THE RELATION BETWEEN LEIBNIZ AND WALLIS: AN OVERVIEW FROM NEW SOURCES AND STUDIES

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In memoriam
Walter S. Contro
(1937-2017)

In 2016 the 400th anniversaries of the death of Cervantes and of Shakespeare were commemorated throughout the world. But the same year is also important in the history of mathematics as the year of the 400th anniversary of the birth of John Wallis (1616-1703) and the year of the 300th anniversary of the death of Gottfried Wilhelm Leibniz (1646-1716). Separated by a generation, there was a mutual reception and discussion of the two scholars which comprises a period of more than 30 years in their lives and deals with a variety of scientific, especially mathematical topics, and some other fields of scholarly discussion, too. The exchange between them was first started by the correspondence Leibniz entered with Henry Oldenburg (1618-1677), the secretary of the Royal Society. But Wallis and Leibniz never met in person: When Leibniz visited London early in 1673 and in October 1676 and met Oldenburg, Robert Boyle (1627-1692), John Pell (1610-1685), and other members of the Royal Society, Wallis was not present, and the same holds for Isaac Newton (1643-1727). A direct correspondence between them was started only in 1695 by Leibniz, but as is clear from their letters and other writings, they read each other's publications carefully, and Leibniz even wrote reviews of books published by Wallis. The last letter from Leibniz dates from 1701, nearly two years before Wallis died.

Since the publication of the first volume of the *Correspondence of John Wallis* in 2003 (Wallis, 2003ff), edited by Christoph J. Scriba (1929-2013) and Philip Beeley, the available sources for the study of the relationship of the two scientists have been growing rapidly, mainly by the now four volumes of the *Correspondence of John Wallis* and several new volumes of the Academy

edition of the *Complete writings and letters* of Leibniz (Leibniz 1923ff)[1]. The most recent volume (III, 8) of the mathematical, scientific and technical correspondence of Leibniz comprises the year 1701, which means that it includes the last letter of the correspondence between Leibniz and Wallis. In addition, there have been published a considerable number of research papers dealing with Leibniz and Wallis, investigating not only their debates concerning the new calculus and priority issues in mathematics but also other topics such as cryptography, calendar reform, logic, linguistics and theology.

The following text consists of two parts, the longer 1st section will present an overview on the relation between Wallis and Leibniz with focus on the early years, and especially on the recent research on that relation: the year 2003 when the 300th anniversary of the death of John Wallis was commemorated, will be the starting point. The 2nd section will present some results of an investigation in the origins of the term "infinitesimal", where Wallis and Leibniz play a prominent role[2].

1.- Leibniz and Wallis.

Concerning the research on the relation between Leibniz and Wallis, it is helpful to take a look at a few publications of the 1970's: When in 1972 the reprint of the three volumes of the *Opera Mathematica* (Wallis, 1693-1699) of John Wallis was published with a preface by Christoph J. Scriba, Joseph E. Hofmann (1900-1973), preparing the first volume of the mathematical, scientific and technical correspondence of Gottfried Wilhelm Leibniz from the years 1672-1676 for publication in the Academy edition (III, 1 – published in 1976), took the opportunity to give a comprehensive overview of the relationship between Leibniz and Wallis from printed and unpublished sources (Hofmann, 1973). The essay review grew into an article of 35 pages in print and was published posthumously[3] in 1973 in the journal *Studia Leibnitiana*.

1 The volumes of LEIBNIZ (1923ff) will subsequently be quoted by series (Roman numerals) and volume (Arabic numerals).

2 The paper is mainly based on a German talk at the Scriba Memorial Meeting in Hamburg 2015, paying tribute to the eminent Wallis scholar Christoph J. Scriba (PROBST, 2017). Some parts were presented earlier at conferences in 2003 (Attendorn), in 2009 (Amsterdam), and afterwards in 2016 (Oxford and Barcelona).

3 Hofmann had been severely injured in a car accident in 1972 and died the following year. Hofmann was primarily interested in the material concerning the priority dispute about the

Hofmann's article provides a wealth of information from the correspondence and from printed as well as unpublished papers and still is a valuable starting point for any serious investigation of the relationship between Leibniz and Wallis. The material concerning the young Leibniz was also included in Hofmann's English monograph on *Leibniz in Paris* (Hofmann, 1974).

As mentioned before, in 2003, the first volume of the *Correspondence of John Wallis*, edited by Christoph J. Scriba and Philip Beeley was published. It happened to be the year of the 300th anniversary of the death of the erudite Oxford mathematician, and at the biennial conference of the section for history of mathematics of the German Mathematical Union (DMV), three talks commemorated his works. The contributors actually worked or had worked for some time on the edition of the correspondence of John Wallis. All papers dealt to some extent with different aspects of the reception of Wallis's writings by Leibniz. Thus, in 2004 Beeley analysed the relationship of mathematics and logic in Wallis and emphasized Leibniz's approval of the treatment of logic by Wallis, especially for didactic purposes (see below). Mayer (2004) compared the treatment of imaginary numbers in Wallis, Tschirnhaus (1651-1708) and Leibniz: The manuscript notes recording the Parisian conversations from the years 1675-1676 of Leibniz and Tschirnhaus concerning imaginary numbers as solutions of algebraic equations show that their discussions were possibly influenced by a meeting of Tschirnhaus with Wallis in Oxford. Probst (2004) reported on the new material concerning Leibniz's early reception of the mathematical writings of John Wallis. Many of the unpublished sources used by Hofmann and additional sources had been published in the meantime: Besides the volumes of the historical-critical edition of the writings and letters of Leibniz, the edition of the *Correspondence of Henry Oldenburg* by Rupert Hall and Mary Boas Hall has to be mentioned in first place (Oldenburg 1965-1986). The three papers presented used the new material available, and there followed a considerable number of further

invention of the calculus and arranged the contents of his study with regard to this topic: the first part deals with the period before Leibniz's answer to the first letter sent by Newton (the famous "Epistola prior") was passed on to Wallis in autumn 1676. The second part ends with Leibniz's review (LEIBNIZ, 1696) of the first two volumes of Wallis's *Opera* in the *Acta Eruditorum* (1696); the third part is devoted to the prehistory of the edition of Leibniz's and Newton's letters in volume III of Wallis's *Opera* and to Leibniz's review of this volume (LEIBNIZ, 1700).

publications in the following years[4]. In addition to the contemporary debate about the calculus and priority issues in mathematics other topics, such as cryptography, calendar reform, logic, linguistics and theology are covered. My overview will start with the early reception, then briefly mention the various different topics before addressing the calculus.

1.1.-The Early Reception: Leibniz in Germany (until Spring 1672).

Already J. E. Hofmann pointed out (Hofmann, 1974, 4) that Leibniz's first information about Wallis was based on the writings of Thomas Hobbes (especially Hobbes, 1655 and Hobbes, 1668). Hobbes in turn was for 20 years engaged in a bitter controversy with both Savilian Professors in Oxford, John Wallis and the astronomer Seth Ward (1617-1689)[5]. Leibniz first had some trouble to assign the indirect references of Hobbes to his opponents correctly and mistook the two occasionally. For example, in one of his earliest publications, the *Disputatio arithmetica de complexionibus* (Leibniz, 1666; VI, 1, 229), Leibniz correctly paraphrased a statement of Ward concerning the question of the infinite, quoted by Hobbes in Latin translation from Wards *Philosophicall Essay* (Ward, 1652), but he erroneously referred to a publication of Wallis, the *Arithmetica infinitorum* (Wallis, 1656). Apparently, Leibniz had at that time no direct access to publications of Wallis[6].

When in 1669 an acquaintance informed Leibniz about an article by Christiaan Huygens (1629-1695) on the laws of motion in the *Philosophical*

4 The secondary literature concerning the relation between Leibniz and Wallis is now easily accessible by searching the online-database of the Leibniz-Bibliography: http://www.leibniz-bibliographie.de/

5 The royalist Anglican divine Seth Ward had helped in publishing political writings of the philosopher Thomas Hobbes (1588-1679) during the English Civil War. But when Hobbes published the *Leviathan* (HOBBES, 1651), Ward, now Savilian professor of astronomy at Oxford, attacked him for theological reasons in his *Philosophicall Essay* (WARD, 1652). A few years later, sparked by the counter attacks by Hobbes in *De Corpore* (HOBBES, 1655), Ward joined efforts with his colleague John Wallis (WALLIS, 1655b; WARD, 1656) and engaged again in a controversy with Hobbes (PROBST, 1993).

6 Nevertheless, Leibniz probably soon had the opportunity to study at least a book of Seth Ward (WARD, 1656): Ursula Goldenbaum discovered a copy of this work that Leibniz's patron Johann Christian von Boineburg (1622-1672) acquired in 1669. The book contains a list of supporters and opponents of Hobbes written by Boineburg: Wallis appears in first place of the opponents, while in the list of followers a capital L marks Leibniz (GOLDENBAUM, 2008, 93).

Transactions from April 1669 (Huygens, 1669), he probably could not yet read the related articles by Wallis and Christopher Wren (1632-1723), published in an earlier issue from January 1669 (Wallis, 1669; Wren, 1669). Shortly afterwards Leibniz received the fifth volume of the *Philosophical Transactions* that Henry Oldenburg forwarded to him via the bookseller Gottfried Schultze (1643-1686) in Hamburg (I, 1, 161). But this volume starts with the issue of March 1669 and therefore doesn't contain the articles of Wallis and Wren. Leibniz then elaborated his own theory and stayed in contact with Henry Oldenburg, but still he could not obtain copies of the main publication of Wallis in this field, the *Mechanica* (Wallis, 1670-1671). So Leibniz probably did not yet have access to the publications of Wallis, when in 1671 he was completing his tracts *Hypothesis physica nova* and *Theoria motus abstracti* (Leibniz, 1671a and 1671b), he knew them only from second hand sources, from hearsay: "Wallisii edita audivi tantum" (VI, 2, 275; see also II, 1, 99). Nevertheless, this didn't prevent the good reception of his two treatises at the Royal Society: When Oldenburg presented parts of it to members of the Society in April 1671, it was Wallis, who undertook to draw up a detailed report, and he noted with satisfaction some similarities in their respective theories. The positive reaction of Wallis was probably crucial for the reprint of the two treatises by the official printer of the Royal Society in the same year (Beeley, 2006). Leibniz received some information on the reaction of Wallis by a letter from Oldenburg of 12 (22) March 1671: The long quotations from the letters of Wallis in this letter were perhaps the first original texts by Wallis that Leibniz ever read[7]. Finally, Leibniz received the issue of the *Philosophical Transactions* from August 1671 where the report of Wallis was printed (Wallis, 1671) in 1672, it was sent to him by Schultze in January 1672 (I, 1, 182). A few weeks later Leibniz travelled to Paris where he stayed for more than four years (except a short journey to London).

In his monograph on the early mathematical development of Leibniz Hofmann states that Leibniz gradually withdrew from the influence of Hobbes in Paris and finally sided with the modern mathematics of Wallis (Hofmann, 1974, 7 f., 20). However, an analysis of the sources shows that already in the previous years in Mainz Leibniz recognized the errors of Hobbes in mathematics and rejected the mathematical methods of the philosopher. This new interpretation is supported by the Leibniz marginalia in

7 II, 1, 217-221; see WALLIS (2003ff), volume 3, 443-447 and 453.

copies of philosophical works of Hobbes (Hobbes, 1655; Hobbes, 1668) from the collection of Leibniz's patron Boineburg. These notes were discovered and published by Ursula Goldenbaum. They contain several clear indications that Leibniz recognized the mathematical errors of Hobbes (Goldenbaum, 2008, 80-92).

1.2.-The Early Reception: Leibniz in Paris (Spring 1672 – October 1676).

Did Leibniz change his views concerning Wallis in these years? During his stay in Paris, Leibniz had the opportunity to spend a few weeks in London, in January and February 1673, participating in a diplomatic mission of the court of Mainz. In a manuscript with notes from this period, *Observata Philosophica in itinere anglicano sub initium anni 1673*, Wallis (whom he didn't meet) is mentioned by name only once, but at least two additional notes refer to articles by Wallis in the *Philosophical Transactions*[8]. But there is a wealth of references to Wallis in Leibniz's mathematical manuscripts of this period: They mostly deal with various problems of infinitesimal mathematics, as one would expect[9]. Besides their mathematical contents[10], they provide a good example for investigating how Leibniz studied publications of mathematical authors that were important for him. He usually started with the most recent publications available to him, quite often articles and reviews from the *Philosophical*

8 VIII, 1, 3-19; see page 5, line 7f.: "Tangentes omnium figurarum. Figurarum geometricarum explicatio per motum puncti in moto lati": Leibniz may refer to WALLIS, 1672; in any case, Leibniz knew this article in the spring of 1673 (VII, 4, 360); page 9, line 4: "Wallisius observat." refers to WALLIS, 1666; page 12, line 3: "formatio loquelae" refers to WALLIS, 1670, where WALLIS, 1653, is mentioned. Leibniz in later years investigated the linguistic theories of Wallis, see. eg. I, 10, 602, and I, 18, 284; further references can be found in SCHULENBURG, 1973, 10.

9 The greatest part (17 references) deals with the methods of the *Arithmetica Infinitorum* (WALLIS, 1656), including the use of induction and interpolation by Wallis. In addition, there are references to several mathematical topics: Decyphering (VII, 3, 253), casting out nines (VII, 1, 530 f.), pointwise construction of several curves using the trigonometric curves (VII, 4, 337), theory of proportion and the controversy with Thomas Hobbes (1588-1679) concerning algebraic methods in geometry (VII, 6, 556).

10 Topics include the quadrature of the hyperbola by Nicolaus Mercator (1620-1687), a result concerning the determination of the center of gravity of the hyperbola by Wallis, and the tangent methods of Wallis. Based on the references in these articles Leibniz analyses, from spring 1673, the mathematical results of Wallis's *Mechanica* (about a year before the closer investigation of the physical parts of this work). In addition to the parabola and hyperbola, these results relate to several transcendental curves (spirals, cissoid, and cycloid).

Transactions of the Royal Society (as we already saw in the case of the laws of motion). In 1672 and 1673 Leibniz used at least 12 mathematical articles or reviews from this journal, three of them written by Wallis. In addition, now he made excerpts of Wallis' most recent book, the aforementioned *Mechanica*: the mathematical parts of this work contain many of Wallis's results printed first in his earlier publications[11]. Later, when Leibniz knew more about the subject, he also consulted some of the earlier original publications.

From 1674 on the references to Wallis's papers and reviews in the *Philosophical Transactions* become rare, and there are several indications that Leibniz now did consult the earlier books from the 1650's, especially the *Arithmetica infinitorum* (1656) with regard to transcendent curves, problems and method of interpolation, infinite fractions.

Leibniz acknowledged the achievements of Wallis but stayed critical with regard to the lack of rigorous demonstrations in Wallis's texts. Generally speaking, in Paris Leibniz emancipated himself from the methods of Wallis step by step in mathematics[12]. The same holds for mechanics, in which he also criticized the lack of proper demonstrations[13].

When Leibniz moved to Hannover in December 1676, and Oldenburg died some months later in 1677, the scientific contacts from Leibniz to England became quite sporadic for some years. We don't know very much about the contents of his mathematical manuscripts of this time, but there are still references to Wallis. Especially, Leibniz made excerpts from Wallis's *A treatise of algebra* (Wallis, 1685), and published a review of the book in the *Acta Eruditorum* (Leibniz, 1686). Ten years later, he started a direct correspondence with Wallis and reviewed the first two volumes of Wallis's *Opera mathematica* (Leibniz, 1696).

1.3.-Topics of later years (1677-1703): logic, linguistics, theology, calendar reform, and cryptography.

In his review (Leibniz, 1700) of the 3rd volume of Wallis's *Opera mathe-*

11 There are also excerpts from the physical parts of this work, but they were written about a year later. They have been published for the first time in 2016 in volume VIII, 2.

12 See PROBST, 2005, BEELEY, 2008, and especially the volumes 3-6 in series VII.

13 See BEELEY, 2006, on the basis of the manuscripts now published in volume VIII,2.

matica, Leibniz mentioned briefly the *Institutio Logicae*: He praised Wallis for transferring the exercise of logic from scholastic disputes to the use in common life and remarked that in this work the chief aim of Wallis had been to teach student readers the use of right reasoning in judging and critically examining the arguments of others and in arguing and structuring the matter treated according to the nature of the subject. He further acknowledged that Wallis provided a strict mathematical foundation of the rules of syllogistic. Thereby, wrote Leibniz, the consequences become irrefragable and hidden errors are detected (Beeley, 2004).

Leibniz also studied the *Grammatica linguae Anglicanae* of Wallis, he made excerpts and mentioned the linguistic concepts and theories of Wallis in his correspondence with Thomas Smith (1638-1710) in 1694 (I, 10, 603) and Johan Gabriel Sparwenfeld (1655-1727) in 1700 (I, 18, 284)[14].

The areas of logic and linguistic sciences are but two of the areas in which both Wallis and Leibniz were active beyond mathematics and natural philosophy. It is therefore not surprising that they occasionally discussed the views of each other in other fields: Leibniz had already dealt intensively with the theological discussions about the Trinity at a young age (VI, 1, 115). In the end of the 1680's, a controversy arose in England, in which Wallis was a major contributor. Wallis pursued a double strategy, on the one hand he declared the Trinity as a matter of faith, which was fundamentally inexplicable, and on the other hand he asserted that the Trinity was not logically inconsistent (Beeley; Probst 2005, 450). The controversy was first received by Leibniz when he read an anonymous publication (Nye, 1693), which he excerpted and commented on (IV, 5, 504–518), and he generally approved the views of Wallis[15]. Moreover, Leibniz explicitly defended Wallis' strategy concerning the doctrine of Trinity in his correspondence with Thomas Burnett of Kemney (1656-1729) in 1696 and in 1702[16].

In the beginning of 1700 Leibniz wrote to the Royal Society concerning the calendar reform recently implemented in the Protestant states of the Empire (III, 8, 319-324). He hoped that England would join the reform (and in fact there were quite influential supporters in England). Wallis on the other hand

14 Another topic in the exchange with Wallis is the connection between migrations of people and the evolution of languages (III, 8, 260).

15 See: I, 10, 602, 654; I, 11, 123, 234; I, 12, 368, 407; I, 20, 812.

16 See: I, 12, 347-348; I, 20, 812.

was one of the most determined opponents of reform and criticized expressly Leibniz's interpretation of the Easter rule in a letter to Hans Sloane (1660-1753), the secretary of the Royal Society. Sloane sent this review in a letter to Leibniz (Beeley; Probst, 2005)[17].

A theme in the spotlight of current research is Cryptography[18]: Wallis was a key decipherer for the English government for several decades. Leibniz tried for years –and ultimately in vain– to acquire the methods of Wallis and successively approached the courts of Brunswick-Luneburg, Brandenburg, Tuscany and of Sweden for financing the project. Particularly enlightening for this topic is the correspondence between the two in the year 1700. The letter, in which Wallis finally declined to proliferate his methods abroad for political reasons, was immediately printed in the *Philosophical Transactions*, but the section concerning his deciphering methods was left out[19]. Starting from notes in Leibniz's manuscripts about a planned encryption and deciphering machine, Nicholas Rescher had a prototype constructed from the data provided by Leibniz (Rescher, 2012). His thesis that Leibniz himself had such a machine built and at his disposal, has met with contradiction (Beeley, 2014b).

1.4.- Topic of History of Mathematics.

Overall mathematics was the central field of the exchanges between Wallis and Leibniz. A group of articles investigates the questions in the history of mathematics both scholars discussed. Most of them are related to controversial topics, especially to problems of foundation, priority issues or accusations of plagiarism. In the case of the cycloid there was a controversy between French and Italian mathematicians concerning the "invention" of the curve (Galileo or Roberval). Wallis took the view that the curve is to be found already in the writings of Nicolaus Cusanus (1401-1464) and of Charles de Bovelles (1479-1567), and supplied Leibniz with some material (Beeley, 2003).

Another case was the question as to whether René Descartes (1596-1650) had used the results of the British mathematician Thomas Harriot (1560-1621)

17 The letter is for the first time printed after the original manuscript in volume III, 8, 448-450.

18 See BREGER, 2006; BEELEY, 2007; RESCHER, 2012; BEELEY, 2014.

19 The complete letter was first published by Gerhardt in 1859 (LEIBNIZ, 1849-1863, subsequently quoted as GM; vol. 4, 75-78) and is critically edited in volume III, 8, 397-402.

in the further development of the symbolic algebra and in drawing up the rule of signs. The critical attitude towards Descartes is a feature common to Leibniz and Wallis. Based on reports, above all from the mathematician John Pell, Wallis claimed in his *Treatise of Algebra* (Wallis, 1685) that Descartes had plagiarized the results of Harriot. Leibniz agreed with Wallis, he had already argued for this in his own papers concerning Descartes. Neither Wallis nor Leibniz was impartial to Descartes, and they spread not only reproaches against Descartes, but Leibniz, on the basis of the arguments of Wallis, made Harriot the real author of the rule of signs[20].

The nature of the angle of contact or contingency had been a controversial topic in the history mathematics since ancient debate concerning the proof of theorem III, 16 in Euclid's *Elements*. Wallis took part in the debate with two publications, in the second work (Wallis, 1684) his position is modified. Beeley and Scriba relate this change of mind with his confrontation with the new calculus of Leibniz (Beeley; Scriba, 2008a)[21].

1.5.- Infinitesimal Calculus and quadrature of the circle.

The reception and critique of the infinitesimal methods of Wallis by Leibniz in his Paris years has been mentioned earlier. There was in the aftermath of the publication of Leibniz's articles concerning his calculus also a direct exchange between the two scientists. As is well known Wallis pursued to establish the priority of the British mathematicians in calculus with the publication of Newton's texts and parts of the letters between Leibniz and his correspondents in England. This is also visible in his correspondence with Leibniz[22].

Beeley compares the methods of Wallis with those of his contemporaries and analyses the discussion between Leibniz and Wallis in the 1690's. He states that Wallis argued that the methods of Newton and Leibniz didn't provide anything fundamentally new, but were but merely refined versions of the received methods of squaring curves. Leibniz on the other hand tried to emphasize that the traditional methods covered only a limited portion of

20 See BEELEY; SCRIBA, 2005; BEELEY; SCRIBA, 2008b; STEDALL, 2012.

21 For a similar conclusion, but with reference to Wallis's study of Newton's writings, see LOGET, 2002, 207 and 256. A different opinion is voiced in WAHL, 2011.

22 See especially volumes III, 6-8; the introductions to these volumes present summaries on the subject.

the results gained by the new calculus (Beeley, 2014a, 54-57).

Analysing the sources mentioned in the correspondence of Leibniz, Charlotte Wahl investigated how Tschirnhaus, Jacob Bernoulli (1655-1705), and Wallis compared Leibniz's and Barrow's methods (Wahl, 2011, 1173-1174). In addition, a comprehensive study on the priority dispute between Newton and Leibniz has been published (Sonar, 2016). There is also a chapter on Wallis in the book: Wallis's edition of several letters exchanged between Leibniz and himself and especially through Henry Oldenburg with other members of the Royal Society aimed a securing the priority claims of the mathematicians in England. Only after his death in 1703 they were used for accusing Leibniz of plagiarism.

Jesper Lützen recently published a paper on "17th century arguments for the impossibility of the indefinite and the definite circle quadrature", in his article he analyses the arguments of Wallis, James Gregory (1638-1675), and Leibniz against the possibility of squaring the circle (Lützen, 2014)[23].

1.6.- Last contacts[24].

In May 1702 Leibniz asked in a letter to Hans Sloane, the secretary of the Royal Society, whether the rumours that Wallis had died were true:

> *"I do not know who told me that the glorious Wallis had died, although I am afraid that considering his age the message may be true, yet I would rather hope for the best as long as possible".*

Some weeks later he received a letter from Sloane that Wallis had been ill but that he had recovered. Leibniz answered in April 1703: "I am glad that Wallis, this most perfect man, is still alive and even, as far as his age allows, in good health." The message that Wallis had died in November 1703 finally was received by Leibniz in a letter sent to him by Johann Bernoulli (1667-1748) in February 1704[25].

23 This topic is also amply discussed in a thesis: CRIPPA, 2014.

24 I would like to thank Michael Kempe, who is currently editing the Leibniz-Sloane correspondence for volume III, 9 of the academy edition, for informing me of some details, partly in unpublished letters, concerning Wallis.

25 Leibniz to Hans Sloane, Hannover, 5 May 1702, London, British Library, Sloane Ms 4038 fol.

2.- Remarks on the introduction of the term "pars infinitesima".

The prehistory of the *terminus technicus* "infinitesimalis" is still not fully investigated. As to my knowledge the first examples published in print (see below) use the expression "pars infinitesima". Leibniz has used "infinitesima" in some mathematical manuscripts as early as spring 1673 (in his mathematical publications since 1692), in summer 1673 there are already many occurrences[26]. Apparently Leibniz took the term from the writings of Nicholas Mercator (1620?-1687), as he was to recall much later in a letter to John Wallis from March 30/[April 9], 1699[27]:

> *"[…] for the calculus it is useful to imagine infinitely small quantities, or, as Nicolaus Mercator called them, infinitesimals, such that when at least the assignable ratio between them is sought, they precisely may not be taken to be nothings."*

But in his book *Logarithmotechnia* (Mercator, 1668a, 30-34), Mercator did not use the term "infinitesima", instead he wrote "pars infinitissima"). Mercator used the same expression also in his article "Some illustration of the Logarithmotechnia" (Mercator, 1668b, 759-764). Mercator's expression signi-

339–340: "Nescio quis mihi dixit inclytum Wallisium obiisse, ego etsi ea aetate ejus verear ne verus sit nuntius, malim tamen optima quaeque sperare dum licet." ("I do not know who told me that the glorious Wallis had died, although I am afraid that considering his age the message may be true, yet I would rather hope for the best as long as possible."); Hans Sloane to Leibniz, London, 11 (22) August 1702, Hannover, GWLB, LBr. 871 fol. 18–19: "Dr Wallis has been ill but is recovered."; Leibniz to Hans Sloane, Berlin, 17 April 1703, London, British Library, Sloane Ms 4039 fol. 116–117 (see: AITON, E. J. (1981) "An unpublished letter of Leibniz to Sloane", *Annals of science* 38, 103-107, (with Engl. transl.,: "Gaudeo Wallisium consummatissimum virum adhuc vivere, imo quantum ea aetate licet, valere." – "I am glad that Wallis, this most perfect man, is still alive and even, as far as his age allows, in good health"; Johann Bernoulli to Leibniz, Groningen, 9 February 1704, Hannover, GWLB, LK-MOW Bernoulli20 fol. A104.A117 (formerly: LBr. 57,2 fol. 104.117): "Didici ex relationibus publicis, Wallisio defuncto successisse Hallejum, qui proinde iter quod meditabatur in mare pacificum exequi non poterit." (GM 3,2, 1856, 743 f.); Leibniz to Johann Bernoulli, Hannover, 25 March 1704, Basel, Universitätsbibl. L Ia 19 fol. 221–222: "Vellem viveret adhuc Wallisius nec morte sua Hallaei iter in mare pacificum non parvo rei magneticae cognoscendae adjumento futurum abrupisset." (GM 3,2, 1856, 744 f.)

26 See for example VII, 4 N. 16, 22, 26, 27, 34, 38, 40, 44.

27 "Sed pro calculo utile est fingere quantitates infinite parvas, seu ut Nicolaus Mercator vocabat infinitesimas: quales cum ratio earum inter se utique assignabilis quaeritur, jam pro nihilis habere non licet."; III, 8, 91; English translation from BEELEY, 2008, 49.

fies a minimal quantity, a concept not compatible with Leibniz's own views. So my first guess was that Leibniz changed the expression to "infinitesima"[28], which effectively paraphrases the symbolic expression 1/∞ introduced by John Wallis[29].

Shortly afterwards I found out that Leibniz in fact had taken the expression from Mercator although the latter did not use it himself[30]. What do I mean here? Mercator had added a note at the end of his article, indicating that readers offended by "infinitissima" should change the expression to "infinitesima". Perhaps this was a reaction to the critique in (Wallis, 1668, 753) at the beginning of his review of the book, printed in the same issue of the *Philosophical Transactions*. Wallis refers to his critique of Mercator's terminology in a letter to John Collins (1625-1683) from 8/[18] September 1668[31], but his remarks concern the composition of ratios, which is not relevant for the question of using "infinitissima" or "infinitesima". This seems to lead away from Wallis, but in the same volume of the *Correspondence of John Wallis* there is a letter from Wallis to the French mathematician Vincent Léotaud (1596-1672), dating from 17/[27] February 1667/[1668], a few months before the publication of Mercator's *Logarithmotechnia*. There Wallis proposes a change of terminology to Léotaud: "Say the angle of contact is an infinitesimal part [pars infinitesima] of two right angles" or "2/∞ R" (Wallis, 2003ff, vol. 2, 426). This letter has been printed for the first time by Wallis himself some years later (Wallis, 1684, 87). Unfortunately, no manuscript of this letter from 1668 seems to exist.

Beeley and Scriba (2008a, 448) remark in an article on the discussion concerning the angle of contact that Wallis inserted the term "infinitesima" in the 2nd edition of his *Arithmetica infinitorum* (Wallis, 1693-1699, vol. 1), where the

28 See: PROBST, 2008, 95-106, especially 103. − My cautious formulation "Leibniz seems to have coined the term" was −without any discussion of my arguments− distorted into "Some scholars have claimed that Leibniz was the first to coin the term (e.g., PROBST 2008, 103)" by (KATZ; SHERRY, 2013, 573); and similarly to "Some contemporary scholars hold that Leibniz coined the term infinitesimal in 1673 (see PROBST 2008 and 2010)" in KATZ; SCHAPS; SHNIDER, 2013, 316. The mysterious publication PROBST 2010 referred to by the authors is missing in their bibliography, and such a publication does not exist (except the short online entry on infinitesimals mentioned below in note 30).

29 WALLIS, 1655, 4 = WALLIS, 1693-1699, vol. I, 297.

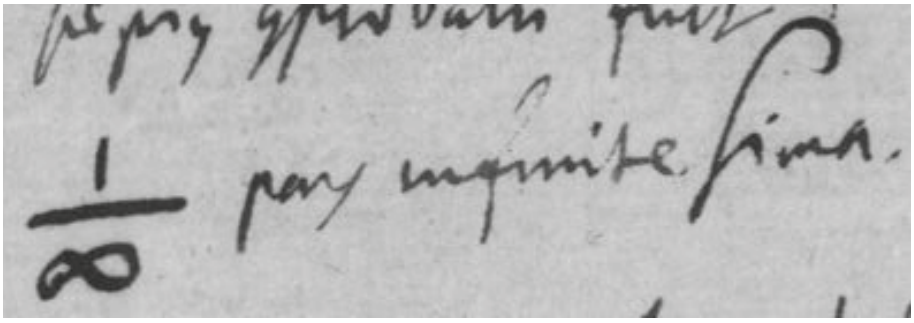
30 See the note (from February 2010, updated 2016) in the entry "infinitesimal" in Jeff Miller's webpage *Earliest Known Uses of Some of the Words of Mathematics*, http://jeff560.tripod.com/mathword.html

31 WALLIS, 2003ff, vol. 2, 597-599.

1st edition (Wallis, 1656) only has "pars infinite parva". A similar insertion or change in the text of the letter after 1668 cannot be excluded. On the other hand, I discovered that Wallis used "infinitesima" in print already in 1670, in his *Mechanica*, explaining his definition of the continuum, where he states that a continuous line is to be thought as consisting of elements of infinitesimal length and height:

> *"For example; a line consisting of infinitely many points, that is, infinitely small lines of equal length or height; whose length or height is taken as 1/∞ (an infinitesimal part) of the length or height of the whole line"* [32].

Considering this early print and the reluctant remark of Nicolaus Mercator together, it seems plausible to me that the term "infinitesima pars" was proposed to Mercator either by Wallis himself or by some intermediary. More than a year after his first use of "infinitesima", Leibniz took a note of this passage of the *Mechanica* in excerpts of the book he made in winter 1674/5 (VIII, 2, 67): "1/∞ pars infinitesima".



LH 35 XIV 2 fol. 118 r° (detail), by kind permission of GWLB Hannover.

32 Puta; Linea, ex infinitis punctis, hoc est, Lineolis infinite exiguis, longitudine aequalibus, vel aeque altis; quarum cujusvis longitudo vel altitudo sit (pars infinitesima) longitudinis vel altitudinis totius lineae (WALLIS, 1670-1671, 110).

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- LK-MOW: Leibniz-Korrespondenz, Memory of the World, Hannover, Gottfried Wilhelm Leibniz Bibliothek – Niedersächsische Landesbibliothek.

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