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Forecasting Malaysian Gold Using a Hybrid of ARIMA and GJR-GARCH Models

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Abstract

An effective way to improve forecast accuracy is to use a hybrid model. This paper proposes a hybrid model of linear autoregressive moving average (ARIMA) and non-linear GJR-GARCH model also known as TARCH in modeling and forecasting Malaysian gold. The goodness of fit of the model is measured using Akaike information criteria (AIC) while the forecasting performance is assessed using mean absolute percentage error (MAPE), bias proportion, variance proportion and covariance proportion.

Keywords: ARIMA-GJR, TARCH, hybrid model, heteroscedasticity, volatility clustering

1 Introduction

Malaysian gold bullion coins called *Kijang Emas* are legal tender coins whose market price depends on their gold content. The price depends on the prevailing international gold price. They are investment coins where the daily selling and buying prices of these coins are important to investors in order to make an investment decision.

For forecasting purposes, Autoregressive Integrated Moving Average (ARIMA) models have been widely used to capture the long term trend in a time series. In time series where volatility clustering, the situation when large changes in the data tend to cluster together and resulting in persistence of the amplitudes of the changes are prevalent, ARCH based models have been used. In the case of Malaysian gold prices, a hybrid model was considered an effective way to improve forecast accuracy [1]. ARIMA-GARCH model was developed and it outperformed ARIMA model. However, in the study of symmetric and asymmetric Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models for forecasting Malaysian gold prices, a variant of GARCH, called TGARCH was shown to outperform GARCH, GARCH-M and EGARCH models [2].

This paper proposes a hybrid of linear autoregressive moving average (ARIMA) and a variant of non-linear generalized autoregressive conditional heteroscedasticity (GARCH) called GJR-GARCH in modeling and forecasting Malaysian gold price.

In this study, the goodness of fit of the model is measured using Akaike information criteria (AIC) while the forecasting performance is assessed using mean absolute percentage error (MAPE), bias proportion, variance proportion and covariance proportion. All analyses are carried out using a software called E-views.

In the next section, the methodology of the study is presented. This is followed by data analysis in Section 3. The study is concluded in Section 4.

2 Methodology

Hybrid ARIMA-GJR Models

Box and Jenkins developed a general class of models called ARIMA for forecasting non-stationary time series [3]. Non-stationarity exists in mean and/or in variance. To remove non-stationarity in mean, transformations such as differencing can be applied. Non-stationary in variance on the other hand, can be removed by a proper variance stabilizing transformation introduced by Box and Cox [4]. The ARIMA (*p*,*d*,*q*) can be written as

$$\phi_p(B)(1-B)^d y_t = \delta + \theta_q(B)\varepsilon_t$$

where $\phi_p(B) = 1-\phi_1 B - ... - \phi_p B^p$ is the autoregressive operator of order *p*; $\theta_q(B) = 1-\theta_1 B - ... - \theta_q B^q$ is the moving average operator of order *q*; $(1-B)^d$ is the d^{th} difference; *B* is backward shift operator; and ε_t is the error term at time *t*.) Using a sample data, the orders are identified through the autocorrelation function (ACF) and the partial autocorrelation function (PACF). The error terms are generally assumed to be independent identically distributed random variables (i.i.d.) sampled from a normal distribution with zero mean, $\varepsilon_t \sim N(0,\sigma^2)$ where σ^2 is the variance. At this point, the model can be used for forecasting.

Not all time series errors satisfy the assumption of common variance. Sometimes, the variances are time-varying and conditional. Engle in 1982 developed autoregressive conditional heteroskedasticity (ARCH) class of models to describe a series with time-varying conditional variance. These models were generalized by Bollerslev in 1986 and are known as GARCH models [5]. The GARCH models are able to capture volatility clustering or the periods of fluctuations, and predict volatilities in the future [6]. In the GARCH model, past variances and past variance forecasts are used to forecast future variances. The standard GARCH model is symmetric in response to past volatility and variance. The GARCH (*p*, *q*) model is

$$y_t = \mu + \varepsilon_t$$

where y_t = time series data; $u_t = \varepsilon_t \sigma_t^2 = \varepsilon_t \sqrt{h_t}$, $\varepsilon_t \sim N(0,1)$

$$h_t = \delta + \left(\alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots\right) + \left(\beta_1 h_{t-1} + \beta_2 h_{t-2} + ..\right)$$
$$= \delta + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_i \sigma_{t-j}^2$$

where $\delta = \alpha_0(1-\beta_1)$, $h_t = \sigma_t^2$, $\alpha_1 + \beta_1 < 1$ for stationarity, $\alpha_i, \beta_j > 0$

The GARCH term is σ^2, where the last period forecast variance is of order *p*,
The ARCH term is ε^2, which is the information about volatility from the previous period measured as the lag of squared residual from the mean equation. It is of order *q*.

Good news and bad news have different effects on volatility [7]. Between good and bad, bad news is said to have more effect on future volatility of returns. When this happens, symmetric GARCH models are unable to capture the asymmetry of volatility response. A characteristic of asymmetric volatility is leverage effect. Leverage effect is asymmetry in volatility induced by big ‘positive’ and ‘negative’ asset returns. Asymmetric GARCH models are able to explain the leverage effects by enabling conditional variance to respond asymmetrically to rises and falls in volatility returns. A model that treats positive and negative news symmetrically as proposed by Glosten, Jagannathan and Runkle is Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) which is also known as TARCH [8]. With positive or good news, $\varepsilon_{t-i} < 0$ and with negative or bad news, $\varepsilon_{t-i} > 0$. TARCH can capture the phenomenon of positive news hitting on the financial market with the market being in a calm period; and the negative news hitting on the financial market with the market entering into a fluctuating period and high volatility. The model is as follows:

$$h_t = \delta + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1} + \sum_{j=1}^{p} \beta_j h_{t-j}^2$$

where $h_t = \sigma_t^2$, γ is the leverage term and α_i, β_j and γ are constant parameters. d_t is an indicator imitation variable where

$$d_t = \begin{cases} 1, & \varepsilon_t < 0 \text{ (bad news), there is a leverage effect} \\ 0, & \varepsilon_t \geq 0 \text{ (good news), vice versa} \end{cases}$$

The GJR (p,q) model has p GARCH coefficients associated with lagged variances, q ARCH coefficients associated with lagged squared innovations, and q leverage coefficients associated with the square of negative lagged innovations.

Augmented Dickey-Fuller (ADF)

A unit-root test called ADF can be used to determine stationarity of a time series. The null hypothesis states that the series is non-stationary. The testing procedure is applied to the model $\Delta y_t = \alpha_0 + \beta t + \theta y_{t-1} + \sum_{i=1}^{k} \alpha_i \Delta y_{t-1} + \varepsilon_t$ where y_t = the tested time series, Δ = the first difference, k = the lag order of the autoregressive process and $\varepsilon_t = y_t - \mu_t$ are the series residual.

Akaike Information Criterion (AIC)

The goodness of fit of a model can be assessed using AIC = $2k - 2\ln(L)$, where L = the maximized value of the likelihood function for the estimated model and k = the number of free and independent parameters in the model.

Breusch-Godfrey Lagrange Multiplier Test (BG-LM)

Autocorrelation is tested using BG-LM test. Rejection of the null hypothesis state that there exists serial correlation of any order up to a certain order lag.

ARCH Lagrange Multiplier Test (ARCH-LM)

The presence of heterocedasticity is determined by using ARCH-LM test. The squared series, ε_t^2 defined as $\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + ... + \alpha_p \varepsilon_{t-p}^2$ is used to check the presence of ARCH effects where p is the length of ARCH lags and ε_t is the residual of the series. Test statistic for LM test is the usual F statistics for the squared residuals regression. The null hypothesis states that ARCH effects do not exist.

Jarque-Bera Test

The Jarque–Bera test is a test of whether sample data have the skewness and kurtosis matching a normal distribution. The null hypothesis states that the sample data follows a normal distribution. The test statistic is defined as

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right)$$

where n = the number of observations, S = the sample skewness and K = the sample kurtosis.

Mean Absolute Percentage Error (MAPE)

The accuracy of forecasts (measured in terms of percentage) is measured using MAPE with the following formula:

$$\text{MAPE} = \left(\left(\sum_{t=1}^{n} \left| \frac{y_t - \hat{y}_t}{y_t} \right| \right) / n \right) \times 100\%$$

where y_t = the actual value, $\hat{y}_t$ = the forecast value and n = the number of periods.

3 Data Analysis and Results

The daily selling prices of 1 oz Malaysian gold recorded from 3 January 2011 until 20 January 2015 were used. The data are plotted in Figure 1.

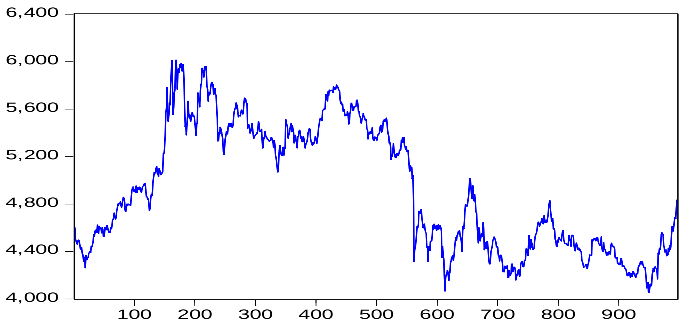


Figure 1: Daily 1 oz Malaysian Gold Prices from 3 Jan 2011 to 20 Jan 2015

Returns were used since a downward trend exists in the data. The return on the t^{th} day is defined as $r_t = \ln(y_t) - \ln(y_{t-1})$. The stationarity of the returns was confirmed by using ADF unit-root test.

Ninety percent of the observations, that is from 3 January 2011 until 20 August 2014 which account for 90% of the data were used for modeling to obtain an ARIMA model. Using ordinary least squares method to estimate the parameters, an appropriate ARIMA model for this series is ARIMA (2, 1, 2) with an AIC value of 10.88681. When the model was used for forecasting, the MAPE value for in-sample forecast is 0.759026. Out-sample forecasts were produced for observations in the period from 21 August 2014 until 20 January 2015 with MAPE value of 0.693575.

Breusch-Godfrey Serial Correlation LM Test was performed on ARIMA (2, 1, 2) and the model was confirmed to not suffer from serial correlation as illustrated in Table 1.

Table 1: Breusch-Godfrey Serial Correlation LM Test

F-statistic	0.202335	Prob. F(2, 887)	0.8169
Obs*R-squared	0.407648	Prob. Chi-Square(2)	0.8156

Figure 2 presents the descriptive statistics of the residuals where the mean of the residuals is close to zero and the residuals have excess kurtosis. Based on the Jarque-Bera statistic, the null hypothesis of residuals following the normal distribution is rejected.

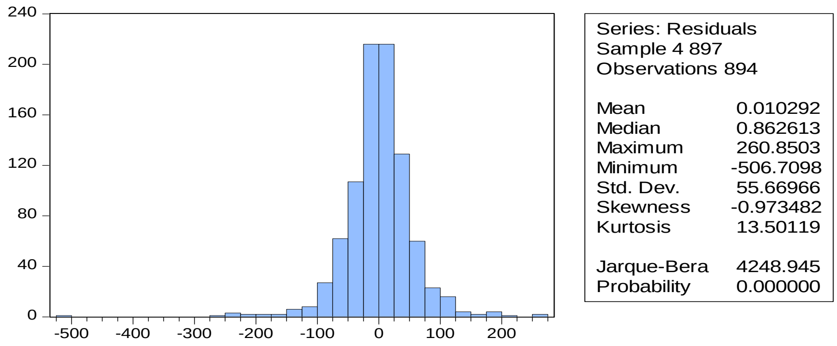


Figure 2: Descriptive Statistics of the Residuals for ARIMA(2, 1, 2)

Figure 3 presents the plot of the residuals where there exists clear volatility clustering in the residuals.

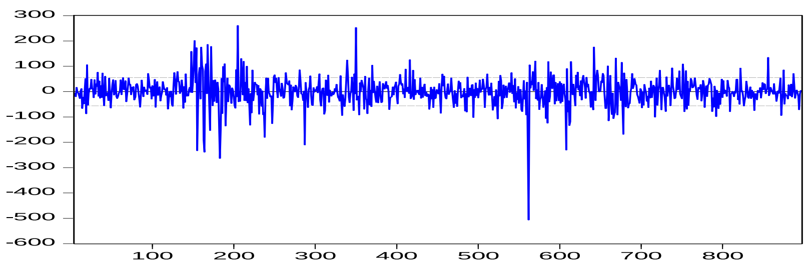


Figure 3: Volatility Clusterings in the Residuals for ARIMA(2, 1, 2)

Using ARCH-LM test, ARIMA(2, 1, 2) residuals were tested for ARCH effects. The results as presented in Table 2 indicate that at 5% significance level, the null hypothesis of ARCH effects do not exist is rejected.

Table 2: Heteroskedasticity Test for ARIMA(2, 1, 2)

F-statistic	34.88902	Prob. F(1,891)	0.0000
Obs*R-squared	33.64971	Prob. Chi-Square(1)	0.0000

Based on the presence of volatility clustering in the residuals and the ARCH-LM test result, it can be concluded that the model was not a good fit. A better model for forecasting Malaysian gold was deemed necessary. A hybrid model was considered an effective way to improve forecast accuracy [1]. In the study of symmetric and asymmetric Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models for forecasting Malaysian gold prices, a variant of GARCH, called TGARCH was shown to outperform GARCH, GARCH-M and EGARCH models [2]. The TGARCH model is a GARCH variant that includes leverage terms for modeling asymmetric volatility clustering. Hence, the current study proposes using ARIMA-GRJ model to analyze the series understudied. Table 3 presents the estimation results for variance equation of the hybrid ARIMA (2, 1, 2)-GJR (1, 1) model as applied to Malaysian gold.

Table 3: Estimation Results for Variance Equation of ARIMA (2, 1, 2)-GJR (1, 1)

Variance Equation

C	422.8536	82.39860	5.131806	0.0000
RESID(-1)^2	0.125027	0.028159	4.440029	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.133392	0.035153	3.794573	0.0001
GARCH(-1)	0.669151	0.046196	14.48512	0.0000
R-squared	0.010391	Mean dependent var		-0.166667
Adjusted R-squared	0.005938	S.D. dependent var		56.05145
S.E. of regression	55.88477	Akaike info criterion		10.68289
Sum squared resid	2776443.	Schwarz criterion		10.73117
Log likelihood	-4766.252	Hannan-Quinn criter.		10.70134
F-statistic	1.166840	Durbin-Watson stat		1.937997
Prob(F-statistic)	0.316238			

In Table 3, since the coefficient of RESID (-1)^2*(RESID(-1)<0) is positive and significant, we can conclude that the model has leverage effects. This means that bad news can have more impact on the conditional variance than good news. The AIC value of the model is 10.68289. The residuals of the model are tested for ARCH effects using ARCH-LM test, with the results presented in Table 4.

Table 4: Heteroskedasticity Test for ARIMA (2, 1, 2)-GJR (1, 1)

F-statistic	0.091229	Prob. F(1,891)	0.7627
Obs*R-squared	0.091425	Prob. Chi-Square(1)	0.7624

Based on Table 4, at significance level of 5%, the null hypothesis of no ARCH effects cannot be rejected. The hybrid model is then tested for serial correlation as presented in Table 5. From the results in Table 5, the null hypothesis of no serial correlation cannot be rejected.

Table 5: Ljung-Box Q-statistics on squared residuals for ARIMA(2,1,2)-GJR(1,1)

lags	AC	PAC	Q-Stat	Prob	lags	AC	PAC	Q-Stat	Prob
1	0.010	0.010	0.0918		19	0.011	0.010	5.5901	0.986
2	-0.015	-0.015	0.3027		20	0.016	0.015	5.8144	0.990
3	0.007	0.007	0.3425		21	0.006	0.007	5.8495	0.994
4	0.030	0.030	1.1481		22	0.055	0.055	8.6658	0.967
5	-0.022	-0.023	1.5919	0.207	23	0.040	0.037	10.112	0.950
6	-0.002	-0.001	1.5951	0.450	24	0.014	0.016	10.283	0.963
7	0.011	0.010	1.6972	0.638	25	0.015	0.016	10.490	0.972
8	0.006	0.005	1.7319	0.785	26	-0.038	-0.041	11.802	0.961
9	-0.016	-0.015	1.9645	0.854	27	-0.011	-0.010	11.915	0.972
10	-0.021	-0.021	2.3454	0.885	28	-0.028	-0.028	12.649	0.972
11	0.026	0.025	2.9654	0.888	29	-0.005	-0.003	12.671	0.980
12	-0.011	-0.012	3.0730	0.930	30	0.015	0.017	12.890	0.985
13	-0.027	-0.025	3.7310	0.928	31	0.012	0.010	13.035	0.989
14	0.008	0.008	3.7840	0.957	32	-0.014	-0.011	13.208	0.992
15	0.025	0.021	4.3331	0.959	33	0.035	0.034	14.366	0.989
16	0.027	0.029	4.9740	0.959	34	0.082	0.081	20.679	0.898
17	0.023	0.025	5.4628	0.964	35	0.011	0.013	20.785	0.917
18	-0.004	-0.006	5.4756	0.978	36	-0.041	-0.042	22.391	0.897

The descriptive statistics of the residuals from ARIMA-GJR model are presented in Figure 4.

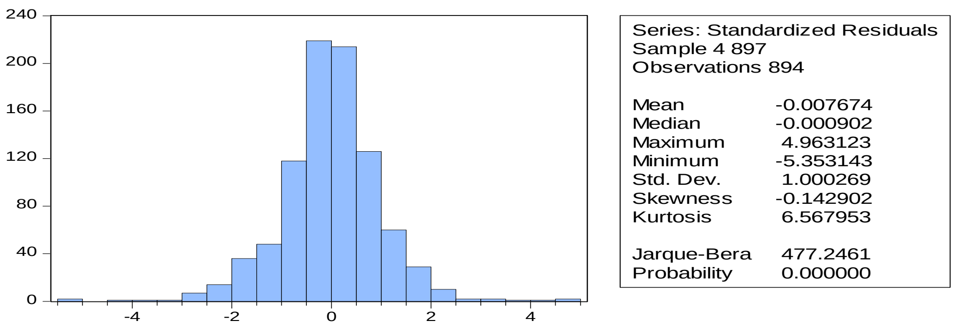
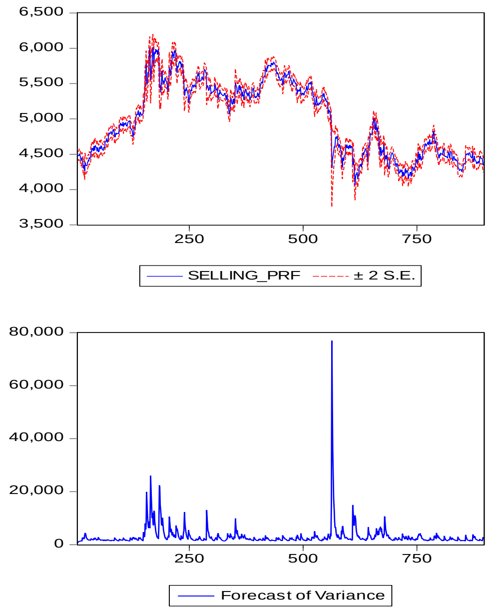


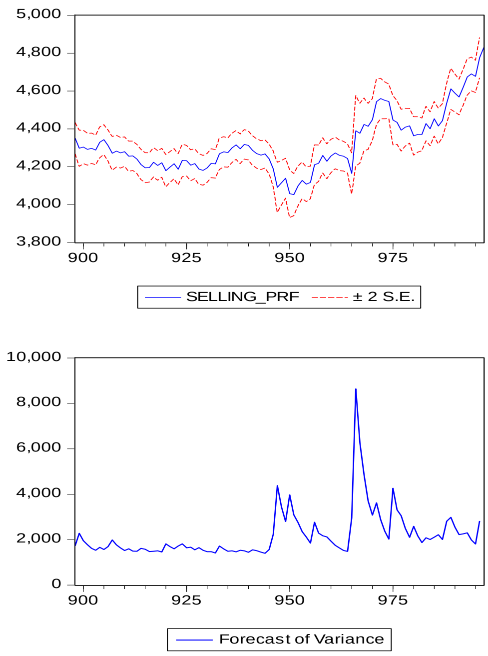
Figure 4: Descriptive Statistics of the Residuals for ARIMA (2, 1, 2)-GJR (1, 1)

The residuals are not normally distributed as implied by Jarque-Bera statistic in Figure 4. However, the hybrid model is used for forecasting. The results of in-sample and out-sample forecasting are presented in Figure 5 and Figure 6 respectively.



Forecast: SELLING_PRF	
Actual: SELLING_PRICE	
Forecast sample: 1 897	
Adjusted sample: 4 897	
Included observations: 894	
Root Mean Squared Error	55.72828
Mean Absolute Error	37.74254
Mean Abs. Percent Error	0.758841
Theil Inequality Coefficient	0.005560
Bias Proportion	0.000037
Variance Proportion	0.000008
Covariance Proportion	0.999955

Figure 5: In-Sample Forecasting Results of ARIMA (2, 1, 2)-GJR (1, 1)



Forecast: SELLING_PRF	
Actual: SELLING_PRICE	
Forecast sample: 898 997	
Included observations: 100	
Root Mean Squared Error	43.25805
Mean Absolute Error	29.96296
Mean Abs. Percent Error	0.689450
Theil Inequality Coefficient	0.005001
Bias Proportion	0.011790
Variance Proportion	0.037129
Covariance Proportion	0.951081

Figure 6: Out-Sample Forecasting Results of ARIMA (2, 1, 2)-GJR (1, 1)

4 Conclusion

The results of modelling and forecasting of 1 oz Malaysian gold daily prices recorded from 3 January 2011 until 20 January 2015 using ARIMA-GJR are tabulated in Table 5. The results are compared with the results obtained by using ARIMA model.

Table 5: Modelling and Forecasting Results

Models	ARIMA	ARIMA-GJR
AIC	10.88681	10.68289
MAPE of in-sample	0.759026	0.758841
MAPE of out-sample	0.693575	0.689450
Bias Proportion of in-sample	0.000000	0.000037
Variance Proportion of in-sample	0.000013	0.000008
Covariance Proportion of in-sample	0.999987	0.999955
Bias Proportion of out-sample	0.013627	0.011790
Variance Proportion of out-sample	0.036969	0.037129
Covariance Proportion of out-sample	0.949404	0.951081

Based on AIC values, ARIMA-GJR is a better model. In terms of forecasting, MAPE of both in-sample and out-sample using ARIMA-GJR are lower than using ARIMA only. There are not much differences in bias proportion which measures how far the mean of the forecast is from the mean of the actual series and in the variance proportion which measures how far the variation of the forecast is from the variation of the actual series. There is also not much difference in the remaining unsystematic forecasting errors as measured by covariance proportion. However, it can be concluded that a hybrid model of ARIMA-GJR is a better forecasting model since even though the residuals do not follow a normal distribution, the model does not suffer from serial correlation and there are no ARCH effects.

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