Chapter 3

Problem Formulation

In Section 3.1, we revisit the algorithm proposed by Gale & Shapley[7] to attain stable matching for a given bipartite sets of men and women. Then in Section 3.2, we introduce the dynamic of MAS to represent the individual of man and woman, and our problem formulation. Next, we introduce the new definition of Dynamical Stable Matching in Section 3.3 . We discuss the formulation of the Lyapunov function based on the preference lists in Section 3.4. The chapter is summarized in Section 3.5

3.1 Gale and Shapley algorithm

The terminology of SMP in the matching theory was first coined in year 1962 in the seminal work of Gale and Shapley[7]. The original optimization problem involved on finding the most stable partners for a given sets of men and women such that the established partnerships do not constitute any blocking pairs. Refer Defs. 2.3.3 and 2.3.4 on the description of *blocking pair* and *stability* in terms of SMP framework.

We assume that there exists bipartite sets of $\mathcal{M} := \{m_1, m_2, \ldots, m_P\}$ and $\mathcal{W} := \{w_1, w_2, \ldots, w_P\}$, where $\mathcal{M}$ and $\mathcal{W}$ stand for the men and women sets, respectively. Meanwhile, P is the number of pairs. Each of the individuals in these two sets rank orderly their preferred partners in the preference lists. We denote m as the current man to propose to the woman w in his list. If the proposed woman w is already engaged, then $\bar{m}$ is to denote her current partner. On the hand, we use $\bar{w}$ as the next-woman to be proposed from of the m's preference list. The procedure of the G-S algorithm [7] is presented in Algorithm 1.

Algorithm 1 Gale & Shapley Algorithm (Men-proposer)

procedure STABLEMATCHING(m, w)
    Initialization: set all men and women to be free
    while $\exists$ free m **do**
        assign $w = m$'s not-yet-proposed woman of the highest rank
        if w is free **then**
            $m = p_M(w)$ & $w = p_M(m)$
            $(m, w) \rightarrow M$ ▷ m, w become *partner*
        else if w is already engaged with $\bar{m}$ **then**
            if m precedes $\bar{m}$ in w's preference list **then**
                $w = p_M(m)$
                $(m, w) \rightarrow M$ ▷ w choose new *partner*
                $\bar{m}$ becomes free
            else
                $w = p_M(\bar{m})$ remain engaged
                $(\bar{m}, w) \rightarrow M$ ▷ $(\bar{m}, w)$ remain *partner*
                m proposes to $\bar{w}$ in his preference list
            end if
        end if
    end while
    Final matching is established.
end procedure

This algorithm is guaranteed to terminate at $O(P \log P)$ iterations[13], and upon termination, stable pairs M will be established. The stable pairs established in M is said to be *Men-Optimal, Women-Pessimal* which favors men over women, since men are the proposer and women are the receiver. The results will be the opposite such that to favors women over men, if women is the first party to give the proposal [7, 33].

Notice that G-S algorithm utilizes the sequential steps of optimizing in seeking for the stable pairs between the bipartite sets. In the following chapters, we attempt to address the same optimization problem as before, but utilize the multi-agent system to achieve the objective.

3.2 Agents dynamics

We consider MAS consists of N number of agents that move on n-dimensional Euclidean space. Each of the agents is described by a single integrator as

$$\dot{x}_i = u_i, \tag{3.1}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ are the position vector and (velocity) control input to be designed, respectively. To represent the dynamic of the total system, we express the total state and control input vectors as

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \in \mathbb{R}^{nN} \text{ and } u = \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} \in \mathbb{R}^{nN} \tag{3.2}$$

respectively. Therefore, we may further rewrite the total system dynamic in the form of

$$\dot{x} = u. \tag{3.3}$$

By taking the average positions of all agents, the group's center of formation can be determined, such that

$$x_c = \frac{1}{N} \sum_{i=1}^{N} x_i. \tag{3.4}$$

On the other hand, it is also assumed that the desired state trajectory $x_d \in \mathbb{R}^n$ for the group center x_c can be achieved by $u_d \in \mathbb{R}^n$, that is

$$\dot{x}_d = u_d. \tag{3.5}$$