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Conference Paper · September 2016

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5th World Conference on
Applied Sciences, Engineering & Technology
02-04 June 2016, HCMUT, Vietnam

Effects of Newtonian Heating and Inclined Magnetic Field on Two Dimensional Flow of a Casson Fluid over a Stretching Sheet

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Abstract: It is known that the Navier-Stokes equations cannot describe the behaviour of fluids having high molecular weights. Due to the variety of such fluids it is very difficult to suggest a single constitutive equation which can describe the properties of all non-Newtonian fluids. Therefore many models of non-Newtonian fluids have been proposed. In this study, the steady two dimensional heat and mass transfer flow of a non-Newtonian Casson fluid over a linear stretching sheet in presence of an inclined magnetic field and radiation effects are considered. The sheet is subjected to Newtonian heating as well as convective boundary conditions. The governing partial differential equations are transformed to nonlinear ordinary differential equation by using similarity transformation. The solutions of these simplified coupled nonlinear equations are calculated using an analytical technique. The effects of various parameters on velocity, temperature and concentration profiles are presented through graphs and discussed.

Keywords: *Newtonian Heating, Magnetic Field, Casson Fluid, Stretching Sheet, Thermal Radiation*

Introduction:

During the past few decades, the study of real life problems dealing with flow models of non-Newtonian fluids has received a special attention. Perhaps, this is mainly due to their several possible applications in engineering and industries such as oil and gas well, drilling, food stuffs, polymer processing, physiology, paper production, blood and cosmetic products. In several areas of science and engineering the researchers have generated their increased interest to study the characteristics of non-Newtonian fluids. Due to a great diversity in the physical structure of non-Newtonian fluids, there is no single constitutive equation available in the literature which describes all the properties or rheological behavior of non-Newtonian fluids. Therefore, many models have been proposed to describe the rheological behavior of non-Newtonian fluids. Some relevant contributions dealing with the flows of non-Newtonian fluids are made by [1-4].

A number of empirical relations have been proposed to account for the behavior of viscoplastic materials. One of the extension of Bingham plastic linear model is a Casson fluid model. This model was originally introduced by Casson [5] for the prediction of the flow behavior of pigment-oil suspensions. A substantial study has been done on the boundary layer flow of Casson fluid because of its important practical applications. Mustafa et al. [6] studied the unsteady flow and heat transfer of a Casson fluid past a moving flat plate. They solved the governing equations analytically using homotopy analysis method. Hayat et al. [7] studied the mixed convection Casson fluid stagnation point flow over a stretching sheet and obtained the solution by homotopy analysis method. In subsequent papers, Mukhopadhyay [8, 9] studied the heat transfer flow of Casson fluid over a unsteady stretching and nonlinearly stretching surfaces, respectively. Rao et al. [10] considered thermal and hydrodynamic slip conditions on heat transfer flow of a Casson fluid past a semi infinite vertical plate. Heat transfer flow of a Casson fluid past a permeable shrinking sheet with viscous dissipation was considered by Qasim and Noreen [11]. Forced convection flow of a Casson fluid with surface heat flux over a symmetric porous wedge was investigated by Mukhopadhyay and Mandal [12]. MHD viscous Casson fluid flow and heat transfer with second-order slip velocity and thermal slip over a permeable stretching sheet in the presence of internal heat generation/absorption and thermal radiation was studied by Megahed [13]. Recently, Khalid et al. [14] considered the unsteady MHD flow of a Casson fluid in a porous medium and obtained the closed form solutions for both types of plate oscillations by Laplace transform method. Few other attempts for the Casson fluid can also be found in [15-17].

In all these studies mentioned above, the effect of Newtonian heating on the boundary was neglected. This situation may arise in the heat transfer flow problems dealing with heat exchanger, bounding surfaces absorb heat by solar radiation and conjugate heat transfer around fins. This idea was first initiated by Merkin [18] and later on, several others incorporated it in their works [19-21]. This motivates us to consider the Newtonian heating phenomenon in the present work. More exactly, our aim is to investigate the effects of Newtonian heating and inclined magnetic field on two dimensional flow of a Casson fluid over a stretching sheet.

Mathematical Formulation:

Consider the steady two-dimensional boundary layer flow of an incompressible Casson fluid over a linear stretching sheet. The x-axis is taken along the stretching sheet and the y-axis is normal to the sheet. The flow is generated due to stretching of the sheet caused by the simultaneous application of two equal and opposite forces along the x-axis. The sheet is subject to Newtonian heating. It is assumed that the fluid is electrically conducted and magnetic field is applied perpendicular to the sheet. The magnetic Reynolds number is considered to be small enough to neglect the induced magnetic field. The stretching velocity varies linearly with the distance from origin. Under above conditions the boundary layer equations of Casson fluid flow over a stretching sheet may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}\sin^2(\psi)u, \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}, \quad (4)$$

subject to the boundary conditions

$$u=U=cx, v=0, \frac{\partial T}{\partial y}=-h_s T, C=C_w \text{ at } y=0, \quad (5)$$

$$u\to 0,\ T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty, \quad (6)$$

where u and v are the velocity components in the directions of x and y, ν is the kinematic viscosity, β is the Casson parameter, B_0 is strength of the magnetic field, ρ is the density, σ is the electrical conductivity, T is the fluid temperature, k is the thermal conductivity, c_p is the heat capacity at a constant pressure, q_r is the radiative heat flux, D_m is the mass diffusivity, h_s is the heat transfer coefficient, T_∞ is the ambient temperature, C_w and C_∞ are the species concentration near and far away from the plate, respectively. Introduce the following dimensionless variables

$$u=cxf'(\eta),\ v=-\sqrt{c\nu}\,f(\eta), \eta=y\sqrt{\frac{c}{\nu}},$$
$$\theta(\eta)=\frac{T-T_\infty}{T_\infty}, \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \quad (7)$$

into equations (2) to (6), we obtain the following system of equations

$$\left(1+\frac{1}{\beta}\right)f'''+ff''-f'^2-M^2\sin^2(\psi)f'=0, \quad (8)$$

$$(1+R)\theta''+\Pr f\theta'=0, \quad (9)$$

$$\frac{1}{Sc}\phi''(\eta)+f\phi'(\eta)=0, \quad (10)$$

$$f'=1, f=0, \theta'=-\gamma(1+\theta), \phi=1 \text{ at } \eta=0, \quad (11)$$

$$f'\to 0,\ \theta\to 0, \phi\to 0 \text{ as } \eta\to\infty, \quad (12)$$

where

$$M^2=\frac{\sigma B_0^2}{\rho c},\ \Pr=\frac{\rho\nu c_p}{k}, R=\frac{16\sigma^* T_\infty^3}{3kk^*},$$
$$Sc=\frac{\nu}{D_m},\ \gamma=h_s\sqrt{\frac{\nu}{c}},$$

are the magnetic parameter, Prandtl number, radiation parameter, Schmidt number and Newtonian heating parameter, respectively. The exact solution of equation (8), subjected to the boundary conditions (11) and (12) is obtained as

$$f(\eta)=\frac{1}{\zeta}\left(1-e^{-\zeta\eta}\right), \quad (13)$$

where $\zeta=\sqrt{\left(\frac{\beta}{1+\beta}\right)\left(1+M^2\sin^2(\psi)\right)}$. Substituting equation (13) into equations (9) and (10), we get

$$\theta''(\eta)+\frac{\Pr_{\text{eff}}}{\zeta}\left(1-\exp(-\zeta\eta)\right)\theta'(\eta)=0, \quad (14)$$

$$\phi''(\eta)+\frac{Sc}{\zeta}\left(1-\exp(-\zeta\eta)\right)\phi'(\eta)=0. \quad (15)$$

In order to find the solutions of equations (14) and (15), we suppose that $t=\exp(-\zeta\eta)$, then equations (14) and (15) become

$$\zeta^2\left(t\theta''(t)+\theta'(t)\right)-\Pr_{\text{eff}}(1-t)\theta'(t)=0, \quad (16)$$

$$\zeta^2\left(t\phi''(t)+\phi'(t)\right)-Sc(1-t)\phi'(t)=0, \quad (17)$$

with the boundary conditions

$$\theta(0)=0,\ \theta'(1)=\frac{\gamma}{\zeta}\left(1+\theta(1)\right),$$
$$\phi(0)=0,\ \phi(1)=1. \quad (18)$$

Solution of equation (16) is obtained

$$\theta(t)=C_1+C_2\left(\text{WhittakerM}\left(-\frac{m-1}{2m},\frac{2m+1}{2m},\frac{t}{m}\right)\right)$$
$$\times\left(\frac{m^3(m+t+1)}{(m+1)(2m+1)}\right)F(t)+C_2F(t)\left(\frac{m^2(m+1)}{(2m+1)}\right)$$
$$\times\left(\text{WhittakerM}\left(\frac{m+1}{2m},\frac{2m+1}{2m},\frac{t}{m}\right)\right), \quad (19)$$

where WhittakerM is the Whittaker functions and C_1 and C_2 are constants and

$$F(t)=\left[t^{-\left(\frac{m+1}{m}\right)}\left(\frac{1}{m}\right)^{\frac{1}{m}}\left(\frac{t}{m}\right)^{-\left(\frac{m+1}{2m}\right)}e^{-\frac{t}{2m}}\right], m=\frac{\zeta^2}{\Pr_{\text{eff}}}.$$

It is noted from the above solution (19) that the boundary condition $\theta(0)=0$ gives $C_1=0$. Besides $\theta'(1)=\frac{\gamma}{\zeta}\left(1+\theta(1)\right)$ implies

$$C_2=\frac{p\left(2m^2+3m+1\right)}{2m^4\mathrm{W}_1F(1)}-\frac{p\left(2m^2+3m+1\right)}{m^4\mathrm{W}_2F(1)}$$
$$-\frac{\left(2m^2+3m+1\right)}{m^4F(1)\mathrm{W}_3}-\frac{p\left(2m^2+3m+1\right)}{m^4\mathrm{W}_3F(1)}$$
$$-\frac{\left(2m^2+3m+1\right)}{m^4\mathrm{W}_2F(1)}-\frac{p\left(2m^2+3m+1\right)}{m^3\mathrm{W}_3F(1)}$$
$$-\frac{p\left(2m^2+3m+1\right)}{m^3\mathrm{W}_2F(1)}-\frac{\left(2m^2+3m+1\right)}{2m^3\mathrm{W}_2F(1)}$$
$$+\frac{p\left(2m^2+3m+1\right)}{5m^3\mathrm{W}_1F(1)}-\frac{\left(2m^2+3m+1\right)}{2m^3\mathrm{W}_3F(1)}$$
$$+\frac{p\left(2m^2+3m+1\right)}{4m^2\mathrm{W}_1F(1)}-\frac{\left(2m^2+3m+1\right)}{m^2\mathrm{W}_2F(1)}$$
$$+\frac{p\left(2m^2+3m+1\right)}{m\mathrm{W}_1F(1)},\ p=\frac{\gamma}{\zeta}.$$

The final analytic solution for $\theta(t)$, can be easily obtained by substituting the values of C_1 and C_2 into equation (19). The analytic solution of equation (17) in term of independent variable t is

$$\phi(t)=C_3+C_4\left(\mathrm{WhittakerM}\left(-\frac{n-1}{2n},\frac{2n+1}{2n},\frac{t}{n}\right)\right)$$
$$\times\left(\frac{n^3\left(n+t+1\right)}{\left(n+1\right)\left(2n+1\right)}\right)G(t)+C_4\left(\frac{n^2\left(n+1\right)}{\left(2n+1\right)}\right)G\left(t\right)$$
$$\times\left(\mathrm{WhittakerM}\left(\frac{n+1}{2n},\frac{2n+1}{2n},\frac{t}{n}\right)\right),\qquad(20)$$

where C_3 and C_4 are constants and

$$G\left(t\right)=\left[t^{-\left(\frac{n+1}{n}\right)}\left(\frac{1}{n}\right)^{\frac{1}{n}}\left(\frac{t}{n}\right)^{-\left(\frac{n+1}{2n}\right)}e^{-\frac{t}{2n}}\right],n=\frac{\zeta^2}{Sc}.$$

By using conditions (18), we get the values of $C_3=0$ and

$$C_4=\frac{1}{G(1)}\left[\frac{n^3\left(n+2\right)}{\left(n+1\right)\left(2n+1\right)\mathrm{W}_4}+\frac{n^2\left(n+1\right)}{\left(2n+1\right)\mathrm{W}_5}\right],$$

where

$$\mathrm{W}_1=\mathrm{WhittakerM}\left(\frac{3m+1}{2m},\frac{2m+1}{2m},\frac{1}{m}\right),$$
$$\mathrm{W}_2=\mathrm{WhittakerM}\left(\frac{m+1}{2m},\frac{2m+1}{2m},\frac{1}{m}\right),$$
$$\mathrm{W}_3=\mathrm{WhittakerM}\left(-\frac{m-1}{2m},\frac{2m+1}{2m},\frac{1}{m}\right),$$
$$\mathrm{W}_4=\mathrm{WhittakerM}\left(-\frac{n-1}{2n},\frac{2n+1}{2n},\frac{1}{n}\right),$$
$$\mathrm{W}_5=\mathrm{WhittakerM}\left(\frac{n+1}{2n},\frac{2n+1}{2n},\frac{1}{n}\right).$$

The above solutions for temperature $\theta(t)$ and concentration $\phi(t)$ fields are obtained in the form of independent variable t. However temperature and concentration fields are function of independent variable η. So, in that case, the solutions for temperature and concentration fields given in equations (19) and (20), respectively, can be easily obtained by replacing $t=\exp(-\zeta\eta)$ in equations (19) and (20).

Results and Discussion:

In this section, we have discussed the velocity temperature and concentration fields for different physical parameters such as Casson parameter β, magnetic parameter M and Newtonian heating parameter γ. Figure 1 shows the effects of magnetic parameter M on the velocity field when $\beta=0.3$ and $\psi=\pi/2$. This figure shows that an increase in magnetic parameter M decreases the velocity. The reason is that the applied magnetic field normal to the flow direction induces the drag force also called Lorentz force which provides resistance to flow and fluid velocity is decreased. The effect of the Casson parameter β on the velocity field is shown in Figure 2, when $M=2$ and $\psi=\pi/2$. The increase of Casson parameter β means decrease of the yield stress (large values of Casson parameter means fluid behaves like Newtonian fluid). From this figure it is seen that velocity decreases with an increase in Casson parameter β, so that boundary layer thickness also decreases. The effect of the magnetic parameter M on the temperature field is illustrated in Figure 3, when $\beta=0.3,\mathrm{Pr}=1,R=0.5,\gamma=0.1$ and $\psi=\pi/2$. This figure shows that an increase in magnetic parameter M increases the temperature of the fluid. The temperature field for different values of the Casson parameter β is plotted in Figure 4, when $M=2,\mathrm{Pr}=1,R=0.5,\gamma=0.1$ and $\psi=\pi/2$. From this figure it is found that temperature increases with an increase in Casson parameter β. Figure 5 depicts that an increase of Newtonian heating parameter γ with fixed values of other parameters $M=2$, $\beta=0.3,\mathrm{Pr}=1,R=0.5$ and $\psi=\pi/2$, temperature also increases and thermal boundary layer becomes thicker. Finally, the effect of the magnetic parameter M on the concentration field is presented in Figure 6, when $\beta=0.3,Sc=0.5$ and $\psi=\pi/2$. The similar type of results seen in this figure as observed in Figure 3 for temperature field.

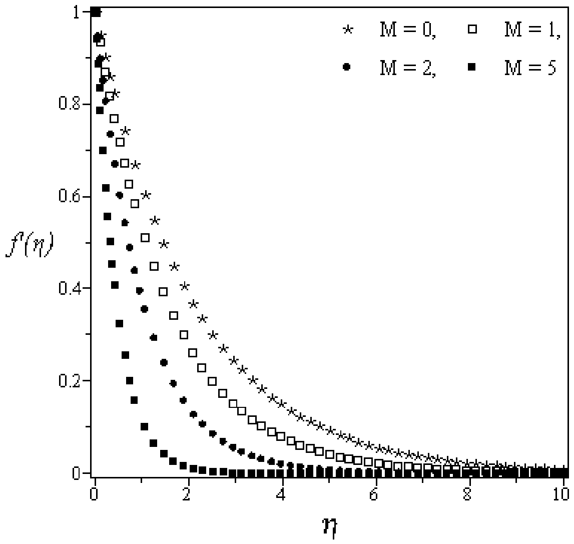

Figure 1: Velocity field for various values of M

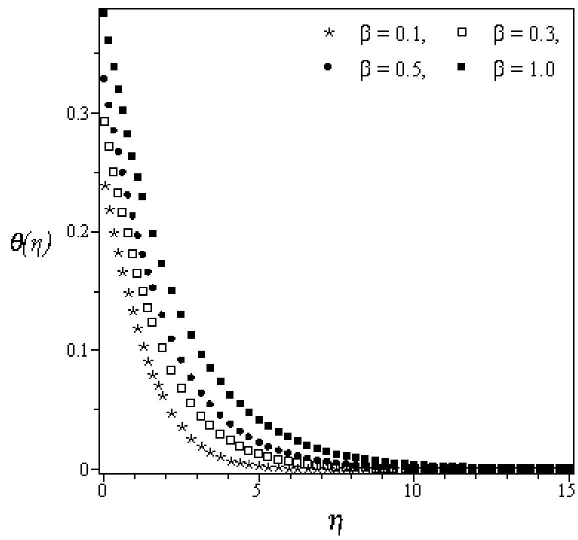

Figure 4: Temperature field for various values of β

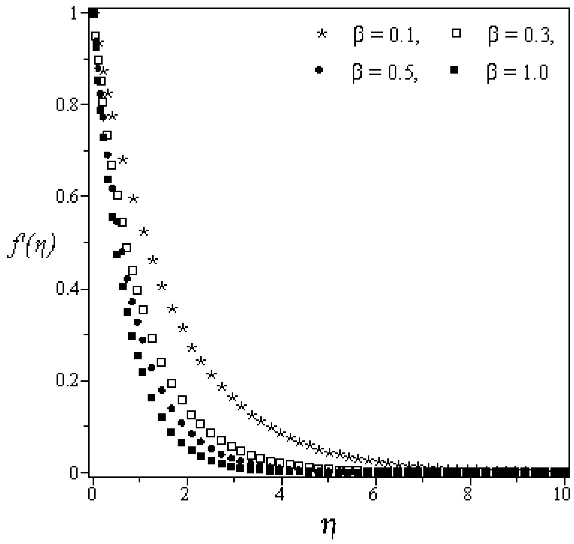

Figure 2: Velocity field for various values of β

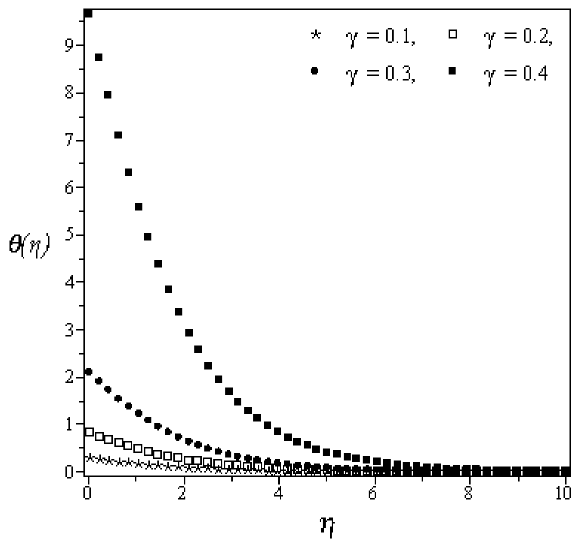

Figure 5: Temperature field for various values of γ

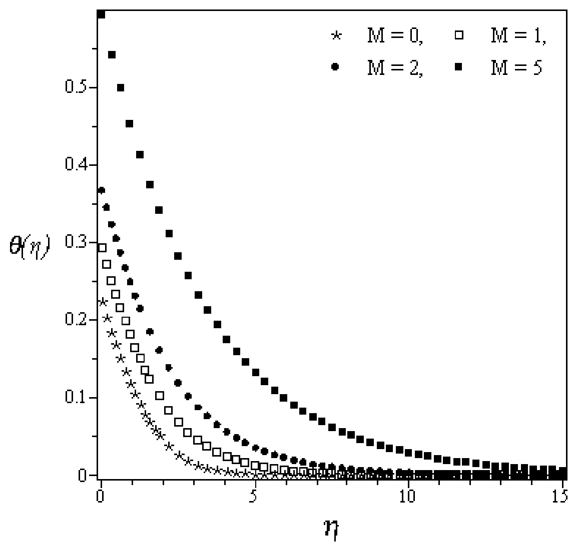

Figure 3: Temperature field for various values of M

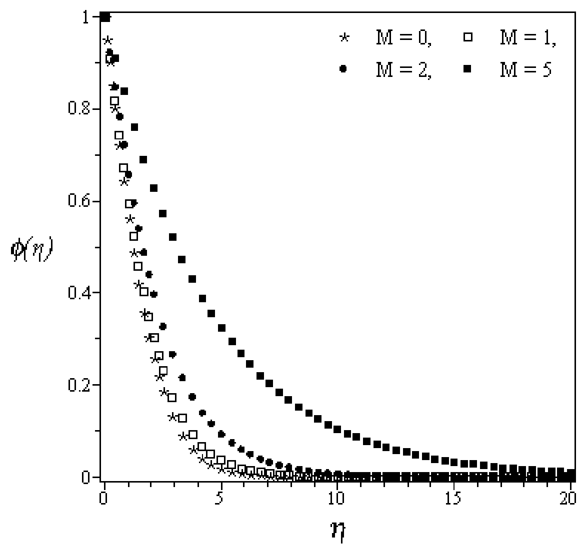

Figure 6: Concentration field for various values of M

Conclusion:

This article addresses the heat and mass transfer flow of a Casson fluid over a linear stretching sheet in presence of an inclined magnetic field. It is found that the temperature and concentration are increasing functions of β and M. However, these parameters have opposite effects on the velocity field.

Acknowledgement:

The authors would like to acknowledge Universiti Malaysia Pahang, Malaysia for the financial support through vote numbers GRS150334 and RDU140111.

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