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Kinematic collapse load calculator: Circular arches

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ABSTRACT

Masonry arches and their typical failure do not fall elegantly into standard design and analysis methods. The system is highly dependent on geometry and failure is dominated by mechanization, not material strength. Focusing directly on the mechanized failure, this work presents the kinematic collapse load calculator (KCLC) for circular arches. The KCLC, a MATLAB® based graphical user interface, provides a simple interactive limit analysis of any ideal semi-circular masonry arch subjected to either an asymmetric point load or constant horizontal acceleration. After defining key geometric factors, the KCLC analyses the arch for any selected and kinematically admissible hinge configuration. For a selected configuration, an equilibrium approach to the upper bound theorem of limit analysis is used to calculate the collapse load multiplier and hinge reactions. The resulting collapse condition values are displayed and used to plot the thrust line that maintains a zero moment at the hinges. Designed primarily as an educational tool, the KCLC also provides a simple and efficient foundation for adapting to different arch geometries and loading conditions.

Code metadata

Current code version	V1
Permanent link to code/repository used of this code version	https://github.com/ElsevierSoftwareX/SOFTX_2018_40
Legal Code Licence	MIT licence
Code versioning system used	None
Software code languages, tools, and services used	MATLAB
Compilation requirements, operating environments & dependencies	MATLAB
If available Link to developer documentation/manual	None
Support email for questions	gabriellee.stockdale@polimi.it

1. Motivation and significance

It has been recently argued that the stability-based design techniques of structural masonry have the potential to become a sustainable building method for modern constructions [1]. Several obstacles must be overcome to reintroduce structural masonry, and specifically, curved masonry as a viable design alternative. Among these, the need for simplified design and analysis techniques as well as the training of engineers are the most relevant.

Additionally, masonry arches encompass a significant portion of European cultural heritage with countless examples in churches and buildings. Italy, Spain and the United Kingdom together have more than 52,000 masonry arch bridges in active use [2–5]. Several strategies, analysis techniques, and experimental investigations for the assessment of existing structures are available for masonry arches [2,6,7]. Limit analysis is prominently used both on its own [8–11] and as a tool for assessments on a larger scale [12–16]. More refined techniques are also employed for detailed investigations on masonry arch bridges, especially when they deal with more complex aspects of their structural behaviour [17–22]. Together, these techniques and their appropriate applications provide a sound foundation for the understanding of most arch conditions, but the labour and computational costs are often very high. Thus, the need for simplified design and analysis techniques.

The purpose of the Kinematic Collapse Load Calculator (KCLC) is to provide an interactive tool to simplify and aid in the understanding of the mechanized failure of masonry arches, and to provide a platform with the potential to become an efficient and robust structural analysis tool for masonry arches. Moreover, the KCLC is specifically developed for assessing kinematic admissibility and, unlike other software dealing with masonry arches, it adopts

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a simple approach aimed at controlling instead of identifying the collapse mechanism.

2. Software description

The KCLC is a stand-alone interactive graphical user interface developed in MATLAB® for the limit analysis of semi-circular masonry arches subjected to two of the most common loading conditions: asymmetric point load and constant horizontal acceleration. The KCLC takes the user-specified geometric parameters of a circular arch and constructs an interactive analysis through a combination of displayed output data and the ability to change the kinematic mechanism through adjusting the hinge locations. Fig. 1 shows the KCLC user interface.

2.1. Software architecture

As seen in Fig. 1, the interface consists of two main interactive components, output data display, plot window, and instructions. The interactive components are the input data and hinge selections. The displayed instructions are:

i. Provide input data

    N — Number of blocks
    R — Intrados radius [m]
    T_R — Thickness-to-radius ratio
    Depth — Block depth [m]
    Density — Block density [kg/m^3]

ii. Choose load type

    PL — Asymmetric point load
    H_acc — Constant horizontal acceleration

iii. Press "RUN"
iv. Adjust hinge locations to determine collapse load
v. Press "RUN" to reset plot if changes are made to input data
vi. Press "CLEAR" to clear all data

NOTE: Thrust line established from eccentricity required to maintain zero moment at hinge 1.

2.1.1. Input data and programme initiation

As stated in the instructions, the input data include the arch input parameters and load type selection. The arch data required are the number of blocks, intrados radius, thickness-to-radius ratio, depth, and density. The load type is chosen by selecting the appropriate check box.

When the "RUN" button is pressed, the input data is checked against the allowed parameters of the programme. The first parameter is that all input data must be specified. Next, the number of blocks must be an odd number greater than five. The block restrictions arise from the requirement of a keystone and the ability to adjust hinge locations. The thickness-to-radius ratio must be between 0.11 and 0.33 which establish the general limits of stability and the potential for mechanization failure respectively. If the input data does not meet these requirements an error message is displayed indicating the error. Fig. 2 shows the various error messages with the input data that produced the error.

If there are no errors in the input data, then after pushing the "RUN" button, then the block boundary points are established; the initial value and limits of the hinges are defined; the arch, hinges and loading condition are drawn in the plot window; and the resulting conditions are passed to the developed evaluation function ("Eval") in the MATLAB® script. The "Eval" function performs the limit analysis on the set arch-hinge-load combination, displays the output data, and adds the thrust line to the plot. Fig. 3 shows the KCLC after the "RUN" button is pushed for both load types.

2.1.2. Hinge selection

After the KCLC is initiated, the hinge sliders are activated with the established limits. The limits of the hinge locations are established to maintain the requirements of a kinematically admissible hinge set. These requirements include the alternation of adjacent hinge positions on the intrados and extrados, starting with hinge 1 on the extrados for the given load cases. Additionally, hinges cannot cross the keystone.

Two hierarchies are established for the hinge limits to maintain admissibility. First, the limits of hinges 1 and 4 control the boundaries of the mechanism and are set as the primary hinges, or base hinges. These hinges have limits set as the base of the arch and one joint short of the keystone on their perspective sides. Hinges 2 and 3 are the secondary hinges. They are restricted to exist between the keystone and their respective base hinge. If the position of hinges 1 or 4 changes, then the limits of hinge 2 or 3 are consequently updated. If the change in hinges 1 or 2 place their position at the same joint as the secondary hinge, then the secondary hinge is moved one joint towards the keystone. Finally, if a base hinge is positioned at its limit near the keystone, then the slider of the respective secondary hinge is deactivated until the base hinge is moved.

2.1.3. Output data

The output data include the calculated reaction forces at each hinge and the value of collapse load multiplier, which are displayed for each admissible mechanism. Since the hinge selection method ensures that the hinge set geometry is kinematically admissible, the admissibility of a given mechanism is only dependent on the calculated results. If a defined mechanism is determined to be inadmissible, then the reaction forces are not displayed, and the output of the collapse load multiplier reads "Not Admissible". The "Eval" function contains the evaluation of admissibility.

2.1.4. Plot window

The plot window is where the arch and hinges are plotted. The thrust line is also added if the mechanism is admissible as evaluated in the "Eval" function. The plot window is updated each time a hinge position is changed or the "RUN" button is pushed.

2.2. Software functionalities

As previously mentioned, the "Eval" function houses the limit analysis calculations and evaluations, displays the results and plots the thrust line. Whenever the "RUN" button is pressed, or a hinge position is changed the "Eval" function is called. In order to effectively understand the "Eval" function, it is first necessary to understand the applied limit analysis approach. Note that this knowledge is not required to operate the interface, only to understand how the results are obtained. The same holds for the thrust line.

2.2.1. Limit analysis

Limit analysis is regarded as the most reliable tool for masonry arch analysis [5,23]. The upper bound theorem of limit analysis, the rigid-no-tension model and assumption of no block slip provide the framework for the approach applied in the KCLC [24]. The upper bound theorem is also known as the kinematic approach and states that an arch will collapse if there exists a kinematically admissible mechanism producing zero or positive work. For both the point load and constant horizontal acceleration conditions a kinematically admissible mechanism requires four hinges that alternate between the intrados and extrados. Fig. 4 shows an example of an admissible mechanism. Fig. 4 also shows the mechanical arch represented as three rigid elements connected by four pins for both loading conditions.

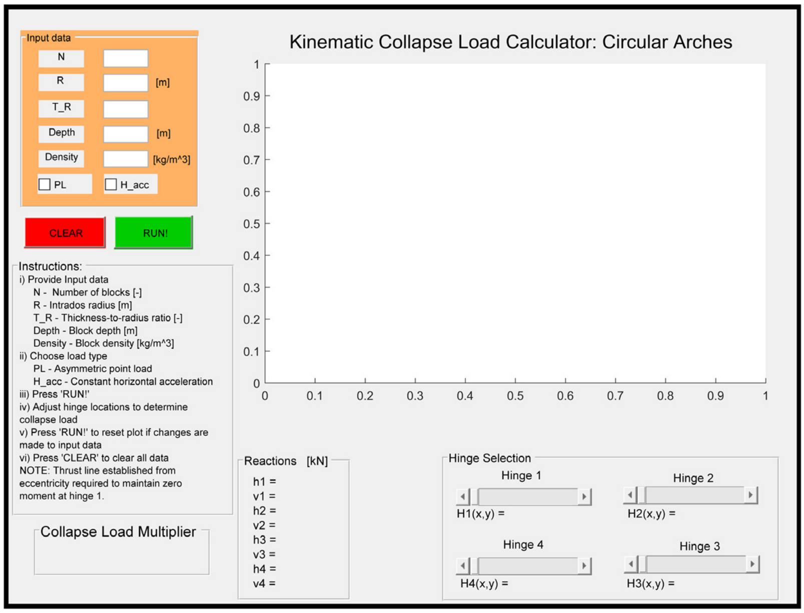


Fig. 1. Image of the KCLC interface upon execution of the MATLAB® script.

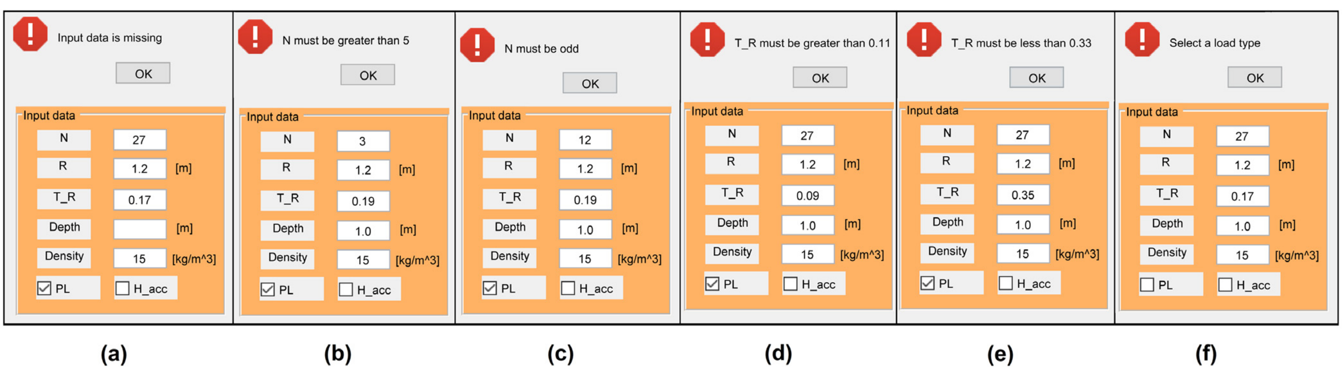


Fig. 2. The error messages and the input data for (a) missing data, (b) too few blocks, (c) even block count, (d) too thin of an arch, (e) too thick of an arch and (f) no selected load type.

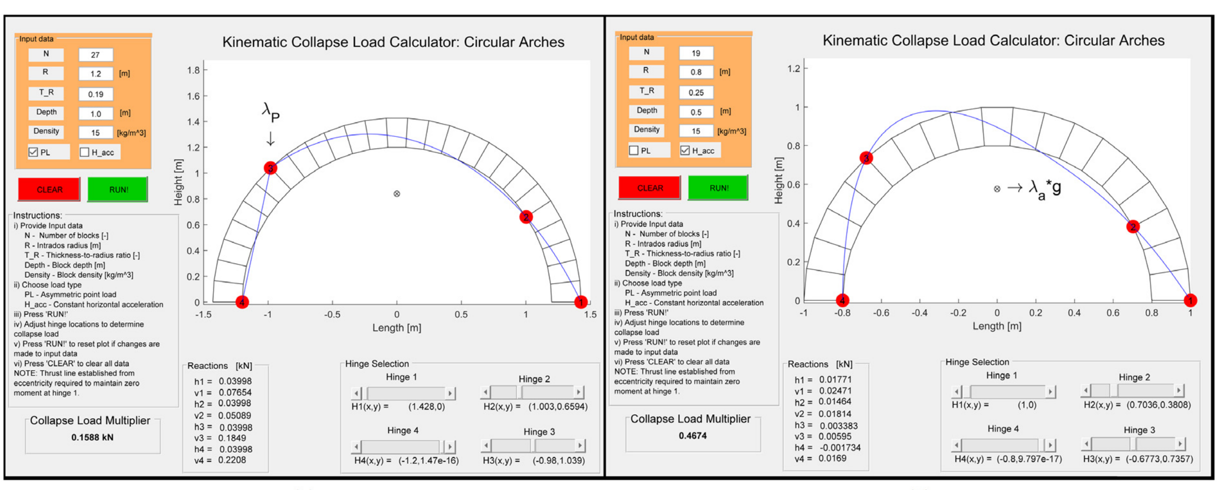
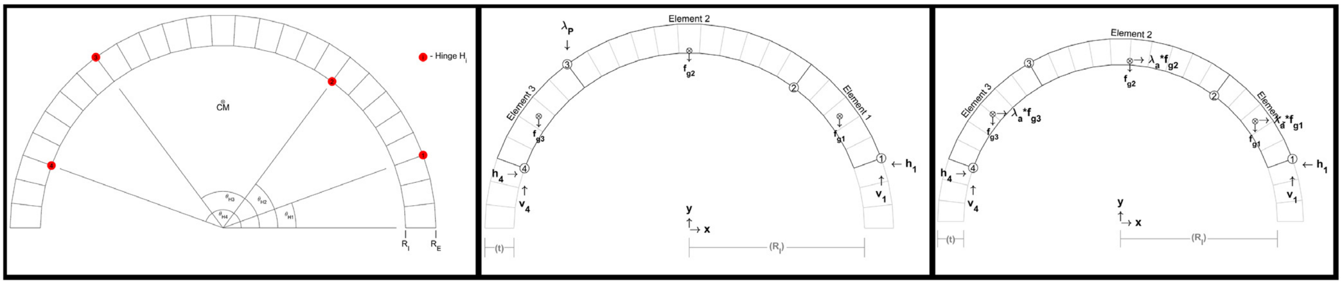


Fig. 3. Initial display of the KCLC for the (a) point load and (b) horizontal acceleration loading condition after pressing the "RUN" button.

A redundant system of nine equations and eight unknowns is established by the equilibrium conditions of Fig. 4. However, the inclusion of the loading condition as a pseudo-reaction produces a determinate system. The collapse condition and hinge reactions



(a) (b) (c)

Fig. 4. An (a) example of a kinematically admissible mechanism and the mechanism represented by three rigid elements connected by four hinges for the (a) constant horizontal and (b) asymmetric point loading conditions.

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & f_{g1} \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\Delta y_{2,1} & \Delta x_{1,2} & 0 & 0 & 0 & 0 & -f_{g1}\Delta y_{CM1,1} \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & f_{g2} \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta y_{3,2} & \Delta x_{2,3} & 0 & 0 & f_{g2}\Delta y_{2,CM2} \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & f_{g3} \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Delta y_{3,4} & -\Delta x_{3,4} & f_{g3}\Delta y_{3,CM3} \end{bmatrix} \begin{bmatrix} h_1 \\ v_1 \\ h_2 \\ v_2 \\ h_3 \\ v_3 \\ h_4 \\ v_4 \\ \lambda_a \end{bmatrix} = \begin{bmatrix} 0 \\ f_{g1} \\ -f_{g1}\Delta x_{1,CM1} \\ 0 \\ f_{g2} \\ f_{g2}\Delta x_{2,CM2}0 \\ f_{g3} \\ -f_{g3}\Delta x_{3,CM3} \end{bmatrix} \quad (1)$$

Box I.

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\Delta y_{2,1} & \Delta x_{1,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & \Delta y_{3,2} & \Delta x_{2,3} & 0 & 0 & -\Delta x_{2,3} \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Delta y_{3,4} & -\Delta x_{3,4} & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ v_1 \\ h_2 \\ v_2 \\ h_3 \\ v_3 \\ h_4 \\ v_4 \\ \lambda_a \end{bmatrix} = \begin{bmatrix} 0 \\ f_{g1} \\ -f_{g1}\Delta x_{1,CM1} \\ 0 \\ f_{g2} \\ f_{g2}\Delta x_{2,CM2} \\ 0 \\ f_{g3} \\ -f_{g3}\Delta x_{3,CM3} \end{bmatrix} \quad (2)$$

Box II.

can then be solved for a given hinge set through the equilibrium equations. With perfect hinges and by summing the moments of elements 1, 2, and 3 at hinges 1, 2, and 3, respectively, the equilibrium equations can be expressed in matrix form as the equation given in Box I for the constant horizontal acceleration condition and the equation given in Box II for the asymmetric point load. In Eqs. (1) and (2), h_i and v_i are the horizontal and vertical reaction forces at the ith hinge, respectively, and f_{gj} is the body force of the jth element applied at the element's centroid. The horizontal lever arms, Δx, and the vertical lever arms, Δy, have subscripts that denote the hinge number or the element's centre of mass location (i.e. $\Delta y_{2,1}$ is $(y_2–y_1)$, $\Delta x_{1,CM1}$ is $(x_1 - x_{CM1})$, etc.). Finally, λ_a and λ_P are the collapse load multipliers for the acceleration in terms of the gravitational constant g and for the point load respectively.

Eqs. (1) and (2) can be expressed in general terms as

$$[C]\{r\} = \{b\} \quad (3)$$

hence the hinge reaction forces and collapse multiplier for a given kinematically admissible hinge set can be determined through simple matrix manipulation

$$\{r\} = [C]^{-1}\{b\}. \quad (4)$$

Also note that for the case of the point load the multiplier is actually the collapse load value, and the location of the point load is at the location of hinge three.

The final stage in this limit analysis procedure is to check for violations in the solutions themselves. The equilibrium conditions do not consider the no-tension assumption, nor the rules applied to an admissible mechanism. As such, these rules must be enforced after the reactions and collapse load are calculated. An admissible mechanism requires that the collapse load multiplier is positive. The no-tension assumption is enforced by comparing the direction of the net reaction force against the orientation of the block boundary line. If the net reaction crosses the boundary line, then the no-tension rule is violated, and the mechanism is not admissible.

2.2.2. Thrust line

The thrust line is a line that represents the flow of concentrated compressive forces through the arch. For the arch to be stable, the line of thrust must exist within the material of the arch. A hinge forms once this line reaches an arch boundary. Since the hinge locations are known and the reactions calculated, the thrust line can be established at each section along the arch by evaluating the equilibrium condition at each section against hinge 1. The zero moment at hinge 1 and lack of tensile capacity in the arch result in the ability to identify the eccentricity required in the concentrated load to maintain the zero moment at hinge 1. Eccentricity is taken from the centreline of the arch and adjusted radially. For the circular arch, the radial eccentricity equation is

$$e_R = \frac{\Pi_R - (A_R + B_R) R_{cl}}{A_R - B_R} \tag{5}$$

with the values Π_R, A_R and B_R being variables established by the combination of arch geometry and load type, and R_{cl} is the centreline of the semi-circular arch. For the constant horizontal acceleration load type, the eccentricity variables of Eq. (5) are

$$\begin{aligned} \Pi_R &= f_g' \left(\Delta x_{1,CM'} - \lambda_a \Delta y_{CM',1}\right) + \left(v_1 - f_g'\right) x_1 + \left(h_1 - \lambda_a f_g'\right) y_1 \\ A_R &= \left(h_1 - \lambda_a f_g'\right) \sin\left(\theta'\right) \\ B_R &= \left(v_1 - f_g'\right) \cos\left(\theta'\right) \end{aligned} \tag{6}$$

where f_g' and the subscript *CM'* are the gravitational force and centre of mass location of the arch segment between hinge one and the evaluation section, respectively. The values x_1 and y_1 are the cartesian coordinates of hinge 1, and θ' is the polar angle of the arch section under evaluation.

For the asymmetric point load condition, two sets of eccentricity variables for Eq. (5) must be established to account for the discontinuity imposed by the point load. Therefore, for the first set is

$$\begin{aligned} \Pi_R &= f_g' \left(\Delta x_{1,CM'}\right) + \left(v_1 - f_g'\right) x_1 + (h_1)\, y_1 \\ A_R &= (h_1) \sin\left(\theta'\right) \\ B_R &= \left(v_1 - f_g'\right) \cos\left(\theta'\right) \end{aligned} \tag{7}$$

and is valid for arch sections between hinges 1 and 3. The eccentricity variables for the arch section between hinges 3 and 4 are

$$\begin{aligned} \Pi_R &= f_g' \left(\Delta x_{1,CM'}\right) + \left(v_1 - \lambda_P - f_g'\right) x_1 + (h_1)\, y_1 + \lambda_P \Delta x_{1,3} \\ A_R &= (h_1) \sin\left(\theta'\right) \\ B_R &= \left(v_1 - \lambda_P - f_g'\right) \cos\left(\theta'\right) \end{aligned} \tag{8}$$

3. Illustrative examples

The collapse condition of the masonry arch is determined by finding the minimum collapse load multiplier that contains the thrust line entirely within the boundaries of the arch. Therefore, the collapse condition can be established with the KCLC by examining the thrust line and comparing different collapse load multipliers. To demonstrate this and validate the KCLC, two examples are presented, one for each load type.

3.1. Asymmetric point load

For the asymmetric point load a 27-block arch with an internal radius of 1.806 m, a thickness to radius ratio of 0.1661, a depth of 0.250 m, a block density of 1530 kg/m^3, and a vertical point load applied eight joints up from the left base is evaluated with the KCLC. The minimum configuration is determined by inputting the arch data, adjusting hinge 3 (i.e. the point load) to the specified location and then adjusting the remaining three hinges to find the minimum collapse load with the thrust line entirely inside the arch. This process can be seen in Fig. 5. The resulting collapse load is 2.751 kN.

The results are compared directly with those obtained through the principle of virtual powers. The principle of virtual powers allows the collapse load to be determined by

$$\lambda_p = \frac{f_{g1}u_1 + f_{g2}u_2 + f_{g3}u_3}{u_p} \tag{9}$$

where u_i and u_P are the vertical displacements of the ith element's centre of mass and point load, respectively. A graphical approach is utilized to obtain the vertical displacements and can also be seen in Fig. 5. The resulting collapse load from the virtual powers approach is 2.756 kN.

Table 1
Comparison of collapse condition obtained from the KCLC and by Como [25].

	Hinge 1 [°]	Hinge 2 [°]	Hinge 3 [°]	Hinge 4 [°]	λ_a [%]
Como	0	39	101	155	14.17
KCLC	0	40	98	155	13.87

3.2. Constant horizontal acceleration

The constant horizontal acceleration load type is compared against the arch analysed by Como with an internal radius of 7.5 m and a thickness to radius ratio of 0.16 [25]. The block count is set to 181 to obtain an approximately 1° block angle. The applied density is 1530 kg/m^3 and the depth at 1 m. The minimum collapse condition is determined by adjusting the hinges until the minimum collapse load multiplier with an enclosed thrust line is determined. The KCLC initialization and the determined minimum collapse condition are shown in Fig. 6. Table 1 shows the comparison of the KCLC results with those obtained by Como with the hinge locations given in polar form [25].

The comparison of the results is good with the KCLC resulting in a slightly more conservative result and only one noteworthy variation in hinge location at hinge 3. Additionally, adjusting the KCLC to match the Como hinge configuration produces a multiplier of 14.06% as can also be seen in Fig. 6.

4. Impact

The KCLC provides a stand-alone analysis tool for the upper bound limit analysis of semi-circular masonry arches. It requires no understanding of the analysis techniques used, which enables it to become an effective educational tool for teaching the concepts of the kinematic theorem as well as the thrust line and stability. The ability to control the arch input parameters allows the effects these parameters have on the analysis to be studied. Finally, the ability to control the hinge positions provides the opportunity to gain an insight into the relationships between stability, kinematically admissible mechanisms and the overall strength of the system.

The KCLC is specifically designed as an educational tool to aid in understanding and visualizing the failure behaviour of masonry arches. Masonry arches behave distinctly different than more modern structural systems and this tool is developed directly from that behaviour. It is not an adaptation of methods developed for steel or reinforced concrete. Additionally, the equilibrium approach is simple, geometry based, and produces closed form solutions. For practitioners, the KCLC provides a simple and effective foundation for masonry arch analysis. Equilibrium conditions are relatively easy to construct for different load conditions, arch geometries can potentially be imported from drafting software, and the closed form solutions allow for easy modifications resulting from experimentations.

5. Conclusions

Given the potential for structural masonry to become a sustainable building method and considering the extensive amount of masonry arches used in building and infrastructures, there is a need for simplified design and analysis techniques and more engineers need to be trained in these techniques. The purpose of the KCLC is to address this need and provide an interactive tool that simplifies and aids in the understanding of the mechanized failure of masonry arches. Additionally, the KCLC provides a foundation with the potential to become an efficient and robust structural analysis tool for masonry arches.

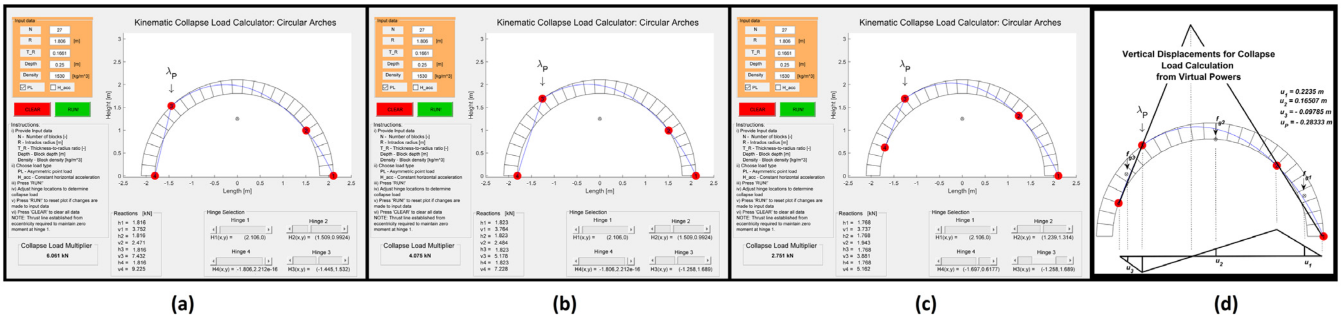


Fig. 5. The KCLC (a) initialization, (b) point load placement, (c) minimum collapse configuration and (d) the graphical approach used to compare results for the point load condition.

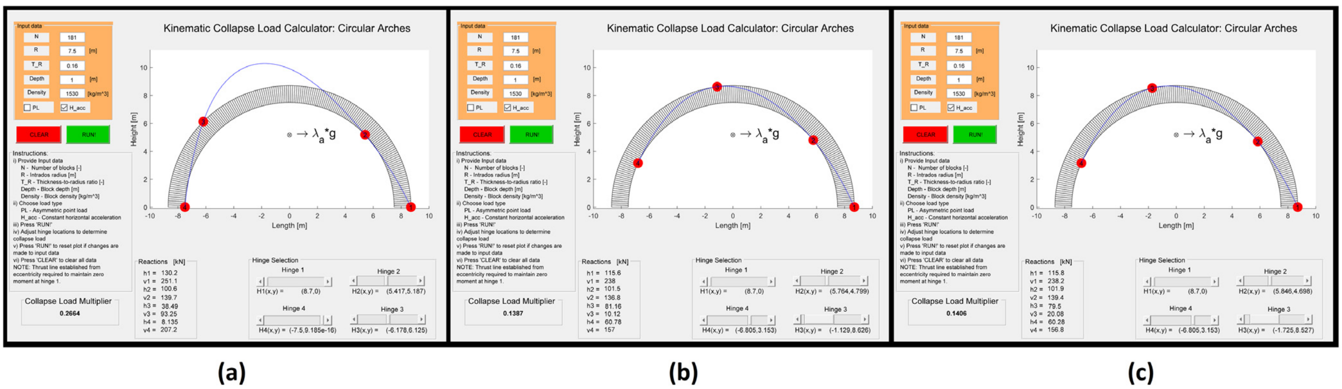


Fig. 6. The KCLC (a) initialization, (b) determined minimum collapse condition and (c) the configuration from the Como arch [25].

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