and dyscalculia is due to common underlying processes or to divergent processes that have joint risks of malfunctioning (e.g., due to genetic influences; Moll, Goebel, Gooch, Landerl, & Snowling, 2016).

Second, high-performing university students with dyslexia are slower at naming digits and at doing elementary arithmetic (Callens, Tops, & Brysbaert, 2012; De Smedt & Boets, 2010). The effect sizes are large (Cohen's $d \approx 1.0$), although not as large as those seen in word naming speed and spelling accuracy ($d \approx 2.0$; Callens et al., 2012). This again suggests an overlap of the processes involved in verbal and arithmetical skills. One element of overlap could be that the addition and multiplication tables are stored in verbal memory.

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THINGS WE ARE STILL TRYING TO DECIDE I: WHAT IS THE NATURE OF THE NUMBER QUANTITY SYSTEM?

Dehaene (1992) put forward a few strong hypotheses about the number quantity system. The first was that it was an analog system, based on a combination of summation and place coding (for computational implementations, see Dehaene & Changeux, 1993; Verguts & Fias, 2004). The activations of the various elements in the input were summed and then translated into activation patterns on an ordered and compressed (e.g., logarithmic) "number line."

The second element was that the number quantity system was not really a magnitude system but an abstract number system (ANS), based on modality-independent, discrete amounts. Dehaene used the term "numerosity" to refer to the number of elements perceived, rather than to the summed magnitude (mass, density, surface,…) of the elements. When all items to be counted are of the same size, magnitude and numerosity are perfectly correlated. However, this is no longer the case if the items differ in magnitude: Two big items can have a bigger mass than three small elements. The ANS was supposed to respond to the discrete number of elements and not to the continuous magnitude correlates (mass, density, surface covered).

The third element proposed by Dehaene was that the number line was oriented along the reading direction. Therefore, for languages read from left to right, the small numbers were located on the left side of the number line and the large numbers on the right side.

It is fair to say that all three hypotheses are still heavily contested. First, the compressed nature of the number magnitude system has been questioned. Other explanations are: more noise for large numbers than for small numbers (Gallistel & Gelman, 1992), differences in frequency of occurrence between numbers (Piantadosi, 2016), and asymmetries because of the task rather than the nature of the number line (Cohen & Quinlan, 2016; Verguts, Fias, & Stevens, 2005).

Second, the idea of the ANS being unresponsive to magnitude differences between the discrete elements has been questioned as well, given that in real life there are virtually no situations in which numerosity and mass are uncorrelated (Cantrell & Smith, 2013; Gebuis, Kadosh, & Gevers, 2016; Reynvoet & Sasanguie, 2016). Some authors have proposed that a discrete number system may have evolved next to a continuous magnitude system (Leibovich, Vogel, Henik, & Ansari, 2015).

Still related to the issue of the true nature of the ANS system, other authors have argued that the system may be order related rather than (or in addition to) numerosity related (Berteletti, Lucangeli, & Zorzi, 2012; Goffin & Ansari, 2016; Merkley, Shimi, & Scerif, 2016; Van Opstal, Gevers,

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De Moor, & Verguts, 2008). Just like there is a high correlation between numerosity and magnitude, there is a high correlation between numerosity and order. The main difference is that order applies to more stimuli than to numbers.

Finally, the left–right orientation of the number line has been questioned as well, based on the finding that the orientation is mostly observed when numbers must be kept in working memory, leading to the proposal that the orientation is limited to numbers held in working memory (van Dijck, Abrahamse, Acar, Ketels, & Fias, 2014; but see Huber, Klein, Moeller, & Willmes, 2016). There is also some evidence that the spatial-numerical association of response codes effect may not be reversed in people with a language read from right to left (Zohar-Shai, Tzelgov, Karni, & Rubinsten, 2017).

THINGS WE ARE STILL TRYING TO DECIDE II: HOW DOES KNOWLEDGE OF NUMBER SYMBOLS AFFECT/ SHARPEN THE NUMBER MAGNITUDE SYSTEM?

Given that there are differences because of number notation, a straightforward question is to what extent the use of number symbols alters the meaning of numerosities. To what extent do the semantic representations of educated human adults differ from those of preverbal children and animals? Some authors have suggested that the use of symbols makes the number line linear rather than compressed (Siegler & Opfer, 2003), but others have doubted the empirical evidence for this claim (Huber, Moeller, & Nuerk, 2014). Others have argued that number symbols make the semantic representations sharper so that there are less confusions between numerosities (Verguts & Fias, 2004). Still others have proposed that symbolic numbers form a separate type of representations, as indicated earlier (Leibovich et al., 2015; Sasanguie, De Smedt, & Reynvoet, 2017).

THINGS WE ARE STILL TRYING TO DECIDE III: WHAT IS THE RELATIVE IMPORTANCE OF THE ANS TO MATHEMATICAL PERFORMANCE?

A third topic of discussion is to what extent the approximate number system contributes to mathematical achievement. Dehaene saw the ANS as the core of number knowledge from which all other number-related information emerged. A similar view was defended by Butterworth (2005; see also Landerl, Bevan, & Butterworth, 2004), and some authors found evidence in line with this hypothesis (Schleepen, Van Mier, & De Smedt, 2016; Zhang, Chen, Liu, Cui, & Zhou, 2016). Others, however, failed to find evidence (Cipora & Nuerk, 2013; Geary & Vanmarle, 2016) or found a stronger effect for symbolic comparison rather than nonsymbolic magnitude comparison (Fazio, Bailey, Thompson, & Siegler, 2014;

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Honoré & Noël, 2016; Vanbinst, Ansari, Ghesquière, & De Smedt, 2016; Vanbinst, Ghesquière, & De Smedt, 2012).

All in all, it seems unlikely that the ANS is strongly related to mathematical achievements in healthy participants. A remaining possibility is that ANS malfunctioning is rare but with grave consequences so that people with a deficient ANS have severe dyscalculia but are too rare to influence correlations in large-scale population studies.

THINGS WE ARE STILL TRYING TO DECIDE IV: DOES LANGUAGE HAVE AN EFFECT ON HOW MATHEMATICAL OPERATIONS ARE PERFORMED?

A basic question about cognitive performance is to what extent language affects thought (also known as linguistic relativity or the Whorfian hypothesis). Is it possible to think without language and is thinking different in languages that carve reality in dissimilar ways? Importantly, the language differences should point to fundamental differences in processing, not simply to differences in strategies to cope with the language difference. For instance, Brysbaert, Fias, and Noël (1998) reported that Dutch-speaking participants are faster to name the solution of problem 4+21 than of the problem 21+4, whereas French-speaking participants show the opposite effect, in line with the observation that two-digit numbers in Dutch but not in French are pronounced in the reverse way: five and twenty instead of twenty-five. Crucially, the language difference disappeared when both groups of participants were asked to type the answers. With this task, both Dutch-speaking and French-speaking participants were faster to solve 21+4 than 4+21, leading Brysbaert et al. (1998) to conclude that the language difference in arithmetic was not a true Whorfian effect.

An argument sometimes used against the idea that language shapes thought is the observation that aphasic people are not obviously deficient in their thinking (e.g., Siegal, Varley, & Want, 2001). This rules out a strong version of the linguistic relativity (thought is impossible without language) but not necessarily a weaker version (language affects thought). Indeed, there is evidence that some nonverbal functions such as picture categorization are hindered in people with aphasia (Lupyan & Mirman, 2013), in line with a weak version of linguistic relativity (Lupyan, 2015; Wolff & Holmes, 2011).

Several researchers have taken issue with the initial negative evidence for Whorfian effects in number processing. For instance, Colomé, Laka, and Sebastián-Gallés (2010) reported that Basque speakers solve problems such as 20+15 faster than Italian or Catalan speakers, both when the solution had to be named and typed, in line with the observation that the Basque language combines multiples of 20 in its number naming system (e.g., 35 is said as "twenty and fifteen"). A language effect between Basque and Spanish was also reported by Salillas and Carreiras (2014).

Pixner, Moeller, Hermanova, Nuerk, and Kaufmann (2011) showed that participants find it harder to decide that 47 is smaller than 62 than that 42 is smaller than 57 because in the former case there is an incongruity between the response required (47 < 62) and the response elicited by the units (7 > 2). Critically, the incongruity effect was larger in German, which names the units before the tens (seven and forty), than in Italian or Czech, which put the tens before the units (forty-seven). Moeller, Shaki, Göbel, and Nuerk (2015) successfully replicated the effect.

A related topic is how bilinguals with contradicting number names cope. It is well documented that bilinguals continue to do arithmetic in the language they used in school, leading to confusions when the school language changes or when people later have a different dominant language (Prior, Katz, Mahajna, & Rubinsten, 2015; Van Rinsveld, Brunner, Landerl, Schiltz, & Ugen, 2015).

NEW THINGS LANGUAGE RESEARCHERS HAVE TO OFFER I: SEMANTIC VECTORS

Research on the commonalities and the differences between language and numerical cognition can take inspiration from new developments on the other side. In the remainder of this Chapter 1 two new developments in language research are mentioned, which may be of interest to colleagues working in numerical cognition.

A first interesting development in language research is that the meaning of a target word can be approximated by studying the words surrounding the target word (Landauer & Dumais, 1997; Lazaridou, Marelli, & Baroni, 2017; Mandera, Keuleers, & Brysbaert, 2017; Sadeghi, McClelland, & Hoffman, 2015). A successful way of doing so is to use a three-layer connectionist network (Mikolov, Sutskever, Chen, Corrado, & Dean, 2013, pp. 3111–3119). The input layer consists of some 100 thousand nodes representing the words found in a billion-word corpus. The output layer consists of the same nodes. In between, there is a hidden layer with 200–300 nodes. The network is trained as follows. A corpus is read word by word. For each target word activated in the output layer, a few words before the target word and a few words after the target word in the corpus are activated at the input layer. The weights are changed so that the prediction of the target word improves, given the surrounding words in the input layer. At the end of the training, the activity vector in the hidden layer activated by a target word represents the semantic representation of that word (see Fig. 1.7).

Such semantic vectors are quite good predictors of semantic distance judgments, semantic priming data, and synonym judgment (Mandera et al., 2017). Fig. 1.8 shows the semantic distances between the number

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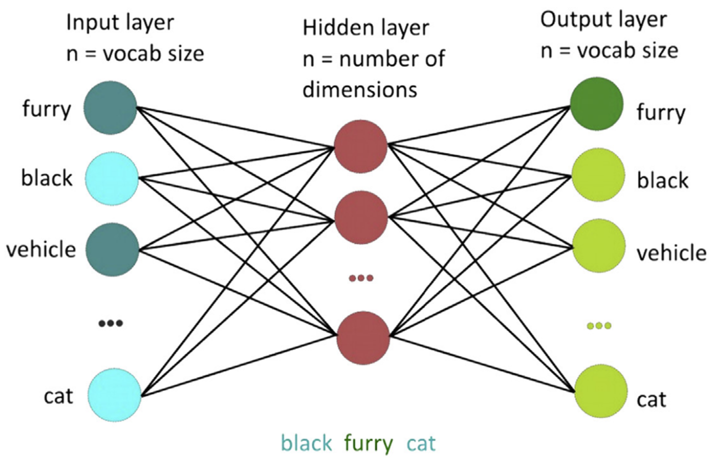

FIGURE 1.7 Network to calculate semantic vectors. A network is trained to predict target words on the basis of the surrounding words. In this example, the weights are changed so that the activity of the word node "furry" in the output layer increases when the words "black" and "cat" are activated in the input layer. The activity patterns in the hidden layer at the end of the training form the words' semantic vectors. *From Mandera, P., Keuleers, E., & Brysbaert, M. (2017). Explaining human performance in psycholinguistic tasks with models of semantic similarity based on prediction and counting: A review and empirical validation.* Journal of Memory and Language, 92*, 57–78.*

	One	two	three	four	five	six	seven	eight	nine	ten
zero	0.66	0.69	0.70	0.66	0.59	0.65	0.63	0.65	0.59	0.66
one		0.35	0.40	0.42	0.48	0.50	0.51	0.51	0.52	0.54
two			0.13	0.18	0.27	0.28	0.33	0.33	0.39	0.38
three				0.11	0.24	0.23	0.29	0.27	0.35	0.36
four					0.20	0.20	0.22	0.21	0.29	0.31
five						0.23	0.23	0.20	0.25	0.15
six							0.20	0.15	0.22	0.29
seven								0.14	0.17	0.29
eight									0.13	0.24
nine										0.29

FIGURE 1.8 Semantic distances between number words based on semantic vectors (0.00 = no distance, 1.00 = maximal distance). The font size stresses the semantic similarity. This shows a distance-related similarity effect: Numbers close in value have a larger similarity than numbers farther away. The number zero is not much related to any other number. *Mandera, P., Keuleers, E., & Brysbaert, M. (2017). From Explaining human performance in psycholinguistic tasks with models of semantic similarity based on prediction and counting: A review and empirical validation.* Journal of Memory and Language, 92*, 57–78.*

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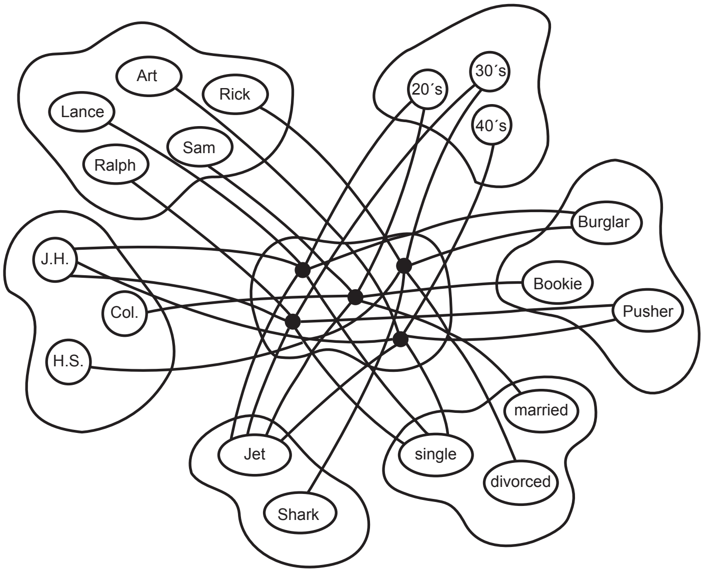

FIGURE 1.9 Hub-and-spoke model to represent information about people in an imaginary world. This model represents information about five people (Lance, Art, Rick, Sam, Ralph) who have different ages (20, 30, 40), have different occupations (burglar, bookie, pusher), have different marital status (married, single, divorced), belong to different groups (jet, shark), and have finished different education levels (junior high school, high school, college). Important in the model is the use of central person nodes (the hub), linking the various aspects belonging to an individual (spokes). *From McClelland, J. L. (1981). Retrieving general and specific information from stored knowledge of specifics. In:* Proceedings of the third annual meeting of the cognitive science society.

words zero to ten (in English). This figure shows that numbers close in value resemble each other more than numbers further apart, simply because they more often co-occur in texts. It will be interesting to examine to what extent the semantic vectors agree with the distance-related effects reported in number comparison and number priming (see also Krajcsi, Lengyel, & Kojouharova, 2016). Another aspect shown in Fig. 1.8 is that the number zero is rather unrelated to the other numbers, in line with the finding that the position of the number zero on the number line is uncertain (Brysbaert, 1995; Pinhas & Tzelgov, 2012).

NEW THINGS LANGUAGE RESEARCHERS HAVE TO OFFER II: THE HUB-AND- SPOKE MODEL OF SEMANTIC REPRESENTATION

Another interesting idea is one originally proposed by McClelland (1981), which has been applied several times in his simulation work (e.g., Rogers et al., 2004). Fig. 1.9 illustrates the idea. Whenever various characteristics of a stimulus must be combined, it makes sense to represent the stimulus as a central, amodal node (hub) in a network connected to modal feature nodes within separate layers (spokes). Such a model successfully predicted the neuroscientific finding that the meaning of stimuli can involve brain areas as diverse as the visual cortex and the motor cortex. It also successfully accounts for the progression of meaning loss in semantic dementia (Rogers et al., 2004). A hypothesis is that the hub of the system is situated in the anterior temporal lobes (Ralph, Jefferies, Patterson, & Rogers, 2017).

It is not difficult to reformulate Dehaene's (1992) triple-code model into a hub-and-spoke model. Rather than having the mutual interactions between the three codes, each code would send activation to a central hub of amodal nodes representing the various numbers. In this way, various types of information can be integrated, including new information (such as that coming from semantic vectors and historical facts). As a matter of fact, one of the first models proposed for number representations—the encoding-complex model—came very close to such a hub-and-spoke model. The encoding-complex model (Campbell & Clark, 1988) stated that numerals become associated with a variety of numerical functions (number reading or transcoding, number comparison, estimation, arithmetic facts,…), which interact with each other and are activated to various extents depending on the task to be performed. One of the factors that have hindered acceptance of the model was that it seemed difficult to implement. The hub-and-spoke model may be a way forward.

CONCLUSIONS

In this chapter, I have reviewed the developments of the past 25 years in our knowledge about number processing and its relationship to language proficiency. First, I showed that arithmetic in intelligence tests initially was seen as part of the verbal scale but later became part of a different (though correlated) factor. Then, I discussed the cognitive models from the 1990s, which focused on the differences between numerical and verbal performance. I argue that research on this topic has led to five established findings and four issues that are still hotly debated. Finally, I presented two new findings from psycholinguistics, which may be of interest to researchers on number processing.

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