Journal of Structural Engineering
Vol. 40, No. 5, December 2013 - January 2014 pp. 507-511 No. 40-T7

Mechanical postbuckling behavior of circular plates through concept of equivalent radial tensile load

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Received: 06 August 2012; Accepted: 10 September 2012

Aerospace and automobile structural systems are assemblages of simple structural members like, columns, circular and rectangular plates etc., designed with lesser margins of safety and also have minimum mass, are subjected to severe mechanical or thermal loads during the service conditions. As such, these structural members are relatively more flexible, compared to the same used in the other fields of engineering. Due to these requirements, the study of the thermal or mechanical postbuckling behavior of these structural members gained importance. To predict the mechanical or thermal postbuckling behavior of these structural members, a more rigorous geometric nonlinear (large deflections) analysis is required. The present study deals with the concept of the effective tensile load developed, due to large deflections, to derive a simple heuristic formula, called simply as the formula, to predict the mechanical postbuckling behavior of circular plates, called simply as plates, without going for complex mathematical treatment[1,2]. The circular plate problem, formulated in the polar coordinate system, is relatively more involved, as the radial and circumferential strains are coupled by the radial displacement, compared to the other structural members, which are formulated in the Cartesian coordinate system. Such formulas are handy and useful to the designers/analysts to quickly evaluate the mechanical postbuckling behavior of the plates, in the iterative design/analysis cycles.

A detailed study of the thermal postbuckling of the structural members was presented by Ziegler and Rammerstorfer[3]. The difference between the mechanical and thermal postbuckling study of the plates is that the radial displacement is constrained in the case of thermal postbuckling and there is no constraint on this displacement in the case of mechanical postbuckling. The constraint on the radial displacement of the plate facilitates the development of the formula easily for the thermal postbuckling[4]. The development of such a similar formula to predict the mechanical postbuckling behavior of the plates is not that straight forward, as the evaluation of the radial tensile load with moving edges is difficult, though not impossible.

It is the hunch of the authors that a similar formula, derived for the thermal postbuckling exists, to predict the mechanical postbuckling behavior of the plates. This motivated the authors to propose intuitively, the concept of an equivalent radial tensile load that can be used to predict the mechanical postbuckling behavior of plates, though a rigorous mathematical formulation is yet to be developed. It is demonstrated here that the mechanical postbuckling behavior of the plates can be easily and quickly predicted by considering the effects like the shear deformation, taper, edge elastic rotational restraints, elastic foundation etc., once the corresponding results are available for thin plates with the same complicating effects. The numerical results, obtained from the proposed concept to predict the mechanical postbuckling behavior of the plates, match very well with those available in the literature on this topic. The method followed to derive the formula for thermal postbuckling is cursorily presented in the following section, for the sake of clarity.

Formula for thermal postbuckling load

The logical sequence of deriving the formula for thermal postbuckling of plates of thickness 't' and radius 'a', for the axisymmetric case, is presented in detail in Ref.[4]. If the temperature of the plate increases, from its stress free temperature, a radial compressive load N is developed and if its value is equal to the buckling load/temperature N_L, the plate just buckles. If the temperature is further increased beyond the buckling temperature, the corresponding incremental edge compressive radial load ΔN_{NL} is increased. This increased load (ΔN_{NL}) is balanced by the radial tensile load T developed due to the large deflections. As such, the mechanical equivalent of the thermal postbuckling load N_{NL} is given by

$$N_{NL} = N_L + T \tag{1}$$

The relation between the equivalent mechanical edge radial load N developed due to a uniform temperature rise is given by the following standard equation, as

$$N = 12(1+\nu)\alpha T_r \frac{a^2}{t^2} \tag{2}$$

where ν is the Poisson ratio, α is the coefficient of linear thermal expansion and T_r is the temperature rise.

The formula to predict the thermal postbuckling behavior of the plate, in the nondimensional form and by rearranging the terms of Eq. (1), is obtained, as

$$\frac{\lambda^t_{NL}}{\lambda_L} = 1 + \frac{\lambda^t_T}{\lambda_L} = 1 + \gamma^t_T \left(\frac{b}{t}\right)^2 \tag{3}$$

where λ_{NL} is the mechanical equivalent of the thermal postbuckling load parameter $= \frac{N_{NL}a^2}{D}$, λ_L is the linear buckling load parameter $= \frac{N_L a^2}{D}$,

λ_T is the radial tensile load parameter $= \dfrac{Ta^2}{D}$,

λ_T is a constant for the thermal postbuckling,

D is the plate flexural rigidity $= \dfrac{Et^3}{12(1-\nu^2)}$

E is the Young's modulus, ν is the Poisson ratio. The superscript 't' in Eq. (3) denotes the thermal postbuckling problem.

EQUIVALENT RADIAL TENSILE LOAD FOR MECHANICAL POSTBUCKLING

As it is not straight forward to evaluate the radial tensile load developed in the plates with radially movable edges, the concept of the equivalent radial tensile load developed, is followed here to arrive at the formula for the mechanical loads. It is to be noted here that this formula can be used for the plates with complicating effects, once the mechanical postbuckling behavior for the basic configuration of the plate is known. The complicating effects considered in this study are, moderately thick linearly tapered plates, uniform thick plates with the edge elastically restrained against rotation and thin plates resting on an elastic foundation, as shown in Fig. 1.

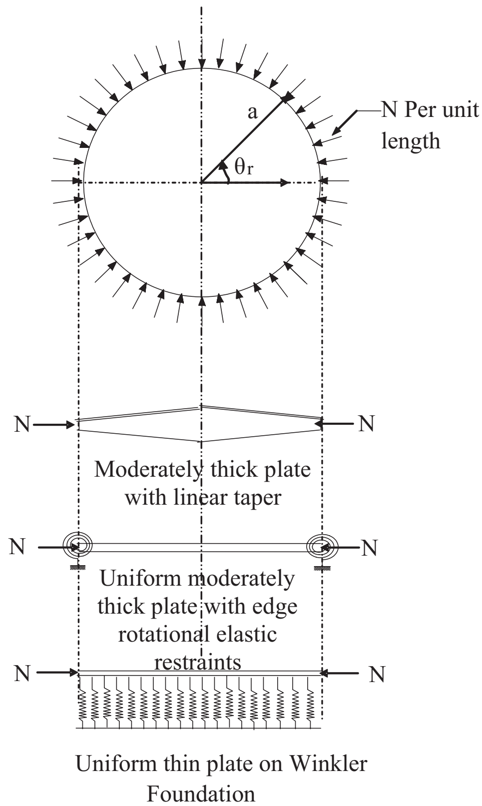

Fig. 1 Different configurations of circular plate

A formula, similar to that of the thermal postbuckling, is proposed here to obtain the ratios of the mechanical postbuckling to buckling loads in terms of $\dfrac{\lambda^m_{NL}}{\lambda_L}$ in the case of the plate subjected to a radial edge uniform compressive load N, as

$$\frac{\lambda^m_{NL}}{\lambda_L} = 1 + \frac{\lambda^m_T}{\lambda_L} = 1 + \gamma^m_T \left(\frac{b}{t}\right)^2 \tag{4}$$

with the same definitions of λ_{NL}, λ_L, λ_T, γ_T, b and t appearing in Eq. (3). The superscript 'm' denotes that these quantities correspond to the mechanical load.

The following steps explain the procedure to obtain the mechanical postbuckling load ratios $\frac{\lambda_{NL}^{m}}{\lambda_L}$ for a specified central deflection ratio *b*/*t* through the formula represented by Eq. (4).

Step 1: Accurate analytical or numerical solutions for the mechanical postbuckling behavior of plates having the basic configuration in terms of $\frac{\lambda_{NL}^{m}}{\lambda_L}$ are available in the open literature[1, 2, 5-7]. The details of the basic configurations of the plate with the complicating effects are given while presenting the numerical results.

Step 2: Knowing the value of $\frac{\lambda_{NL}^{m}}{\lambda_L}$ for the specified amplitude ratio (*b*/*t*) the value of for the basic plate can be evaluated.

Step 3: From the value of $\gamma_\tau{}^m$ evaluated, the mechanical postbuckling behavior of plates with complicating effects can be easily evaluated.

NUMERICAL RESULTS AND DISCUSSIONS

The concept of the equivalent radial tensile load, developed due to large deflections, to predict the mechanical postbuckling behavior of the plates with the complicating effects, is used here to study the mechanical postbuckling behavior of plates. Throughout the present study the value of Poisson ratio ν is taken as 0.3. The actual configurations and the basic configurations of the plates considered here are given below:

i. Moderately thick linearly tapered plate.
ii. Moderately thick plate with edge elastic rotational restraint.
iii. Thin plate with the elastic foundation.

For the above configurations of the plates, the basic configurations of the plates considered are the corresponding thin plates for the problems i and ii and for the problem iii, a thin plate without elastic foundation.

Table 1 presents the values of $\gamma_\tau{}^m$, from which the mechanical postbuckling behavior of moderately thick, linearly tapered simply supported plates is evaluated by using Eq. (4). The similar results for the moderately thick, linearly tapered clamped plate are given in Table 2. A very good agreement for the values of $\gamma_\tau{}^m$ obtained for the various taper parameters α, defined as, $\frac{(t_c - t_e)}{D}$ where the subscripts '*c*' and '*e*' represent the central and edge thicknesses of the plate, from the proposed concept and evaluated from the results of Ref.[5], is observed from these tables. Obviously, $\alpha = 0$ ($t_c = t_e$) corresponds to a uniform plate, representing the simplest basic configuration and the present values of $\gamma_\tau{}^m$ and those obtained from Ref.[5] are exactly the same, as it should be.

TABLE 1

VALUES OF λ_L AND $\gamma_\tau{}^m$ OR MODERATELY THICK LINEARLY TAPERED SIMPLY SUPPORTED CIRCULAR PLATES WITH VARYING VALUES OF α

tc /*a*	α	-0.2	-0.1	0.0	0.1	0.2
0.05	λ_L	5.668	4.891	4.185	3.546	2.970
	$\gamma_\tau{}^m$	0.2404	0.2552	0.2710	0.2876	0.3046
		(0.2406)*	(0.2554)	(0.2710)	(0.2875)	(0.3043)
0.1	λ_L	5.609	4.844	4.148	3.517	2.948
	$\gamma_\tau{}^m$	0.2429	0.2577	0.2734	0.2899	0.3068
		(0.2432)	(0.2579)	(0.2734)	(0.2899)	(0.3068)
0.15	λ_L	5.514	4.767	4.087	3.470	2.911
	$\gamma_\tau{}^m$	0.2471	0.2618	0.2775	0.2939	0.3107
		(0.2475)	(0.2622)	(0.2775)	(0.2937)	(0.3105)
0.2	λ_L	5.386	4.664	4.006	3.405	2.861
	$\gamma_\tau{}^m$	0.2530	0.2676	0.2830	0.2995	0.3162
		(0.2537)	(0.2680)	(0.2830)	(0.2992)	(0.3156)

*Values in parentheses are from Ref[5]

In Table 3, the values of $\gamma_\tau{}^m$ for the uniform moderately thick plates with edge elastic rotational restraints, represented by a rotational spring, are presented for various values of the rotational spring parameter β, defined as, $k_s a/D$, where k_s is the stiffness of the rotational spring. It is clear from this table that the values of $\gamma_\tau{}^m$ obtained from the present concept matches well with that taken from Ref.[6], for the ratios t_c /a = 0.1 and 0.2 for the values of β = 0.1, 1, 10, 100 and 1000. It may be noted that $\beta = 0$ corresponds to the simply supported and $\beta \rightarrow \infty$ corresponds to the clamped plate.

Table 4 shows the values of for the uniform thin simply supported and clamped plates, for several values of the elastic foundation parameters defined as, $\frac{k_F a^4}{D}$ where k_F is the stiffness of the elastic foundation. Once again the values of $\gamma_\tau{}^m$ obtained in the present study match very well with those taken from Ref.[7].

The values of the linear buckling load parameters λ_L are also included in these Tables to readily evaluate the value of the mechanical postbuckling load λ_{NL}, once the value of λ_L is known for the plates with the complicating effects. It is to be noted that the values of can be obtained from Refs.[8, 9] and if not readily available in these references, this parameter can be evaluated by

TABLE 2

VALUES OF λ_L AND $\gamma_\tau{}^m$ OR MODERATELY THICK LINEARLY TAPERED SIMPLY SUPPORTED CIRCULAR PLATES WITH VARYING VALUES OF α

t_c /a	α	-0.2	-0.1	0.0	0.1	0.2
0.05	λ_L	20.42	17.33	14.53	12.00	9.745
	$\gamma_\tau{}^m$	0.1833	0.1936	0.2049	0.2176	0.2320
		(0.1834)*	(0.1935)	(0.2049)	(0.2176)	(0.2318)
0.1	λ_L	19.66	16.75	14.09	11.68	9.515
	$\gamma_\tau{}^m$	0.1903	0.2003	0.2112	0.2236	0.2376
		(0.1908)	(0.2004)	(0.2112)	(0.2330)	(0.2370)
0.15	λ_L	18.51	15.86	13.42	11.18	9.155
	$\gamma_\tau{}^m$	0.2022	0.2116	0.2218	0.2336	0.2469
		(0.2031)	(0.2118)	(0.2217)	(0.2330)	(0.2458)
0.2	λ_L	17.10	14.76	12.57	10.55	8.694
	$\gamma_\tau{}^m$	0.2188	0.2273	0.2368	0.2475	0.2600
		(0.2203)	(0.2278)	(0.2364)	(0.2464)	(0.2579)

*Values in parentheses are from Ref[5]

TABLE 3

VALUES OF $\wedge_L$ AND $\gamma_\tau{}^m$ OR MODERATELY THICK LINEARLY TAPERED SIMPLY SUPPORTED CIRCULAR PLATES WITH VARYING VALUES OF β

t /a	β	0.1	1.0	10.0	100.0	1000.0
0.1	λ_L	4.3959	6.2669	11.7859	13.8242	14.0641
	$\gamma_\tau{}^m$	0.2623	0.2109	0.1934	0.2088	0.2110
		(0.2627)	(0.2136)	(0.1957)	(0.2090)	(0.2110)
0.2	λ_L	4.2447	5.9946	10.7147	12.3598	12.5510
	$\gamma_\tau{}^m$	0.2716	0.2205	0.2127	0.2335	0.2365
		(0.2734)	(0.2287)	(0.2163)	(0.2335)	(0.2361)

*Values in parentheses are from Ref[6]

TABLE 4 VALUES OF Λ_L AND $\gamma_T{}^m$ OF UNIFORM THIN CIRCULAR PLATES ON ELASTIC FOUNDATION							
γ_F	0.1	0.2	0.5	1.0	2.0	5.0	10.0
λ_L(SS)	4.2150	4.2322	4.2839	4.3700	4.5424	5.0591	5.9203
λ_L (C)	14.6962	14.7098	14.7507	14.8187	14.9548	15.3624	16.0396
$\gamma_T{}^m$ (SS)	0.2691	0.2680	0.2647	0.2595	0.2497	0.2241	0.1915
	(0.2691)*	(0.2680)	(0.2646)	(0.2595)	(0.2494)	(0.2237)	(0.1906)
$\gamma_T{}^m$ (C)	0.2026	0.2025	0.2019	0.2009	0.1991	0.1938	0.1856
	(0.2026)	(0.2025)	(0.2020)	(0.2013)	(0.1998)	(0.1954)	(0.1888)

*Values in parentheses are from Ref [7]

applying either the Rayleigh-Ritz or the finite element methods, as the evaluation of λ_L is relatively easier compared to the direct evaluation of $\frac{\lambda^m_{NL}}{\lambda_L}$ through the rigorous mathematical formulations. The results obtained in the present study, show the effectiveness of the proposed concept of the equivalent radial tensile load developed, due to the large deflections, to predict the mechanical postbuckling behavior of plates with complicating effects.

CONCLUSIONS

In this note, the concept of the equivalent radial tensile load is developed, due to large deflections, to study the mechanical postbuckling behavior of plates with complicating effects, by using a simple formula. The equivalent tensile load parameter evaluated from the basic configuration of the plate is successfully used to predict the mechanical postbuckling behavior of plates with the complicating effects. Though, the results obtained, through the formula, in the present investigation match very well with those available in the literature, a necessity still exists to develop such a formula by using a rigorous mathematical formulation, to study the mechanical postbuckling behavior of plates. The authors are presently working to develop such formulas, when the plate is subjected to both the mechanical and the combined thermo – mechanical loads.

ACKNOWLEDGEMENTS

The authors are thankful to the management of the respective Institutes for their encouragement during the course of this work.

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(Discussion on this article must reach the editor before March 31, 2014)