



Title	Quantized Event-Triggered Control of Discrete-Time Linear Systems with Switching Triggering Conditions
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Citation	IEICE transactions on fundamentals of electronics communications and computer sciences, E101-A(2), 322-327 https://doi.org/10.1587/transfun.E101.A.322
Issue Date	2018-02
Doc URL	http://hdl.handle.net/2115/70884
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Type	article
File Information	Quantized event-triggered control of discrete-time linear systems with switching triggering conditions.pdf



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Quantized Event-Triggered Control of Discrete-Time Linear Systems with Switching Triggering Conditions

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SUMMARY Event-triggered control is a method that the control input is updated only when a certain triggering condition is satisfied. In networked control systems, quantization errors via A/D conversion should be considered. In this paper, a new method for quantized event-triggered control with switching triggering conditions is proposed. For a discrete-time linear system, we consider the problem of finding a state-feedback controller such that the closed-loop system is uniformly ultimately bounded in a certain ellipsoid. This problem is reduced to an LMI (Linear Matrix Inequality) optimization problem. The volume of the ellipsoid may be adjusted. The effectiveness of the proposed method is presented by a numerical example. **key words:** event-triggered control, quantization, linear matrix inequality (LMI), networked control systems

1. Introduction

The IoT (Internet of Things) has attracted much attention in many research fields such as control engineering and communication engineering. The IoT is one of the platforms in systems consisting of software, sensors, actuators, and network connectivity that enables these objects to collect and exchange data (see, e.g., [6]). A networked control system (NCS) plays an important role in the IoT. An NCS is a control system where components such as plants, sensors, and actuators are connected through communication networks. Hence, theory of NCSs may be regarded as that of the IoT.

In NCSs, it is important to decrease the number of sent and received messages without degradation of control performance. From this viewpoint, event-triggered and self-triggered control methods have been studied as an aperiodic control method (see e.g., [1], [2], [5], [9]–[21], [23]–[26]). The basic idea of event-triggered control is that transmissions of the measured signal and the control input are executed, only when a certain triggering condition on the measured signal is satisfied (i.e., the event occurs). The basic idea of self-triggered control is that the next sampling time at which the control input is recomputed is computed based on predictions.

In this paper, we propose a new method for quantized event-triggered control. In NCSs, it is important to consider quantization errors via A/D conversion. Some results have been obtained so far (see, e.g., [7], [8], [22]). In these results, asymptotic stabilization using a quantized event-triggered

controller has been mainly considered. However, under the existence of quantization errors, other control performance may be better. In [26], the notion of uniformly ultimately boundedness [3] has been utilized under the existence of disturbances. This notion is utilized in also this paper (see the next section for further details), but the method proposed in [26] cannot be directly applied to quantized event-triggered control. This is because quantization errors are included in a triggering condition, and it is difficult to model the state error.

In the proposed method, the controller is given by a state-feedback controller, where the measured state is quantized by a uniformed quantizer. In the triggering condition, the difference between the current quantized state and the quantized state sent in past times is evaluated. When this difference is greater than a certain threshold value, the current quantized state is sent to the controller, and the control input is updated. In [26], the threshold value is given using the state (see also Remark 1). In the proposed method, two threshold values are given by a constant. These values are switched according to a certain condition. For example, it is desirable that the threshold value becomes smaller at a neighborhood of the origin. In such case, a switch of the threshold value is effective. Using the result in [26], the problem of finding a state-feedback gain is reduced to multiple LMI feasibility problems (or LMI optimization problems). The effectiveness of the proposed method is presented by a numerical example.

This paper is organized as follows. In Sect. 2, the problem of finding a quantized event-triggered controller is formulated. In Sect. 3, a solution method for this problem is derived. First, the problem is reduced a BMI (bilinear matrix inequality) feasibility problem. After that, it is reduced to LMI feasibility/optimization problems. In Sect. 4, a numerical example is presented. In Sect. 5, we conclude this paper.

Notation: Let \mathcal{R} denote the set of real numbers. Let I and 0 denote the identity matrix with the appropriate size and the zeros matrix with the appropriate size, respectively. For a scalar $a \in \mathcal{R}$, let $\lceil a \rceil$ denote the ceiling function of a . Let $\mathbf{1}_n$ denote the n -dimensional vector whose elements are all one. For a vector x , let $\|x\|$ denote the Euclidean norm of x . For a matrix M , let M^T denote the transpose matrix of M . For a matrix M , let $\text{tr}(M)$ denote the trace of M . For matrices M_1, M_2, \dots, M_n , let $\text{diag}(M_1, M_2, \dots, M_n)$ denote the block-diagonal matrix. For a positive definite matrix P and a scalar γ , the ellipsoid $\mathcal{E}(P, \gamma) := \{x \in \mathcal{R}^n \mid x^T P x \leq \gamma\}$

Manuscript received April 14, 2017.

Manuscript revised August 28, 2017.

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DOI: 10.1587/transfun.E101.A.322

is defined. The symmetric matrix $\begin{bmatrix} A & B^\top \\ B & C \end{bmatrix}$ is denoted by $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$.

2. Problem Formulation

Consider the following discrete-time linear system:

$$x(k+1) = Ax(k) + Bu(k), \quad (1)$$

where $x(k) \in \mathcal{R}^n$ is the state, $u(k) \in \mathcal{R}^m$ is the control input, and $k \in \{0, 1, 2, \dots\}$ is the discrete time. We assume that all states are measured, but are quantized using the uniform quantizer $q(\cdot) : \mathcal{R}^n \rightarrow \mathcal{R}^n$ satisfying the following condition:

$$q(x(k)) - \frac{d}{2}\mathbf{1}_n \leq x(k) < q(x(k)) + \frac{d}{2}\mathbf{1}_n, \quad (2)$$

where $d > 0$ is a given scalar. We remark here that the maximum value of the quantization error $\|x(k) - q(x(k))\|$ is given by $d\sqrt{n}/2$. The event-triggering condition is given by

$$\|\hat{x}(k-1) - q(x(k))\| > \sigma, \quad (3)$$

where $\sigma > 0$ is a given scalar. Then, for the system (1), consider the following quantized event-triggered state-feedback controller:

$$u(k) = K\hat{x}(k), \quad (4)$$

where

$$\hat{x}(k) = \begin{cases} q(x(k)) & \text{if (3) holds,} \\ \hat{x}(k-1) & \text{otherwise,} \end{cases} \quad (5)$$

and $\hat{x}(0)$ is given in advance.

In this paper, the parameter σ in (3) is switched as follows:

$$\sigma = \begin{cases} \sigma_1 & \text{if } q(x(k)) \notin \mathcal{E}(P, 1) \text{ holds,} \\ \sigma_2 & \text{if } q(x(k)) \in \mathcal{E}(P, 1) \text{ holds,} \end{cases} \quad (6)$$

where σ_1 and σ_2 are given scalars satisfying $\sigma_1 \geq \sigma_2 \geq 0$. The positive-definite matrix P should be designed. See Remark 2 for implementation of (6).

Next, we introduce the following definition [3].

Definition 1: The system (1) with the controller (4) is said to be uniformly ultimately bounded (UUB) in a convex and compact set \mathcal{S} containing the origin in its interior, if for every initial condition $x(0) = x_0$, there exists $T(x_0)$ such that for $k \geq T(x_0)$ and $T(x_0) \in \{0, 1, 2, \dots\}$, the condition $x(k) \in \mathcal{S}$ holds.

Under these preparations, we consider the following problem.

Problem 1: For the discrete-time linear system (1), find a quantized event-triggered state-feedback controller (4) such

that there exists an ellipsoid $\mathcal{E}(P, 1)$, in which the system with the controller (4) is UUB.

Remark 1: In conventional event-triggered control, the event-triggering condition is given by the form of $\|\hat{x}(k-1) - x(k)\| > \sigma\|x(k)\|$ (see, e.g., [11], [20], [25], [26]). In this paper, for considering quantization errors, the simple condition (3) is utilized. However, it is not practical to evaluate $\|\hat{x}(k-1) - x(k)\|$ by only a constant. In addition, precise control is required within the ellipsoid $\mathcal{E}(P, 1)$. Hence, we introduce switching of σ .

3. Solution Method

As a preparation, the error variable is defined as follows:

$$e(k) := \hat{x}(k) - x(k).$$

Then, noting that $\sigma_1 \geq \sigma_2 \geq 0$, the following relation holds:

$$\|e(k)\| \leq \sigma_1 + \frac{d\sqrt{n}}{2}. \quad (7)$$

Using $e(k)$, the closed-loop system is given by

$$x(k+1) = \Phi x(k) + BK e(k), \quad \Phi = A + BK. \quad (8)$$

In the solution method, we consider the following two cases: i) $x(k) \notin \mathcal{E}(P, 1)$ and ii) $x(k) \in \mathcal{E}(P, 1)$.

First, consider the case of $x(k) \notin \mathcal{E}(P, 1)$. In this case, we introduce the following Lyapunov function:

$$V(k) = x^\top(k)Px(k).$$

We consider the problem of finding a controller satisfying the following condition:

$$V(k+1) - V(k) < -\beta V(k), \quad (9)$$

where $\beta \in [0, 1)$ is a given scalar. Then, we can obtain the following lemma.

Lemma 1: (9) holds if the following relation holds:

$$P_0 - \kappa_1 P_1 - \kappa_2 P_2 > 0, \quad (10)$$

where

$$P_0 = \begin{bmatrix} \bar{\beta}P - \Phi^\top P\Phi & * & * \\ -K^\top B^\top P\Phi & -K^\top B^\top PBK & * \\ 0 & 0 & 0 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 0 & * & * \\ 0 & -I & * \\ 0 & 0 & (\sigma_1 + \frac{d\sqrt{n}}{2})^2 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} P & * & * \\ 0 & 0 & * \\ 0 & 0 & -1 \end{bmatrix},$$

$\bar{\beta} = 1 - \beta$, and $\kappa_1, \kappa_2 \geq 0$ are design parameters.

Proof : From (8) and (9), we can obtain

$$\begin{aligned} & (\Phi x(k) + BKe(k))^\top P(\Phi x(k) + BKe(k)) \\ & - x(k)^\top Px(k) < -\beta x(k)^\top Px(k). \end{aligned}$$

This inequality can be transformed into

$$\begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix}^\top P_0 \begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix} > 0. \quad (11)$$

(7) can be transformed into

$$\begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix}^\top P_1 \begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix} \geq 0. \quad (12)$$

The condition $x(k) \notin \mathcal{E}(P, 1)$ can be transformed into

$$\begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix}^\top P_2 \begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix} > 0. \quad (13)$$

By applying \mathcal{S} -procedure [4] to (11), (12), and (13), we have (10). \square

Next, consider the case of $x(k) \in \mathcal{E}(P, 1)$. In this case, if $x(k) \in \mathcal{E}(P, 1)$ holds, then $x(k+1) \in \mathcal{E}(P, 1)$ must hold. From this fact, we can obtain the following lemma.

Lemma 2: Both $x(k) \in \mathcal{E}(P, 1)$ and $x(k+1) \in \mathcal{E}(P, 1)$ hold if the following relation holds:

$$\bar{P}_0 - \bar{\kappa}_1 P_1 - \bar{\kappa}_2 \bar{P}_2 > 0, \quad (14)$$

where

$$\bar{P}_0 = P_0 + \begin{bmatrix} -\bar{\beta}P & * & * \\ 0 & 0 & * \\ 0 & 0 & 1 \end{bmatrix},$$

$$\bar{P}_2 = -P_2,$$

and $\bar{\kappa}_1, \bar{\kappa}_2 \geq 0$ are design parameters.

Proof : The condition $x(k+1) \in \mathcal{E}(P, 1)$ can be transformed into

$$\begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix}^\top \bar{P}_0 \begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix} \geq 0. \quad (15)$$

The condition $x(k) \in \mathcal{E}(P, 1)$ can be transformed into

$$\begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix}^\top \bar{P}_2 \begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix} \geq 0. \quad (16)$$

By applying \mathcal{S} -procedure to (12), (15), and (16), we have (14). \square

According to the result in [26], we can obtain the following theorem from Lemma 1 and Lemma 2.

Theorem 1: Problem 1 is reduced to the following BMI

feasibility problem.

Problem 2: Find scalars $\kappa_2 \in [0, \bar{\beta})$ and $\alpha_1 > 0$, the positive-definite matrix $S \in \mathcal{R}^{n \times n}$, unrestricted matrices $G \in \mathcal{R}^{n \times n}$ and $W \in \mathcal{R}^{m \times n}$ satisfying

$$\begin{bmatrix} (\bar{\beta} - \kappa_2)(\bar{G} - S) & * & * & * & * \\ 0 & \bar{G} - \alpha_1 I & * & * & * \\ 0 & 0 & \kappa_2 & * & * \\ AG + BW & BW & 0 & S & * \\ 0 & 0 & 1 & 0 & \frac{\alpha_1}{(\sigma_1 + \frac{d\sqrt{n}}{2})^2} \end{bmatrix} > 0, \quad (17)$$

where $\bar{G} = G^\top + G$.

Using the solution of this problem, the state-feedback gain K and the matrix P in the Lyapunov function and the ellipsoid are given by

$$K = WG^{-1}, \quad P = S^{-1},$$

respectively.

Proof : By setting $\bar{\kappa}_1 = \kappa_1$ and $\bar{\kappa}_2 = 1 - \kappa_2$, the left hand side of (14) can be rewritten as

$$\begin{aligned} & \bar{P}_0 - \bar{\kappa}_1 P_1 - \bar{\kappa}_2 \bar{P}_2 \\ & = P_0 + \begin{bmatrix} -\bar{\beta}P & * & * \\ 0 & 0 & * \\ 0 & 0 & 1 \end{bmatrix} - \kappa_1 P_1 + (1 - \kappa_2) P_2 \\ & = P_0 - \kappa_1 P_1 - \kappa_2 P_2 + \begin{bmatrix} \beta P & * & * \\ 0 & 0 & * \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

Hence, if (10) holds, then (14) also holds. Hereafter, based on this fact, we consider only (10).

Since (10) can be rewritten as

$$\begin{bmatrix} (\bar{\beta} - \kappa_2)P & * & * \\ 0 & \kappa_1 I & * \\ 0 & 0 & \kappa_2 - \kappa_1(\sigma_1 + \frac{d\sqrt{n}}{2})^2 \end{bmatrix} - \begin{bmatrix} \Phi \\ BK \\ 0 \end{bmatrix}^\top P \begin{bmatrix} \Phi & BK & 0 \end{bmatrix} > 0,$$

we can obtain

$$\begin{bmatrix} (\bar{\beta} - \kappa_2)P & * & * & * \\ 0 & \kappa_1 I & * & * \\ 0 & 0 & \kappa_2 - \kappa_1(\sigma_1 + \frac{d\sqrt{n}}{2})^2 & * \\ \Phi & BK & 0 & P^{-1} \end{bmatrix} > 0 \quad (18)$$

by using the Schur complement [4]. Moreover, (18) can be rewritten as

$$\begin{bmatrix} (\bar{\beta} - \kappa_2)P & * & * & * \\ 0 & \kappa_1 I & * & * \\ 0 & 0 & \kappa_2 & * \\ \Phi & BK & 0 & P^{-1} \end{bmatrix}$$

$$-\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}^\top \kappa_1 \left(\sigma_1 + \frac{d\sqrt{n}}{2} \right)^2 \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} > 0.$$

By using the Schur complement again, we can obtain

$$\begin{bmatrix} (\bar{\beta} - \kappa_2)P & * & * & * & * \\ 0 & \kappa_1 I & * & * & * \\ 0 & 0 & \kappa_2 & * & * \\ \Phi & BK & 0 & P^{-1} & * \\ 0 & 0 & 1 & 0 & \frac{1}{\kappa_1(\sigma_1 + \frac{d\sqrt{n}}{2})^2} \end{bmatrix} > 0. \quad (19)$$

Left-/right-multiplying (19) by the matrix $\text{diag}(G^\top, G^\top, I, I, I)$, we can obtain

$$\begin{bmatrix} (\bar{\beta} - \kappa_2)G^\top P G & * & * & * & * \\ 0 & \kappa_1 G^\top G & * & * & * \\ 0 & 0 & \kappa_2 & * & * \\ \Phi G & BKG & 0 & P^{-1} & * \\ 0 & 0 & 1 & 0 & \frac{1}{\kappa_1(\sigma_1 + \frac{d\sqrt{n}}{2})^2} \end{bmatrix} > 0. \quad (20)$$

Finally, from the following two condition:

$$\begin{aligned} (\kappa_1^{-1}I - G)^\top \kappa_1 I (\kappa_1^{-1}I - G) &\geq 0, \\ (P^{-1} - G)^\top P (P^{-1} - G) &\geq 0, \end{aligned}$$

we can obtain the following two inequalities:

$$\kappa_1 G^\top G \geq G^\top + G - \kappa_1^{-1}I, \quad (21)$$

$$G^\top P G \geq G^\top + G - P^{-1}. \quad (22)$$

By substituting (21) and (22) into (20), and by defining $S := P^{-1}$, $W := KG$, and $\alpha_1 := 1/\kappa_1$, we can obtain (17). Furthermore, since (21) is applied to (20), $G^\top G > 0$ holds, which implies that G is invertible automatically under (17). Finally, consider $T(x_0)$ in Definition 1. From (9), we can obtain $V(k) < \bar{\beta}^k V(0)$. Then, from $\bar{\beta}^k V(0) = 1$, $T(x_0)$ can be obtained as $T(x_0) = \lceil -\log x_0^\top P x_0 / \log \bar{\beta} \rceil$. \square

We remark here that by fixing κ_2 , Problem 2 becomes the LMI feasibility problem. Since the parameter κ_2 must be included in the interval $[0, \bar{\beta}]$, we can obtain the solution of Problem 2 by using e.g., the grid search method.

In addition, it is desirable that the volume of the obtained ellipsoid $\mathcal{E}(P, 1)$ is small. We may add an objective function $\text{tr}(S)$. By minimizing $\text{tr}(S)$, it is expected that the volume of $\mathcal{E}(P, 1)$ becomes small.

Remark 2: In implementation, it is necessary to modify (6), because even if $q(x(k)) \in \mathcal{E}(P, 1)$ holds, $x(k) \in \mathcal{E}(P, 1)$ does not hold necessarily. (6) is modified to

$$\sigma = \begin{cases} \sigma_1 & \text{if } q(x(k)) \notin \mathcal{E}(P, 1 - \delta) \text{ holds,} \\ \sigma_2 & \text{if } q(x(k)) \in \mathcal{E}(P, 1 - \delta) \text{ holds,} \end{cases} \quad (23)$$

where the scalar $\delta > 0$ is a given parameter satisfying the following condition: if $q(x(k))$ is included in the ellipsoid

$\mathcal{E}(P, 1 - \delta)$, then $x(k)$ is also included in the ellipsoid $\mathcal{E}(P, 1)$. From the obtained P , we can determine δ .

4. Numerical Example

In this section, we present a numerical example to demonstrate the proposed method. The matrices A and B in the plant (1) are given by

$$A = \begin{bmatrix} 1.0 & 0.9 \\ 0 & 1.1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

respectively. Other parameters are given as follows:

$$\sigma_1 = 0.56, \quad \sigma_2 = 0.3, \quad d = 1.0, \quad \beta = 0.3.$$

We present the computation result. By solving Problem 2, where $\text{tr}(S)$ is minimized, we can obtain

$$K = [0.1529 \quad 1.3300], \quad P = \begin{bmatrix} 0.1113 & 0.1776 \\ 0.1776 & 0.5842 \end{bmatrix}.$$

Based on P , δ in (23) is set to $\delta = 0.01$. The initial state is given by $x(0) = [10 \quad 10]^\top$. Figure 1, Fig. 2, Fig. 3, and Fig. 4 show the time response of the state, the state trajectory, the time response of the control input, and the time response of the event, respectively. From Fig. 1, we see that the state converges to a neighborhood of the origin. From Fig. 2, we see that once the state reaches to the ellipsoid, the state stays within it. In this example, even if the quantization errors are ignored, the state stays within the obtained ellipsoid. However, this property does not hold in general. From Fig. 4, we see that update of the control input is skipped three times. In addition, $T(x_0)$ in Definition 1 can be obtained as $T(x_0) = 14$. From Fig. 2, we see that the state reaches to the ellipsoid at time 13. Since Problem 1 is reduced to Problem 2 based on sufficient conditions, $T(x_0)$ may be larger than the true value.

Finally, under the above setting, consider two cases, i.e., (i) $\sigma_1 = \sigma_2 = 1$ and (ii) $\sigma_1 = 1$ and $\sigma_2 = 0.1$, where only β is changed to $\beta = 0.2$. By solving Problem 2, where $\text{tr}(S)$ is minimized, we can obtain

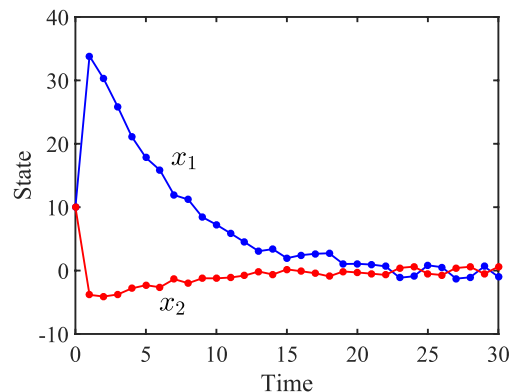


Fig. 1 Time response of the state.

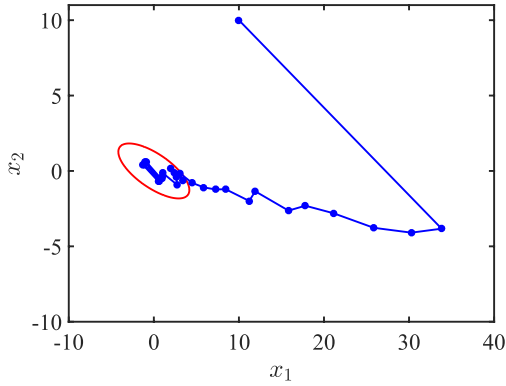


Fig. 2 State trajectory and the obtained ellipsoid.

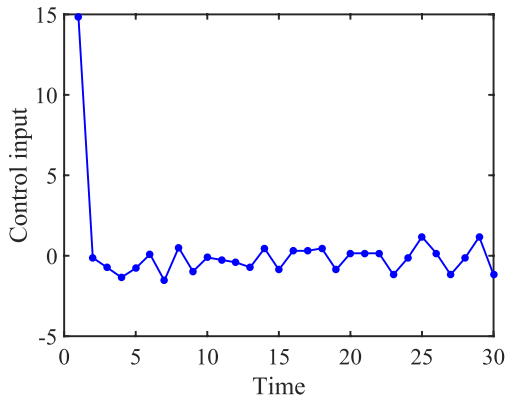


Fig. 3 Time response of the control input.

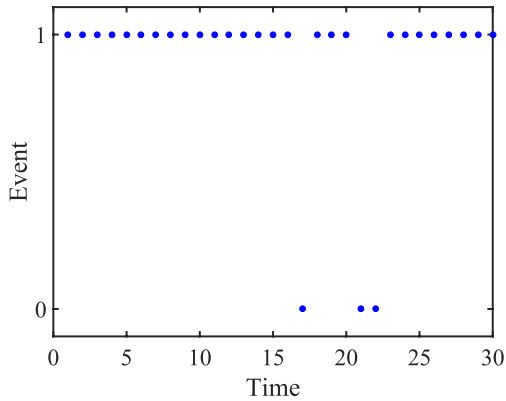


Fig. 4 Time response of the event. “1” implies the event occurs (i.e., the triggering condition is satisfied). “0” implies the event does not occur.

$$K = [0.2703 \ 1.6312], \quad P = \begin{bmatrix} 0.0663 & 0.1174 \\ 0.1174 & 0.3523 \end{bmatrix}.$$

Based on P , δ in (23) is set to $\delta = 0.01$. We remark here that in both cases, K and P are the same, because Problem 2 does not depend on σ_2 . Figure 5 and Fig. 6 show the state trajectories for the case (i) and the case (ii), respectively. Comparing these figures, we see that in the case (ii), the convergence property of the state is improved. Thus, the performance is improved by switching σ . It is one of the

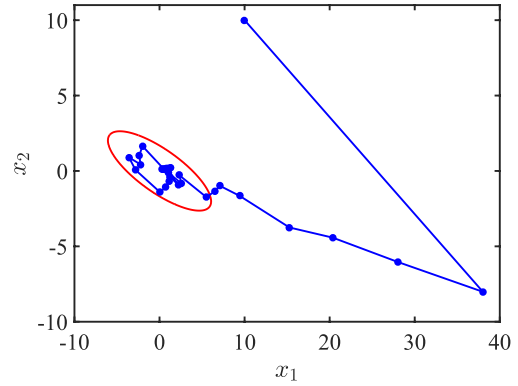


Fig. 5 State trajectory in the case of $\sigma_1 = \sigma_2 = 1$.

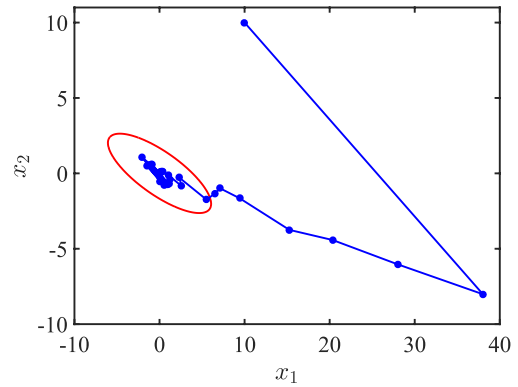


Fig. 6 State trajectory in the case of $\sigma_1 = 1$ and $\sigma_2 = 0.1$.

future efforts to design the ellipsoid $\mathcal{E}(P, 1)$ depending on σ_2 .

5. Conclusion

In this paper, we proposed a new method for quantized event-triggered control of discrete-time linear systems. In the proposed event-triggered controller, the threshold value σ is switched. The switching condition (6) is simple. However, the threshold value may be set to a given time-varying parameter that is smaller than σ_1 . The state-feedback gain can be obtained by solving multiple LMI feasibility/optimization problems.

One of the future efforts is to extend the proposed method to decentralized event-triggered control [18]–[20], [23]. It is also significant to develop a method to decide the threshold value σ appropriately.

This work was partly supported by the Telecommunications Advancement Foundation and JSPS KAKENHI Grant Numbers 17K06486, 16H04380.

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