SAPIENZA UNIVERSITÀ DEGLI STUDI DI ROMA

Doctoral School of Economics

MODELLI PER L'ECONOMIA E LA FINANZA

Fewer is better: the cases of portfolio selection and of operational risk management

Supervisor:

Prof. Fabio Tardella

Candidate

Jacopo Moretti

Academic Year 2016-2017

Alla mia Famiglia, che ha sempre saputo ascoltare senza che io parlassi.

A Francesco, per tutte le volte che mi ha sopportato e perchè il suo entusiasmo per la ricerca mi ha spinto a intraprendere questa bellissima esperienza.

A Marco e Lorenzo, per tutti gli anni e i momenti importanti condivisi insieme.

A Matteo e Enrico, che sono sempre stati, indistintamente, in tutti i momenti al mio fianco.

A Virgilio, Carlo e Andrea, che hanno sempre creduto in me e che mi hanno sempre spinto a migliorare sotto il profilo professionale e umano.

A chi vorrà condividere con me il momento in cui sarà concluso questo lavoro.

Contents

1 Optimally chosen small portfolios are better than large ones **1**

1.1 Introduction 2

1.2 The portfolio models 8

1.2.1 The Equally-Weighted portfolio 9

1.2.2 Fixed-Size Minimum Variance Equally-Weighted portfolios 10

1.2.3 Fixed-Size Minimum Variance portfolios . . 12

1.2.4 Fixed-Size Minimum CVaR portfolios . . . 13

1.2.5 Fixed-Size Minimum Semi-MAD portfolios 15

1.3 Empirical behavior of the models 16

1.3.1 In-sample analysis 19

1.3.2 Out-of-sample analysis 28

1.4 Conclusions 40

1.5 Further research 42

2 Operational risk assessment of a new product using AHP 45

2.1 Introduction 47

2.2 Risk assessment for a new product 52

2.3 The Analytic Hierarchy Process 56

2.3.1 Assessing operational risks of a new product with AHP 58

2.3.2 80/20 rule 69

2.4 Optimally choosing the mitigation actions of the intervention plan 72

2.4.1 Optimally choosing a subset of mitigation actions 75

2.5 Case study 78

2.6 Conclusions 87

Abstract

This thesis aims to show that in some applications the appropriate selection of a small number of available items can be beneficial with respect to the use of all available items.

In particular, we focus on portfolio selection and on operational risk management and we use operations research techniques to identify the few important elements that are needed in both cases.

In the first part of this work - based on an article published in Economics Bulletin [Cesarone et al (2016)], we show that, for several portfolio selection models, the best portfolio which uses only a limited number of assets has in-sample performance very close to that of an optimized portfolio which could include all assets, but generally obtains better out-of-sample performance. This is true for various performance measures, and it is often possible to identify a "golden range" of sizes where the best performances are obtained. These general empirical findings are consistent with theoretical results obtained by Kondor and Nagy (2007) under

very restrictive assumptions. We also note that small portfolios are preferable for several practical reasons including monitoring, availability for small investors, and transaction costs.

In the second part of the thesis, we develop an operational risk management framework for the assessment of the exposure of a company (with particular reference to a financial institution) to potential risk events arising from the launch of a new product. This framework is based on the Analytic Hierarchy Process and on the 80/20 rule which allows one to rank and to identify the most relevant risk events, respectively.

By means of appropriate integer programming models we then address the problem of identifying the mitigation actions that secure the internal processes of a company with minimum cost. This corresponds to the primary goal of an operational risk manager: reducing the exposure to potential risk events. An alternative approach, when the budget is fixed, consists in selecting the subset of mitigation actions that provide the greatest reduction in operational risk exposure for that budget. A parametric analysis with

respect to the budget level provides additional information for the management to take decisions about possible budget adjustments.

Keywords: Asset Management; Risk Diversification; Size Constraints; Small Portfolios; Analytic Hierarchy Process; New product; Operational Risk Assessment; 80/20 rule.

Chapter 1

Optimally chosen small portfolios are better than large ones

One of the fundamental principles in portfolio selection models is minimization of risk through diversification of the investment. However, this principle does not necessarily translate into a request for investing in all the assets of the investment universe. Indeed, following a line of research started by Evans and Archer almost fifty years ago, we provide here further evidence that small portfolios are sufficient to achieve almost optimal in-sample risk reduction with respect to variance and to some other popular risk measures, and very good out-of-sample performances.

While leading to similar results, our approach is significantly different from the classical one pioneered by Evans and Archer. Indeed, we describe models for choosing the portfolio of a prescribed size with the smallest possible risk, as opposed to the random portfolio choice investigated in most of the previous works. We find that the smallest risk portfolios generally require no more than 15 assets. Furthermore, it is almost always possible to find portfolios that are just 1% more risky than the smallest risk portfolios and contain no more than 10 assets. Furthermore, the optimal small portfolios generally show a better performance than the optimal large ones.

Our empirical analysis is based on some new and on some publicly available benchmark data sets often used in the literature.

1.1 Introduction

Since the start of Modern Portfolio Theory with the seminal Mean-Variance (MV) model of Markowitz (1952, 1959), the main aim of portfolio selection models was that of reducing the risk of an investment in the stock market through diversification while trying to achieve a satisfactory return. However, Markowitz also realized that, due to high correlations in the stock market, the benefit of diversification would rapidly decline with the size of the portfolio. In his fundamental book Markowitz (1959) he observed that: "To understand the general properties of large portfolios we must consider the averaging together of large numbers of highly correlated outcomes. We find that diversification is much less powerful in this case. Only a limited reduction in variability can be achieved by increasing the number of securities in a portfolio."

The first empirical evidence of the sufficiency of small portfolios to achieve almost complete elimination of the diversifiable risk in a market is probably due to a very influential work by Evans and Archer (1968) where, for any given size K from 1 to 40, they randomly picked subsets of K assets from a market of 470 securities and computed some statistics on the standard deviations of the Equally-Weighted portfolios formed with each subset of assets. They found that the *average* standard deviation for each size K

was decreasing and rapidly converging to an asymptote and they concluded that no more than around 10 assets were needed to almost completely eliminate the unsystematic variation of a portfolio return.

Thenceforth, several authors contributed to the debate about the right size of a portfolio that almost completely eliminates the diversifiable risk in a market (see, e.g., Newbould and Poon (1993) and references therein). Furthermore, based on Evans and Archer's and on other similar findings, such *magic* size, or size range, has been recommended in several textbooks on investment management and on corporate finance, as reported by Tang (2004).

There are several reasons for preferring small portfolios to large portfolios. The first and more obvious one concerns the infeasibility of holding large portfolios for small investors. However, even big investors should consider the opportunity cost of holding large portfolios and should identify the threshold where the costs exceed the benefit of risk reduction. Statman (1987) identifies such costs with the transaction costs and, using the cost of holding an index fund that replicates the market as a proxy, finds a threshold around 30-40 assets. Furthermore, there are other sources of cost that depend on the size such as those for monitoring the behavior and fundamentals of all the companies involved in the portfolio. Another important advantage of small portfolios seems to be that of reducing the estimation errors for variances and covariances thus leading to better out of sample performance (see, e.g., Cesarone et al (2014); DeMiguel et al (2009a)).

In this work we provide further evidence of the benefits of small portfolios both in terms of in-sample risk reduction and in terms of out-of-sample performance. However, our approach is significantly different from the mainstream approach pioneered by Evans and Archer. Indeed, we overcome one of the main weaknesses of their approach which consists in stating results that are valid only on average. In other words, if one picks an arbitrary Equally-Weighted portfolio of a given size in a market, there is no guarantee that its risk will not be much larger than the average risk of all portfolios of the same size in that market.

The conceptually simple solution that we propose here is just to choose the *best* Equally-Weighted portfolio for each given size with respect to variance, and, furthermore, the *optimal* portfolios for each given size with respect to three different and complementary risk measures. In this way for each size we clearly obtain a portfolio which has a risk not greater (and typically quite smaller) than the average risk. The reason why this simple idea was not investigated before is probably due to the computational hardness of the models required to find such best portfolios. Indeed, some of these models have been solved exactly only recently for small to medium size markets (see, e.g., Angelelli et al (2008); Cesarone et al (2015) and references therein), and one model is solved here for the first time. Once we have obtained the optimal size of the minimum risk portfolio, we proceed with a sensitivity analysis that allows us to find the smallest size of a portfolio whose risk is not more than 1% larger than that of the minimum risk portfolio, thus finding even smaller portfolios with satisfactory risk level.

Another difference between our approach and the one of Evans and Archer consists in the possibility of using general weights in the selected portfolio instead of equal weights only. For each portfolio size, this clearly allows one to find portfolio with even lower in-sample risk. However, since optimizing weights might also cause the maximization of estimation errors DeMiguel et al (2009b); Michaud (1989), this choice does not necessarily implies better out-of-sample performance. For both weighting schemes and for all risk measures we find results comparable to those of Evans and Archer. More precisely, we identify some ranges of (typically small) sizes where the portfolio risks are minimized and ranges of even smaller sizes where the portfolio risks do not exceed the minimum by more than 1%. The out-of-sample performance of the selected portfolios for each specified size is another important feature of our analysis which is rarely found in previous works on the subject. Also in this case we find that the best performances are generally obtained by portfolios with no more than 15 assets.

As an interesting complement to our findings, we mention that, in a recent and detailed analysis on the empirical behavior of investors and on the performance of their portfolios, Ivković et al (2008) show that portfolios of small investors with low diversification exhibit superior performance with respect to the ones with high diversification.

1.2 The portfolio models

In this section we describe the models analyzed and we provide an integer or a mixed-integer linear or quadratic formulation for all models. We first need to introduce some notation. Let $T+1$ be the length of the in-sample period used to estimate the inputs for the models. We use p_{it} to denote the price of the i-th asset at time t, with $t = 0, ..., T$; $r_{it} = \dfrac{p_{it} - p_{i(t-1)}}{p_{i(t-1)}}$ is the i-th asset return at time t, with $t = 1, ..., T$; x is the vector whose components x_i are the fractions of a given capital invested in asset i in the portfolio we are selecting; y is a boolean vector whose components y_i are equal to 1 if asset i is selected, and 0 otherwise. We assume that n assets are available in a market and, adopting linear returns, we have that $R_t(x) = \sum_{i=1}^{n} x_i r_{it}$ is the portfolio return at time t, with $t = 1, ..., T$. The n-dimensional vector μ is used to denote the expected returns of the n risky asset, while Σ denotes their covariance matrix, and u denotes an n-dimensional vector of ones.

1.2.1 The Equally-Weighted portfolio

The most intuitive way to diversify a portfolio is to equally distribute the capital among all stocks available in the market. In terms of relative weights we have $x_i = 1/n$. This is known as the Equally-Weighted (also called naïve or uniform) portfolio. Clearly the choice of the Equally-Weighted (EW) portfolio does not use any in-sample information nor involve any optimization approach. However, some authors claim that its practical out-of-sample performance is hard to beat on real-world data sets DeMiguel et al (2009b). Furthermore, from the theoretical viewpoint, Pflug et al (2012) show that when increasing the amount of portfolio model uncertainty, i.e., the degree of ambiguity on the distribution of the assets returns, the optimal portfolio converges to the EW portfolio. We will thus use this portfolio as a benchmark to compare the performances of the portfolios obtained by the models.

1.2.2 Fixed-Size Minimum Variance Equally-Weighted portfolios

As already observed, the EW portfolio is the most robust choice when there is a great uncertainty about the distribution of the asset returns. However, the EW portfolio has the drawback of using all available assets, which might be too numerous and not all desirable. A first proposal to overcome this drawback is due to Jacob (1974), who proposes to select a small EW portfolio (with a specified number K of assets) that has minimum variance among all EW portfolios of the same size. The model by Jacob is a nonlinear 0-1 optimization model that has not yet been tested in practice due to its computational complexity. Thanks to the recent advances in solution methods and computing power, we can propose here an empirical study of such Fixed-Size Minimum Variance Equally-Weighted (FSMVEW) model formally described below.

$$
\begin{aligned}
\min \quad & y^T \Sigma y \\
s.t. \quad & u^T y = K \\
& y \in \{0,1\}^n
\end{aligned}
\tag{1.1}
$$

This is probably the simplest Fixed-Size portfolio model and has the advantage of not requiring the problematic estimates of the assets expected returns. Furthermore, the effects of the possible estimation errors of the covariance matrix Σ do not result in very large or small weights for some assets, but only influence the choice of the subset of selected assets in the portfolio. From the optimization viewpoint, it falls into the class of pseudoBoolean Quadratic Programming problems which are known to be theoretically hard to solve in the worst case (NP-hard) Boros and Hammer (2002). However, due to its special structure, practical problems of this type with several hundreds variables can be actually solved fairly efficiently with available free or commercial codes.

Note that the vector x of weights of the optimal FSMVEW portfolio selected by model (1.1) is obtained as $x = \frac{1}{K}y$. When $K = n$ the FSMVEW portfolio coincides with the EW portfolio.

1.2.3 Fixed-Size Minimum Variance portfolios

Another model that does not require the estimates of the assets expected returns is the extreme case of the Markowitz model where we only seek to minimize variance. Within our framework we thus consider the following Fixed-Size Minimum Variance (FSMV) model where only K assets are allowed in the selected portfolio

$$\begin{aligned} \min \quad & x^T \Sigma x \\ s.t. \quad & u^T x = 1 \\ & u^T y = K \\ & \ell y \leq x \leq y, \\ & y \in \{0,1\}^n \end{aligned} \tag{1.2}$$

The first constraint above is the budget constraint; the second one represents the portfolio fixed-size constraint; u is an n-dimensional vector of ones; y is an n-dimensional vector of binary variables used to select the assets to be included in the portfolio; x is the vector of portfolio weights, and ℓ is a minimum threshold (often called buy-in threshold) for the weights of the selected assets which must be greater than zero (in our experiments we chose $\ell = 0.01$).

Without these thresholds, Problem (1.2) could generate portfolios with less than K assets, which is equivalent to replacing the constraint $u^T y = K$ with $u^T y \leq K$. Note that Problem (1.2) is a Quadratic Mixed Integer Programming (QMIP) problem that falls again in the class of NP-hard problems. However, also in this case problems with a few hundred variables can be solved fairly efficiently with available free or commercial codes. Furthermore, a recently proposed Cesarone et al (2009, 2013) specialized algorithm can solve problems of this type with up to two thousand variables.

1.2.4 Fixed-Size Minimum CVaR portfolios

The Fixed-Size Minimum CVaR (FSMCVaR) model is a minimum risk model like the previous one, but instead of variance it measures risk with Conditional Value-at-Risk at a specified confidence level ε ($CVaR_\varepsilon$), namely the average of losses in the worst $100\varepsilon\%$ of the cases Acerbi and Tasche (2002). In our analysis losses are defined as negative outcomes, and we set ε equal to 0.05. The

FSMCVaR model can be written as follows:

$$
\begin{aligned}
\min \quad & CVaR_\varepsilon(x) \\
s.t. \quad & u^T x = 1 \\
& u^T y = K \\
& \ell y \leq x \leq y, \\
& y \in \{0,1\}^n
\end{aligned}
\tag{1.3}
$$

where ℓ plays the same role as in (1.2).

Using a classical approach introduced by Rockafellar and Uryasev (2000) (see also Cesarone et al (2014)), Problem (1.3) can be reformulated as a Mixed Integer Linear Programming (MILP) problem with $n + T + 1$ continuous variables, n binary variables and $T + n + 3$ constraints. Some recent computational experiences reported in Cesarone et al (2015) on the solution of this model with state-of-the-art commercial solvers show that models with more than a few hundreds variables are hard to solve with general purpose solvers and would probably benefit from more specialized methods.

1.2.5 Fixed-Size Minimum Semi-MAD portfolios

The last risk measure that we take into account in our analysis is the downside Mean Semi-Absolute Deviation (Semi-MAD):

$$SMAD(x) = E[\min(0, \sum_{i=1}^{n}(r_{it} - \mu_i)x_i)], \tag{1.4}$$

This is a concise version of the more famous Mean Absolute Deviation (MAD) risk measure, which is defined as the expected value of the absolute deviation of the portfolio return from its mean Konno and Yamazaki (1991). Indeed, Speranza (1993) showed that Semi-MAD leads to a portfolio selection model that is equivalent to the MAD model, but with half the number of constraints. We thus consider the following Fixed-Size Minimum Semi-MAD (FSMSMAD) model

$$\begin{aligned} \min \quad & SMAD(x) \\ s.t. \quad & u^T x = 1 \\ & u^T y = K \\ & \ell y \leq x \leq y, \\ & y \in \{0,1\}^n \end{aligned} \tag{1.5}$$

where ℓ plays the same role as in (1.2). Using the linearization approach described in Speranza (1993), we can reformulate this problem as a MILP problem with $n+T$ continuous variables, n binary variables and $n+T+3$ constraints (see Cesarone et al (2014)). From the computational experiences reported in Cesarone et al (2015) it appears that also this model, although slightly easier than the previous one, cannot easily be solved with general purpose state-of-the-art solvers when more than a few hundreds variables are involved.

1.3 Empirical behavior of the models

In this section we test the models described above on some publicly available data sets.

The analysis consists of two parts. First, we examine the behavior of the portfolios selected by the models on the in-sample window where we obtain the input parameters of the models. The second part consists in evaluating the out-of-sample performance of the portfolios, which is the aspect that matters most to investors.

Since the markets are in continuous evolution, it seems appropriate to rebalance the portfolio from time to time in order to take new information into account. For this purpose, we use a Rolling Time Window procedure (RTW), i.e., we shift the in-sample window (and consequently the out-of-sample window) all over the time length of each data set. More specifically, we consider a time window (in-sample period) of 200 observations for the data sets with weekly frequency, and of 120 observations for the data sets with monthly frequency. The choice of the lengths of the in-sample and of the out-of-sample windows is based on typical settings of portfolio selection problems (see, e.g.,Bruni et al (2012, 2013); Cesarone et al (2015); DeMiguel et al (2009a)). Then we solve the selection problem for overlapping windows built by moving forward in time with step size 4 (for the weekly data sets) or 1 (for the monthly data sets). The optimal portfolio found w.r.t. an in-sample period is held for the following 4 weeks (out-of-sample period of the weekly data sets) or 1 month (out-of-sample period of the monthly data sets).

The out-of-sample performances of the resulting portfolios are evaluated in different ways by computing some performance measures commonly used in the literature Rachev et al (2008). Let $x^* = (x_1^*, \ldots, x_n^*)$ denote the allocation of the selected portfolio and $r_t = (r_{1t}, \ldots, r_{nt})$ denote the assets returns at time t. Then, in our analysis we consider:

- the **Standard Deviation** of the selected portfolio return;
- the **Sharpe Ratio** as $\frac{E[x^* r_t' - r_f]}{Std[x^* r_t' - r_f]}$, where $r_f = 0$;
- the **Rachev Ratio** as $\frac{CVaR_\alpha[r_f - x^* r_t']}{CVaR_\beta[x^* r_t' - r_f]}$, where $r_f = 0$ and $\alpha = \beta = 0.1$;
- the **Max Drawdown** as $-\min x^* r_t'$ which is the maximum loss achieved by a portfolio during the holding period.

In our analysis we use six data sets, summarized in Table 1.1. The monthly data sets (FF25, 48Ind, 100Ind) are taken from Ken French's website[1]. The weekly data sets (Stoxx50, FtseMib,

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

	Data set	♯ of assets	time interval	frequency	source
1	FF25	25	07/1963-12/2004	monthly	K. French
2	48Ind	48	07/1963-12/2004	monthly	K. French
3	100Ind	100	01/1969-12/2011	monthly	K. French
4	Stoxx50	32	01/2007-05/2013	weekly	Yahoo Finance
5	FtseMib	34	01/2007-05/2013	weekly	Yahoo Finance
6	Ftse100	63	01/2007-05/2013	weekly	Yahoo Finance

Table 1.1: List of data sets analyzed.

Ftse100) are downloaded from *http://finance.yahoo.com*, and are publicly available at

http://host.uniroma3.it/docenti/cesarone/DataSets.htm.

1.3.1 In-sample analysis

For each model described in Section 2.2 we study the behavior of its optimal value (minimum risk) when varying the number K of assets in the portfolio.

One of our main empirical findings is the scarce effect of diversification in terms of risk reduction when the portfolio size K does not belong to a certain range of values. Indeed, in all analyzed markets, we find that the risk measures, representing the objective

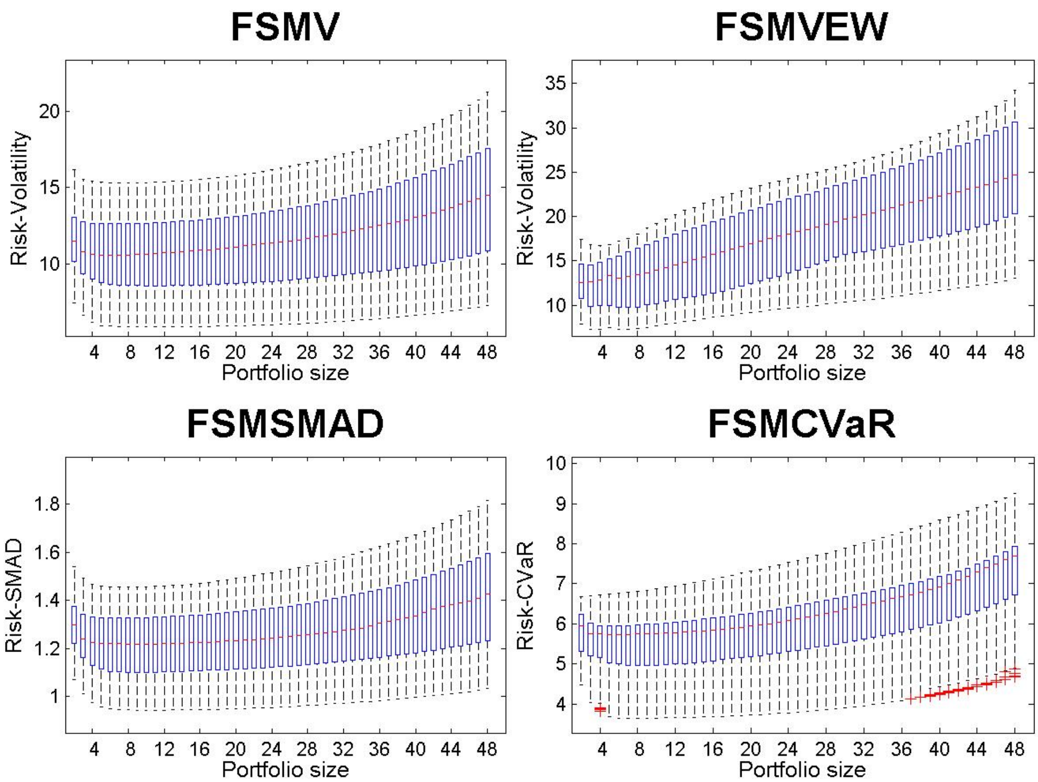


Figure 1.1: Boxplot of the in-sample risk w.r.t. the portfolio size for 48Ind.

functions of the models, achieve minimum values for a range of portfolio sizes corresponding to a significantly limited number of assets w.r.t. the total. Furthermore, these risk measures tend to increase when increasing the portfolio size, thus contrasting the paradigm that the larger the diversification, the lower the risk. In Figs. 1.1 and 1.2 we report some empirical evidences of this phenomenon for monthly (48Ind) and for weekly (Stoxx50) data sets. However, this behavior is similar for each data set analyzed. Fig. 1.1 exhibits the boxplots of the different risk measures w.r.t. all considered in-sample windows by varying the portfolio size K. This means that, e.g., in the case of the 48Ind data set, for a fixed K we have 377 values of risk, one for each in-sample window (i.e., one for each rebalancing of the portfolio). Similarly we obtain Fig. 1.2, where we examine the Stoxx50 data set. Note however that in the cases of weekly data sets for a fixed K we have 32 in-sample windows (i.e., 32 values of risk). As mentioned above, the boxplot of the in-sample volatility generated by the EW portfolios corresponds to that of the FSMVEW portfolios when $K = n$, and

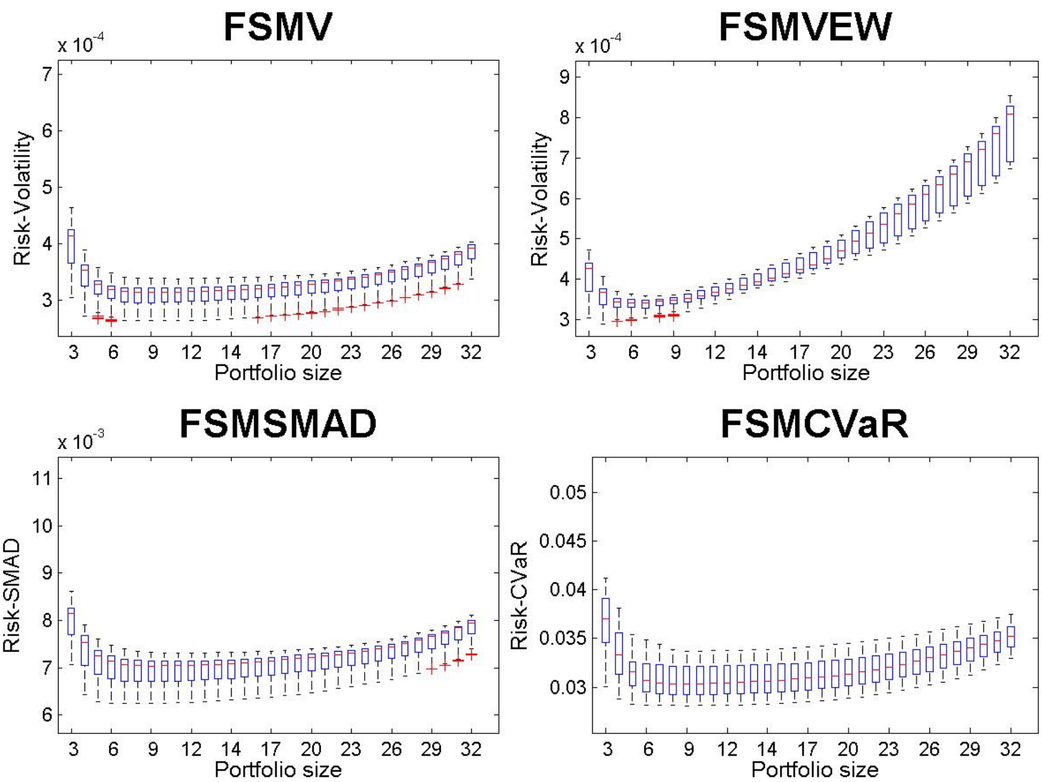


Figure 1.2: Boxplot of the in-sample risk w.r.t. the portfolio size for Stoxx50.

it generally presents the highest median volatility. This feature is common to all data sets, and it suggests that a greater diversification does not always imply a risk reduction, i.e., increasing the number of assets in the portfolio could worsen its in-sample performance in terms of risk.

The empirical results of the FSMVEW portfolios could be compared to the findings obtained by Evans and Archer, and by further influential experiments in the literature such as the well-known Fama's experiment Fama (1976). The author finds that, in a market with 50 stocks, the effect of *naïve diversification* determines a remarkable reduction of the portfolio in-sample volatility, but only when including in the portfolio up to 20 stocks. We refer to *naïve diversification* as an EW strategy with a random selection of K out of n available stocks. Indeed, he observes that adding further stocks in the portfolio does not yield a considerable improvement. More precisely, Fama claims that approximately 95% of the possible reduction deriving from diversification is achieved passing from 1 to 20 assets. However, we point out that our approach is significantly different from that of Fama, as well as from that of Evans and Archer. Indeed, we overcome one of the main weaknesses of their approach which consists in stating results that are valid only on average. In other words, if one picks an arbitrary EW portfolio of a given size in a market, there is no guarantee that its risk will not be much larger than the average risk of all portfolios of the same size in that market. While the results obtained by the FSMVEW portfolios are those corresponding to the best Equally-Weighted portfolios for each given size with respect to volatility. The findings on the FSMVEW model also highlight that when the EW strategy is combined with risk minimization (instead of a randomly selection of K out of n available stocks) the selected small portfolios show an improvement both in terms of volatility and of robustness of its values obtained on all in-sample windows.

In addition, once we have obtained the optimal size of the minimum risk portfolio for each in-sample window, we examine the range spanned by these optimal sizes. In Fig. 1.3 we show for the

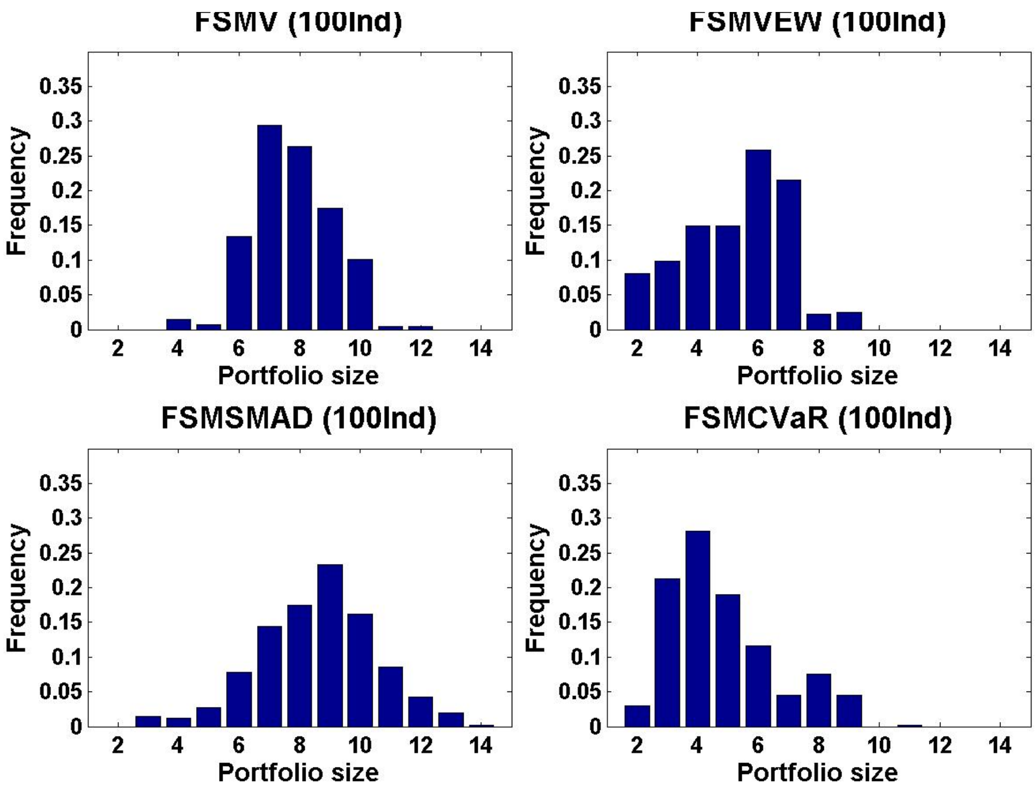


Figure 1.3: Distribution of the portfolio size corresponding to the global minimum risk.

100Ind data set the distribution of the optimal portfolio sizes (i.e., corresponding to the global minimum risk) for all models analyzed w.r.t. all in-sample windows. We can see that the global minimum risk portfolio never exceeds 15 stocks for the 100Ind data set. However, this behavior is almost the same in all the other considered data sets, with the only exception of Ftse100, where the optimal portfolio size is seldom around 20 stocks. Furthermore, given the global minimum risk on an in-sample window, we detect the smallest size of a portfolio whose risk is not more than 1% larger than that of the minimum risk portfolio, thus finding even smaller portfolios with satisfactory risk level. Then, we repeat this procedure for each in-sample window and for each portfolio model. In Fig. 1.4 we report the distribution of these *101% min-risk* optimal portfolio sizes for each model analyzed w.r.t. all in-sample windows. More precisely, for each in-sample window we consider all the cardinalities for which the corresponding portfolio has a risk at most 1% greater than that of the minimum risk portfolio. As highlighted from the four sub-figures (one for each portfolio), the *101% min-risk* portfolios generally show a significant risk reduction with 10 stocks for 100Ind data set. Furthermore, in most of the cases we can achieve it with just 6 stocks. However, the *101% min-risk* portfolio for the other data sets never exceeds a size of 15, and generally are needed at most 10 stocks.

The most compelling result emerging from the in-sample analysis is the existence of a portfolio size range (whose location could

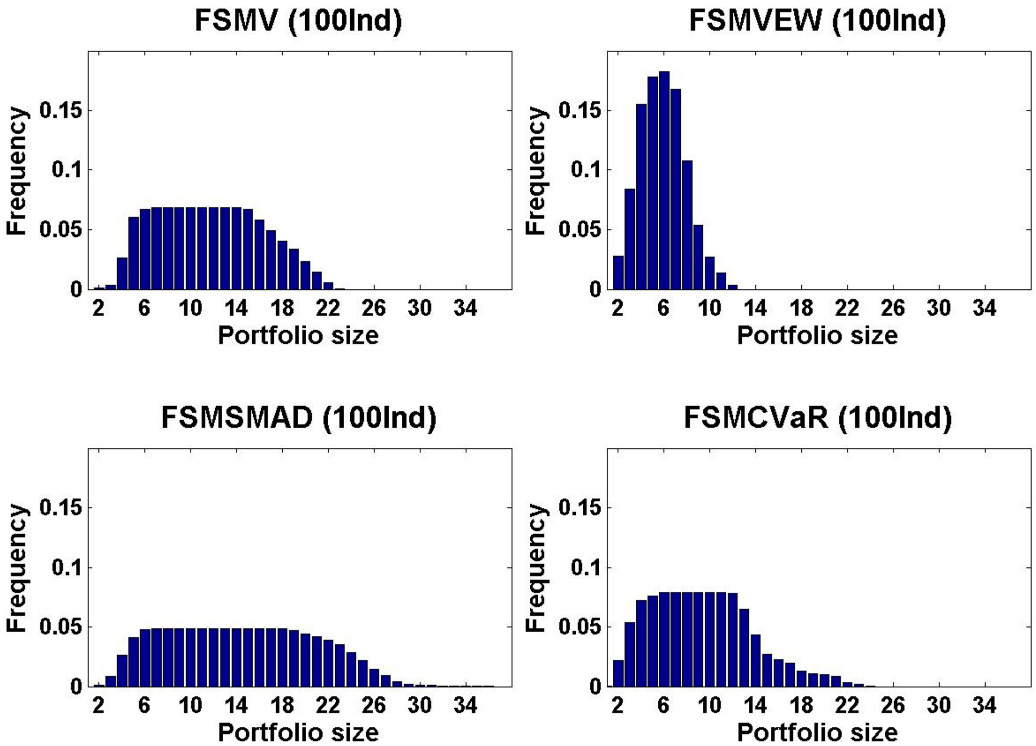


Figure 1.4: Distribution of the *101% min-risk* portfolio.

depend on the number of assets for each market) where one can generally find the lowest values of risk for all models considered. Indeed, we find that the smallest risk portfolios generally require no more than 15 assets. Furthermore, it is almost always possible to find portfolios that are just 1% more risky than the smallest risk portfolios and contain no more than 10 assets.

Further evidences of this phenomenon can be found, e.g., in Fig. 1.5, 1.6, 1.7, and 1.8, where we emphasize the behavior of the optimal range considering all the rolling time windows considered for the portfolios. More precisely, the red lines stand for the minimum number portfolios that are, at most, 1% more risky than the minimum risk portfolio.

1.3.2 Out-of-sample analysis

The second part of our analysis concerns the out-of-sample behavior of the portfolios. Our main goal is to confirm the finding, emerged from the in-sample analysis, that we can improve performances without investing in a large number of stocks.

Again, we consider the EW portfolio as a benchmark and, in-

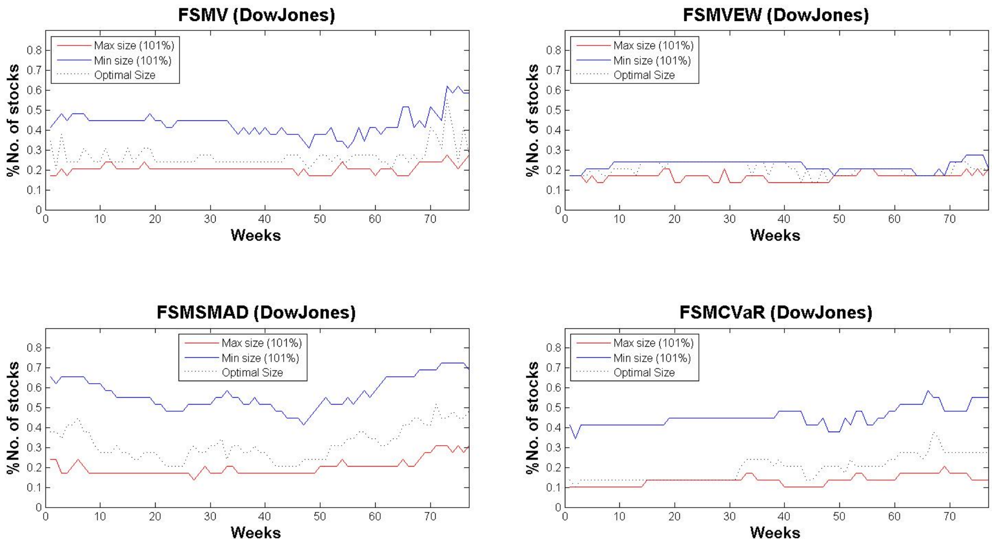


Figure 1.5: Minimum risk portfolios - 101% range (Dow Jones)

stead of focusing only on volatility reduction, we also compute the performance indices described in Section 1.3. We start by verifying the behavior of the out-of-sample standard deviation. More precisely, we check whether this performance measure reaches an optimal value, or at least a good value, for small-size portfolios. We can see in Figs. 1.9a and 1.10a that, both for monthly (48Ind) and for weekly (Stoxx50) data sets, the standard deviations of the portfolios returns reach their minima for small sizes. For larger sizes, the portfolios volatility tends to increase with different growth rates. These increases, except for the FSMVEW, are due

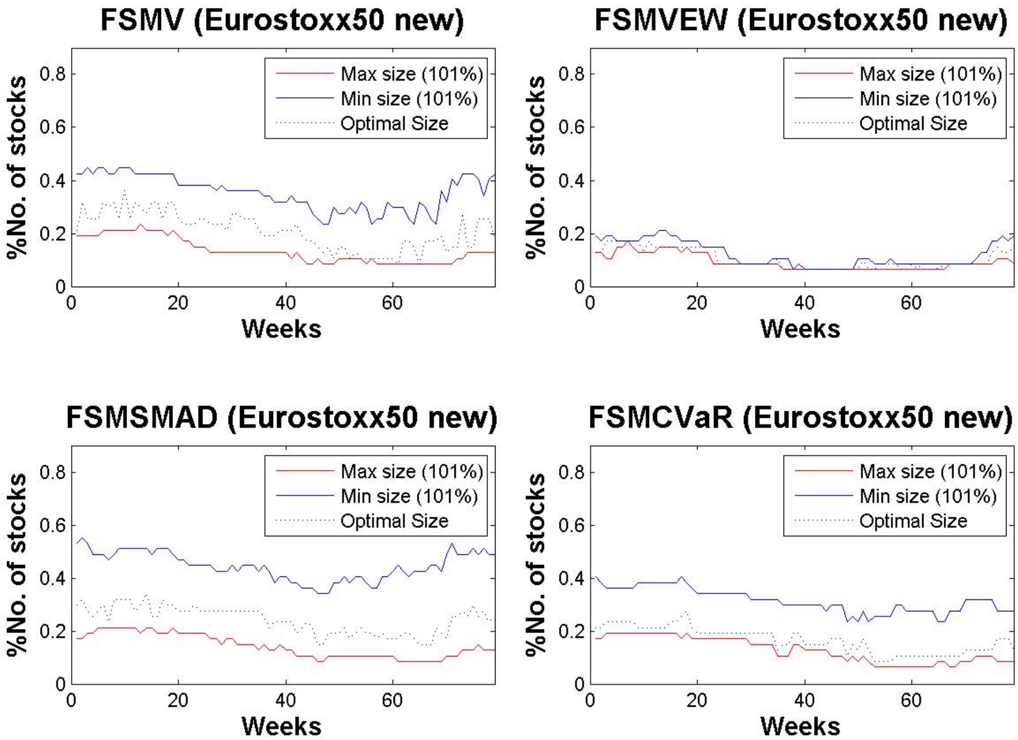


Figure 1.6: Minimum risk portfolios - 101% range (Euro Stoxx 50)

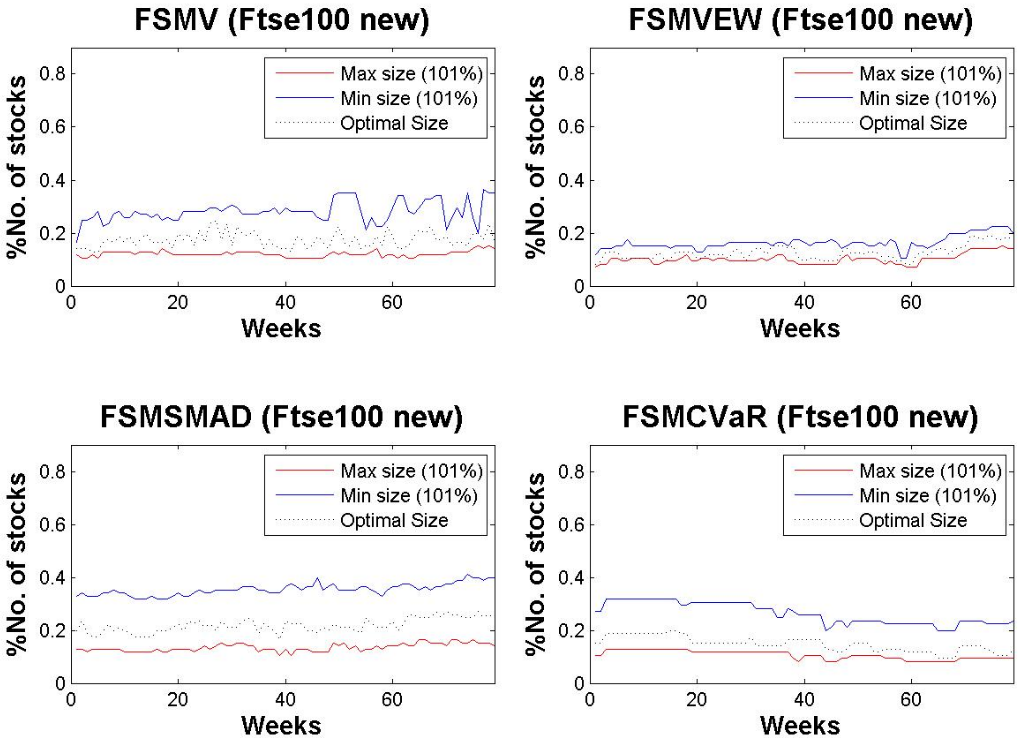


Figure 1.7: Minimum risk portfolios - 101% range (Ftse 100)

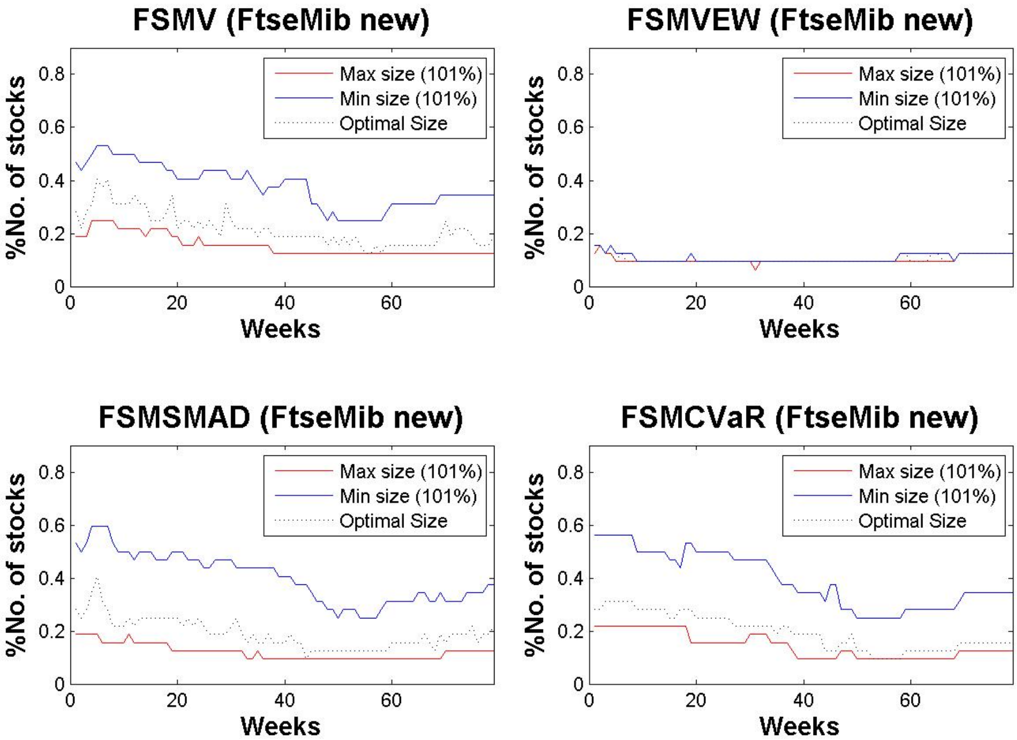
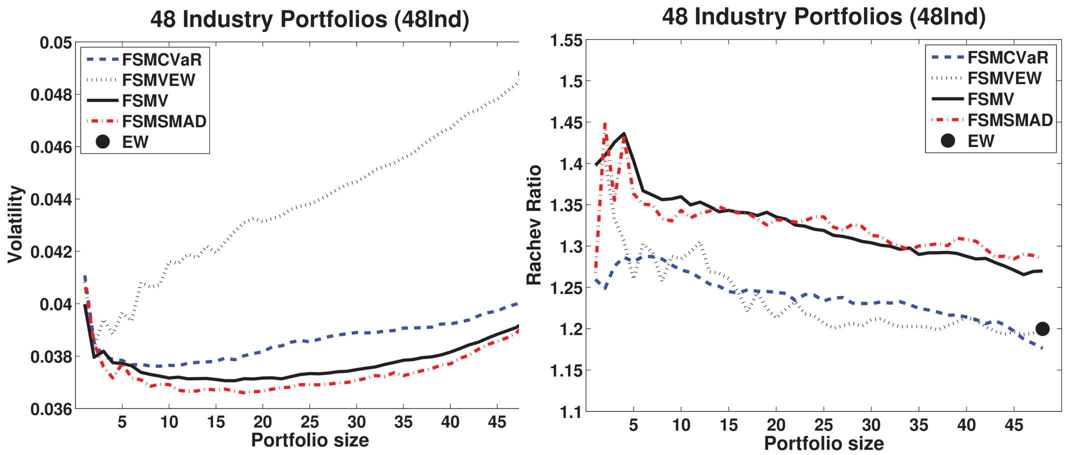


Figure 1.8: Minimum risk portfolios - 101% range (Ftse Mib)

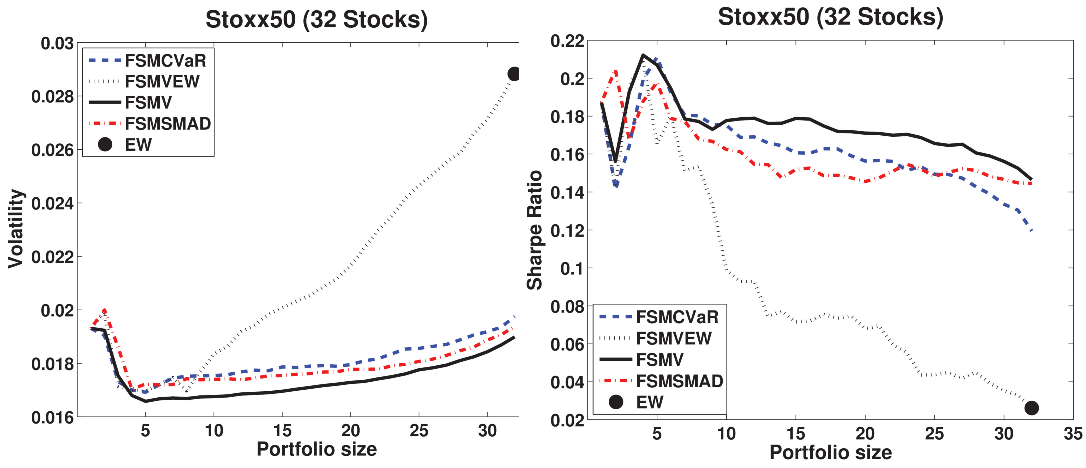


(a) Volatility.

(b) Rachev ratio.

Figure 1.9: Analysis of the out-of-sample portfolio returns for 48Ind.

to the buy-in threshold constraints. Without these constraints we should expect nearly flat curves. However, the buy-in threshold constraints are necessary to eliminate unrealistically small trades that can otherwise be included in an optimal portfolio. In Figs. 1.9b and 1.10b we show the values of two other performance measures, namely the Rachev and Sharpe ratios for the same data sets. As for the standard deviation, each model generally tends to provide the best values of the latter performance measures for small sizes. Furthermore, these values almost always decay when the portfolio size approaches n. This behavior provides a further



(a) Volatility.

(b) Sharpe ratio.

Figure 1.10: Analysis of the out-of-sample portfolio returns for Stoxx50.

support to the idea of improving the performances of a portfolio by limiting the number of its stocks.

In addition to the graphical evidence, where only the most representative results are shown, we also performed an extensive comparative analysis on all data sets considered. Since describing the results for all data sets and for all portfolio sizes is impractical, we report here the out-of-sample analysis for only three fixed sizes: $K = 5, 10, 15$. This choice is based on the observation that $K = 5, 10, 15$ generally belong to the optimal ranges in which the various models achieve the in-sample lowest risk for each data set. In Table 1.2 we provide the standard deviation of the out-of-sample returns for $K = 5, 10, 15$ for each model and data set analyzed. It is remarkable that the EW portfolio has almost always the worst performance, with the single exception of the 100Ind market, where the FSMCVaR portfolios generate the highest standard deviation. In Table 1.3 we report the Sharpe ratio of the out-of-sample returns for the same portfolio sizes of the previous table and for each model and data set analyzed. Note that when the portfolio excess return is negative some gain-to-risk ratios have no meaning, thus we report "-". Again we observe that the EW portfolio yields the worst performances compared with those of the other models, with the exception of the FSMCVaR portfolios for the 100Ind and FF25 markets. Similar considerations can be made about the Rachev Ratio of the out-of-sample returns shown in Table 1.4. Indeed, again the EW portfolio tends to be the worst choice, with the only exception of the 100Ind data set. We also observe that for $K = 10$ the FSMV model seems to be preferable since it provides the best results for 4 data sets

***K*=5**	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	0.0448	0.0378	*0.0556*	**0.0134**	0.0185	0.0169
FSMVEW	0.0431	0.0397	**0.0424**	0.0139	0.0181	0.0170
FSMV	0.0431	**0.0377**	0.0443	0.0141	**0.0169**	**0.0166**
FSMSMAD	**0.0430**	**0.0377**	0.0446	0.0136	0.0177	0.0172
EW	*0.0509*	*0.0488*	0.0512	*0.0210*	*0.0296*	*0.0288*
***K*=10**	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	0.0447	0.0376	*0.0536*	0.0133	0.0189	*0.0175*
FSMVEW	0.0441	0.0416	**0.0428**	**0.0125**	0.0203	0.0184
FSMV	**0.0432**	0.0372	0.0438	0.0126	0.0170	**0.0168**
FSMSMAD	**0.0432**	**0.0369**	0.0440	0.0128	**0.0167**	0.0174
EW	*0.0509*	*0.0488*	0.0512	*0.0210*	*0.0296*	0.0288
***K*=15**	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	0.0448	0.0378	*0.0537*	0.0134	0.0188	*0.0179*
FSMVEW	0.0458	0.0419	**0.0435**	0.0128	0.0218	0.0201
FSMV	**0.0433**	0.0371	0.0439	0.0127	0.0173	**0.0170**
FSMSMAD	0.0434	**0.0367**	0.0436	**0.0126**	**0.0171**	0.0175
EW	*0.0509*	*0.0488*	0.0512	*0.0210*	*0.0296*	0.0288

Table 1.2: Standard Deviation of the out-of-sample returns.

K=5	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	*0.244*	0.279	*0.133*	**0.334**	0.124	**0.211**
FSMVEW	0.267	0.285	**0.249**	0.247	0.119	0.165
FSMV	0.268	**0.293**	**0.249**	0.257	**0.126**	0.207
FSMSMAD	**0.276**	0.293	0.247	0.244	0.086	0.198
EW	0.264	*0.242*	0.215	*0.173*	–	*0.026*
K=10	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	*0.248*	0.278	*0.142*	**0.298**	0.120	0.175
FSMVEW	**0.290**	0.268	**0.256**	0.284	0.091	0.099
FSMV	0.27	0.289	0.242	0.276	0.112	**0.178**
FSMSMAD	0.276	**0.294**	0.253	0.266	**0.122**	0.162
EW	0.264	*0.242*	0.215	*0.173*	–	*0.026*
K=15	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	*0.250*	0.274	*0.143*	**0.281**	**0.124**	0.161
FSMVEW	**0.296**	0.260	0.249	0.278	0.042	0.072
FSMV	0.272	0.289	0.242	0.269	0.098	**0.179**
FSMSMAD	0.280	**0.293**	**0.257**	0.260	0.109	0.152
EW	0.264	*0.242*	0.215	*0.173*	–	*0.026*

Table 1.3: Sharpe Ratio of the out-of-sample returns.

***K*=5**	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	1.216	1.281	*0.877*	**1.618**	1.162	1.167
FSMVEW	1.201	1.260	**1.049**	1.614	1.048	1.101
FSMV	1.245	**1.403**	1.043	1.514	**1.164**	1.205
FSMSMAD	**1.25**	1.363	1.032	1.382	1.07	**1.218**
EW	*1.150*	*1.200*	0.992	*1.007*	*0.861*	*0.910*

***K*=10**	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	1.215	1.271	*0.892*	1.459	1.145	1.104
FSMVEW	1.199	1.284	**1.047**	1.477	0.963	1.007
FSMV	**1.244**	**1.360**	1.027	**1.514**	**1.147**	1.150
FSMSMAD	1.234	1.343	1.033	1.333	1.147	**1.185**
EW	*1.150*	*1.200*	0.992	*1.007*	*0.861*	*0.910*

***K*=15**	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	1.217	1.245	*0.896*	1.397	1.123	1.064
FSMVEW	1.195	1.261	**1.052**	1.347	0.861	*0.907*
FSMV	**1.244**	**1.343**	1.024	**1.426**	**1.135**	**1.135**
FSMSMAD	1.234	1.343	1.047	1.320	1.103	1.132
EW	*1.150*	*1.200*	0.992	*1.007*	*0.861*	0.910

Table 1.4: Rachev Ratio of the out-of-sample returns.

***K*=5**	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	0.209	0.150	*0.578*	0.027	0.042	0.031
FSMVEW	0.214	0.175	**0.201**	**0.029**	0.045	0.036
FSMV	**0.198**	0.123	0.206	0.030	**0.037**	**0.031**
FSMSMAD	0.201	**0.121**	0.206	0.039	0.041	0.032
EW	*0.261*	*0.259*	0.262	*0.052*	*0.064*	*0.061*
***K*=10**	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	0.210	0.145	*0.501*	0.033	0.042	0.034
FSMVEW	0.219	0.201	**0.201**	0.029	0.041	0.041
FSMV	**0.198**	0.126	0.209	**0.023**	**0.039**	**0.031**
FSMSMAD	0.203	**0.125**	0.212	0.031	0.039	0.034
EW	*0.261*	*0.259*	0.262	*0.052*	*0.064*	*0.061*
***K*=15**	**FF25**	**48Ind**	**100Ind**	**Ftse100**	**FtseMib**	**Stoxx50**
FSMCVaR	0.212	0.141	*0.493*	0.029	0.040	0.036
FSMVEW	0.235	0.206	0.213	**0.027**	0.050	0.045
FSMV	**0.199**	0.129	**0.210**	0.027	**0.039**	**0.031**
FSMSMAD	0.203	**0.127**	**0.210**	0.027	0.040	0.035
EW	*0.261*	*0.259*	0.262	*0.052*	*0.064*	*0.061*

Table 1.5: Max drawdown of the out-of-sample returns.

out of 6, while for $K = 15$ it presents the best performances for 5 data sets out of 6.

The last performance measure considered in our analysis is the Max drawdown, which is the worst out-of-sample loss achieved by a portfolio, as described in Section 1.3. Table 1.5 shows that, again, the EW portfolio always has the worst performance for the prescribed sizes $K = 5, 10, 15$, with the exception of the 100Ind market, where the FSMCVaR portfolios provide the worst loss. On the other hand, although there is not a clear superiority of a single model, we observe that the FSMV portfolios present the best values for 3 data sets out of 6 for $K = 5$, and for 4 data sets out of 6 for $K = 10$ and for $K = 15$.

1.4 Conclusions

The concept of diversification is not well-defined and the measures of diversification are continuously evolving (see, e.g., Fragkiskos (2013); Meucci (2009) and references therein). However, the qualitative idea of diversification is to not overly concentrate the investments in very few stocks. Indeed, the role of diversification is to reduce risk by diversifying it as much as possible.

In this work we investigated the possible benefits and disadvantages of enlarging the portfolio size in several portfolio selection models with respect to various measures of performance. Similar to various previous findings, but with a substantially different approach, our empirical results show that in most cases limiting the size of the selected portfolio improves both the in-sample and the out-of-sample performance. We might call this a "*small portfolio effect*". These results are somewhat in line with the tendency described by DeMiguel et al (2009a), where an improved out-of-sample performance is often observed for the 1-norm-constrained minimum-variance portfolios. The analogy is based on the observation that the 1-norm is often regarded as an approximation of the 0-norm, i.e., the size of the portfolio.

Further studies are underway to investigate the validity of this *small portfolio effect* with respect to other risk and performance measures and in larger markets.

1.5 Further research

To verify the existence of a "small portfolio effect", we use the results of Kondor and Nagy (2007), who study the estimation error arising from a large portfolio and limited time series. The authors claim that the noise of the minimal risk portfolio is greater than that of the Equally Weighted portfolio. More precisely, to measure the effect of noise on portfolio selection, the authors use the following metrics:

$$q_0^2 = \frac{\Sigma_{ij} w_i^{opt} \sigma_{ij}^{(0)} w_j^{opt}}{\Sigma_{ij} w_i^{(0)} \sigma_{ij}^{(0)} w_j^{(0)}} \tag{1.6}$$

where the superscript *opt* refers to the optimal weights w and 0 to the "true" weights and the true covariance matrix elements σ_{ij}. For a portfolio with returns that are (i) standard, (ii) independent, and (iii) normal variables, one can demonstrate that,

$$\overline{q}_0 = \frac{1}{\sqrt{1 - \frac{N}{T}}} \ , \tag{1.7}$$

where N and T are the number of stocks in a market and the length of the returns time series, respectively. In Fig. 1.11 we show

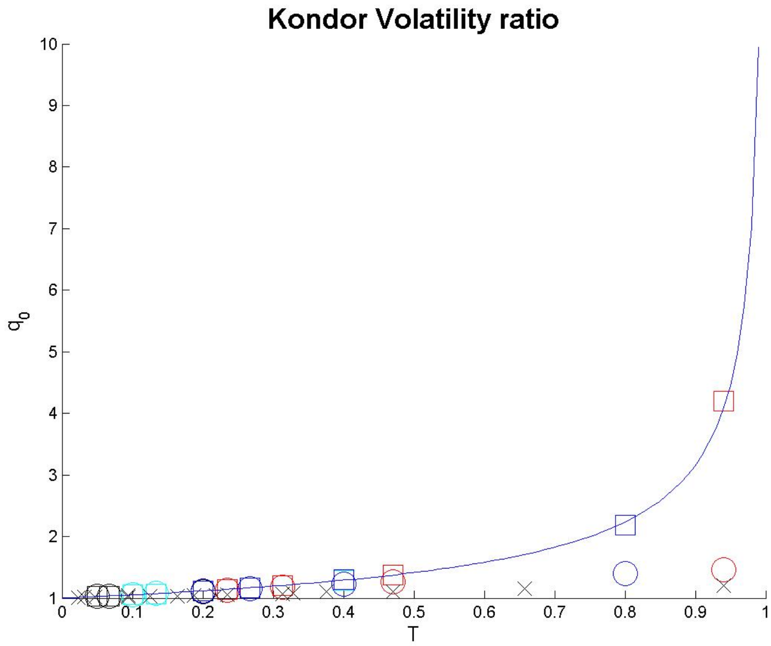


Figure 1.11: Cumulative distribution function of risk events' priorities

some preliminary empirical results related to the effect of noise on portfolio selection considering a market in which the returns follows a multivariate standard normal distribution (of dimension N). Here, the blue line is the theoretical behavior of q_0 while, for various combination of N and T, the squares are the ratios for the Global Minimum Variance portfolios with short selling, the circles are the ratios of the Global Minimum Variance portfolios without short selling, and the Xs are those of the Fixed-Size Minimum Variance portfolios.

As one can see from the figure, there is a huge benefit - in terms of noise reduction - when investing in few stocks when the ratio N/T approaches to 1. This benefit can be measured by the vertical segment which divides the circles and the Xs from the blue line. However, we need to verify this behavior with empirical market data to justify a noise reduction for the portfolios with few stocks. This last investigation is left for further research.

Chapter 2

Operational risk assessment of a new product using AHP

Risk assessment of a new product is one of the most critical activities performed by the Operational Risk Management (ORM) of a company operating in the financial sector. When introducing a new product, there are few reference points to assess its riskiness for ORM, due both to the lack of operational loss data and to the inexperience of the process owners in handling the new operation. To overcome these two limitations, we propose an operational risk framework that is able to identify and prioritize the most dangerous operational risk events with respect to the introduction of a new product in a bank. In this paper, we apply a methodology based both on the Analytic Hierarchy Process (AHP) approach to prioritize operational risk events, and on the "80/20 rule" to allocate them in appropriate risk rating classes. The aim of ranking and assigning risk rating classes is to select the mitigation actions to protect the most exposed internal processes of a company with respect to operational risks.

2.1 Introduction

A product is defined new when one or more factors such as, for instance, product complexity and/or target customers, represent a point of attention during the feasibility evaluation of the product itself Ingber (2016). Although it is difficult to define uniquely a new product, one can consider the cases in which a company significantly modifies the features of an existing product (e.g. new distribution channel, new geographic market). For instance, offering a loan to different customer segments that current ICT systems fail to handle (as it requires a new software or an upgrade of the existing one) is a new product. Again, one can define as new a product that requires a modification of the risk tolerance thresholds; even an existing product distributed in a geographical area with different regulations from those of the registered office of the company can be considered new. More generally, we can define a product new when the current operational context of a financial institutions[1] (e.g., pricing models, ICT systems, and organi-

[1] In any case, companies operating in other areas may also adopt this framework.

zational models) does not allow to develop, distribute, manage, and control it. Conversely, products that only modify contractual terms (e.g., modification from a fixed to a floating interest rate) are not defined as new.

According to Girling (2013), in the development phase of a new product a company should try to identify its critical points, including the potential operational risk exposure. For this reason, a company should question itself “around the operational practicalities, accounting and tax practices, legal and regulatory requirements, and any other areas that should be addressed before the launch”.

A new product can cause an exposure to both known and new operational risk events for a bank. Such events may give rise to several types of impacts, which are difficult to quantify in terms of economic losses, especially because there are no historical data of losses. “Senior management should ensure that there is an approval process for all new products, activities, processes and systems that fully assesses operational risk” Basel Committee et al

(2011). Thus, the operational risk managers of a company must establish a methodology to perform a risk assessment before the launch of a new product or business.

Generally, risk assessments are based on expert judgments Cooke (2004). Thus, it seems reasonable to use the judgements expressed by process owners to assess the operational risks of a new product. Since generally $risk = likelihood \cdot impact$ Anthony Tony Cox (2008), classical operational risk assessments require that process owners should identify the likelihood and the severity of the potential operational risk events[2]. However, this estimation can be very difficult due to the lack of historical losses data and of experience in managing the new operation. Furthermore, process owners' judgments can embody cognitive biases Skjong and Wentworth (2001). For instance, typical examples of bias are (i) the tendency to overestimate the likelihood of the most recent events and to underestimate that of the oldest events (availability bias); (ii) to ignore events that rarely occurred (threshold heuristic); (iii) to

[2]Potential operational risk events are identified during a specific mapping activity (see Section 2.3.1).

maintain previous judgments (anchoring bias).

We apply a strategy to assess the operational risk of a new product in the financial sector, which is able to overcome the limitations of the traditional assessment methodologies, based on the combination of the likelihood and the impact of risk events by means of a heat map (see, e.g., Anthony Tony Cox (2008)). Indeed, we stress that our approach does not require any information about the frequency and the severity of the operational risk events.

Note that there literature about the operational risk management of a new product is scarce, even though the problem of estimating the operational risk exposure of a new product is definitely one of the main duties for operational risk managers. Furthermore, the existing bibliographical references (see, e.g., Scandizzo (2010)) focus on what should be done instead of how to practically deal with the operational risk management of a new product. Indeed, in previous articles and books we can find the elements that a risk manager should consider. However, there is no trace of any technique or tool to implement an operational risk assessment. Our contribution, in this context, is to provide a framework whose tools are already widely tested in other fields (particularly in the field of engineering). Indeed, our framework allows to assess the riskiness of any new product. Thus, although we present a case study related to a new financial product, one can implement our technique in companies operating in many different fields.

More precisely, in this work we provide an operational risk framework based on the Analytic Hierarchy Process (AHP), by which we are able to prioritize operational risk events (in a decreasing order of importance). For such a ranking, process owners only have to perform pairwise comparisons between the elements belonging to the same level of a decisional hierarchy Saaty (1987). In addition, combining the results obtained from the AHP model with the well-known "80/20 rule" (also called Pareto principle, see Pareto (1964)), we can divide all risk events into 4 classes, each with a specific degree of relevance.

The chapter is organised as follows. In Section 2.2 we present the various phases that characterize a risk assessment process for a new product. In Section 3 we briefly discuss the main features of the AHP model, also describing the structure of a suitable hierarchy for the operational risk assessment of a new product. Section 2.3.2 shows how to apply the "80/20 rule" to cluster operational risk events in 4 rating categories, in order to facilitate the planning of the mitigation actions to protect the most exposed internal processes of a company. In Section 2.3 we provide a capital budgeting model to optimally prioritize risk mitigation actions, while in Section 2.5 we illustrate a numerical example of the entire framework. Conclusions are drawn in Section 2.6, where we also describe some issues left for future research.

2.2 Risk assessment for a new product

As mentioned above, the launch of a new product makes it necessary to establish a strategy to identify, evaluate and mitigate operational risk events that can arise during or after its launch. The European Banking Authority claims that "The Risk Control function should be involved in approving new products or significant changes to existing products. Its input should include a full and objective assessment of risks arising from new activities under a variety of scenarios, of any potential shortcomings in the institution's risk management and internal control frameworks, and of the ability of the institution to manage any new risks effectively" European Banking Authority (2011).

According to Scandizzo (2010), an operational risk manager should consider several risk factors related to the introduction of a new product, such as

- new market;
- characteristics of the new product;
- new way of doing business;
- new laws;
- change in regulatory requirements;
- change in market beliefs.

The analysis of these factors can allow to depict their causal link with operational risk events (cause-event-effect). In other words, risk factors can be seen as detonators for the occurrence of operational risk events. Scandizzo (2010) also states that the departments of a bank should highlight the operational risk aspects of a new product, which include - inter alia - (i) potential new reporting requirements, (ii) accounting treatment, (iii) representation within the ICT software, (iv) reinforce requirements. These aspects should be thoroughly analyzed to detect any issues to be solved. For example, if the system of inserting and managing data of a product requires changes of the ICT system, then it would be necessary to verify whether their timing is coherent with the launch of the new product.

An operational risk manager should define a taxonomy of operational risk events with a sufficient degree of detail, so that the output of AHP can better represent the real risk profile. For instance, in the case of an internal fraud event, a possible taxonomy could be

- level 1: internal fraud;
- level 2: unauthorised activity
- level 3: transactions not reported (intentional);
- level 4: transfer to a bank account of the same operator not reported (intentional).

Furthermore, an operational risk manager should focus on the existing/potential internal processes to manage the operation of the new product. Then, given the taxonomy of operational risk events and the flowchart of the existing/potential internal processes, an operational risk manager can perform the risk mapping into the internal processes. Once the risk mapping is completed, process owners can assess each potential operational risk event. As mentioned earlier, following the classical methodologies for operational risk assessments, the inherent risk mainly depends on two parameters, namely the likelihood of loss occurrence and the expected loss in the case of occurrence. More precisely, once process owners have expressed their views about these two parameters, one can estimate the inherent risk of each operational risk event by means of a heat map, where different levels of criticality are generally represented by an appropriate color scale. However, during the launch of a new product process owners have often very limited information to carefully estimate the likelihood and the expected loss of each operational risk event and, generally, they do not have experience in managing the new operation. These are the main reasons that lead us to develop a new strategy, based on the Analytic Hierarchy Process (AHP). As we shall explain in the next section, following our methodology, process owners need only pairwise comparisons between elements of the same nature, thus significantly simplifying the evaluation phase. The main steps for applying the AHP method to the operational risk evaluation are listed in Section 2.3.1.

2.3 The Analytic Hierarchy Process

As discussed above, our strategy for the evaluation of a new product is based on the Analytic Hierarch Process (AHP) technique, which is a widely used model for multi-criteria decision analysis (a branch of Operations Research), introduced by Saaty (1977). AHP aims to analyze and solve complex problems, especially those related to the cognitive distorsion of human decisions. More generally, AHP is a method to determine a ranking among a set of elements.

According to Forman and Selly (2001)[3], AHP provides the following advantages:

- a decrease in the time needed to find an agreed solution[4] ;
- an increase in the level of detail of the analysis;
- an increase in the level of participation and consensus of all process owners;
- the resolution of conflicts between process owners;
- a reduction of cognitive biases during the Risk Assessment.

In our framework, AHP allows us to evaluate the relative level of criticality for all potential operational risk events without know-

[3]The authors report some applications of the AHP at the Inter-American Development Bank, among which some that concern supplier/vendor selection.

[4]Note that this feature is particularly useful in case of internal cross-processes involving different owners.

ing their likelihood of occurrence and their expected loss, historical loss data, and any experience in managing the new operation. In particular, we emphasize that AHP allows an operational risk manager to mitigate the process owners' cognitive bias. This model is able to quantify, by means of a semantic scale (see Section 2.3.1 for a complete description), the process owners' judgments (which are the only input required by AHP), through pairwise comparisons of the analyzed elements Saaty (2005). Furthermore, we point out that AHP can handle multiple judgments (expressed by different process owners) on the same couple of elements. Indeed, AHP can aggregate these multiple judgments through a geometric mean Forman (2001). AHP is also able to quantify cognitive biases by means of simple calculations (see Section 2.3.1).

2.3.1 Assessing operational risks of a new product with AHP

Our implementation of the AHP method for the evaluation of the operational risks related to the introduction of a new product is similar to that of Mustafa and Al-Bahar (1991), who propose the assessment of the project risks for the construction of a bridge through AHP.

Our analysis consists of the following steps:

- risk mapping;
- construction of the decisional hierarchy;
- evaluation phase.

Risk Mapping

The first step involves mapping the potential operational risk events into the existing/potential processes of the company. To identify all the possible events related to the new operation, one should examine all available documentation on the new product. From this documentation, one could derive roles and responsibilities for each department of the company involved into the launch of the new product and into the management of the new operation.

Process owners, who perform the mapping activity under the supervision of operational risk managers, have the responsibility of identifying all potential operational risk events.

Construction of the decisional hierarchy

The second step of the methodology requires the representation of a multi-criteria decision problem, which consists in prioritizing the operational risk events due to the introduction of a new product by means of a decisional hierarchy. A decisional hierarchy is a multilayer structure to organize factors and actors of a problem. Each layer is composed of homogeneous elements, namely a given level of the hierarchy must contain a set of pairwise comparable elements.

The decisional hierarchy of our approach consists of the following layers:

- Layer 1: (goal) prioritize operational risk events arising from the introduction of a new product;
- Layer 2: (decisional criteria) identify the departments of the company involved in the management of the new operation;
- Layer 3: (sub-criteria) establish the existing/potential processes that characterize the new operation;

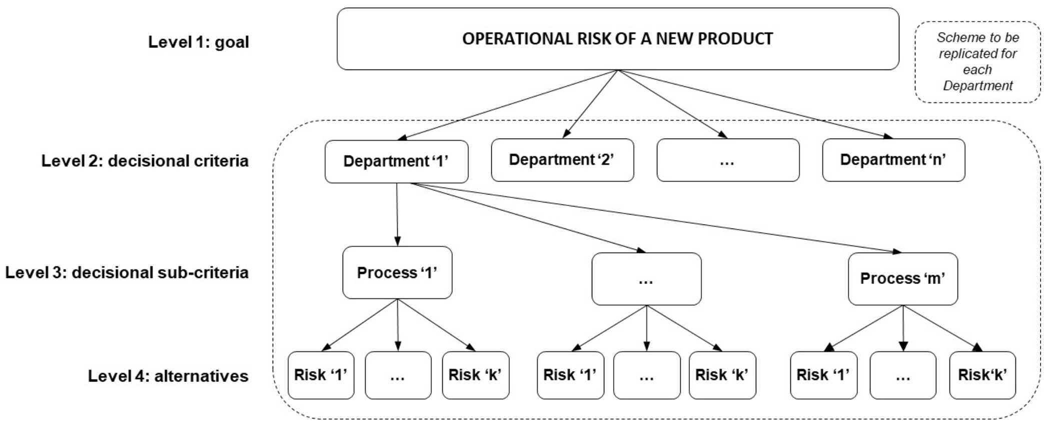


Figure 2.1: Decisional hierarchy

- Layer 4: (alternatives) identify operational risk events in each process.

To support intuition, In Fig. 2.1 we provide a scheme of the decisional hierarchy. Note that the modularity of this hierarchy could also allow us to add a further layer. For instance, this fifth layer could contain the loss effects due to an operational risk event, or the risk factors that can determine a given loss event.

The elements of a layer of the hierarchy can be clustered into groups. Each group is a set of homogeneous elements that a process owner must pairwise compare and that are connected to the same element of the upper layer of the hierarchy. For example, in Fig.2.1 the set of risks (Risk "1", Risk "2", ..., Risk "k") related to Process “1” is a group. Niemira and Saaty (2004) suggest to create groups with few elements to simplify the decision making process.

The output provided by AHP is a scalar (called a priority) for each operational risk event, which enables to rank the risk events with respect to their level of criticality. As shown more in detail in next section, one can compare these priorities locally, namely among elements of a cluster (so-called “Local Scale”), or globally, namely among elements of a layer (so-called “Global Scale”). Note that all the elements of Layer 2 are considered as a single group. Thus, in Layer 2 the Global and the Local Scale comparisons coincide.

Evaluation phase

The last step of the analysis consists in assessing the relative importance of the elements belonging to the same layer by means of process owners’ judgments. More in detail, this phase is generally performed by the operational risk managers, who can use several ways to collect this information, e.g., through a survey, a workshop, or interviews. We believe that the most convenient way to gather information is an interview with process owners, so that an operational risk manager can directly control their cognitive biases. Indeed, operational risk managers can report in real-time to a process owner any inconsistencies of his judgments by means of the Consistency Ratio, which quantifies cognitive biases (see Expression (2.1)).

One of the key properties of AHP is to reduce possible cognitive biases by simplifying the decision-making process, namely by pairwise comparing elements rather than comparing them all together.

In practice, for pairwise comparisons required by AHP, operational risk managers could ask process owners, during the interviews, questions of this kind[5]: "with reference to the operational risk level of the new product, what is the department that manages the most risky processes between Department "1" and Department

[5]Probably, the most suitable person to assess the riskiness of each organizational unit is the project manager who is responsible for monitoring the launch of the new product. Note that the project manager is, in turn, a process owner.

Intensity of importance on an absolute scale	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Moderate importance of one over another	Experience and judgment slightly favour one activity over another
5	Essential or strong importance of one over another	Experience and judgment strongly favour one activity over another
7	Very strong importance of one over another	An activity is favoured very strongly over another; its dominance demonstrated in practice
9	Extreme importance of one over another	The evidence favouring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values between the two adjacent judgments	When compromise is needed

Table 2.1: Semantic scale

"2"? To what extent? ". Process owners could answer these questions through the semantic scale proposed by Saaty (1988) and reported in Table 2.1. Clearly, the purpose of this scale, whose values range from 1 to 9, is to convert qualitative judgments into ordinal numbers[6]. A justification for the interval $[1, 9]$ of the semantic scale is due to Dehaene (2011), who states that "introspection suggests that we can mentally represent the meaning of

[6]According to Dantzig (1954), "number sense should not be confused with counting, which is probably of a much later vintage, and involves, as we shall see, a rather intricate mental process". Indeed, through number sense it is possible to compare a plurality of objects.

	Element1	Element2	...	ElementN
Element1	1	a_{12}	$\cdots$	a_{1N}
Element2	$1/a_{12}$	1	$\cdots$	a_{2N}
$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
ElementN	$1/a_{1N}$	$1/a_{2N}$	$\cdots$	1

Table 2.2: Generic pairwise comparison matrix

numbers 1 through 9 with actual acuity. Indeed, these symbols seem equivalent to us".

The judgements on elements of a given cluster are collected within a so-called pairwise comparison matrix (see Table 2.2).

For a practical construction of the pairwise comparison matrix, see the case study described in Section 2.5. However, for sake of clarity, we can say that if a process owner assigns the value of 3 (i.e., moderate importance of one over another) when comparing elements "1" and "2" of the same group, then $a_{12} = 3$. In addition, in order for the comparison between the elements "1" and "2" to be consistent, we have that $a_{21} = \frac{1}{a_{12}} = \frac{1}{3}$. Summarizing, the pairwise comparison matrices have the following properties:

1. by construction, the elements of the diagonal are all equal to one (i.e., each element is equally important to itself);

2. for consistency the elements of the upper triangle are reciprocal with respect to those of the lower triangle.

Note that, thanks to these properties, a process owner must only express $N(N-1)/2$ comparisons to fulfil a pairwise comparison matrix. With this matrix it is possible to compute the priorities of the elements belonging to the same cluster. A strategy to obtain the priorities of the elements from their pairwise comparison matrix is the principal eigenvalue method Saaty (2003), where the priorities are given by the components of the normalized eigenvector ω_{max} corresponding to the maximum eigenvalue λ_{max}. One of the advantages of the principal eigenvector method is its capability of easily quantifying the process owner's bias through the so-called Consistency Ratio (CR). This ratio is calculated as follows:

$$CR = \frac{CI}{RI} \tag{2.1}$$

where CI and RI are the Consistency Index and the Random In-

N	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Table 2.3: Random Index values

dex, respectively. The Consistency Index is

$$CI = \frac{\lambda_{max} - N}{N - 1} \tag{2.2}$$

The Random Index is a parameter that depends on the number of elements of the pairwise comparison matrix, as reported in Table 2.3 (see Saaty (2003)). According to Saaty (1990) a pairwise comparison matrix is *consistent* when $CR \leq 10\%$. Consistency is closely linked to the following requirements: (a) $a_{ij} = \frac{1}{a_{ji}}$ and (b) $a_{ij}a_{jk} = a_{ik}$ for all $i, j, k = 1, ..., n$. As described above, the priorities of a cluster of potential operational risk events, calculated by the principal normalized eigenvector ω_{max}, represent the relative importance of each risk event with respect to the others belonging to the same cluster (the so-called "Local Scale"). Once we have calculated the priorities for all clusters of two contiguous layers in the hierarchy (see Fig.2.1), we can compute the "Global Scale" priority of each element of the lower layer with a multiplicative approach, as described from the following example. Let us assume that *Layer 2* consists of 2 elements, Department "1" and Department "2", having local scale priorities $\omega_1 = 70\%$ and $\omega_2 = 30\%$, and that *Layer 3* provides two clusters: c_1 linked to Department "1" and containing three elements (γ, δ, and ε); and c_2 linked to Department "2" and containing two elements (ϕ, η). The local scale priorities of the three elements of c_1 are $\omega_\gamma^{LS} = 50\%$, $\omega_\delta^{LS} = 20\%$, $\omega_\varepsilon^{LS} = 30\%$, while those of the remaining two are $\omega_\phi^{LS} = 50\%$, $\omega_\eta^{LS} = 50\%$. Then, the global scale priorities of the elements belonging to *Layer 3* are $\omega_\gamma^{GS} = 70\% \times 50\% = 35\%$, $\omega_\delta^{GS} = 14\%$, $\omega_\varepsilon^{GS} = 21\%$, $\omega_\phi^{GS} = 15\%$, and $\omega_\eta^{GS} = 15\%$. The same mechanism is true for the remaining layer of the hierarchy.

The global scale priorities of the risk events (i.e., the elements of *Layer 4* of the hierarchy) represent the relative contribution of each potential operational risk event to the overall risk profile of the new product. Thus, through the Global Scale priorities, one can prioritize operational risk events in terms of relevance (in a decreasing order).

2.3.2 80/20 rule

Once the ranking of the potential operational risk events is obtained from the AHP method, we can group this set into four rating clusters to better represent their criticality when choosing the risks to be mitigated. For this purpose, we use Juran's "80/20 rule"[7], which is a widespread empirical principle (used in several business fields) that states that roughly 80% of a phenomenon is explained by roughly 20% ("vital few") of its causes Juran (1951). This 80/20 rule could facilitate risk managers to interpret the output provided by AHP. In view of this rule, we cluster the operational risk events, identified by the process owners, into four rating categories in a decreasing order of relevance: "critical", "high", "medium", "low".

To determine the four classes of rating, we first identify three classes by the cost-benefit analysis of Lysons and Farrington (2006), and then we split the first class into two sets, as shown in Table

[7] Juran was inspired by Pareto's results (Pareto, 1896).

ABC ANALYSIS Lysons and Farrington (2006)			Risk rating classes		
Class of elements	Elements	Relevance	Rating	Elements cumulative distribution range	Relative contribution to overall risk of the new product
Class A	First 20%	$\simeq$ 80%	Critical	[0%, 5%]	$\simeq$ 50%
			High	(5%,20%]	$\simeq$ 30%
Class B	Second 30%	$\simeq$ 15%	Medium	(20%, 50%]	$\simeq$ 15%
Class C	Remaining 50%	$\simeq$ 5%	Low	(50%, 100%]	$\simeq$ 5%

Table 2.4: Risk rating classes for the operational risk events of a new product.

2.4. In a nutshell, these four classes are obtained by fixing appropriate cut-offs on the cumulative distribution function of the operational risk events.

In our framework, we can interpret the "80/20" rule by stating that roughly the 80% of the operational risk related to the introduction of a new product is due to roughly 20% of the risk events identified by the process owners. To establish the above mentioned cut-offs, we refer to the quantiles of the cumulative distribution function of the operational risk events (sorted in descending order) with respect to their ranking obtained by the AHP model. This means that our "vital few" elements are both the "critical" and the "high" risks.

Note that, using this additional clustering into four classes, the Management of a company can choose to focus on mitigation actions especially for “critical” and “high” risks, coherently with principles of parsimony and efficiency. However, this choice has to be coherent with the risk appetite and the risk tolerance thresholds defined by the same Management. Conversely, “medium” and “low” risks, as a matter of principle, should be constantly monitored in order to ensure that these risks remain below appropriate tolerance thresholds established by the Management. Indeed, in the case of risk events related to strategic processes, the Management can choose to invest in additional mitigation actions also for “medium” and “low” risks.

Although the primary goal is to guarantee the core processes, it is reasonable to assume that the Management of a company wants to try to maximize efficiency for a given money budget and other available resources. In this context, grouping these two categories of risks (“medium” and “low”) is fundamental to save money and other resources.

A further support to these evidences can be obtained from Fig.

2.2, where we report a theoretical cumulative distribution of the priorities of the risk events. Indeed, one can directly observe the importance of the contribution to risk with respect to the number of operational risks. Note that the closer the cdf of the operational risk events to the Pareto distribution is, the more the use of the "80/20 rule" is justified.

Furthermore, we point out that the risk map should provide at least 20 operational risk events to apply the "80/20 rule". Indeed, if the number of risk events is less than 20, then the first 5% of elements is less than a single risk event.

2.4 Optimally choosing the mitigation actions of the intervention plan

Once the most relevant risk events are identified by means of the AHP model and of the 80/20 Rule, process owners must sketch proper mitigation actions, which generally concern the development of existing/new ICT systems, staffing review, drawing of internal processes, and policy formalization. Note that a mitigation

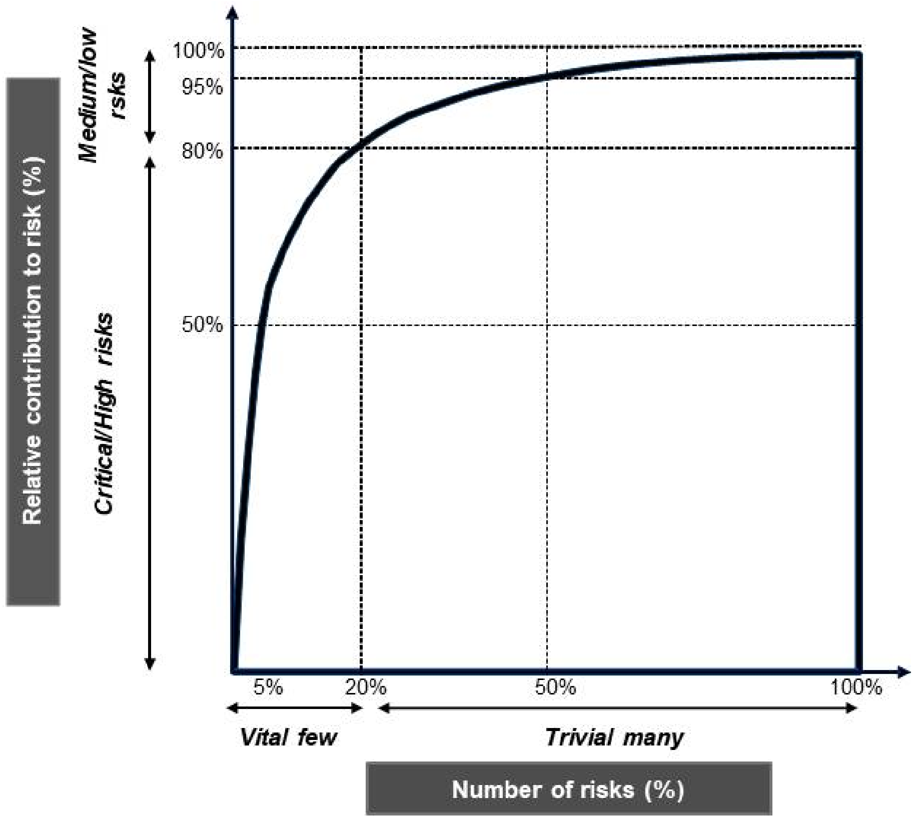


Figure 2.2: Cumulative distribution function of risk events' priorities

action can cover one or more risk events, while each risk event can be covered by one or more mitigation actions. A company should first estimate the cost of each mitigation action. Then one can search for an optimal intervention plan to reduce the operational risk exposure arising from a new product.

A typical aim in operational risk management is to cover the operational risk exposure minimizing the cost of the mitigation actions. Thus, considering n risk events (i.e. those *critical* and *high*) and m mitigation actions, we solve the following set covering problem:

$$\begin{aligned} \min \quad & c^T x \\ s.t. \quad & Ax \geq \mathbf{1} \\ & x \in \{0,1\}^m \end{aligned} \tag{2.3}$$

where $\mathbf{1}$ is the all-ones vector of dimension m, c is the vector of costs of the mitigation actions, and A is a $n \times m$ boolean matrix with rows representing the risk events and the columns representing the mitigation actions. Element a_{ij} is 1 if the action i mitigates risk event j, and 0 otherwise.

The output of the model (for a practical application, see Sec. 2.5) provides the mitigation actions that secure the internal processes of the company, minimizing the cost of the intervention plan. One of the most interesting features of the model is that one can handle overlaps among mitigation actions by minimizing their total cost.

2.4.1 Optimally choosing a subset of mitigation actions

As previous mentioned, operational risk managers together with process owners must rank the potential risk events. Furthermore, critical and high risk events should be all mitigated by means of suitable corrective actions. However, the Management of a company can adjust the aforementioned ranking, in coherence with its risk tolerance. Accordingly to this faculty, the Management can choose to mitigate only a subset of these events or rather extend the set of the risks upon which it is necessary to identify an intervention. In other words, they can either judge that the company has bigger fishes to fry or that a medium/low risk events could put the company on thin ice.

The company is safe and sound only when mitigating all the most relevant risks (better safe than sorry!). However, the budget might not suffice to cover all critical and high risks. In this context, the Management should choose a subset of mitigation actions to get the best sub-optimal situation, aiming at not throwing the baby out with the bathwater. An operational risk manager could help the Management providing a model to achieve the highest reduction in operational risk exposure under a budget constraint. Note that, if one looks more closely, we have already established a rule to prioritize risk events (i.e. the Analytic Hierarchy Process) and to identify the subset of risk events to mitigate (i.e. the 80/20 rule): now we deal with an additional problem related to the unavailability of an appropriate budget.

The following optimization problem represents a possible solution to choose the best subset of mitigation actions:

$$\begin{aligned} \max \quad & p^T y \\ s.t. \quad & cx \leq b \\ & y \leq Ax \leq my \\ & y \in \{0,1\}^n \\ & x \in \{0,1\}^m \end{aligned} \tag{2.4}$$

where m and n are the number of mitigation actions and risk events, respectively. Furthermore, p is the reduction of operational risk exposure correlated to each mitigant[8] and b is the available budget. Here $x_i = 1$ if the i-th mitigation action is chosen, and $x_i = 0$ otherwise. On the other hand, note that the constraint $y \leq Ax \leq my$ implies that $y_j = 1$ if at least one mitigation action covers the j-th risk event, and $y_j = 0$ otherwise. Thus, the model correctly aims at maximizing the score of all mitigated risks.

However, as previously mentioned, one should take the output of this model with a grain of salt. Indeed, the model provides

[8]We assume here that the reduction of operational risk exposure of a given mitigation action, is equal to the sum of the priorities (obtained with the Analytic Hierarchy Process) of the risk events that the action covers.

the optimal combination of mitigation actions in terms of priority, without caring about to the processes they secure. However, in a more refined version of the model, one can also include constraints on the minimum reduction for each process (considering the location of the risk events).

One of the most interesting feature of Model 2.4 is the possibility of performing a parametric analysis for different budgets. More precisely, one can obtain the optimal sets of mitigation actions obtained by increasing the available financial resources b. This can be used by the Management to decide whether an increase in the budget to mitigate operational risk exposure is justified by an increase in the score of the risks covered.

2.5 Case study

Imagine a bank wants to launch a new type of loan. The goal of the problem is to prioritize the most relevant operational risk events arising from the new product. The three departments (i.e. criteria) taken into account are (i) Front Office, (ii) Back Office,

	(i)	(ii)	(iii)
(**i**)**FrontOffice**	1	3	5
(**ii**)**BackOffice**	1/3	1	3
(**iii**)**MiddleOffice**	1/5	1/3	1

Table 2.5: Departments pairwise comparison matrix

and (iii) Middle Office. Given the following pairwise comparison matrix

one can easily calculate the normalized maximum eigenvector $\omega_{max}^{Dep} = (0.64 \;\; 0.26 \;\; 0.10)$.

The processes (i.e., sub-criteria) are: (a) Inquiry; (b) Deliberation; (c) Disbursement; (d) Recovery; (e) Monitoring; (f) Renewal. The alternatives are the possible breakdowns that can arise during the operations, identified by the process owners, which are the operational risk events. In this case study they are:

#1 Missed verification of prejudice absence and of previous/ pendants competition proceedings

#2 Data entry errors regarding customers/guarantees/other data for the evaluation of the credit worthiness

#3 Missing/ incomplete/ not updated documentation, related to third parties and required for the inquiry by internal procedures

#4 Missed analysis of the links between subject to inquiry and its legal/ economic membership group

#5 Deliberation of a loan beyond delegated powers

#6 Unavailability of ICT systems that support the deliberation of a loan

#7 Documentation for the execution of the contract falsifified by internal resources

#8 Missing/ incomplete/ not updated contracts

#9 Disavowed signature and/or signature of people without proper power

#10 Failure to notify credit transfer

#11 Hacking of ICT systems that support the delivery of a loan

#12 Delay or failure to activate credit recovery actions (unintentional)

#13 Failure to observe the terms for insinuations for concourse procedures

#14 Failure to activate credit recovery actions (intentional)

#15 Lack of preservation of physical goods given as collateral

#16 Failure to control or reintegrate value of collateral over time

#17 Errors in the preparation of the report on the status of mortgages granted and disbursed

#18 Unavailability of ICT systems that support the renewal of a loan

#19 Data entry errors regarding customers/guarantees/other data for the evaluation of the credit worthiness

#20 Errors in archiving customer documentation for the renewal of a loan (intentional)

Given the above cited elements for the problem, one can build the decisional hierarchy of Fig. 2.3 To complete the hierarchy, on one hand, process owners must evaluate the processes with

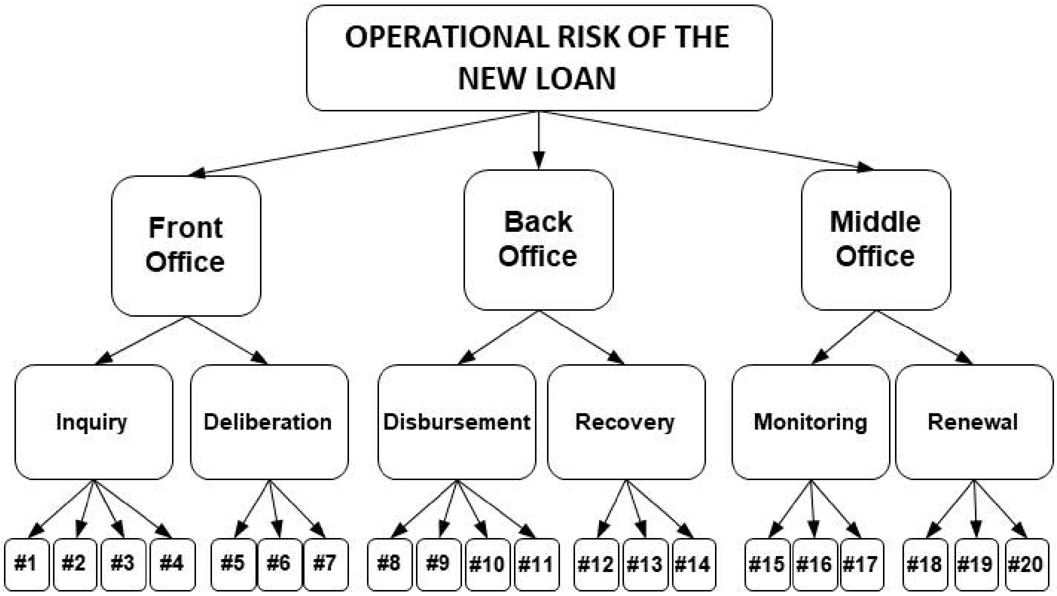


Figure 2.3: Decisional hierarchy

respect to their reference department and, on the other hand, the risk events with respect to their reference process. In both cases, process owners must follow the steps below

1. fulfil the pairwise comparison matrices;
2. check the consistency of judgments by means of the Consistency Ratio;
3. calculate the normalized eigenvector corresponding to the maximum eigenvalue;
4. convert the global scale priorities into the local scale.

	(i)	(ii)
(i)**Inquiry**	1	1/7
(ii)**Deliberation**	7	1

Table 2.6: Processes protected by the Front Office

	(i)	(ii)
(i)**Disbursement**	1	5
(ii)**Recovery**	1/5	1

Table 2.7: Processes protected by the Middle Office

Here we list the remaining pairwise comparison matrices needed to solve the problem. First, we compare the processes related to their reference departments:

Second, we list the pairwise comparison matrices of the risks potentially arising from the processes.

We report the local scale priorities of the elements considered in the analysis (for simplicity, we do not include all the pairwise comparison matrices) in Fig. 2.4: Applying the 80/20 rule we find

	(i)	**(ii)**
(i)Monitoring	1	9
(ii)Renewal	1/9	1

Table 2.8: Processes protected by the Back Office

	#1	**#2**	**#3**	**#4**
#1	1	1	3	1
#2	1	1	3	1/3
#3	1/3	1/3	1	1/3
#4	1	3	3	1

Table 2.9: Risks of the inquiry

	#5	**#6**	**#7**
#5	1	1/5	1/7
#6	5	1	1/3
#7	7	3	1

Table 2.10: Risks of the inquiry

	#8	**#9**	**#10**	**#11**
#8	1	3	1	1/9
#9	1/3	1	1	1/7
#10	1	1	1	1/9
#11	9	7	9	1

Table 2.11: Risks of the disbursement

	#12	**#13**	**#14**
#12	1	1	1/5
#13	1	1	1/5
#14	5	5	1

Table 2.12: Risks of the recovery

	#15	**#16**	**#17**
#15	1	1/7	1/5
#16	7	1	3
#17	5	1/3	1

Table 2.13: Risks of the monitoring

	#18	#19	#20
#18	1	1	3
#19	1	1	3
#20	1/3	1/3	1

Table 2.14: Risks of the renewal

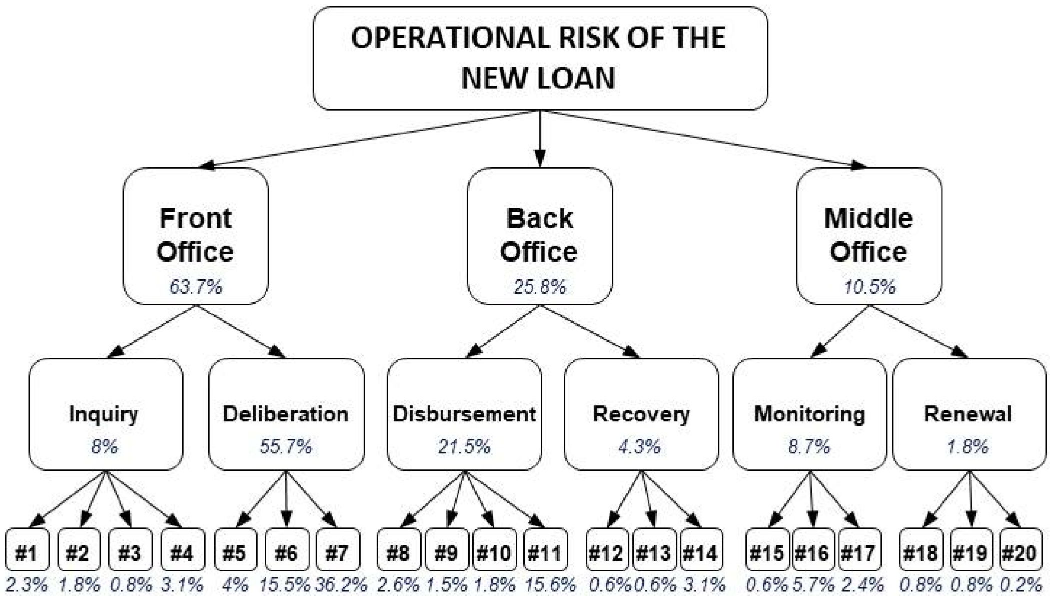


Figure 2.4: Local scale priorities of the elements

a unique "critical" risk (i.e. #7 *Documentation for the execution of the contract falsified by internal resources*) and four "high" risks (i.e. #11, #6, #16, and #5). To mitigate these five risk events, process owners hypothesize the following mitigation actions (into the square brackets we show their cost):

A Drawing a new process for handling the operation [50];

B Recruitment of 4 junior analysts [60];

C Recruitment of 2 senior analysts [100];

D Purchasing an external software to support the operation [120];

E Development of an internal tool to support the operation [180].

Into the boolean matrix 2.15 we show what risks are covered by each mitigation action (1 if the risk is covered, 0 otherwise) Solving the model of Sec. 2.4 one can optimize the cost of mitigation actions necessary to secure the most critical processes. In this case study, the optimal choice is the implementation of mitigation actions "A" and "D". Indeed, thanks to these two actions, one can mitigate all the "critical" and "high" risk events.

	A	**B**	**C**	**D**	**E**
#7	1	0	1	0	0
#11	0	0	0	1	1
#6	0	0	0	1	1
#16	1	1	1	0	1
#5	0	1	1	1	1

Table 2.15: Mitigation actions vs risk events

2.6 Conclusions

Our approach to assess the operational risk exposure arising from the launch of a new product provides several advantages. First, AHP allows to address the lack of information through a simplification of the decision-making process. Furthermore, we can control the process owner's cognitive bias. We are still able to estimate a rating for each operational risk event, thus making the output of the AHP more intuitive. Indeed, the main goal of the proposed methodology is to identify the most exposed internal process of a company and, consequently, to reduce the gaps of the control environment. In this context, the application of the "80/20 rule" helps to narrow down the environment of the most urgent mitigation actions.

The impossibility to implement appropriate mitigation actions, against the most significant operational risk events, can cause several impacts of different types, including material operating losses. These impacts can affect the success of the new product implementation project and, in extreme cases, can compromise the business continuity.

Thanks to the approach shown in this paper, the operational risk managers of a company operating in the financial sectors can carry out an operational risk assessment for the new product. This analysis falls into the broader perimeter of the new product approval process which involves the entire risk management department and should provide a useful tool to express an informed opinion on its feasibility.

Further studies are underway to investigate the possibility of improving the flexibility of the two optimization models shown above to be able to consider also partial mitigation actions (e.g., if a given mitigation action involves hiring 4 people, we want to investigate what can be the effect of hiring only 2 of them).

Bibliography

Acerbi C, Tasche D (2002) On the coherence of expected shortfall. J Bank Financ 26:1487–1503

Angelelli E, Mansini R, Speranza M (2008) A comparison of MAD and CVaR models with real features. J Bank Financ 32:1188–1197

Anthony Tony Cox L (2008) What's wrong with risk matrices? Risk analysis 28(2):497–512

Basel Committee, et al (2011) Principles for the sound management of operational risk. Basel, Switzerland: Bank for International Settlements Communications

Boros E, Hammer PL (2002) Pseudo-boolean optimization. Discrete Appl Math 123:155–225

Bruni R, Cesarone F, Scozzari A, Tardella F (2012) A new stochastic dominance approach to enhanced index tracking problems. Econ Bull 32:3460–3470

Bruni R, Cesarone F, Scozzari A, Tardella F (2013) No arbitrage and a linear portfolio selection model. Econ Bull 33:1247–1258

Cesarone F, Scozzari A, Tardella F (2009) Efficient algorithms for mean-variance portfolio optimization with hard real-world constraints. G Ist Ital Attuari 72:37–56

Cesarone F, Scozzari A, Tardella F (2013) A new method for mean-variance portfolio optimization with cardinality constraints. Ann Oper Res 205:213–234

Cesarone F, Moretti J, Tardella F (2014) Does Greater Diversification Really Improve Performance in Portfolio Selection? Available at SSRN: http://ssrncom/abstract=2473630

Cesarone F, Scozzari A, Tardella F (2015) Linear vs. quadratic portfolio selection models with hard real-world constraints. Comput Manage Sci 12:345–370

Cesarone F, Moretti J, Tardella F (2016) Optimally chosen small portfolios are better than large ones. Economics Bulletin 36(4):1876–1891

Cooke GLH R M (2004) Expert judgement elicitation for risk assessments of critical infrastructures. Journal of risk research 7(6):643–656

Dantzig T (1954) Number: The language of science: A critical survey written for the cultured non-mathematician. George Allen and Unwin

Dehaene S (2011) The number sense: How the mind creates mathematics. OUP USA

DeMiguel V, Garlappi L, Nogales F, Uppal R (2009a) A generalized approach to portfolio optimization: Improving performance by constraining portfolio norms. Manage Sci 55:798–812

DeMiguel V, Garlappi L, Uppal R (2009b) Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? Rev Financ Stud 22:1915–1953

European Banking Authority (2011) EBA Guidelines on Internal Governance (GL 44) London, 27 September 2011

Evans J, Archer S (1968) Diversification and the Reduction of Dispersion: An Empirical Analysis. J Financ 23:761–767

Fama E (1976) Foundations of finance: Portfolio decisions and securities prices. Basic Books New York

Forman EH, Selly MA (2001) Decision by objectives: how to convince others that you are right. World Scientific

Forman S E & Gass (2001) The analytic hierarchy process-an exposition. Operations research 49(4):469–486

Fragkiskos A (2013) What is Portfolio Diversification? Available at SSRN: http://ssrncom/abstract=2331475

Girling PX (2013) Operational risk management: a complete guide to a successful operational risk framework. John Wiley & Sons

Ingber J (2016) Managing the new product approval process. The RMA Journal pp 58–62

Ivković Z, Sialm C, Weisbenner S (2008) Portfolio concentration and the performance of individual investors. J Financ Quant Anal 43:613–655

Jacob N (1974) A limited-diversification portfolio selection model for the small investor. J Financ 29:847–856

Juran JM (1951) Quality-control Handbook. New York, McGraw-Hill

Kondor S Imre; Pafka, Nagy G (2007) Noise sensitivity of portfolio selection under various risk measures. Journal of Banking & Finance 31(5):1545–1573

Konno H, Yamazaki H (1991) Mean-absolute deviation portfolio optimization model and its application to Tokyo stock exchange. Manage Sci 37:519–531

Lysons K, Farrington B (2006) Purchasing and supply chain management. Pearson Education

Markowitz H (1952) Portfolio selection. J Financ 7:77–91

Markowitz H (1959) Portfolio selection: Efficient diversification of investments. Cowles Foundation for Research in Economics at Yale University, Monograph 16, John Wiley & Sons Inc., New York

Meucci A (2009) Managing diversification. Risk 22:74–79

Michaud R (1989) The Markowitz Optimization Enigma: Is 'Optimized' Optimal? Financ Anal J 45:31–42

Mustafa MA, Al-Bahar JF (1991) Project risk assessment using the analytic hierarchy process. IEEE transactions on engineering management 38(1):46–52

Newbould G, Poon P (1993) The minimum number of stocks needed for diversification. Finan Practice Educ 3:85–87

Niemira MP, Saaty TL (2004) An analytic network process model for financial-crisis forecasting. International Journal of Forecasting 20(4):573–587

Pareto V (1964) Cours d'économie politique, vol 1. Librairie Droz

Pflug G, Pichler A, Wozabal D (2012) The 1/N investment strategy is optimal under high model ambiguity. J Bank Financ 36:410–417

Rachev S, Stoyanov S, Fabozzi F (2008) Advanced stochastic models, risk assessment, and portfolio optimization: The ideal risk, uncertainty, and performance measures, vol 149. Wiley

Rockafellar R, Uryasev S (2000) Optimization of Conditional Value-at-Risk. J Risk 2:21–42

Saaty RW (1987) The analytic hierarchy processï£¡what it is and how it is used. Mathematical modelling 9(3-5):161–176

Saaty T (2005) The analytic hierarchy and analytic network processes for the measurement of intangible criteria and for decision-making. Multiple criteria decision analysis: state of the art surveys pp 345–405

Saaty TL (1977) A scaling method for priorities in hierarchical structures. Journal of mathematical psychology 15(3):234–281

Saaty TL (1988) What is the analytic hierarchy process? In:

Mathematical models for decision support, Springer, pp 109–121

Saaty TL (1990) How to make a decision: the analytic hierarchy process. European journal of operational research 48(1):9–26

Saaty TL (2003) Decision-making with the ahp: Why is the principal eigenvector necessary. European journal of operational research 145(1):85–91

Scandizzo S (2010) The Operational Risk Manager's Guide: Tools and Techniques of the Trade. Risk books

Skjong R, Wentworth BH (2001) Expert judgment and risk perception. In: The Eleventh International Offshore and Polar Engineering Conference, International Society of Offshore and Polar Engineers

Speranza M (1993) Linear programming models for portfolio optimization. Finance 14:107–123

Statman M (1987) How many stocks make a diversified portfolio? J Financ Quant Anal 22:353–363

Tang G (2004) How efficient is naive portfolio diversification? an educational note. Omega 32:155–160