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DEVELOPING FRAGILITY CURVES & ESTIMATING FAILURE PROBABILITIES FOR PROTECTED STEEL STRUCTURAL ELEMENTS SUBJECT TO FULLY DEVELOPED FIRES

Danny Hopkin[1] Ruben Van Coile[2] & Ian Fu[3]

ABSTRACT

Reliability methods are at the core of ambient Eurocode design. Realising exceptional / complex buildings necessitates that an adequate level of safety be demonstrated. Rationally demonstrating adequate safety can only be achieved through the application of probabilistic risk assessment (PRA). This paper presents a novel application of PRA in a structural fire engineering context. It first proposes a generalised limit state for protected steel members undergoing failure modes dictated by yielding. Subsequently, fragility curves describing failure likelihood in function of protection specification and mean fire load are presented for a 1,000 m² compartment, subject to fully developed fires (parametric and travelling fires). The presented fragility curves have subsequently proven to be of value for further life-time-cost-optimisation applications, with the intent of arriving at explicit safety targets.

1 INTRODUCTION

Realising exceptional buildings necessitates that an adequate level of fire safety be explicitly demonstrated. This requires an evaluation of all foreseeable consequences, and the probability of their manifestation [1]. Unlike ambient temperature evaluations of structural reliability, further sources of uncertainty exist at high temperature that relate to: the characterisation of fires, their implications for structural element temperatures, the degradation of material properties, permanent and imposed loading at the time of the fire, and further (potential) model uncertainties. Previous studies by Hopkin, et. al. [2], have evaluated failure probabilities for protected steel elements with different protection specifications (i.e. 30, 60, 90 and 120 minutes) by comparing the probability density functions (PDFs) for maximum temperatures attained during fire to a deterministic limiting temperature. Uncertainties regarding the structural loading, degradation of material properties at elevated temperatures and model uncertainties were, however, not considered. This paper builds upon the work by Hopkin, et al. [2] by introducing these additional uncertainties noted above, resulting in a more complete estimation of failure probabilities for isolated protected steel elements.

2 OVERVIEW OF THE PROBABILISTIC STUDY

2.1 Probabilistic factors leading to a fire induced structural failure

The events that lead-up to a potential structural failure in case of fire all have a probability of occurring. In the first instance a fire must develop, subsequently there must be a compound failure

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[1] Associate Director, Olsson Fire and Risk. Visiting Professor, University of Sheffield, UK.
e-mail: Danny.Hopkin@OlssonFire.com and D.J.Hopkin@Sheffield.ac.uk

[2] Assistant Professor, University of Ghent, Belgium.
e-mail: Ruben.VanCoile@UGent.be

[3] Graduate Fire Engineer, Olsson Fire and Risk, UK.
e-mail: Ian.Fu@OlssonFire.com

of early intervention by the occupants, active measures and the fire brigade. From this point, the fire may become fully developed (or 'significant'). Allied to this, the structure must be sufficiently affected by the fully developed fire such that it undergoes damage and, potentially, fails. Broadly, within this series of events, two domains can be identified – (i) the event instigation domain, and (ii) the response domain. This differentiation allows benchmarking of performance against two safety targets – (a) an overall reliability index β which includes the likelihood of the fire event, and (b) a reliability index given a significant fire (β_{fi}). Discussion on the meaning of β in fire can be found elsewhere [3]. While β refers to the (annual) probability of fire-induced failure and can be compared with failure rates due to other unforeseen events, the conditional reliability index β_{fi} relates to the robustness of the structure in the unlikely event of a fully developed fire.

2.2 The event instigation domain

The event instigation domain simply describes the ignition likelihood, and consequently the occurrence rate of structurally significant fires, i.e. those that have the potential to undermine structural integrity. Studies, such as those in the Natural Fire Safety Concept Valorisation Project [4], provide means for estimating the structurally significant fire occurrence rate in function of e.g. compartment size, occupancy and various system and management based intervention possibilities.

2.3 The response domain

The response domain is the principal focus of this paper, and in the specific application of protected steel structures. Evaluating the stochastic response of a structural element subject to a significant fire occurrence necessitates that: (a) the probabilistic manifestations of potential significant fire scenarios be evaluated, and (b) subsequently, subject to a given fire manifestation, an evaluation is made of structural response. (a) requires appropriate fire models, with corresponding stochastic inputs for relevant parameters describing a fire's development. (b) needs an evaluation of both the: (stochastic) applied action at the time of the fire (cognisant of the variability in the permanent and variable components), and the available resistance of the structure or structural component (impacted by uncertainty in fundamental material properties, how they degrade with temperature, etc.). Common to (a) and (b) are further (potential) model uncertainties.

3 STOCHASTIC FIRE MANIFESTATION

The procedure for arriving at probability density functions for the maximum temperature attained by a protected steel element subject to a fully developed fire is subject to wider discussion in Hopkin, et. al. [2] and is summarised herein. First, there must be an idealisation of the fire's development (i.e. a fire model), and second, the stochastic inputs for that model are assigned distributions through a combination of measurements or judgement. Two fire models are adopted, in recognition of two likely outcomes should a significant fire occur. The first is the Eurocode parametric fire for cases where flashover can reasonably be expected to occur. The second, is a travelling fire model proposed by Hopkin [5]. The decision as to whether to adopt a post-flashover fire vs. a travelling fire is informed by: (a) spread rates, and (b) the ventilation conditions. That is, flashover is only considered viable if: (i) the fire has spread to the far end of the compartment before the origin starts to decay; and (ii) the ventilation conditions lead to an opening factor, as defined in BS EN 1991-1-2, of between 0.02 and 0.2 $m^{0.5}$. The above conditions naturally lead to more post-flashover fires in smaller, relative to larger compartments. Albeit, conditions to support flashover can occur in moderate sized compartments, where the spread rate is rapid. Subject to a thus defined fire time-temperature curve and protection specification, the maximum temperature of a protected element is simply derived from the bulk-capacitance methodology given in EN 1993-1-2 [6]. The stochastic variables required to describe fire development are discussed in Section 6.2.

4   PROTECTION SPECIFICATIONS FOR PROTECTED ELEMENTS

For the steel section and insulation properties specified further in *Table 4*, the insulation thickness required based on current deterministic design procedures is given in *Fig. 1* (left) in function of the fire utilisation u_{fi}, for different ISO 834 standard fire exposures. The fire utilisation u_{fi} is given by *Eq. (1)*, where the global resistance factor γ_R equals unity when $\gamma_s = 1$. η_{fi} is the reduction factor of the design load for the fire situation as specified in the Eurocodes (e.g. EN 1993-1-2 [6]), given by *Eq. (2)* for the EQU limit state in normal design conditions. ψ_{fi} is the imposed load combination factor for the fire design and χ the load ratio defined by *Eq. (3)*. *Fig. 1* (right) visualizes *Eq. (2)* for different ψ_{fi} and load ratio χ.

$$u_{fi} = \frac{E_{fi,d}}{R_{fi,d}\left(t=0\right)} = \frac{\eta_{fi}E_d}{\gamma_R R_d} = \frac{\eta_{fi}u}{\gamma_R} \tag{1}$$

$$\eta_{fi} = \frac{1+\psi_{fi}\dfrac{\chi}{1-\chi}}{\gamma_G + \gamma_Q\dfrac{\chi}{1-\chi}} = \frac{1+\psi_{fi}\dfrac{\chi}{1-\chi}}{\phi} \tag{2}$$

$$\chi = \frac{Q_k}{Q_k + G_k} = \frac{M_{Qk}}{M_{Qk} + M_{Gk}} \tag{3}$$
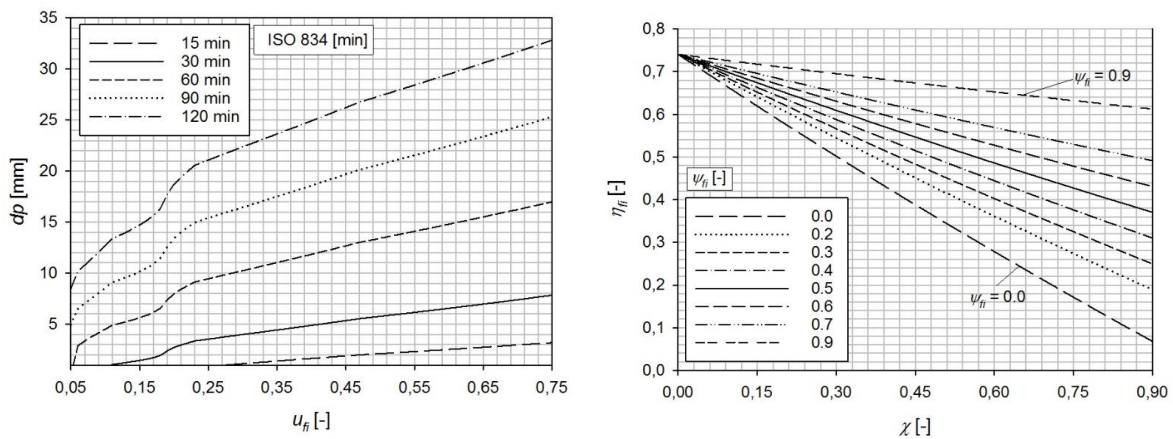
Fig. 1. (left) Required insulation thickness in function of utilisation u_{fi} and fire resistance period, (right) Fire design load reduction factor in function of the load ratio χ, for different ψ_{fi}, considering the EQU limit state in normal design.

5   A GENERALISED LIMIT STATE FOR STEEL ELEMENTS UNDER YIELDING

5.1 Basis – a specific limit state for a bending element

In the case of beams subjected to pure bending, the limit state defining failure is given by *Eq. (4)*, with parameters as listed further in *Table 1*. For elements subjected to pure tension, an equivalent limit state can be formulated. The beam is assumed to have a utilization ratio $u \leq 1$ in normal design conditions, in accordance with the ultimate limit state (ULS) design requirements of EN 1990 [7], i.e. *Eq. (5)*, with M_{Ed} the design value of the bending moment induced by the load effect and M_{Rd} the design value of the bending moment capacity.

$$Z = K_R M_R - K_E\left(M_G + M_Q\right) \tag{4}$$

$$M_{Ed} = u M_{Rd} \tag{5}$$

Considering a uniform temperature of the steel beam in case of fire exposure (as obtained through the bulk-capacitance model of EN 1993-1-2 [6]), the bending moment capacity M_R is given by *Eq. (6)*, with k_{fy} the temperature dependent reduction factor for the steel yield strength. Consequently, the general limit state of *Eq. (4)* can be rewritten as *Eq. (7)*, where all temperature-dependent and temperature-independent variables have been grouped.

$$M_R = W_{pl} k_{fy} f_y \tag{6}$$

$$Z_{fi} = k_{fy} - \frac{K_E}{K_R} \frac{(M_G + M_Q)}{W_{pl} f_y} = k_{fy} - k_{fy,req} \tag{7}$$

As indicated by the right-hand equation in *(7)*, the temperature-independent variables define the (minimum) required value for k_{fy}, *Eq. (8)*. This allows the splitting of the reliability analysis in a fire-dependent evaluation of k_{fy} and a generally applicable, fire-independent evaluation of $k_{fy,req}$.

$$k_{fy,req} = \frac{K_E}{K_R} \frac{(M_G + M_Q)}{W_{pl} f_y} \tag{8}$$

5.2 A generalised limit state – required residual yield strength

Starting from *Eq. (8)* and analytically combining both lognormal model uncertainties in a single total model uncertainty K_T, *Eq. (8)* is rewritten as *Eq. (9)* by introducing scaled variables $X^* = X / c$, with c a constant factor. The scaled variables applied in *Eq. (9)* are listed in *Table 2*.

$$k_{fy,req} = \frac{K_E}{K_R} \frac{(M_G + M_Q)}{W_{pl} f_y} = \frac{\mu_{KT} M_{Gk}}{W_{pl} f_{yk}} K_T^* \frac{(G^* + Q^*)}{f_y^*} = \frac{\mu_{KT} u}{\gamma_s \phi} K_T^* \frac{(G^* + Q^*)}{f_y^*} \tag{9}$$

Simplifying *Eq. (9)*, $k_{fy,req}$ is found to be independent of the beam properties (W_{pl}, f_{yk}). The formulation can be further generalized by introducing $nk_{fy,req}$ in Eq. (10), allowing a stochastic representation of the minimum required strength factor independent of the utilization u and the choice of μ_{KT} (as this parameter is not clearly defined for fire applications). For a given utilization and μ_{KT}, $nk_{fy,req}$ can be scaled directly to $k_{fy,req}$. Monte Carlo results for $nk_{fy,req}$ are visualized in *Fig. 2*, indicating that $nk_{fy,req}$ can be approximated by a lognormal distribution.

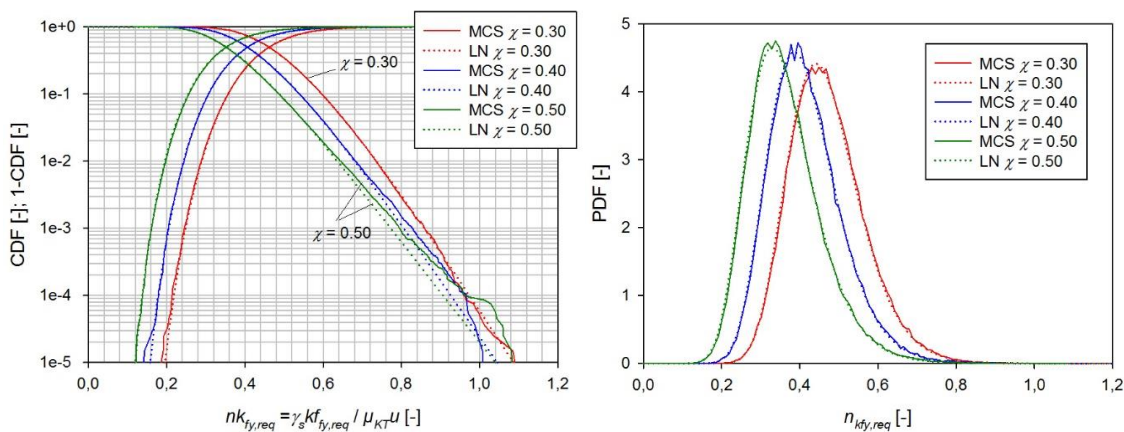$$nk_{fy,req} = \frac{\gamma_s k_{fy,req}}{\mu_{KT} u} = \frac{K_T^* (G^* + Q^*)}{\phi f_y^*} \tag{10}$$

Fig. 2. (left) CDF and cCDF for $nk_{fy,req}$, (right) PDF for $nk_{fy,req}$. Monte Carlo simulations and lognormal approximation.

6 STOCHASTIC PARAMETERS

6.1 Parameters describing the bending limit state

The distributions in *Table 1* describing the stochastic variables are based on the literature review by Holicky and Sykora [8] and the JCSS Probabilistic Model Code [9]. A 5-year reference period is considered for the imposed load effect to take into account the imposed load present at the time of fire exposure. For normal design conditions, lognormal model uncertainties K_R and K_E apply. Appropriate model uncertainties for the fire condition are currently not clearly defined. In the following it is assumed that also in case of fire, the model uncertainties can reasonably be described by a lognormal distribution. To account for increased uncertainty and reduced redundancy during fire, the parameters for K_R have been modified relative to the (bracketed) normal design situation.

Table 1. Parameters for the bending limit state

Symbol	Name	Dimension	Distribution	Mean (μ)	COV (V)
K_R	Model uncertainty for the resistance effect	-	Lognormal (LN)	1.10 (1.15)	0.10 (0.05)
M_R	Bending moment capacity	kNm	TBD		
K_E	Model uncertainty for the load effect	-	Lognormal (LN)	1.00	0.10
M_G	Bending moment induced by the permanent load effect	kNm	Normal (N)	M_{Gk}	0.10
M_Q	Bending moment induced by the imposed load effect	kNm	Gumbel (G)	0.2 M_{Qk} (5-year reference)	1.1 (5-year reference)

Table 2. Original and scaled variables. Distributions given as distribution type (mean value, coefficient of variation)

Original variable	Original distribution	Constant	Scaled variable	Scaled distribution
K_T	LN $(\mu_{KT}; V_{KT})$	μ_{KT}	K_T^*	LN $(1; V_{KT})$
M_G	N $(M_{Gk}; 0.1)$	M_{Gk}	G^*	N $(1; 0.1)$
M_Q	$G\left(0.2\dfrac{\chi}{1-\chi}M_{Gk};1.1\right)$	M_{Gk}	Q^*	$G\left(0.2\dfrac{\chi}{1-\chi};1.1\right)$
f_y	$LN\left(\dfrac{f_{yk}}{1-2V_{fy}};V_{fy}\right)$	f_{yk}	f_y^*	$LN\left(\dfrac{1}{1-2V_{fy}};V_{fy}\right)$

6.2 Fire development parameters

Fire development inputs vary according to the fire dynamics model adopted, i.e. post-flashover parametric fires or travelling fires. Input distributions are provided for e.g. fire load density, percentage glazing failure and combustion efficiency. *Table 3* summarizes the input distributions for all fire development metrics. These are generally adopted from the research literature, e.g. [4]. However, in instances, it has been necessary to apply judgement, and simply present an upper and lower bound, assuming a uniform distribution in-between (indicated via a min and max).

6.3 Yield strength retention parameters

Khorasani [10] proposed probabilistic models for the steel yield stress retention (reduction) factor at elevated temperatures. In the following, the 'no base' logistic model is applied, as given by *Eq. (11)*, with ε a standard normally distributed error term. Note that also at 20°C there is considerable variability of k_{fy} in this model. This is because part of the uncertainty regarding f_y (*Table 2*) is also considered in the k_{fy} model, thus the reliability at low temperatures will be underestimated. Further research is underway to further evaluate its impact. At elevated temperatures this effect disappears.

$$k_{fy}(\theta) = 1.2 \frac{\exp\left(1.61 - 1.68 \cdot 10^{-3}\theta - 3.36 \cdot 10^{-6}\theta^2 + 0.35\varepsilon\right)}{\exp\left(1.61 - 1.68 \cdot 10^{-3}\theta - 3.36 \cdot 10^{-6}\theta^2 + 0.35\varepsilon\right) + 1} \tag{11}$$

Table 3. Fire development parameters

Symbol	Name	Dim	Distribution	Mean (μ)	COV (V)
q_F	Fire load density	MJ/m^2	Gumbel	$q_{F,nom}$	0.3
t_{lim}	Limit time fuel controlled fire (EN 1991-1-2)	min	Deterministic	20	-
HRR_pua	Heat release rate per unit area	MW/m^2	Deterministic	0.25	-
φ_w	Fraction of glazing failure	-	Uniform	min: 0.1250	max: 0.9999
φ_b	Beam position relative to compartment length	-	Uniform	min: 0.6	max: 0.9
φ_c	Combustion efficiency	-	Uniform	min: 0.75	max: 0.999
sr	Spread rate	m/s	Uniform	min: 0.0035	max: 0.0193
T_{nf}	Near field fire temperature	°C	Normal	1050	$\mu_{Tnf} \cdot \left(1.939 - 0.266 \cdot \ln\left(\mu_{Tnf}\right)\right)$

7 PILOT STUDY – GENERATATING FRAGILITY CURVES IN FUNCTION OF DP

7.1 Overview

Obtaining fragility curves for protected steel elements in function of insulation thickness and fire characteristics necessitates three prerequisites: (1) the generation of maximum temperature PDFs, i.e. as reported by Hopkin, et. al. [2], (2) the translation of the maximum temperature PDFs into corresponding CDFs for achieved yield strength retention (k_{fy} in *Eq. (7)*), and (3) benchmarking of the achieved yield strength retention factors against $k_{fy,req}$, with failure corresponding with a condition whereby the right-hand side of equation *(7)* is negative. Core deterministic parameters for the pilot study are given in *Table 4*. Probabilistic variables are as discussed previously in Section 6.

Table 4. Deterministic parameters for pilot study

Symbol	Name	Dim	Metric	Symbol	Name	Dim	Metric
w	Room width	m	22.36	k_p	Insulation thermal conductivity	W/mK	0.2
l	Room depth	m	44.72	c_p	Insulation specific heat	J/kgK	1700
h	Room height	m	3.40	ρ_p	Insulation density	kg/m^3	800
v_w	Total window width	m	129.45	H_p	Heated perimeter	m	2.14
h_w	Window height	m	3.06	A_p	Cross-section area	m^2	0.017
d_p	Insulation thickness	mm	$d_{p,nom}$	Δt	Time step temperature calculation	s	15

7.2 Maximum temperature PDF

Considering the parameters listed in *Table 4*, the complementary CDF describing the probability of the steel section exceeding a specified maximum temperature is given in *Fig. 3* (left) for different nominal (mean) fire load density q_F (insulation thickness d_p = 5.7 mm) and in *Fig. 3* (right) for different nominal insulation thickness d_p (q_F = 400 MJ/m^2). These results were obtained through 10,000 Latin Hypercube Simulations (LHS).
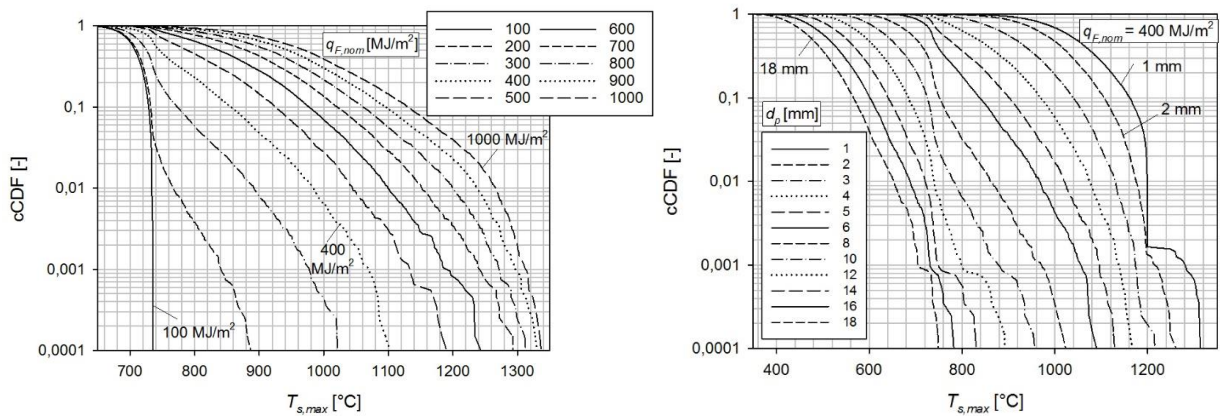
Fig. 3. (left) cCDF of T$_{s,max}$ for different q$_{F,nom}$ (dp = 5.7mm); and (right) of T$_{s,max}$ for different d$_p$ (q$_{F,nom}$ = 400 MJ/m^2)

7.3 Yield strength retention CDF in function of fire characteristics

The steel yield strength retention factor (i.e. minimum residual strength) is visualized in *Fig. 4* taking into account the distribution of the maximum steel temperature T$_{s,max}$ as observed in *Fig. 3*, and the stochastic model for k$_{fy}$ as discussed in Section 6.3.
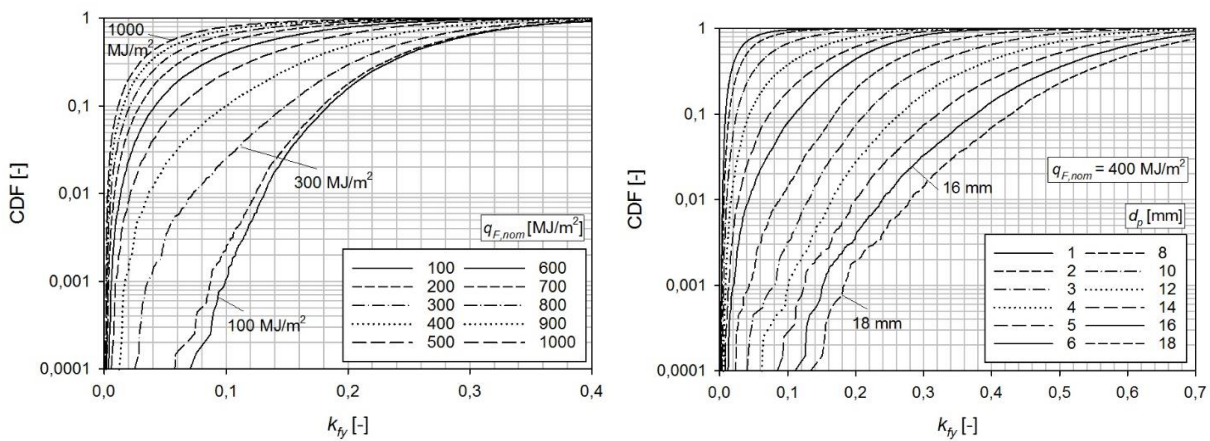

Fig. 4. (left) CDF of k$_{fy,ach}$ for different q$_{F,nom}$ (dp = 5.7mm), (right) CDF of k$_{fy,ach}$ for different d$_p$ (q$_{F,nom}$ = 400 MJ/m^2)
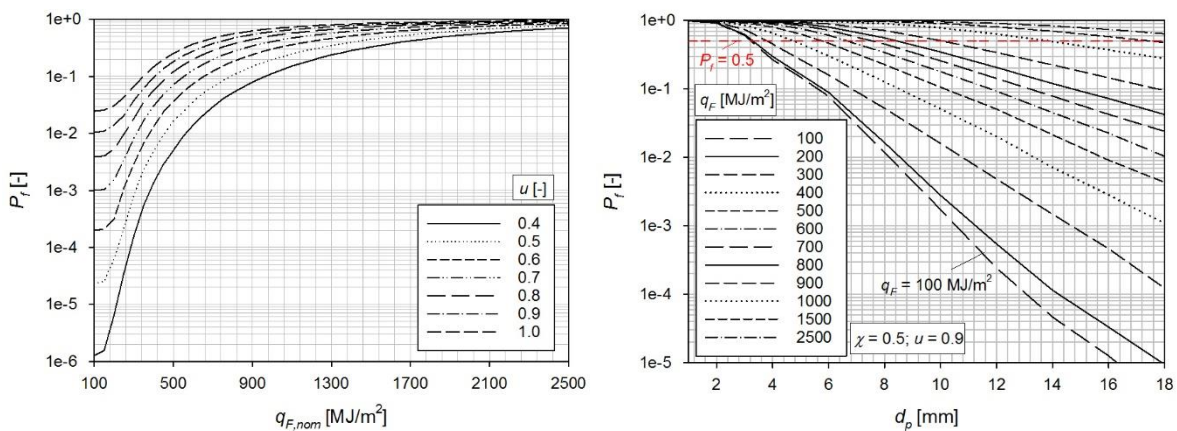

Fig. 5. (left) Fragility curve in function of q$_{F,nom}$, for d$_p$ = 16 mm, χ = 0.40, and different ambient utilization u, and (right) Fragility curve in function of the insulation thickness d$_p$, for u = 0.9, χ = 0.50, and different q$_{F,nom}$.

7.4 Fragility curves for protected elements

Resulting fragility curves visualizing the parameter-dependency of the probability of failure are given in *Fig. 5*, with $\mu_{KT} = 0.92$ and $\gamma_R = 1$. *Fig. 5* (left) visualises the probability of failure in function of the nominal fire load $q_{F,nom}$. *Fig. 5* (right) visualises fragilities for given nominal fire load density q_F, in function of the insulation thickness d_p using logarithmic axes. For failure probabilities below 0.5, this visualization of the fragility curve is approximately linear. This is of particular interest for cost-optimization calculations, as presented in follow up research [11].

8 DISCUSSION & CONCLUSIONS

Currently no simplified reliability-based methods exist for structural fire design, as is the case for ambient design following the Eurocodes. In order to demonstrate adequate structural fire safety, risk-based methodologies should be applied, requiring an assessment of the reliability of structural systems exposed to fire. To this end, a step-wise approach to evaluate the failure probability of insulated steel elements has been presented, taking into account uncertainties with respect to both the fire exposure, and thermal and mechanical properties, with due consideration of the wide range of possible fire behaviours (travelling fires vs. post-flashover fires). The methodology has been applied to derive fragility curves for protected steel beams in function of the insulation thickness, nominal fire load density and ambient utilisation ratio. The results emphasize the strong effect of the nominal fire load density and the insulation thickness on the failure probability, providing input for rationally differentiating investments in structural fire protection between buildings. The presented results have been applied as a basis for cost-optimization in follow-up research, resulting in an assessment of optimum (target) reliability levels for structural fire design of protected steel structural elements.

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