Referee Assignment in the Chilean Football League using Integer Programming and Patterns

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Abstract

This article uses integer linear programming to address the referee assignment problem in the First Division of the Chilean professional football league. The proposed approach considers balance in the number of matches each referee must officiate, the frequency each is assigned to a given team, the distance each must travel over the course of a season, and the appropriate pairings of referee experience or skill category with the importance of the matches. Two methodologies are studied, one traditional and the other a pattern-based formulation inspired by the home-away patterns for scheduling season match calendars. Both methodologies are tested in real-world instances and experimental instances, reporting results that improve significantly on the manual assignments. The pattern-based formulation attains major reductions in execution times, solving real instances to optimality in just a few seconds, while the traditional one takes anywhere from several minutes to more than an hour.

Keywords:
Referee assignment, sports scheduling, football, patterns, integer linear programming.

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1. Introduction

Football is widely recognized as the most popular sport around the world. To date, the Fédération Internationale de Football Association (FIFA), the world governing body for football, has 208 member associations. Events like the FIFA World Cup captivate the attention of whole nations, including players, fans, sponsors and media. Unfortunately, quoting Forrest (2012), "the world's most popular sport is also its most corrupt, with investigations into match fixing ongoing in more than 25 countries". The performance of the referees plays a crucial role in this issue. Scandals such as the *Calciopoli* in the Italian football (Distaso et al., 2012) and the corruption case in the Czech football (Landa, 2006), are examples where referees have been involved in controversial cases. Other sports are not exempt of controversy. For example, the National Basketball League (NBA) in USA suffered from one of its own referees getting involved in a gambling scandal in 2007 (Rodenberg, 2011), and some surveys have even reported the existence of racial discrimination among its referees (Price and Wolfers, 2010). In this context, a relevant problem that sport league managers need to solve is how to carry out the assignment of referees to matches. The Operations Research community can support this task, through the use of sports scheduling techniques.

The attention of sports scheduling has long focussed on the efficient design of sports season match calendars. The use of formal scheduling techniques for calendar planning has spread widely in the last few decades, specially among the world's football leagues. Examples can be found in the literature for The Netherlands (Schreuder, 1992), Germany and Austria (Bartsch et al., 2006), Chile (Durán et al. 2007, 2012), Denmark (Rasmussen, 2008), Belgium (Goossens and Spieksma, 2009), Norway (Flatberg et al., 2009), Honduras (Fiallos et al., 2010), Brazil (Ribeiro and Urrutia, 2012) and Ecuador (Recalde et al., 2013).

More recently, interest has been growing in the related problem of assigning referees to scheduled matches. This problem typically has a combinatorial structure that is practically impossible to solve by manual methods. Published applications for referee assignment are still relatively few in number. The earliest one was reported by Evans (1988), who specifies a multicriteria optimization problem for scheduling the assignment of umpires in a top-tier North American baseball league using a range of heuristic methods to obtain good solutions with reasonable execution times. Zakarya et al. (1989) develop a computer system for assigning umpires to a basketball league in

Switzerland. Wright (1991) also develops a computer system for assigning umpires to professional cricket matches in England. The problem specification includes both hard and soft constraints and various optimization criteria. It is solved by finding an initial solution using only some of the constraints and then improving it by applying local perturbations in which pairs of umpires are swapped. Farmer et al. (2007) formulate an integer programming model to assign umpires to professional tennis tournaments in the U.S. They propose a solution method comprising a two-phase heuristic in which the first phase constructs an initial set of assignments and the second employs a simulated annealing heuristic to improve them. In a very recent paper, Trick et al. (2012) report an application using network optimization and simulated annealing to schedule umpires for Major League Baseball games in North America. Yavuz et al. (2008) develop a model that was used to identify a fair assignment of football referees to the 2005-2006 season of the Turkish Premier League. The main consideration in this formulation was the referee-team assignments, that is, the frequency referees were assigned to matches involving any given team.

Other published works have taken a more theoretical approach. Dinitz and Stinson (2005), for example, discuss the referee assignment problem for a previously scheduled tournament using certain types of Room squares. Trick and Yildiz (2007) propose a specification similar to the well-known Travelling Tournament Problem (Easton et al., 2001) which attempts to minimize the total distance travelled by the referees instead of the teams, thus renaming it the Travelling Umpire Problem. They develop a solution approach based on Benders' cuts and large neighborhood search. In later articles, the authors present an updated version of the previous one (Trick and Yildiz, 2011) and develop another approach based on a genetic algorithm (Trick and Yildiz, 2012).

Duarte et al. (2007) define a referee assignment problem that seeks an efficient assignment of referees to a sports season calendar by minimizing the sum over all referees of the differences between the desired match assignments for each referee and the ones they are actually assigned by the definitive solution. Gil Lafuente and Rojas Mora (2007) suggest an assignment method that uses confidence intervals to incorporate the degree of uncertainty inherent in the assessments of each referee's experience and skill category given the subjective nature of such information.

For a review of the literature on sports scheduling, we refer the reader to the annotated bibliography by Kendall et al. (2010) and to the more recent

survey by Ribeiro (2012).

The present article addresses the referee assignment problem for the First Division of the Chilean professional football league using integer linear programming models. The models incorporate various user-defined criteria that enhance the transparency and objectivity of the assignment process. These criteria include achieving better balances in the number of matches each referee must officiate, in the frequency each is assigned to a given team, and in the distances each must travel over the course of a season. It also include the generation of appropriate pairings of referee experience or skill category with the importance of certain matches.

Two methods for solving the problem are studied: a traditional one in which a single model is run directly, and a novel two-stage approach in which a first model constructs referee patterns for the season and a second model generates the actual assignments. This strategy is inspired by the successful use of home-away patterns in the match scheduling methods of various sports leagues around the world (see, for example, Cain, 1977; de Werra, 1988; Nemhauser and Trick, 1998; Bartsch et al., 2006; Goossens and Spieksma, 2009; Durán et al., 2012; Ribeiro and Urrutia, 2012). To our knowledge this is the first article to develop a pattern-based approach to referee assignment.

The proposed models are implemented for real-world instances of the referee assignment problem and a flexible tool is devised for convenient application by prospective users. The results of the implementation are clearly superior in terms of the defined criteria to those obtained with manual assignment. Furthermore, the pattern-based approach solves the instances significantly faster than the traditional method.

The contribution of this article consists in addressing two principal issues that emerge from the foregoing bibliographic survey: the shortage of studies applying sports scheduling techniques to referee assignment and the potential of such techniques for improving assignments in real-world situations. More particularly, it offers an approach to the referee assignment problem in the context of an actual case (the First Division of Chile's professional football league) and devises a pattern-based method that has already proven to be highly effective in scheduling the season calendars of various sports leagues.

The remainder of this article is organized as follows. Section 2 outlines the context of the assignment problem to be addressed. Section 3 formulates a traditional ILP model to address it, while Section 4 develops a pattern-based solution approach to the problem. Section 5 presents the results obtained by these two approaches. Finally, Section 6 discusses the results and sets out

the conclusions.

2. Referee assignment in the Chilean context

The top league in Chile's professional football league system is the First Division, which is governed and managed by the Asociación Nacional de Fútbol Profesional de Chile (ANFP). One of the ANFP's responsibilities is the assignment of referees to the Division's scheduled matches. This task is handled by a group of experts normally made up of retired professional football referees known as the Referee Committee. Assignments are decided by the Committee week by week based on relatively loose criteria using strictly manual methods and no sophisticated decision tools, with results that are often disadvantageous. Some referees, for example, will typically be assigned significantly fewer games than others despite having similar experience and skill levels. The absence of any official explanation for these discrepancies raises suspicions about the process and suggests a lack of transparency. As can be seen in Figure 1, the differences between referees in the number of match assignments for the 2007 season were as high as 12, or 50%.
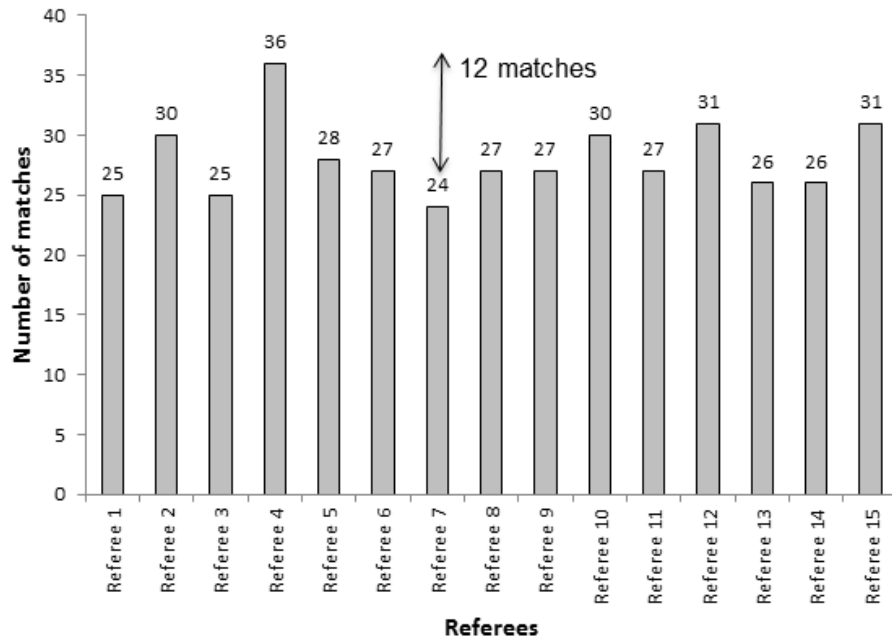

Figure 1: Number of match assignments for each referee in the 2007 season.

It is also common for some referees to be assigned relatively many matches involving the same team, while others are never assigned to certain teams. Figure 2, which shows how frequently two selected referees were assigned to each First Division team's games in 2007, reveals that *Referee 2* officiated at six *Team 14* and seven *Team 18* matches but none featuring *Team 15*, while *Referee 3* never officiated at matches with the former two clubs but did at three involving *Team 15*.
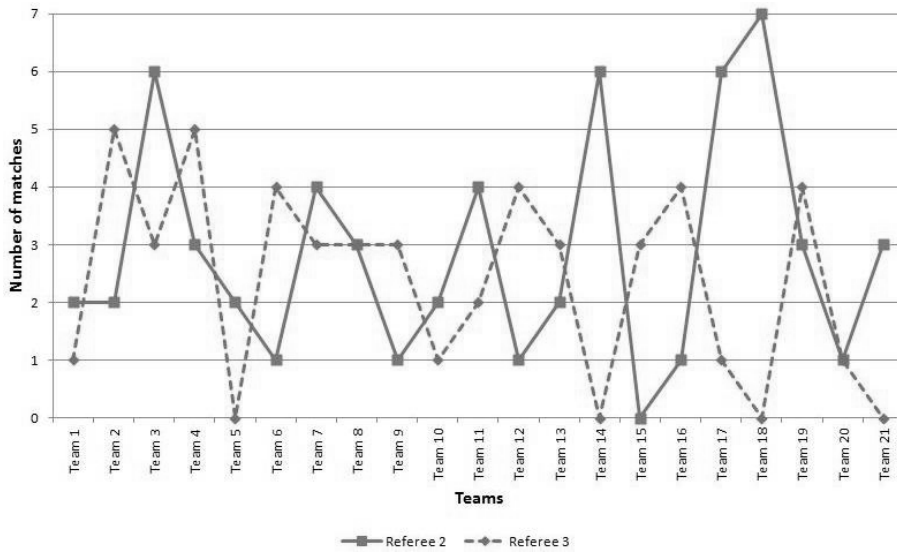

Figure 2: Number of each team's matches assigned to two selected referees in the 2007 season.

In addition, due to the long and narrow shape of Chile's physical territory, assigning referees using manual methods may result in some of them travelling considerably longer distances than others. Given that most referees live in Santiago, the nation's capital, they naturally prefer assignments within the city's greater urban area where they are close to their homes and workplaces, especially since the majority have another job during the week. But as long as there are First Division teams scattered along the 4200 kilometres separating the country's northern and southern extremes, some referees must be willing at any given point in the season to travel to the more outlying venues. Figure 3 illustrates the disparities among the referees during the 2007 season in average travel, some of them racking up more than three times the average mileage per match of certain of their colleagues.
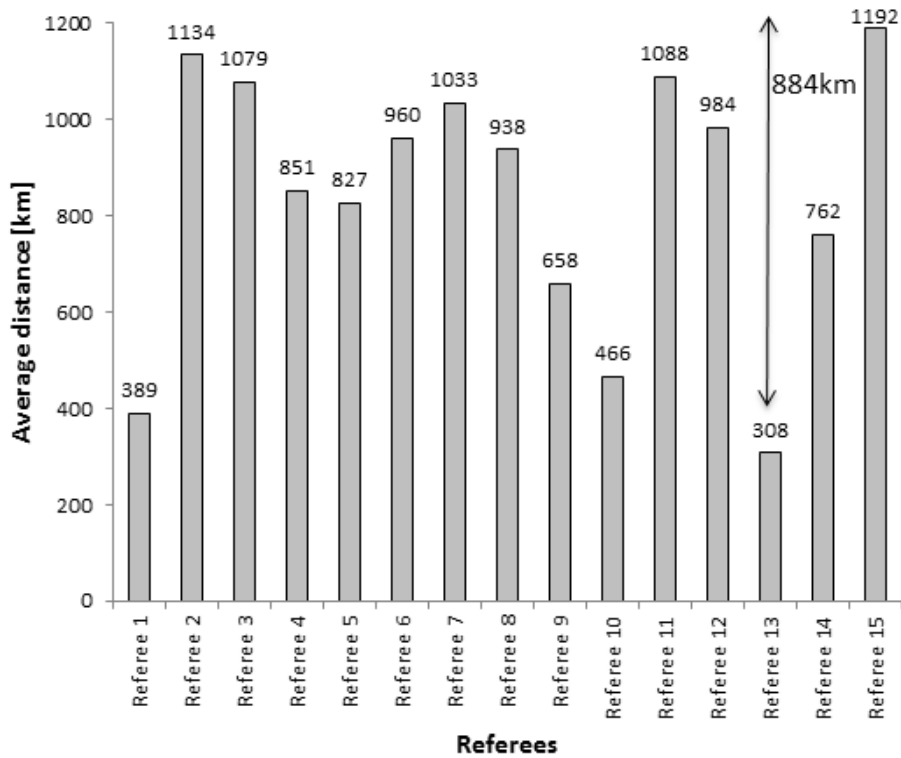
Figure 3: Average travel distance per match for each referee in the 2007 season.

The above factors point up the importance of improving the efficiency of the Chilean league's referee assignment. The dissatisfaction expressed by players, fans and team managers with the current quality of officiating is hardly surprising, and only serves to increase the pressures on the referees. Though the problems just described are not entirely attributable to poor assignment, the establishment of objectively defined criteria implemented by mathematical programming models would do much to ensure the process was both fair and transparent in the eyes of all relevant actors and would raise the league's general level of professionalism.

A previous application of mathematical programming tools to top-level Chilean football was reported by Durán et al. (2007), who developed an optimization model for defining the First Division's annual match calendar. This formulation has had a considerable impact since it was first applied in 2005 and has been used by the league ever since. A modified version was adopted two years later by the Second Division (Durán et al., 2012). Further de-

tails on the organization of Chilean professional football and its competition formats may be found in the two just-cited papers. The referee assignment scheduling described in the present article is one of various projects involving the present authors that have grown out of these earlier experiences.

3. Model 1: A traditional integer linear programming approach

The various conditions that should be satisfied by the First Division referee assignment were defined in the light of conversations with officials of the ANFP and its Referee Committee that focussed on the weaknesses of the manual assignment methods detailed above and various suggestions by the participants.

Since the season match calendar is assumed to be already known, the referee assignment determines which referee will officiate at each scheduled match. In practice, changes may be made as the season progresses if, for example, unforeseen circumstances affect the availability of certain referees. The proposed model is flexible to be re-executed before each round (i.e., match date) using updated information incorporating these eventualities as well as the assignment experience up to that moment.

We now formally set out a traditional formulation of an integer linear programming model for addressing the referee assignment problem, which we call Model 1 or simply the "original" model.

- **Sets**

  M: The set of matches.

  R: The set of referees.

  T: The set of teams.

  K: The set of rounds.

  FIX: The set of pairs (r, m) predetermining that referee r must officiate at match m.

  $NOFIX$: The set of pairs (r, m) predetermining that referee r must not officiate at match m.

- **Parameters**

$$\alpha_{m,k} = \begin{cases} 1 & \text{if match } m \text{ is played in round } k \\ 0 & \sim \end{cases}$$

$$\beta_{m,t} = \begin{cases} 1 & \text{if team } t \text{ plays in match } m \\ 0 & \sim \end{cases}$$

a_r: Minimum number of match assignments for referee r.

$\bar{a}_r$: Maximum number of match assignments for referee r.

$n_{r,t}$: Minimum number of referee-team assignments involving referee r and team t.

$\bar{n}_{r,t}$: Maximum number of referee-team assignments involving referee r and team t.

c: Minimum number of consecutive rounds between assignments of a referee r to the same team ($c \geq 1$).

$\delta_{r,m}$: Distance (round trip) between home town of referee r and city of venue of match m, in kilometres.

$\bar{\delta}$: Maximum difference between any two referees' average match travel distances.

u_r: Maximum number of consecutive rounds for which a referee r is unassigned, that is, has no match assignment.

τ_r: Target number of match assignments for referee r.

- **Variables**

$$x_{r,m} = \begin{cases} 1 & \text{if referee } r \text{ is assigned to match } m \\ 0 & \sim \end{cases}$$

$\Delta_r =$ Absolute value of the difference between target and actual number of match assignments for referee r.

- **Objective function and constraints**

  Objective function. The objective function (OF) is the one suggested by Duarte et al. (2007), which minimizes the sum over all referees of the absolute value of the difference between the target and the actual number of games assigned to each referee.

$$\min f = \sum_{r \in R} \Delta_r \tag{1}$$

  Basic constraints. Each match must be assigned one and only one referee.

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$$\sum_{r \in R} x_{r,m} = 1 \qquad \forall\, m \in M. \tag{2}$$

Referee-round constraints. Each referee may be assigned to a maximum of one match per round.

$$\sum_{m \in M} \alpha_{m,k} \cdot x_{r,m} \leq 1 \qquad \forall\, r \in R,\ k \in K. \tag{3}$$

Season match assignment balance constraints. Minimum and maximum numbers of total season match assignments for each referee, limited by lower and upper bounds (the target number τ_r is thus a value between these two bounds).

$$\sum_{m \in M} x_{r,m} \geq a_r \qquad \forall\, r \in R. \tag{4}$$

$$\sum_{m \in M} x_{r,m} \leq \bar{a}_r \qquad \forall\, r \in R. \tag{5}$$

Referee-team balance constraints. Minimum and maximum referee-team assignments are limited by lower and upper bounds.

$$\sum_{m \in M} \beta_{m,t} \cdot x_{r,m} \geq n_{r,t} \qquad \forall\, r \in R,\ t \in T. \tag{6}$$

$$\sum_{m \in M} \beta_{m,t} \cdot x_{r,m} \leq \bar{n}_{r,t} \qquad \forall\, r \in R,\ t \in T. \tag{7}$$

Note that in some sports leagues such as North American Major League Baseball, upper and lower bounds are applied not only generally to total referee-team assignments but also separately to home and away game assignments (Trick et al., 2012). These additional considerations have not arisen in our work with officials of the Chilean leagues, no doubt because in Chile the most popular clubs have supporters all around the country who follow their teams to every venue, thus limiting the importance of their home-away status in any given match.

Another referee-team balance condition states that in c consecutive rounds a given referee cannot be assigned to the same team more than once.

$$\sum_{d=0}^{c-1} \sum_{m \in M} \beta_{m,t} \cdot \alpha_{m,k+d} \cdot x_{r,m} \leq 1 \qquad \forall \, r \in R, \, t \in T, \, k \leq |K| - c + 1. \tag{8}$$

Note that using a value of $c = 1$ is equivalent to placing no restrictions on the frequency of a referee's assignments to a given team (the resulting inequality is dominated by constraint (3)).

Average travel distance balance constraints. The differences between the referees' average match travel distances are subject to an upper bound. The distances are calculated assuming the number of referee assignments are equal to their target values. This assumption is not necessarily satisfied *a priori*, but since the objective function attempts to satisfy it, we would expect that calculating the distances this way should yield good estimates while allowing us to conserve the problem's linearity.

$$\frac{1}{\tau_r} \sum_{m \in M} \delta_{r,m} \cdot x_{r,m} - \frac{1}{\tau_{\hat{r}}} \sum_{m \in M} \delta_{\hat{r},m} \cdot x_{\hat{r},m} \leq \bar{\delta} \quad \forall \, r, \hat{r} \in R. \tag{9}$$

No assignment constraint. Sets the maximum number of consecutive rounds for which a referee may have no assignment.

$$\sum_{i=0}^{u_r} \sum_{m \in M} \alpha_{m,k+i} \cdot x_{r,m} \geq 1 \qquad \forall \, r \in R, \, k \leq |K| - u_r. \tag{10}$$

Referee category and match importance level. Certain matches during the season must be officiated by more experienced or higher skill "category" referees. To formulate this restriction, the set M of matches is partitioned into three subsets denoted M_V, M_H and M_N containing matches of *very high*, *high* and *normal* importance level, respectively ($M = M_V \cup M_H \cup M_N$). Matches classed as very high level feature teams with a strong historic rivalry while those classed as high level are matches that will be televised or for any other reason are awaited

with particular interest by football fans. Normal level matches are all those not classed in the other two subsets.

In similar fashion, the referees are partitioned into three subsets denoted R_A, R_B and R_C corresponding to skill level categories A, B and C (in decreasing skill level order) as determined by the Referee Committee ($R = R_A \cup R_B \cup R_C$). An R_A referee can officiate any match, an R_B referee can officiate M_H or M_N matches and an R_C referee can only be assigned to M_N matches. These constraints are expressed as follows:

$$\sum_{r \in R_A} x_{r,m} = 1 \qquad \forall\, m \in M_V. \tag{11}$$

$$\sum_{r \in R_A \cup R_B} x_{r,m} = 1 \qquad \forall\, m \in M_H. \tag{12}$$

Note that with the incorporation of these two restrictions, constraint (2) can be specified only for the M_N subset rather than the entire set M.

Special assignments and non-assignments. A referee may be unable to officiate a certain match, for example, due to a suspension or an injury. To accommodate such cases the following constraint is included:

$$x_{r,m} = 0 \qquad \forall\, (r, m) \in NOFIX. \tag{13}$$

The Referee Committee may want to impose the assignment of a given referee to a certain match. This can be done through the following constraint:

$$x_{r,m} = 1 \qquad \forall\, (r, m) \in FIX. \tag{14}$$

Logical constraints for Δ_r***.*** The next two constraints ensure that Δ_r is the absolute difference, for each referee, between the target number of assignments defined *a priori* and the actual number assigned.

$$\sum_{m \in M} x_{r,m} + \Delta_r \geq \tau_r \qquad \forall\, r \in R. \tag{15}$$

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$$\sum_{m \in M} x_{r,m} - \Delta_r \leq \tau_r \qquad \forall\, r \in R. \qquad (16)$$

Nature of the variables.

$$x_{r,m} \in \{0,1\} \text{ and } \Delta_r \in \mathbb{Z}^+ \cup \{0\} \qquad \forall\, r \in R,\, m \in M. \qquad (17)$$

4. Model 2: A pattern-based solution approach

Solving the formulation presented in the previous section is likely to be difficult due to its combinatorial structure, the nature of the data and the size of the instances. To illustrate this point, a football season organized as a double round-robin with 6 teams and 4 referees available for each match (thus requiring 3 of the 4 referees for each round) would have 63 billion possible referee assignments. For the current season format of Chile's First Division, with 18 teams, 34 rounds and about 15 referees, the possibilities would be almost literally endless. As we will see in Section 5, Model 1 is in fact capable of solving real cases of this problem in 14 to 72 minutes using a commercial solver. However, though such execution times are reasonably acceptable, reducing them still further would be desirable so that solutions could be readily generated at meetings of the Referee Committee or when conducting multiple tests with different parameter values.

An alternative approach to producing good solutions in relatively little time that has been widely and successfully used for sports scheduling problems involves the use of an additional formulation to generate structures known as patterns for defining each team's home-away match sequences. Once these patterns have been constructed and assigned to the various teams, the complete season schedule is then determined.

This two-stage approach usually cuts computation times significantly while delivering solutions that, though not necessarily optimal, perform well in terms of the objective function value. Local search procedures can then be utilized to improve solution quality, or exact procedures can be employed using the solutions as a starting point. An excellent survey of pattern-based methods and the diversity of ways they have been applied to solve sports scheduling problems is found in the fourth section of Rasmussen and Trick (2008).

In what follows we briefly describe the home-away pattern approach in sports season scheduling and then set out our adaptation of it to referee assignment.

4.1. The pattern-based approach in season scheduling

A pattern as used in a season scheduling model refers to an ordered array of characters H, A and B denoting "home", "away" and "bye", respectively. The dimension of the array is the number of rounds in the season. In a pattern assigned to a particular team, the jth element indicates whether, in round j, the team plays at home or away or has a bye.

For example, in a season with six rounds the pattern $P(\textit{Team 1}) = (H, A, H, B, A, A)$ indicates that $\textit{Team 1}$ plays at home in the first and third rounds, away in the second, fifth and sixth rounds, and has a bye in the fourth round.

Various strategies have been suggested in the literature for generating these home-away patterns, including logical rules and integer programming models. However they are constructed, they are generally used in a first stage of the solution approach to determine, for each round, which teams play at home and which away, making sure to satisfy the various applicable home-away sequence constraints. In a second stage, the strategies determine which teams play against each other in each round subject to the home-away determinations made by the patterns in the previous stage. The key decision variable in the second stage is usually a binary variable $w_{t,\hat{t},k}$ that takes the value 1 if team t plays at home against team $\hat{t}$ in round k and zero otherwise, defined for all teams and rounds. The patterns are linked to these variables through logical relationships. This application of the patterns thus dispenses with the need in the second stage for the constraints already satisfied in the first stage. The focus then shifts to obtaining feasibility on the remaining constraints and, if there is an objective function, finding an optimal solution.

4.2. A pattern-based approach for referee assignment

In a referee assignment model, the concepts of "home" and "away" obviously have no meaning. The formulation we propose uses the patterns to define the geographical zone each match is played in. For this purpose the country is segmented into $\textit{North}$ (N), $\textit{Centre}$ (C) and $\textit{South}$ (S) zones, and each league team is classed into one of them as determined by the location of its home venue. Since the number of referees is usually greater than the number of matches, we also incorporate a value denoted $\textit{Unassigned}$ (U) that indicates the rounds in which a referee has no game assigned.

A pattern is thus defined as an ordered array of characters in $\{N, C, S, U\}$ whose dimension is equal to the number of rounds in the season.

For example, in a season with nine rounds the pattern $Q(\textit{Referee 1}) = (C, N, S, N, U, C, S, S, N)$, indicates that *Referee 1* officiates somewhere in the North zone in rounds 2, 4 and 9; in the Centre zone in rounds 1 and 6; and in the South zone in rounds 3, 7 and 8. In round 5, he is unassigned. Note also that once the pattern of a referee r is defined, so is the total number of matches that referee will officiate over the course of the season. In our example Q, with only one round unassigned *Referee 1* will officiate in 8 of the 9 rounds.

The geographical segmentation of the teams was motivated by the peculiarities of Chile's physical territory referred to earlier, and also because it allows us to incorporate conditions of the original model. Any other criterion could be used in order to generate the patterns. One alternative might be, for example, to group teams by level of popularity.

With the foregoing definitions and explanations we can now develop our proposed two-stage solution methodology for referee assignment. In the first stage, a formulation known henceforth as Model 2a generates the patterns for each referee considering some of the constraints defined in Model 1, namely, the ones that seem *a priori* particularly relevant to the pattern sequences. In the second stage, a formulation denoted Model 2b incorporating only the remaining constraints generates the definitive assignments of referees to matches. Together, these models make up the two-stage approach we will call pattern-based approach, or simply Model 2.

4.2.1. Model 2a: the pattern-generation model

We begin the development of the pattern-generation model by introducing a family of variables for the construction of the patterns and selecting certain constraints from the original model to be partially or wholly captured in this new specification. The sets and parameters include some also taken from the original model, which retain their definitions, plus a number of additional ones that are set out below.

- **Additional sets**

  $Z = \{N, C, S, U\}$: A set of characters indicating that the referee is either assigned to officiate in the specified zone (North, Centre or South) or is unassigned.

  RN: The set of triples (r, z, k) such that referee r cannot officiate in zone z in round k.

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RY: The set of triples (r, z, k) such that referee r must officiate in zone z in round k.

- **Additional parameters**

$$\gamma_{m,z,k} = \begin{cases} 1 & \text{if match } m \text{ is played in zone } z \text{ in round } k \\ 0 & \sim \end{cases}$$

$$\rho_{t,z,k} = \begin{cases} 1 & \text{if team } t \text{ plays in zone } z \text{ in round } k \\ 0 & \sim \end{cases}$$

$\tilde{\delta}_{r,z}$: Average distance between home town of referee r and cities where home venues of teams in zone z are located.

- **Variables**

$$y_{r,z,k} = \begin{cases} 1 & \text{if pattern of referee } r \text{ indicates that he officiates in} \\ & \text{zone } z \text{ or is unassigned in round } k \\ 0 & \sim \end{cases}$$

Δ_r = Difference between target and actual number of match assignments for referee r.

- **Objective function and constraints**

The objective function is the same as that for Model 1.

$$\min f = \sum_{r \in R} \Delta_r \tag{18}$$

Basic constraints on patterns and season schedule. The number of patterns indicating a match to be officiated in zone z in round k must equal the number of matches specified by the match calendar for that zone in that round.

$$\sum_{r \in R} y_{r,z,k} = \sum_{m \in M} \gamma_{m,z,k} \qquad \forall\, z \in \{N, C, S\},\ k \in K. \tag{19}$$

This family of constraints is similar to constraints (2) of Model 1 except that here it is applied to the variables y.

Referee-round constraints. For every round, each referee must either be assigned to officiate in some zone or be unassigned.

$$\sum_{z \in Z} y_{r,z,k} = 1 \qquad \forall\, r \in R,\ k \in K. \tag{20}$$

This family of constraints is analogous to constraints (3) of Model 1.

Season match assignment balance constraints for each referee.
Echoing the restrictions (4) and (5) in Model 1, lower and upper bounds impose minimum and maximum values for the total number of matches each referee can officiate in a season.

$$\sum_{z \in \{N,C,S\}} \sum_{k \in K} y_{r,z,k} \geq a_r \qquad \forall\, r \in R. \tag{21}$$

$$\sum_{z \in \{N,C,S\}} \sum_{k \in K} y_{r,z,k} \leq \bar{a}_r \qquad \forall\, r \in R. \tag{22}$$

Referee-team balance constraints. To partially capture the restrictions on referee assignments to particular teams expressed in constraints (6) of Model 1, we impose for each team and referee a lower bound on the number of times any pattern may be assigned to the zone in which that team plays.

$$\sum_{z \in \{N,C,S\}} \sum_{k \in K} \rho_{t,z,k} \cdot y_{r,z,k} \geq n_{r,t} \qquad \forall\, r \in R,\ t \in T. \tag{23}$$

However, the upper bound $\bar{n}_{r,t}$ defined in Model 1 is not only unnecessary here but would be undesirable: unnecessary because various matches are normally played in a given zone in a given round so that assigning referee r to a zone does not necessarily mean he officiates team t, and therefore also undesirable because its mere application would reduce the range of assignment options.

Average travel distance balance constraints for each referee.
This constraint, similar to constraints (9) in Model 1, aims at achieving a balance between the referees' average travel distances. We partially capture this in terms of the average distance $\tilde{\delta}_{r,z}$ between the home town of referee r and the cities where the home venues of teams in zone z are located.

$$\frac{1}{\tau_r} \sum_{z \in \{N,C,S\}} \sum_{k \in K} \tilde{\delta}_{r,z} \cdot y_{r,z,k} - \frac{1}{\tau_{\hat{r}}} \sum_{z \in \{N,C,S\}} \sum_{k \in K} \tilde{\delta}_{\hat{r},z} \cdot y_{\hat{r},z,k} \leq \bar{\delta} \qquad \forall\, r,\hat{r} \in R. \tag{24}$$

No assignment constraint. As with constraints (10) in Model 1, this constraint bounds the number of consecutive unassigned rounds for each referee pattern.

$$\sum_{i=0}^{u_r} y_{r,z,k+i} \leq u_r \qquad \forall \, r \in R, \; k \leq |K| - u_r, \; z \in \{U\}. \qquad (25)$$

Referee category and match level. This captures constraints (11) and (12) of Model 1 by imposing that the number of patterns assigned to A category referees officiating in zone z in round k be equal to the number of matches in zone z in round k that require this referee category.

$$\sum_{r \in R_A} y_{r,z,k} \geq \sum_{m \in M_V} \gamma_{m,z,k} \qquad \forall \, z \in \{N, C, S\}, \; k \in K. \qquad (26)$$

The same condition is imposed for matches requiring referees of at least B category (obviously, A category referees can also officiate such matches).

$$\sum_{r \in R_A \cup R_B} y_{r,z,k} \geq \sum_{m \in M_V \cup M_H} \gamma_{m,z,k} \qquad \forall \, z \in \{N, C, S\}, \; k \in K. \qquad (27)$$

Special assignments and non-assignments. To capture constraints (13) of Model 1, we impose the constraint that the referee cannot officiate in the zone where the match in question is to be played.

$$y_{r,z,k} = 0 \qquad \forall \, (r, z, k) \in RN. \qquad (28)$$

Analogously, constraints (14) in Model 1 are expressed here by the constraint imposing that the referee must officiate in the zone where the home team's venue is located:

$$y_{r,z,k} = 1 \qquad \forall \, (r, z, k) \in RY. \qquad (29)$$

Logical constraints to calculate Δ_r**.** The variable Δ_r is calculated in analogous fashion to constraints (15) and (16) of Model 1, except that here it is the variables y in the geographical zones that are summed instead of x.

$$\sum_{z \in \{N,C,S\}} \sum_{k \in K} y_{r,z,k} + \Delta_r \geq \tau_r \qquad \forall \, r \in R. \qquad (30)$$

$$\sum_{z \in \{N,C,S\}} \sum_{k \in K} y_{r,z,k} - \Delta_r \le \tau_r \qquad \forall\, r \in R. \tag{31}$$

Nature of the variables.

$$y_{r,z,k} \in \{0,1\} \text{ and } \Delta_r \in \mathbb{Z}^+ \cup \{0\} \qquad \forall\, r \in R,\ z \in Z,\ k \in K. \tag{32}$$

These patterns do, however, guarantee constraints (3), (4), (5), (10), (13), (15) and (16), dispensing with the need for them here in Model 2b.

4.2.2. Model 2b: the pattern-based assignment model

Once the patterns have been determined by solving Model 2a, the actual assignments of the referees to matches are generated using Model 2b, an integer linear model we develop in what follows. Since the number of matches each referee will officiate is already defined by the patterns, so are the values for the variables Δ and the objective value f. This being the case, Model 2b will solely search for a feasible assignment based on these patterns. Note that constraints (3), (4), (5), (10), (13), (15) and (16) are already satisfied by the patterns generation, therefore Model 2b does not need to incorporate them.

As with Model 2a, the parameters and sets defined for Model 1 retain their previous definitions in Model 2b. However, the optimal values of variables y in the Model 2a solution are incorporated into Model 2b as parameters, now denoted $\tilde{y}$ to avoid confusion. This predefines the pattern that will be assigned to each referee.

- **Parameters**

$$\tilde{y}_{r,z,k} = \begin{cases} 1 & \text{if in round } k \text{ the pattern of referee } r \text{ indicates that he is} \\ & \text{assigned to officiate in zone } z \text{ or is unassigned} \\ 0 & \sim \end{cases}$$

- **Variables**

$$x_{r,m} = \begin{cases} 1 & \text{if referee } r \text{ is assigned to match } m \\ 0 & \sim \end{cases}$$

- **Constraints**

  Constraints on patterns and their logical relationship with variable x***.*** A condition is imposed on the relationship between variable x and parameter $\tilde{y}$ ensuring that a match m is assigned to referee

r only if the corresponding pattern assigns r to the zone z in which the venue of m is located. This restriction is modelled as follows:

$$\sum_{m \in M} \gamma_{m,z,k} \cdot x_{r,m} = \tilde{y}_{r,z,k} \qquad \forall \, r \in R, \; z \in \{N, C, S\}, \; k \in K. \quad (33)$$

Model 2b also explicitly includes the Model 1 constraints (2), (6), (7), (8), (9), (11), (12) and (14), which are not necessarily guaranteed by the patterns generated by Model 2a.

If Model 2b is unable to find a feasible solution with the set of patterns constructed by Model 2a, new pattern sets can be generated and tried iteratively until a solution is arrived at. Various different procedures for generating the new patterns can be used. For example, it is possible to swap patterns, or parts thereof, between a pair of referees. Another possibility is to run Model 2a adding forbidding the use of some patterns already used. Alternatively, a heuristic could be employed that relaxes the pattern specifications iteratively. Thus, if a set of patterns generated by Model 2a for use in Model 2b does not yield a solution, the patterns for two chosen referees are eliminated so that the search space becomes larger. Model 2b is then run again, but this time all of the original problem constraints are imposed on the two referees now without patterns and the objective function expresses only the difference between their target and actual number of match assignments. For the remaining referees, of course, the differences are already fixed. If again no solution is found, the pattern of a third referee is eliminated and the process is iterated, each time stripping the pattern of another referee. In the worst possible case, this heuristic will terminate with all referee patterns eliminated and all the original problem constraints restored, in effect returning to Model 1, the original formulation.

However, as will be reported below our experience solving the four actual instances of the problem for the 2007 through 2010 seasons was that Model 2b always found a feasible assignment using the pattern set generated by Model 2a such that all referees officiate their target number of matches. Thus, the pattern approach delivered an optimal solution to the four actual instances in every case without recourse to additional procedures.

5. Results

In this section we evaluate a range of characteristics of the solutions obtained by our models as well as the solution times. The solution characteris-

tics were derived from data for the regular 2007 season of Chile's First Division. That year there were 21 teams, 15 referees and 420 matches scheduled in 42 rounds across two half-seasons. In each round, 10 matches were played and one team had a bye. Model 1 for this instance contained about 6300 binary variables, 15 non-negative integer variables and 15,500 constraints.

The parameter values were defined in consultation with the Referee Committee. The target number τ_r of match assignments for each referee was confined to an interval ranging from $a_r = 27$ to $\bar{a}_r = 29$, the exact values chosen so as to add up to the total number of matches in the season. Thus, $\sum_{r \in R} \tau_r = 420$. The minimum and maximum numbers of referee-team assignments n_r and $\bar{n}_r$ were fixed at 1 and 4, respectively. The value chosen for maximum travel difference $\bar{\delta}$ was 500 kilometres. The consecutive match parameter c was set at 2, same as the no-assignment parameter u_r for all r.

To assess the solution quality and running times, all four instances of the problem covering the First Division's 2007-2010 seasons were solved. Our implementation is coded in AMPL and uses the solver CPLEX.

5.1. Characteristics of the solution

The results of the pattern-based model are compared with the actual manually produced results for the 2007 season in Table 1. Model 2 solved the problem to optimality and the objective function value was 0, meaning that the solution satisfied the targets regarding the number of match assignments for every referee. As can be seen, the model solution is superior on every point of comparison. In particular, for the number of match assignments, number of assignments to the same team and the average travel distance, the model results are better balanced.

Table 1: Comparison of actual 2007 season assignment with the pattern-based model assignment.

Measure	Actual	Model 2
Min. no. of match assignments for a referee	24	27
Max. no. of match assignments for a referee	36	29
Min. no. of assignments of a referee to the same team	0	1
Max. no. of assignments of a referee to the same team	7	4
Max. difference between the referees' average travel distances	884	496
Max. no. of consecutive rounds in which a referee was unassigned	4	2

A useful indicator of how well an assignment is balanced is its standard deviation. For the number of assignments to a referee, the actual 2007 stan-

dard deviation was 3.01 whereas in our solution it was only 0.63. The lowest and highest absolute numbers of actual matches assigned in 2007 were 24 and 36 respectively whereas in the Model 2 solution they were 27 and 29. These values are indeed well-balanced given that the ratio of the number of matches to the number of referees ($|M|/|R|$) was $420/15 = 28$. The assignments for each referee, shown in Figure 4, differ markedly from the actual 2007 results in Figure 1.
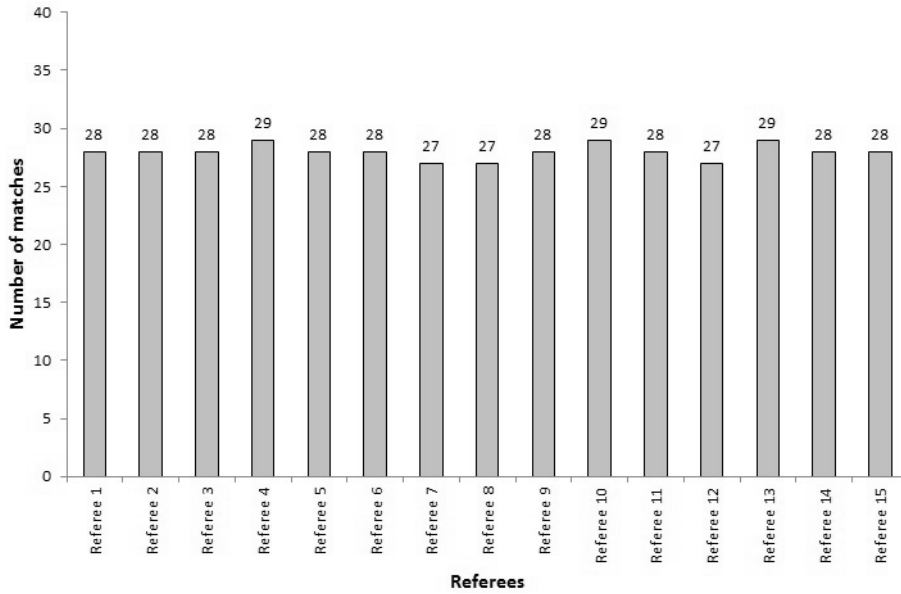

Figure 4: Number of match assignments for each referee in the pattern-based model solution.

The actual 2007 standard deviation in the number of referee assignments to the same team for all referees and teams was 1.55 while the Model 2 value was only 1.11. Since this is a highly sensitive issue for fans and the media, a balanced result on this indicator is particularly important. A good idea of what the average number of referee-team assignments should be in a perfectly balanced scenario is given by the ratio of the number of matches played by each team to the number of referees, which in 2007 was $40/15 = 2.67$. The actual minimum and maximum results that year were 0 and 7 respectively, contrasting sharply with the corresponding model results of 1 and 4. The referee-team assignments generated under the pattern-based approach for the same two referees featured in Figure 2 above are given in Figure 5. As can be

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observed, the model assigns *Referee 2* to just four *Team 14* matches instead of six, while assigning *Referee 3* to three matches with this team instead of none.
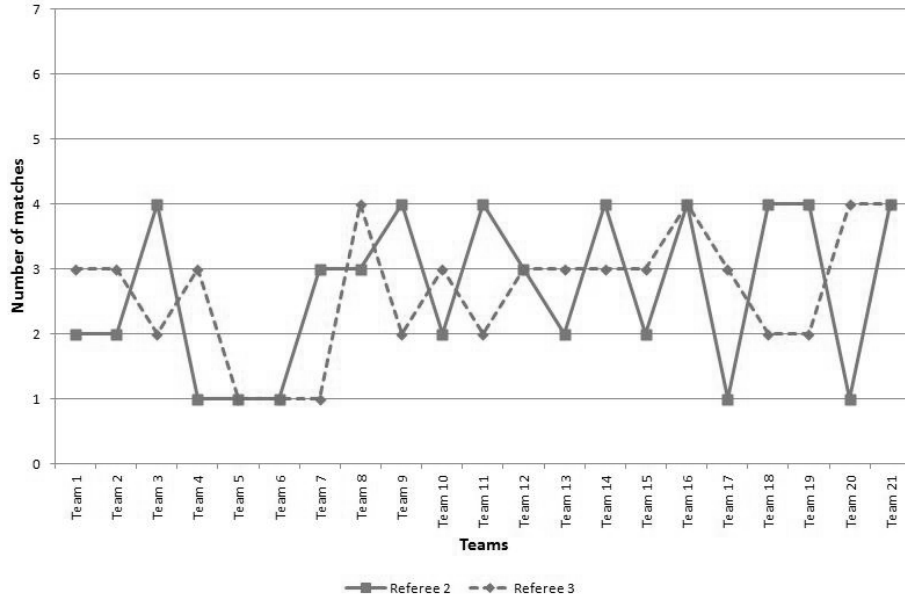

Figure 5: Number of each team's matches assigned to two selected referees in the pattern-based model solution.

As for average travel distance, the maximum difference between two referees was $1192 - 308 = 884$ kilometres in the actual assignment but only $1146 - 650 = 496$ kilometres in the Model 2 solution, as shown in Figure 6. In addition, the actual 2007 standard deviation was 268 whereas the model result was 181.

5.2. Solution times

Two sets of computational experiments were performed to compare the solution times of the original model (Model 1) with those of the pattern-based approach (Model 2). The first set used the real instances of the problem for the four seasons between 2007 and 2010 while the second set utilized instances derived by modifying the values of the parameters for the 2007 season, the largest instance and the one most difficult to solve.
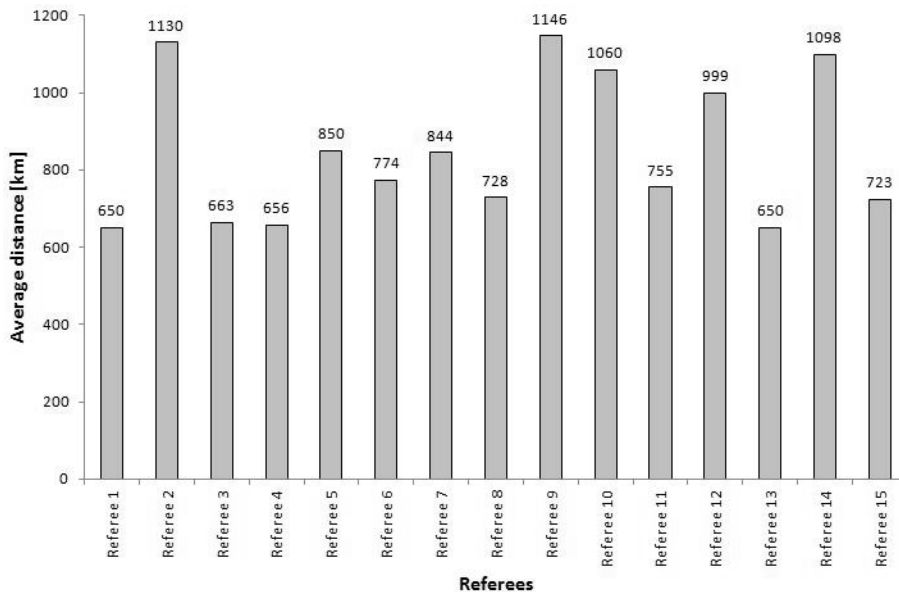
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Figure 6: Average travel distance per match for each referee in the pattern-based model solution.

5.2.1. Real instances, 2007-2010

The solution times obtained for the four seasons 2007 through 2010 are summarized in Table 2. The number of teams was greatest in 2007 at 21, dropping to 20 in 2008 and 18 in 2009 and 2010. The number of matches fell accordingly, from 420 in 2007 to 380 in 2008 and 306 in 2009 and 2010. The number of referees did not follow this trend, increasing from 15 in 2007 to 16 in 2008, back to 15 in 2009 and up again to 17 in 2010.

Table 2: Solution times of real instances, 2007-2010 (in seconds).

Instance	T_1	T_{2a}	T_{2b}	$T_{2a} + T_{2b}$
2007	4314.9	1.3	5.7	7.0
2008	3359.4	1.0	0.7	1.7
2009	827.9	0.7	0.3	1.0
2010	2764.7	0.9	0.8	1.7

As was noted above for the 2007 season, the other 3 real instances were solved to optimality under both approaches and the objective function value was zero, indicating that the solution satisfied the targets regarding the number of match assignments for every referee. Although the pattern-based ap-

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proach does not guarantee it *a priori*, an optimal solution was found for all four instances in a single run of Model 2, with no need to generate a second or further pattern set. The time reduction achieved by the pattern methodology over the traditional model is significant in all of the instances.

The actual solution times T_1 for Model 1 (column 2 in the table) ranged from 827.9 to 4314.9 seconds, or about 14 to 72 minutes. As noted earlier, however, the Chilean league's Referee Committee requires a method that worked considerably faster, and the pattern-based approach clearly satisfies that desire. The solution time for Model 2a was between 0.7 and 1.3 seconds (column 3) while for Model 2b it varied from 0.3 to 5.7 seconds (column 4). Thus, the total time needed to arrive at a referee assignment for the four real instances was just 1.0 to 7.0 seconds, the sum of the two models' individual times (column 5). Compared to the traditional Model 1, the time reduction obtained was more than 99%.

5.2.2. Experimental instances

A series of 14 experimental instances were run to obtain further comparisons of the pattern-based approach's performance with that of the traditional model. The results are summarized in Table 3. These instances were constructed on the basis of the actual 2007 season, which as we just saw was the largest and most difficult to solve in terms of execution time.

Table 3: Solution times of experimental instances, with and without patterns, 2007-2010 (in seconds). NF indicates that the execution time extended beyond 1 hour without finding a feasible solution.

Instance	Description: $(a,\bar{a}),(n,\bar{n}),c,\bar{\delta}$	OF_1	OF_2	T_1	T_{2a}	T_{2b}	$T_{2a}+T_{2b}$
1	(27,29), (1,4), 2, 500	f=2	f*=0	3600.0	1.3	5.7	7.0
2	(26,30), (1,4), 2, 500	f=10	f*=0	3600.0	1.0	1.1	2.1
3	(27,29), (1,4), 1, 500	f*=0	f*=0	1590.9	1.3	0.8	2.1
4	(26,30), (1,4), 1, 500	f*=0	f*=0	1432.7	1.0	1.2	2.2
5	(27,29), (1,4), 3, 500	f=8	f*=0	3600.0	1.3	74.0	75.3
6	(26,30), (1,4), 3, 500	f=12	f*=0	3600.0	1.0	154.7	155.7 **
7	(27,29), (2,5), 2, 500	NF	f*=0	3600.0	0.2	306.0	306.2
8	(26,30), (2,5), 2, 500	NF	f*=0	3600.0	0.9	81.5	82.4 **
9	(27,29), (2,5), 1, 500	f*=0	f*=0	1916.9	0.2	1.7	1.9
10	(26,30), (2,5), 1, 500	f*=0	f*=0	1469.6	0.9	27.3	28.2 **
11	(27,29), (1,4), 2, 300	f=8	f*=0	3600.0	1.5	11.2	12.7
12	(26,30), (1,4), 2, 300	f=2	f*=0	3600.0	1.4	10.3	11.7
13	(27,29), (1,4), 1, 300	f*=0	f*=0	1107.8	1.5	1.2	2.7
14	(26,30), (1,4), 1, 300	f*=0	f*=0	1291.5	1.4	1.1	2.5

Instance 1 is the 2007 season and the other 13 are variations on it in which the values of some parameters have been changed within a range conserving the practical relevance. As detailed in the second column of the table, the parameters that were altered are the minimum and maximum numbers of assignments for a referee, the minimum and maximum numbers of referee-team assignments, the minimum number of consecutive rounds between a referee's assignment to the same team, and the maximum difference between any two referees' average match travel distances. Since for these various upper and lower bounds $a, \bar{a}, n$ and $\bar{n}$, the same values are used for each referee, we have omitted the subindex r. A limit of 3600 seconds (1 hour) was imposed on each run.

As can be seen in column OF_1 of the table, the original model found the optimal solution $f^* = 0$ in only 6 of the 14 instances, with solution times (column T_1) from 1107.8 seconds in Instance 13 to 1916.9 seconds in Instance 9. In another 6 instances the model had generated solutions when the time limit was reached that were feasible but sub-optimal, while in Instances 7 and 8 it managed no feasible solution at all within the time limit.

The value of parameter c appears to play a critical role in these results. In all 6 instances where the model found the optimal solution, c was equal to 1. This is equivalent to omitting constraints (8). In the other 8 instances the parameter took a value of either 2 or 3 (values of $c \geq 4$ being of little practical interest). Although these two higher values increasingly reduce the number of constraints in the model by lowering the upper bound for k in (8), the ones that remain are more restrictive. This is immediately evident from the fact that if $c = \bar{c} + 1$, the sum on the left-hand side of (8) will include all the terms contained when $c = \bar{c}$ plus some additional ones. The resulting problem becomes more difficult to solve, increasing running times and negatively affecting the quality of the solutions, as is experienced in the 8 instances where the original model could not find the optimal solution within the time limit.

The pattern-based approach, on the other hand, performed much better than the traditional approach in the experimental instances, just as it did in the real ones. The time T_{2a} Model 2a required to reach the solution did not exceed 1.5 seconds while the time taken by Model 2b fluctuated between 0.8 and 306.0 seconds. Total solution time using patterns was thus dramatically less than that of the traditional approach in all instances. Furthermore, as is evident in column OF_2 of the table, Model 2 found an optimal solution $f^* = 0$ in every case.

Note that in instances 6, 8 and 10 (indicated by asterisks in the rightmost table column) the pattern set generated by Model 2a produced infeasibility in Model 2b. This was detected by the solver in less than 1 second. A second attempt was made to solve the instance in which the patterns of two referees were eliminated and the Model 1 constraints that had not been included in Model 2b were reimposed for these two referees, and an objective function with the sum of Δ_r for these two referees was reapplied. Under this procedure Model 2b found optimal solutions relatively quickly, with execution times ranging from 27.3 seconds for Instance 10 to 154.7 for Instance 6.

The superiority of the pattern-based approach compared to the traditional method lies in the significant reduction in the size of the solution space the solver needs to analyze and the smaller number of constraints that have to be satisfied. In the pattern-generation stage, the geographical zone each not-unassigned referee is assigned to is defined for each round, thus reducing the number of possible values that various of the $x_{r,m}$ variables may take in the actual assignment stage. As an example, assume 10 matches are played in the first round and 3 of them are located in the North zone. This implies that in the pattern-generation stage, only 3 of the 15 referees will be assigned to that zone in round 1, the remaining 12 being sent either to the Centre or the South zone (except, of course, those who are unassigned). When the assignment model is run with these patterns, only for 3 of the 15 referees in set R can variable $x_{r,m}$ take the value 1 for matches m played in the North zone in round 1. For the other 12 referees, $x_{r,m} = 0$ for those matches so that 36 variables will be 0.

The foregoing is analogous to the home-away pattern assignment in match calendar scheduling mentioned in Section 4.1. There, for a given round k–say, round 1–the home-away patterns set 10 teams at home and 10 away. The binary variables $w_{t,\hat{t},k}$ that can take the value 1 for a pair of teams t and $\hat{t}$ in that round are limited to those whose patterns differ, that is, patterns that indicate a home match for one of the teams and an away match for the other. If the two teams have patterns indicating that both play at home or both away, then $w_{t,\hat{t},k} = 0$ for $k = 1$.

As regards the constraints, since some of them are embodied in the pattern generation process they can be omitted in the assignment stage. For example, the maximum number of consecutive matches a referee can be unassigned, a required condition expressed by constraint (10) in Model 1, is assured by the patterns generated by Model 2a via constraint (25). Sim-

ilarly, in match calendar scheduling problems a typical condition is that no team play more than two consecutive rounds away. This would be achieved by the home-away pattern generation in the first stage.

Finally, we reiterate that although the patterns were constructed in our case to determine the referees' geographical zone assignments, they could well be defined on some other criteria. The pattern approach could thus be applied in contexts other than the Chilean league where very different issues would arise.

6. Discussion and conclusions

This article used integer linear programming for improving the assignment of referees to scheduled matches in the First Division of the Chilean professional football league. The assignments produced by existing manual procedures have various deficiencies, including major differences in the average distances referees must travel to matches, imbalances in the number of matches they officiate, and wide variations in the frequency they are assigned to matches involving a given team.

Two solution approaches were developed. The first is a traditional ILP approach that runs an assignment model directly; the second is a novel two-stage approach in which a first ILP model constructs referee patterns for the season and a second incorporates this information to generate the actual assignments.

The two approaches were tested on the real-world cases of the referee assignments in the First Division of the Chilean league for the years 2007 through 2010. They both delivered significant improvements over the actual manual assignments for those years on the criteria where deficiencies had been observed, while the pattern-based version also achieved major reductions in solution times over the traditional formulation. The models also simplify the assignment process and render it more transparent by establishing clearly defined decision criteria.

The models were used for First Division referee assignment on a trial basis in the 2010 season. For this purpose, a friendly interface was developed in Microsoft Excel so that they could be applied easily and directly as a tool by league officials. They also requested that the model be extended to handle referee assignment for the Under-17 and Under-18 youth leagues, where it was used successfully for much of 2010. However, due to changes in the governing body's referee committee over the last couple of years the

application of the referee assignment model was dropped. Unfortunately, as the Italian and Czech cases mentioned in the introduction, the Chilean football league has recently also been affected by referee scandals. In these scandals, the manual referee assignment has been one of the most criticised issues, as reported by official Cristián Basso to a main newspaper in Chile (Emol, 2012). Efforts of our research group are continuing to have the league employ the Operations Research approach on a permanent basis, as is the case with the match scheduling application which has been used every season since 2005 (Durán et al., 2007, 2012).

A series of useful extensions could be addressed in future work. One of these is the geographical aspect, which was included in the present formulation to improve the balance between the different referees' average travel distances. If the season calendar at a given point schedules a mid-week round followed immediately by a weekend round (e.g., Wednesday and then Saturday) or vice versa, better advantage could be taken of the extensive travel involved by assigning a referee to both rounds within one of the outlying zones, thus obviating the need to return to Santiago between matches. Moreover, when calculating referees' average travel distances in our real-world instances they all lived in Santiago, but if this were not the case the distances could perfectly well be calculated in terms of their corresponding zone of residence.

Note that although football matches require two assistant referees (linesmen) as well as the main referee, our model only assigns the latter. An obvious candidate for further development would therefore be to incorporate the assignment of these assistant referees into the specification.

Yet another valuable extension would be to formulate the model so that it integrates match scheduling and referee assignment in a single problem. Existing developments, including the present one, generate the referee assignment on the basis of a previously defined match calendar. This puts conditions on the setting of the assignment problem in that some of its constraints will be determined by the calendar scheduling. A simple example of this interaction in the case of our model would arise if league officials wanted two particular matches to be officiated by the same referee (implementable by constraint (14)). This would rule out scheduling the two games for the same round since referees are limited to officiating no more than one match in a single round by constraint (3).

The simultaneous generation of match schedules and referee assignments could also be pursued at a theoretical level by combining the Travelling Umpire Problem (Trick and Yildiz, 2007) with the Travelling Tournament Prob-

lem (Easton et al., 2001). This would provide a conceptual benchmark for integrated formulations of the two problems that till now have always been addressed separately.

As regards national team competitions, an interesting topic would be to analyze how often each country's squad is officiated by referees of a given nationality. Strong evidence of referees' national favouritism has been reported by Page and Page (2010), based on data of two major international competitions of rugby. An analysis by the present authors of the South American zone qualifying stages for the 2010 FIFA World Cup revealed that some teams were officiated relatively frequently by referees from certain countries while others were officiated by referees with a greater variety of national origins. Just as our model attempted to equilibrate the frequency of assignments of a given referee to a specific team (by constraints (6) and (7)), referee assignments by nationality in international tournaments could be similarly balanced.

Finally, greater use of sports scheduling techniques for referee assignment could reduce much of the controversy and criticism among referees, players, team officials, fans and the media that often surrounds the choice of referees for sporting events. As the above examples suggest, there are numerous opportunities for research in the application of sports scheduling techniques to referee assignment, but an immediate practical task is simply to achieve the adoption of referee assignment modelling by actual sports leagues. Though the use of OR techniques for match calendar scheduling is now widespread, their implementation for referee assignment apart from the cases mentioned in Section 1 is not yet firmly established.

Acknowledgements

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