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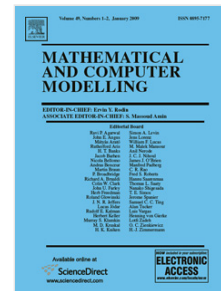Laura V. Pérez, Cristian H. de Angelo, Víctor Pereyra

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Determination of the Adjoint State Evolution for the Efficient Operation of a Hybrid Electric Vehicle

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Abstract

To minimize fuel consumption in hybrid electric vehicles it is necessary to define a strategy for the management of the power flows within the vehicle. Under the assumption that the velocity to be developed by the vehicle is known a priori, this problem may be posed as a non linear optimal control problem with control and state constraints. We find the solution to this problem using the optimality conditions given by Pontryagin Maximum Principle. This leads to boundary value problems that we solve using a software tool named PASVA4. On real time operation, the velocity to be developed by the vehicle is not known in advance. We show how the adjoint state obtained from the former problem may be used as a weighing factor, called "equivalent consumption". This weighing factor may be used to design suboptimal real time algorithms for power management.

Keywords: non linear constrained optimal control, Pontryagin Maximum Principle, boundary value problems solvers, hybrid electric vehicles, equivalent consumption minimization algorithms

1. Introduction

Hybrid Electric Vehicles (HEVs) are those where the power needed to drive the vehicle is provided by one or more electric motors fed by electro-chemical batteries and that in addition have on board an internal combustion

engine. The engine power not only contributes to the traction when necessary but also is used to recharge the batteries. Among the many advantages that these vehicles present concerning fuel economy and reduction of pollutant emissions, it is that of regenerative braking. That means that during braking, electric motors may change their operation mode, turn into generators and recover the kinetic energy accumulated on the vehicle by turning it into electrical energy that is sent back to the batteries. In order to achieve an efficient operation, a sophisticated control system that optimizes all the energy flows that take place within the vehicle must be defined. This energy management problem is usually referred to as the "supervisory control" and is usually stated as a high level control problem, whose commands have to be obeyed at a lower level by the controllers of the particular vehicle devices.

Many real-time supervisory control algorithms for HEVs are based in the so called "Equivalent Consumption Minimization Strategy" (ECMS) ([1], [2],[3]). This strategy consists of multiplying the power supplied by the batteries by a weighing factor in order to turn it into an equivalent power that can be added to the power supplied by the internal combustion engine. The interest of this approach raises from the fact that by means of this equivalence the problem of minimizing consumption during a whole mission (a global problem), may be turned into an instantaneous minimization of this weighted sum of powers, thus allowing its use in a real time algorithm. In this way, the optimal control problem reduces to determining this "equivalent consumption" factor ([1], [3], [4]).

However, this weighing factor varies according to the relative efficiency of the energy sources and to the features of the velocity cycle required of the vehicle. A great effort in current research is devoted to the determination of this parameter ([1], [2], [3]).

In a previous work [5] we formulated the supervisory control problem in HEVs as an optimal control problem with state and control constraints and derived the optimality conditions given by Pontryagin Maximum Principle ([6], [7], [8], [9], [10]). In this approach, it is observed that the equivalent consumption factor is related to the adjoint state of the optimal control problem [1] . Hence, by solving the optimality conditions and obtaining the evolution in time of the adjoint state it would be possible to obtain an estimation of the equivalent consumption factor. This is the purpose of this work.

It is worth noting that the numerical solution of these problems is not straightforward. Generally, the optimality conditions are differential-algebraic

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equations ([11], [12]). Sometimes, the algebraic equations may be solved independently and, after suitable replacements, the problem may be turned into an ordinary differential equations boundary value problem. It is then needed to use a solver for this kind of problems. In addition, in this particular problem, the control and state constraints introduce discontinuities in the solution and in the right hand side (RHS) of the differential equations. Moreover these discontinuities occur at unknown times, since they depend on the solution itself [5].

PASVA4 is a software tool able to solve non linear boundary value problems with discontinuities in the RHS and in the solution, even in unknown locations, multipoint boundary conditions and unknown algebraic parameters ([13],[14]). This tool has been successfully used for problems with the above difficulties that appear in geophysics, electronics, mechanics, etc. and hence, we will use it to solve this problem. We hope the same approach can be used for other optimal control problems with constraints.

In what follows we will first present the model we use (section 2) and the statement of the problem (section 3). Later, by taking a particular case, we will describe how we obtain the off-line solution. We start from two simpler formulations of the problem where the state constraints are not taken into account and then we show some results (sections 4 and 5). With these results we propose an algorithm to implement a primitive suboptimal power management strategy (section 6). Finally we mention some final comments and the work in progress in section 7.

2. Model

2.1. Power flows

In order to consider the supervisory control problem, a simplified scheme for the system that represents the vehicle ([16], [17], [15]) is normally used. In this scheme, the intermediate energy conversion devices of the power-train such as generators and electronic converters are replaced by the net power flow from each energy source (the fuel tank and the electrical storage system). We will use u to indicate the power flow at time t in the fuel tank/engine/generator path (which we shall call fuel-path henceforth), (see Figure 1). We establish the following convention: *a positive power flow means power flowing away from the sources towards the vehicle*. That means that a negative flow will take place in the electrical path during regenerative braking. Besides, the power flow from the fuel tank cannot be negative, as
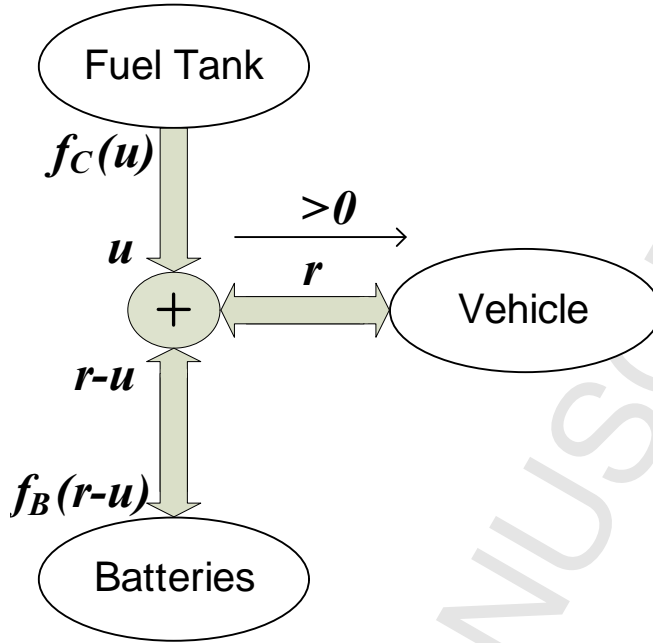
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Figure 1: Simplified scheme of a series hybrid electric vehicle.

it cannot absorb any power. The velocity profile to be performed by the vehicle is considered a given function. The required power is computed from this profile using a model of the inverse longitudinal dynamics of the vehicle and it will be denoted by $r(t)$ (in the real case, the required power is not known a priori since it depends on the transit and road conditions). In order to maintain drivability it is required that the sum of the power from both sources be equal to the required power at all times. Then, the energy flow from the electrical source has to be equal to $r(t) - u(t)$.

2.2. Energy from the sources

Regarding the net energy consumed from each source it must be taken into account that not all the power delivered by the sources can be actually used to supply the demand, since in the intermediate energy conversion processes there are losses. This fact will be represented by means of two functions f_C and f_B that depend on the power flows. We consider that these are known functions that in practice will be obtained by interpolation of a set of values determined by laboratory tests, setting the vehicle at different power

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operating points. This functions are normally increasing and non linear.

The fuel consumption during a time interval $[0, T]$, where T is known, is represented by the net energy consumed from the fuel source in the interval, i.e.:

$$\int_0^T f_C(u)dt. \tag{1}$$

Our control objective will be the minimization of this energy or equivalently, the maximization of the negative of this functional. To compute the net energy in the batteries it has to be considered that the effect of losses implies a power contribution from the batteries greater than that required during acceleration (i. e., for $r - u > 0$), but a power income lower than that produced by regenerative braking (i. e., for $r - u < 0$). This fact is represented by means of the function f_B. Then, the net energy in the batteries at time t is

$$x(t) = x_0 - \int_0^t f_B(r(s) - u(s))ds \tag{2}$$

where x_0 is the initial energy. From (2) we arrive at

$$\dot{x}(t) = -f_B(r(t) - u(t)). \tag{3}$$

This will be considered the state equation with initial condition x_0. The losses in this path will probably also depend on the state, increasing as it deviates from the nominal value, since it is clear that common batteries are less efficient as they get depleted and also when overcharged. So, we will modify the state equation with a new function f_B of the following form. Note that the state equation is non linear.

$$\dot{x}(t) = -f_B(x(t), r(t) - u(t)). \tag{4}$$

It is not necessary to impose a terminal condition to the state. It may be free, which means that it does not matter how much energy is there in the batteries at the end of the cycle (charge depleting operation mode). However, for brevity, we shall limit the exposition to the case of a fixed terminal state x_f. If it is equal to the initial state, it represents a "charge sustaining operation mode" of the vehicle.

2.3. Constraints

Clearly, the power flows are physically limited, hence:

$$0 \leq u(t) \leq u_{max} \tag{5}$$

and

$$K_{min} \leq r(t) - u(t) \leq K_{max}. \tag{6}$$

In addition, the bank of batteries has to be protected from depletion and from overcharge. This implies that the net energy in the electrical storage system has to be maintained between proper limits. Hence,

$$x_{min} \leq x(t) \leq x_{max}. \tag{7}$$

Summarizing, there are constraints on the control action and bounds on the state variable.

3. Optimal control problem statement

Find a piecewise continuous control u that maximizes

$$-\int_0^T f_C(u)dt \tag{8}$$

subject to

$$\dot{x} = -f_B(x(t), r(t) - u(t)) \ \forall t \in [0, T] \tag{9}$$
$$x(0) = x_0, \quad x(T) = x_T \tag{10}$$
$$\underline{U}(t) \leq u(t) \leq \bar{U}(t) \tag{11}$$
$$x_{min} \leq x(t) \leq x_{max} \tag{12}$$

where

$$\underline{U}(t) = max(0, r(t) - K_{max}), \quad \bar{U} = min(u_{max}, r(t) - K_{min}) \tag{13}$$

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and where K_{max}, K_{min}, u_{max}, x_{min} and x_{max} are known.

Much of the difficulty of the problem arises from the form of the function $r(t)$. For the case of our prototype, a neighborhood electric vehicle, it is interesting to consider the case where this function presents a highly non linear form since it follows from the train of successive accelerations and decelerations that such a vehicle must perform in any urban mission required of it. This required power function determines in turn the form of $\underline{U}$ and $\bar{U}$. We based our tests on the Normalized European Driving Schedule (NEDS), scaled to fit the power capabilities of our vehicle. A piece of this driving cycle corresponding to the urban section is shown in the top graph of figure 2. The middle graph of that figure shows the power that this vehicle requires to perform the above driving cycle. A negative power means that the vehicle is braking. The bottom graph shows the resulting functions $\underline{U}$ and $\bar{U}$. Note that the searched control function u must lie between this two limits.

To solve this problem using the optimality conditions given by Pontryagin Maximum Principle, it is necessary to define the corresponding Hamiltonian function which in this case is

$$H(t, x, \lambda, u) = -f_C(u(t)) - \lambda(t) f_B(x(t), r(t) - u(t)),$$

where λ is a new state variable, called the *adjoint* state or *co-state* whose dynamics is inherited from the fact that the state equation is a dynamical constraint. Later, the algebraic constraints will be taken into account in the so called *augmented Hamiltonian* or *Lagrangian*. The optimal solution is the function $u(t)$, $t \in [0, T]$, that maximizes H or equivalently minimizes $-H$, over the set of all functions satisfying the constraints. That means that the optimal u minimizes the sum of the power (including losses) that has to be supplied by the fuel-path plus the complementary power that has to be supplied by the electrical path times the adjoint state. Hence $\lambda(t)$ can be thought of as a weighing factor that scales the "cost" of using power from the electrical path against the "cost" of using power from the fuel path. If this "equivalent consumption factor" were known in advance we could compute the optimal control just by an instantaneous minimization of the Hamiltonian using only the current required power $r(t)$. Unfortunately, the optimality conditions must be solved globally, using the knowledge of the power requirements $r(t)$ in the whole interval $[0, T]$. Nevertheless, the computation off-line of $\lambda(t)$ over intervals with typical forms of $r(t)$ or over short time intervals, previous to the current one, may allow the design of
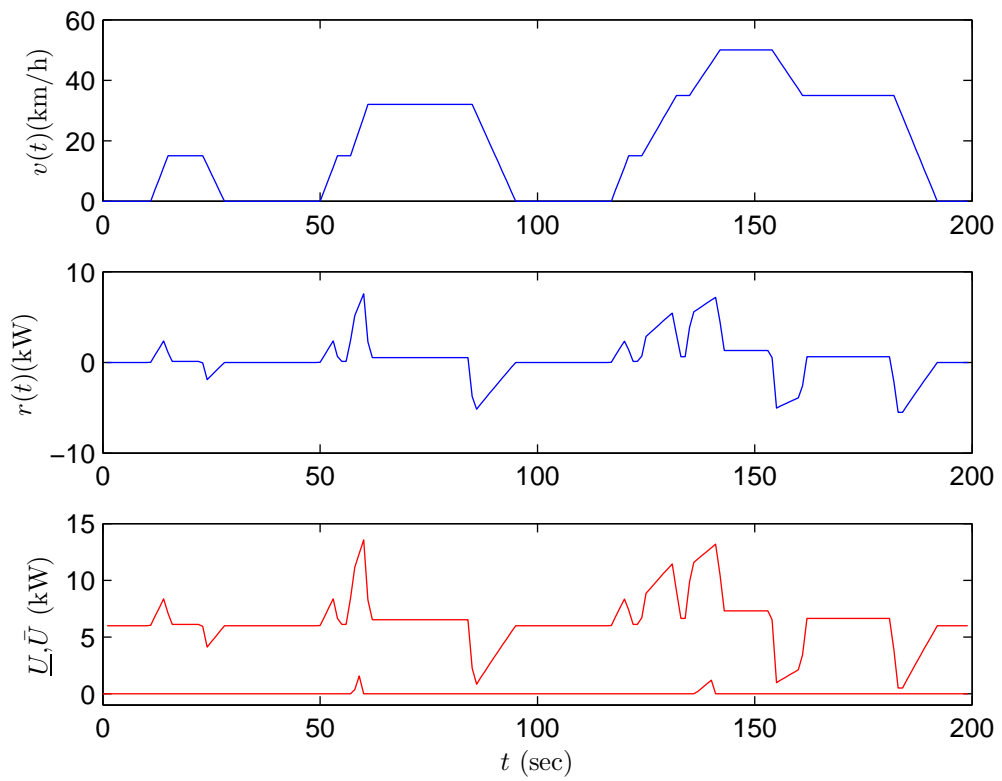
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Figure 2: Required velocity cycle, corresponding required power for our neighborhood electrical vehicle and upper and lower control limiting functions.

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algorithms to compute suboptimal control functions by an instantaneous optimization. This is our purpose.

To address this problem we begin by solving formulations of the problem that, although physically meaningful, are simpler to be treated. Then, as usual in constrained control, we will be progressively adding difficulties step by step. So, at this first step we will ignore the bounds on the state. In addition, we chose to approximate the functions f_C and f_B by quadratic polynomials in u, in order to obtain analytical expressions for the maximizing control. In the general case a numerical constrained maximization algorithm must be used to derive the control function.

4. Linear state equation case

To simplify the treatment of the above problem, we will additionally assume that $f_B(r - u) = r - u$. Note (see (2)) that this means that we are not taking into account the losses in the electrical path. Instead, all the power flow in the electrical path is supposed to enter or to go away from the electrical storage system. This approximation is somehow acceptable since electrical losses are much smaller than losses in the fuel path. Normally losses in the electrical path are less than 30% while those in the fuel-path are about 70%. Note also that this choice of f_B implies that the power flow from the batteries do not depend on their state of energy, represented by the state variable x. We also assume we can fit a quadratic polynomial to f_C. As our prototype has not yet an internal combustion engine, we used an hypothetical efficiency function. To design this function we took into account three typical features of internal combustion engines: 1) their energetic losses are greater than 60%; 2) the few losses occur at an operating point which is a bit to the left of the highest power that they can deliver; 3) losses increase at lower delivered power. These features constraint the set of possible quadratics. In the examples below we used $f_C(u) = a_C u^2 + b_C u + c_C$, $u \in [0, u_{max}]$, $u_{max} = 15kW$, $a_C = .0476$, $b_C = 1.7517$, $c_C = 3.2738$.

Under the previous assumptions, the problem is then the following:
Find a piecewise continuous control u that maximizes

$$- \int_0^T f_C(u) dt \tag{14}$$

subject to

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$$\dot{x} = -(r(t) - u(t)) \ \forall t \in [0, T] \tag{15}$$
$$x(0) = x_0, \quad x(T) = x_T \tag{16}$$
$$\underline{U}(t) \leq u(t) \leq \bar{U}(t). \tag{17}$$

4.1. Optimality conditions

To derive the optimality conditions we need to define the Hamiltonian or Lagrangian [6]

$$H(t, u, x, \lambda, \bar{\theta}, \underline{\theta}) = -(a_C u^2 + b_C u + c_C) - \lambda(r - u) + \tag{18}$$
$$+ \underline{\theta}(u - \underline{U}) + \bar{\theta}(\bar{U} - u).$$

where $\underline{\theta}(t)$ and $\bar{\theta}(t)$ are the Lagrange multipliers functions (see [7] for details). Then the optimality conditions are:

$$u = argmaxH \tag{19}$$
$$u \leq \bar{U} \quad \bar{\theta} \geq 0 \quad \bar{\theta}(\bar{U} - u) = 0 \tag{20}$$
$$\underline{U} \leq u \quad \underline{\theta} \geq 0 \quad \underline{\theta}(u - \underline{U}) = 0 \tag{21}$$

$$\dot{x} = -(r(t) - u(t)) \ \forall t \in [0, T] \tag{22}$$
$$\dot{\lambda} = 0$$
$$x(0) = x_0, \ x(T) = x_T.$$

4.2. Solving the problem

Using (19),(20),(21), we get:

$$u = \begin{cases} \underline{U} & if & \frac{\lambda - b_C}{2a_C} < \underline{U} \\ \frac{\lambda - b_C}{2a_C} & if & \underline{U} \leq \frac{\lambda - b_C}{2a_C} \leq \bar{U} \quad \triangleq sat(\frac{\lambda - b_C}{2a_C}, \underline{U}, \bar{U}) \\ \bar{U} & if & \frac{\lambda - b_C}{2a_C} \geq \bar{U}. \end{cases} \tag{23}$$

Then the ODE system to be solved is

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$$\dot{x} = -(r(t) - sat(\frac{\lambda - b_C}{2a_C}, \underline{U}, \bar{U})) \tag{24}$$
$$\dot{\lambda} = 0$$
$$x(0) = x_0, \ x(T) = x_T.$$

The RHS is piecewise defined according to the adjoint state values. This situation is typical of control constrained fuel optimal problems. Note that, as f_B does not depend on the state, λ results constant.

4.3. Solution

We set up this boundary value problem to be solved using PASVA4. This entails the computation of the Jacobian of the differential equation RHS with respect to the state and to the adjoint state variable and the Jacobian of the boundary values with respect to $x(0)$, $\lambda(0)$, $x(T)$ and $\lambda(T)$. In most cases, the solver could find the solution even if the starting point was not close to the true solution. Particularly, the solution could be found if the starting point was taken as the state trajectory corresponding to the case of pure electric operation mode, i.e., if $u(t) = 0$ for all t. Hence, this state trajectory, not necessarily a feasible solution but able to be computed a priori from the input data, can be taken as starting point for the state variable in all cases.

Concerning the starting point for the adjoint variable, independently of the starting point for the state and for several values for the starting point of the adjoint state, including 0, the solver converged to the same value for the adjoint state.

In figure 3 we show the results obtained for the case considered over a urban driving cycle. In the figures we used $K_{max} = 6kW$, $K_{min} = -6kW$, $T = 200sec$ and $x_0 = 400kWsec$. The first two graphs show respectively the control u that minimizes fuel consumption and the complementary power that has to be supplied by the batteries. The required power r was also included (dotted line) as a reference. In the bottom graph it is represented the corresponding state trajectory, i. e., the evolution of energy in the batteries. Note that in this example it was set $x_0 = x_T$. The objective value and the equivalence consumption factor λ are indicated.

5. Non linear state equation case

Next we solved the problem using $f_B(u) = a_B(r - u)^2 + (r - u)$. This function is fictitious since up to date we had only one experimental value of
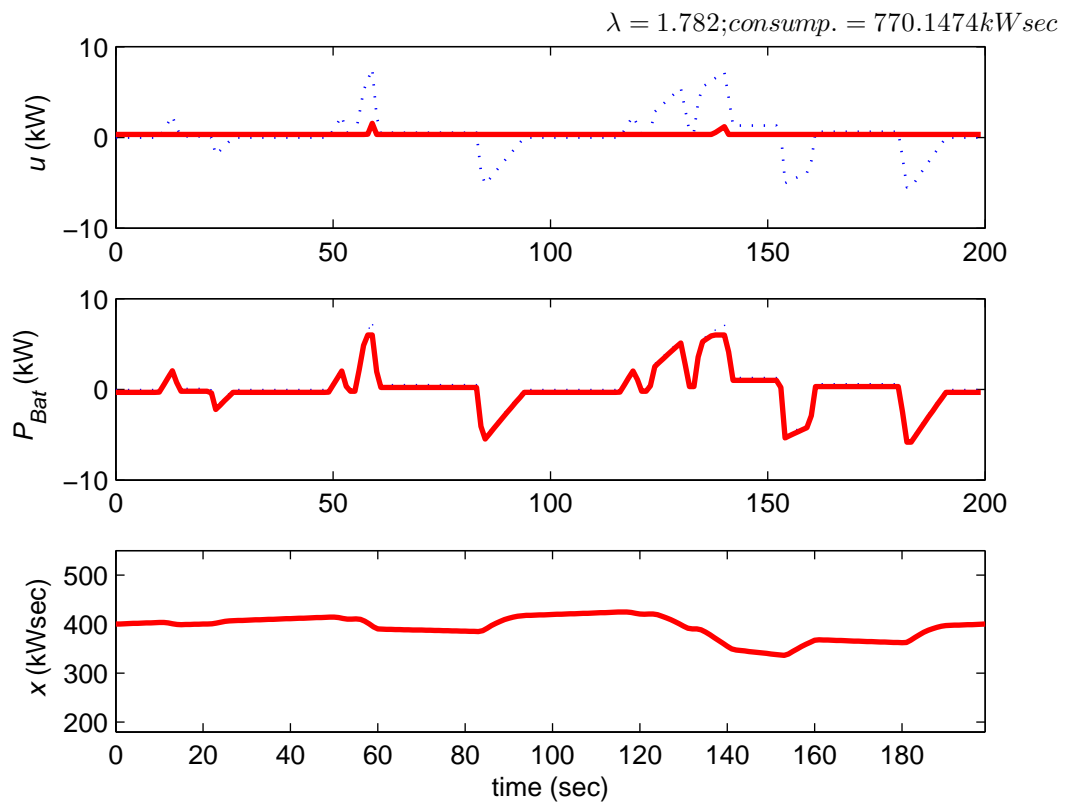
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$\lambda = 1.782; consump. = 770.1474 kWsec$

Figure 3: Results for the case where electrical losses are not considered; urban cycle.

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the efficiency of the electrical path of our vehicle, namely that corresponding to the electrical motors nominal power ($u = 3kW$). The choice of the quadratics fulfils two requirements in order to emulate a typical behaviour of these efficiency functions: a) $f_B(u)$ must be always greater than u, which means that if power is positive, a greater amount must be delivered to compensate for losses; if power is negative, that means regenerative braking, not all the mechanic energy will be sent back to the batteries, since in this case there also will be losses. Then, the absolute value of the power entering the batteries will be less than the originated by regenerative braking. b) It is needed that $f_B(0) = 0$. We set $a_B = 0.0684$

The corresponding expression for the Hamiltonian is now

$$
\begin{aligned}
H(t, u, x, \lambda, \bar{\theta}, \underline{\theta}) = & -(a_C u^2 + b_C u + c_C) - \\
& -\lambda(a_B(r - u)^2 + (r - u)) + \\
& +\underline{\theta}(u - \underline{U}) + \bar{\theta}(\bar{U} - u).
\end{aligned} \tag{25}
$$

The derivation of the maximizing control is analogous to what was done in the previous section, leading to the following optimality conditons

$$
u = argmax H \tag{26}
$$

$$
u \leq \bar{U} \quad \bar{\theta} \geq 0 \quad \bar{\theta}(\bar{U} - u) = 0 \tag{27}
$$

$$
\underline{U} \leq u \quad \underline{\theta} \geq 0 \quad \underline{\theta}(u - \underline{U}) = 0 \tag{28}
$$

$$
\begin{aligned}
\dot{x} & = -(a_B(r(t) - u(t))^2 + (r(t) - u(t)) \ \forall t \in [0, T] \tag{29} \\
\dot{\lambda} & = 0 \\
x(0) & = x_0, \ x(T) = x_T,
\end{aligned}
$$

where

$$
\begin{aligned}
u & = \begin{cases} \underline{U} & if \quad \frac{b_C - \lambda(2a_B r(t) + 1)}{2(a_C + a_B \lambda)} < \underline{U} \\ \frac{b_C - \lambda(2a_B r(t) + 1)}{2(a_C + a_B \lambda)} & if \quad \underline{U} \leq \frac{b_C - \lambda(2a_B r(t) + 1)}{2(a_C + a_B \lambda)} \leq \bar{U} \\ \bar{U} & if \quad \frac{b_C - \lambda(2a_B r(t) + 1)}{2(a_C + a_B \lambda)} \geq \bar{U} \end{cases} \tag{30} \\
& \triangleq sat\left(\frac{b_C - \lambda(2a_B r(t) + 1)}{2(a_C + a_B \lambda)}, \underline{U}, \bar{U}\right).
\end{aligned}
$$

5.1. Solution

To run PASVA4, we took as starting point the solution found for the previous formulation. The optimal power split appears in figure 4. Because of
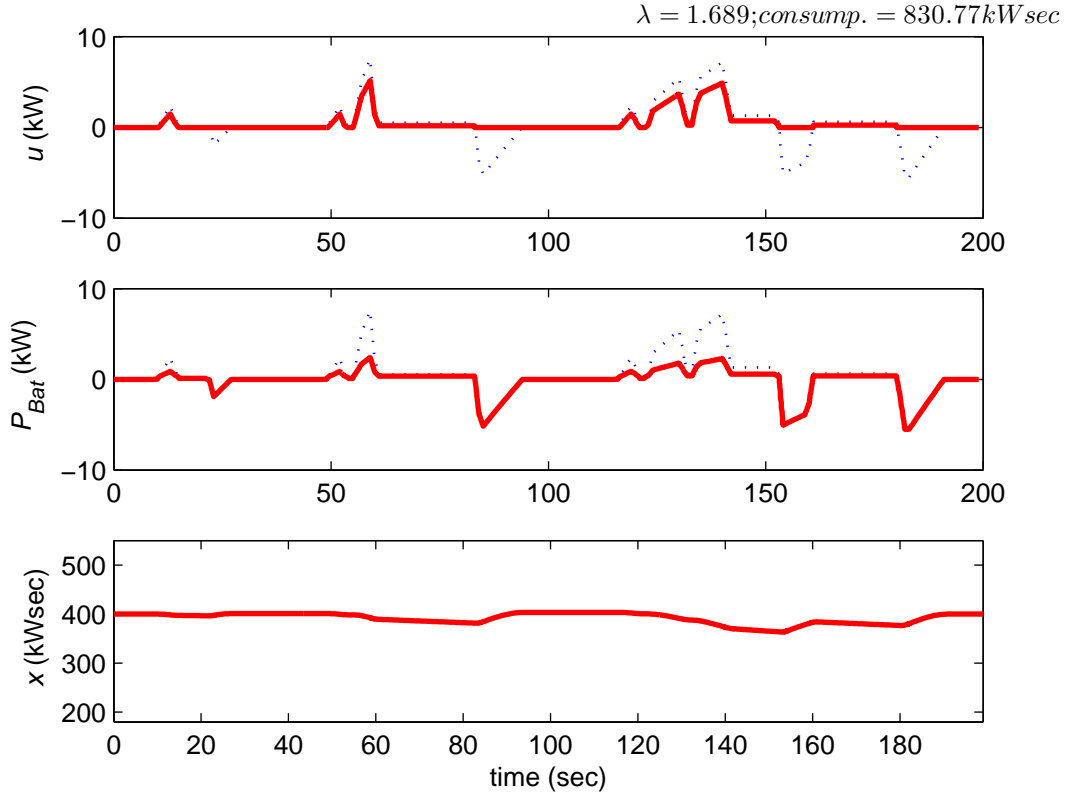
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$\lambda = 1.689; consump. = 830.77kWsec$

Figure 4: Results for the case where electrical losses are considered; urban cycle.

the addition of the electrical losses effect in the model, the power contribution from the fuel-path (top graph) increased with respect to the previous case and, consequently, consumption increased. Instead, the equivalent consumption factor decreased, which means that the cost of using the electrical-path is now comparatively closer to the cost of using the fuel-path. In the middle and bottom graphs, a minor contribution from the batteries is apparent.

6. Example of a real time algorithm

After a number of runs, we observed that λ changes according to

1. the relative size of x_T with respect to x_0,
2. the shape of $r(t)$,

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3. the length of the time interval T,

4. the shape of f_C,

5. the shape of f_B.

Concerning 1) it is found that λ decreases as the difference $x_T - x_0$ decreases. Table 1 shows the values found for the cycle in figures 4 and 3 for different values of that difference.

Table 1

$x_T - x_0$	40 kWsec	0kWsec	-20 kWsec
λ	1.774	1.691	1.631

Concerning 2), if the vehicle is to fulfil a driving cycle where the speed remains constant for a longer period of time, e.g., a path on an avenue or higher speed road, the corresponding required power will be lower, since no power for acceleration or deceleration is needed. Only the necessary power to compensate for the losses in the system will be required in constant speed intervals. In figure 5 such a situation is depicted. It should be expected that the value for λ is greater in this case, since in the absence of regenerative braking situations, the cost of using the electrical path should increase. We found $\lambda = 1.885$ against $\lambda = 1.691$ obtained for a urban path of similar features (equal time length and similar power peaks).

Concerning the remaining aspects, the way λ changes is far from being intuitive. This is why any on-line algorithm must update permanently its value. There are many possibilities to design suboptimal real time algorithms. For instance, we now propose the following simple algorithm to be used on-line. Basically it consists in updating λ each 200 seconds, using as required power the data of the previous 200 seconds and imposing a terminal condition equal to the initial condition in each subinterval.

6.1. Real time algorithm

    Array $r[1...200]$

    Initialize λ

    For $INTERVAL = 1, NINTERVAL$

        For $t = 1, ..., 200$

            $s = t + INTERVAL * 200$

            read $r[s]$

            Compute $u^* = argmin_u(f_C(u) + \lambda * f_B(r[s] - u))$

        end for

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Compute λ using $r[1, ..., 200]$
end for

Application of this algorithm to a modification of the Normalized European Driving Schedule is shown in figure 6. This cycle presents an urban interval [0 sec,800 sec] followed by an interval [800 sec,1200 sec] at approximately constant speed, i. e., without accelerations and decelerations. It was chosen to highlight the behaviour of the algorithm with respect to changes in the driving mode. Figure 6 shows the minimizing control obtained by solving the optimality conditions in the whole interval setting $x_T = x_0$ (top graph) and suboptimal controls given by the algorithm when λ initial was simply set equal to 0 (middle graph) and when a better initial guess for λ is used (bottom graph). It can be seen that for this driving cycle the only significant differences among the resulting control functions appear in the first interval, due to the arbitrary choice of initial λ, and in the last intervals, due to the changes in the driving mode. Fuel consumption and electrical energy consumed from the batteries are also indicated in the graphs. Note that fuel consumption was smaller for the suboptimal controls, but at the expenses of a positive energy consumption from the batteries.

7. Discussion and conclusions

In the above sections we computed the equivalent consumption factor λ under the condition that the final state was equal to the initial state. As it can be seen in the examples of figures 4 and 5, for many driving cycles this condition is sufficient to ensure that the state trajectory does not deviate very much from that initial value and so the bounds for the state are not reached during the whole cycle. This particularly occurs in algorithms like the one presented, where the control problem is to be solved in a short time window. Moreover, the final state value is a parameter of the algorithm that may be modified along the iterative process in order to force a stable trajectory.

Most authors do not take into account the bounds on the state. Usually some function of the difference $x(T) - x(0)$ is added as a penalty term to the objective to penalize battery use (see [1]) and obtain a state trajectory that remains within bounds. This approach led to optimal control problems that can also be solved in a similar form using PASVA4. For example, we tested the case where the penalty term was $\frac{1}{2}(x(T) - x(0))^2$, which leads to a boundary value problem with mixed boundary conditions. Results from this

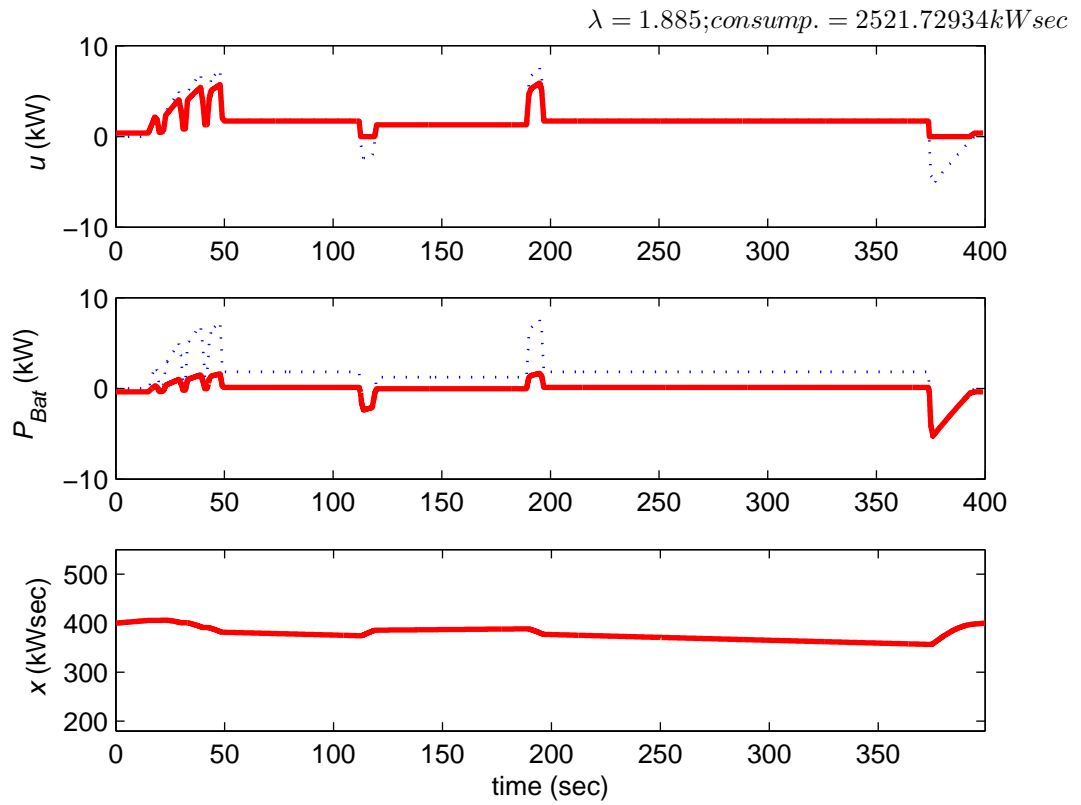17

$$\lambda = 1.885; consump. = 2521.72934 kW sec$$

Figure 5: Results for the case where electrical losses are considered; almost constant speed cycle.
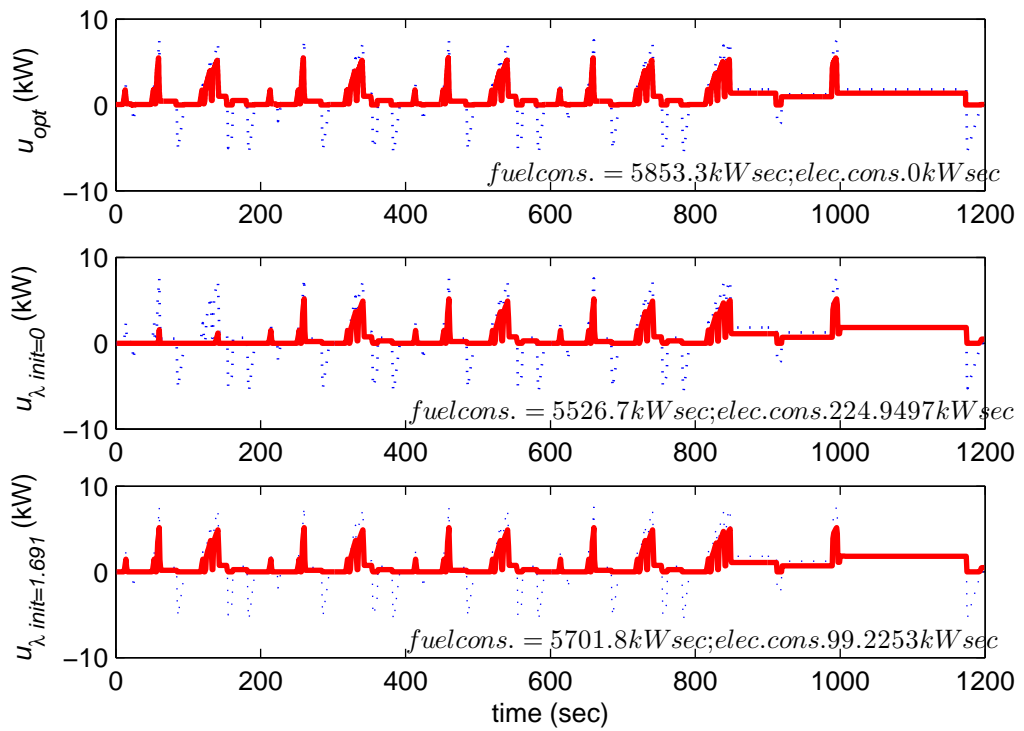
Figure 6: Comparison of the optimal control function with suboptimal control functions computed using a simple equivalent consumption minimization algorithm

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formulation were quite similar to those presented here. The case where the penalty term was $|x(T) - x(0)|$ leads to two different values for λ depending of the sign that is imposed to that difference.

Nevertheless, there may be driving cycles for which including the state constraints in the form (12) is necessary to ensure that the trajectory will remain within the bounds. It is in that case where the facilities of PASVA4 will be most useful. It can be seen that the boundary value problem that results, has switchings in the RHS, that occur at unknown locations. These locations are determined by the junction points, where the state trajectory enters or leaves the binding interval. We are currently working on this problem where these junctions points are also estimated. Tests made by using PASVA4 for the classical linear time invariant fuel optimal state constrained control problem, in which analytical solutions can be obtained and used as a reference, were successful and will be included in detail in a future publication.

Our future work will also be concerned with the use of numerical algorithms to obtain the control from the maximization of the Hamiltonian, without the need of the assumption that a quadratic polynomial may be fit to the experimental efficiency functions. The maximization must be fast enough to allow the whole algorithm to run in real time.

Summarizing, altough this work is still in progress, we think we have done a first step in the design of a real time control strategy for optimizing power management in HEVs. In addition, we gained experience in the numerical solution of constrained control problems, by means of solving the boundary value problem that results from the statement of Pontryagin Maximum Principle optimality conditions.

8. Acknowledgments

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