PRELIMINARY STUDY OF STUDENT PERFORMANCE ON ALGEBRAIC CONCEPTS AND DIFFERENTIATION

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Abstract— *Teaching on differentiation topic is paramount important for science and engineering students at first year tertiary level. However lectures are facing difficulty when students are unable to apply the concepts and basic problem solving skills on differentiation when applied to science and engineering related problems at higher level. The aim of this study is to determine students' performances on algebraic and differentiation concept. An analysis had been carried out to identify the algebraic concepts in solving mathematical equations and to relate the misconceptions with the basic technical errors done in solving the first derivatives. Two sets of questionnaires had been distributed; Questionnaire I for Additional Mathematics teachers and Questionnaire II for form four students (n = 113) in three selected secondary schools (an urban school, two rural schools) in the district of Kuantan, Pahang. Findings of the study revealed that the students' performance in solving algebraic is better than students' performance on differentiation problems.*

Keywords: *algebra concept, first derivative, differentiation problem, student performance, misconception*.

I. INTRODUCTION

Learning mathematics consist of wide range of conceptual and problem solving techniques. Mathematics consists of a range of ideas that connect with each other and is not a collection of isolated facts and algorithms [8]. One of the basic mathematical skills is algebra concept such as simplifying and expanding algebra expression. This algebra skill is needed at various studies of mathematics. If the students are unable to handle algebraic problems, then they may face difficulties in solving other mathematical problems. In other words it was suggested that students need more algebraic scaffolding to build their mathematics skill to solve problems and perform better in mathematics [1].

An important objective in teaching mathematics is to develop students' mathematical problem-solving skills [9]. Result from their analyses indicate that matriculation students have a moderately favourable attitude towards problem solving in algebra which may be due to the students unwillingness to solve problems and lack of perseverance in solving problems. In addition, the result also reveals that the students' problem solving skills in algebra are about average. This seems that a student who are well-versed in mathematic will contribute to strong understanding in mathematical concept. Successful completion of algebra is considered essential for future success in scientific and other postsecondary endeavours and is increasingly being required for high-school graduation in the United States [2].

Apparently, mathematics is an essential subject that must be learned by most students starting from lower until higher education. Students understanding on basic knowledge of mathematics during primary and secondary school will be applied by them at university level especially during first year mathematic course. Student must possess a good mastery of mathematics at the matriculation level, as it used extensively in the fields of science, technology, accounting and economics at the university level [8]. A study by Rasmussen [5] said that the typical engineering or physical science students begin his or her university studies in mathematics with a year and a half of calculus, followed by differential equations in the second year. The framework reported from his study on new direction in differential equations for interpreting students' understanding and difficulties with important mathematical ideas is central to new directions for both theory and practice [5]. On the other research by Sofronas et al. [6] states that many of the experts in this study noted that within the mathematics community, including their own mathematics departments, conversation with respect to articulating what the essential learning points should be for first year calculus is often disputatious, but important. Specifically, they viewed identifying the objectives of the first year calculus as a necessary step in moving toward a leaner curriculum, which they generally agreed is facilitative to development of deeper student understanding [6]. Result from this study showed that all the participants in their study cited student understanding of the derivative as a central skill of the mastery of basic concept of the first year calculus.

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Recently, many methods were done in identifying the difficulties encountered by students during solving mathematics problems were cited. The Diagnostic Algebra Assessment System (DAAS) were developed by Russell et al. [3] in their research which use to assists teacher to identify students within their classroom who struggle with a specific algebraic topic because of misconception specific to that topic. On the other hand, study by Davrajoo et al. [1] used Algebraic Mastery Learning Module (AMaLM) for 50 low achievers in Form Four from a secondary school located in a rural area and the finding revealed that the experimental group improved considerably better than control group. Maat et al. [4] used Maple in exploring students understanding of ordinary differential equation s (ODEs) found that most of the students are seem to lack the basic understanding of calculus, which is a prerequisite for ODEs. Based on the given responses during the interview, the students face difficulties in remembering the methods learnt in integration and differentiation [4]. Clinical interview series were conducted by Tarmizi [7] by giving a set of question related to the course with main objective is to examine problem solving procedures and into the diagnosis of the difficulties experienced by students in learning Calculus I (MTH 3100).

Lead to these literature, it is essential to perceive students misconception in mathematics at the early school level before they pursue their study at higher level. Mathematics is compulsory in accomplishing upper secondary level in Malaysian Education System [1]. Mathematics lecturers who taught Calculus and Ordinary Differential Equation courses for first year engineering students at Universiti Malaysia Pahang noticed and experienced that some of the first year students having difficulties in solving differentiation related problems. Due to this, this research tried to shed some light on the root cause of the problem at school level. According to Sijil Pelajaran Malaysia (SPM) syllabus, the earliest topics on differentiation are taught in form four Additional Mathematic subject. The aim of this study is to identify and determine student problem solving performances on algebraic concepts and differentiation. The study was carried out among selected secondary school students. Those student performances will prior on their misconception and understanding in solving problems regarding algebra and differentiation question.

II. METHODOLOGY

The respondents were Additional Mathematics teacher (n=6) and the total of 113 form four students from three selected secondary schools in district of Kuantan, Pahang as described in Table I. Two set of self-answered questionnaire were distributed to the respondents. Questionnaire I was designed in order to get teachers perception on teaching of Additional Mathematics whereas Questionnaire II is problem solving questionnaire, is a knowledge test of algebra and differential question. As such this study focuses on Additional Mathematics teachers' perception on their experience of teaching Additional Mathematics for form four students and analyzes their student difficulties in problem solving on algebra and differentiation.

TABLE I. NUMBER OF RESPONDENT BASED ON SCHOOL

School	Number of students (n)
Scholl A (urban scholl)	52
Scholl B (rural scholl)	42
Scholl C (rural scholl)	19
Total	113

The Questionnaire I was distributed to Additional Mathematics teacher at those selected schools. This questionnaire is used as a pilot study to identify the difficult topics faced by the instructors in teaching form four Additional Mathematics. From the teachers' feedback, differentiation is one of the difficult topics in their teaching syllabi. Consequently, interview sessions with the Additional Mathematics teachers and '*Guru Pakar Matematik*' were conduct. The discussion focused on the difficulties in teaching this topic, the reasons why students have difficulties to understand and scored this topic and suggestions to improve students in comprehending this topic.

Questionnaire II were constructed and distributed to the students. These questionnaires consist of four sections as depicted in Table II. The reliability of the questions is based on 'Modul Kecemerlangan Sijil Pelajaran Malaysia Additional Mathematic (Edisi 2009) Modul Asas' from Jabatan Pelajaran Negeri Pahang.

TABLE II. DESCRIPTION OF THE TASK

Section	Description	The task
A	Demographic	Respondent background (i) gender (ii) race (iii) previous mathematics grade in Penilaian Menengah Rendah (PMR)
B	Basic Algebra	(i) Question 1 to 4: Structured Algebraic question emphasis on expansion of a given expressions (ii) Question 5 and 6 : Express of equation y in terms of x (iii) Question 7 and 8: express of equation x in terms of y
C	Structured differentiation problems	Respondents are required to solve problem to determine the first derivative of the function using formula $y = ax^n$ and to determine the first derivative of the function involving addition or subtraction of algebraic terms.
D	Differentiation of any variable	Respondent are required to perform differentiation for derivative problem of any variable.

An overall analysis of students solution were done according to the types of error done by respondent which were categorized using the Likert scale ranging from one (1) to five (5) as defined in Table III.

TABLE III. CLASSIFICATION OF ERROR

Score	Description
1	The student is unable to begin the problem or do not answer due to lack of understand the right concept.
2	The student approaches the problem but wrong with error in procedure and unsuccessful algorithm.
3	The student able to demonstrate meaningful work toward rational answer but wrong due to error at the mathematical operations
4	The problem is nearly solved but wrong due to carelessness at final stage of solution.
5	A valid solution with correct and appropriate method applied.

III. FINDING AND DISCUSSION

The findings of this study were mainly based on the number of students' scores in each part of the Questionnaire II and together with discussion on several example of problem solution. The results were presented by sub questions that guided this study as follows:

A. Students performance based on algebraic question

The result shown in Table IV described that most of the students could solve mathematical problem on basic algebra very well because the percentages number of Score 5 is high compare to others. Student showed a better understanding in structured algebraic question emphasis on expansion of a given expressions compared to the other one. Data were clearly show that decreasing number of students Score 5 starting from question 5 to question 8.

TABLE IV. STUDENTS SCORE PRIOR TO THEIR SOLUTION ON ALGEBRAIC QUESTION ($n=113$)

Question no.	Mathematical problem	Score				
		1	2	3	4	5
1	$2x(x+2)$	4	3	2	5	99
2	$6x+2x(x-3)$	4	6	8	7	88
3	$(x+1)^2$	4	1	4	6	98
4	$(x+2)(x-3)$	7	6	6	6	88
5	$2x-y=3$	10	6	5	4	88
6	$3x+4y=6$	8	4	6	14	81
7	$x+3y=2$	10	8	5	3	87
8	$5x-3y=1$	13	5	5	17	73

This result revealed that students face difficulties in solving problem that involved manipulation technique of dependent and independent variable. Figure 1 and Figure 2 displayed below shows the mistakes done by those students in rewriting expressions. Those students failed to distinguish between the dependent and independent variables or they might not understand the questions. These misconceptions will affect their understanding in the related differentiation topics which they will encounter in a later study.

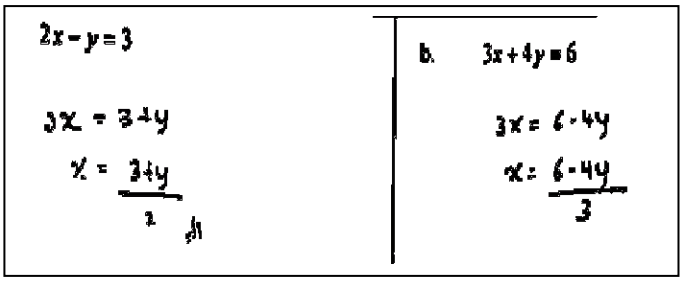

Figure 1. Student Score 3 in solving problem on express of equation y in terms of x .

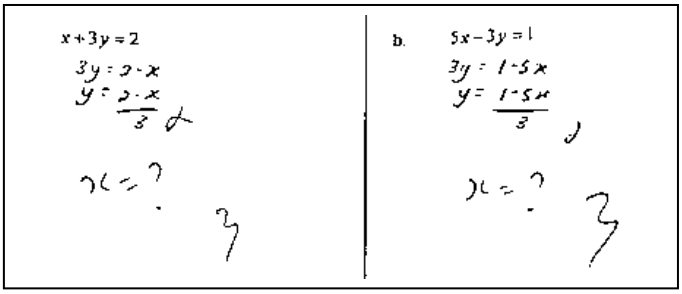

Figure 2. Student Score 3 in solving problem on express of equation x in terms of y

Figure 3 shows a common careless mistake done by students. This student could not perform the algebraic steps correctly. This is mainly due to the misunderstanding of the algebraic concepts. The carelessness happened at the final stage of the solution when student tried to simplify his answer. These finding were supported by Zakaria et al. [9], who states that students failed to simplify, as well as to reform expression; they committed careless mistakes that went against algebraic law.

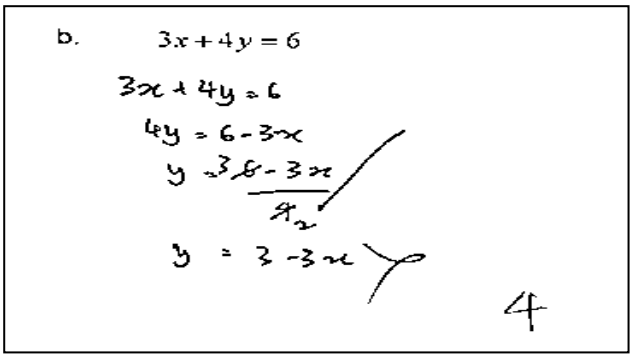

Figure 3. Student Score 4 in solving problem on express of equation in terms of y in terms of x .

B. Students performance based on first derivative question

Result shown in Table V summarized that many students have a good insight in derivative. However, for questions 5, the number of students who answer it correctly is least percentage (48.7%). In this part, students were tested on the basic differentiation followed by the derivative involving addition or subtraction of algebraic terms. Based on the answer analysis, students showed a wrong procedure in solving the question which involves two polynomials.

TABLE V. STUDENTS SCORE PRIOR TO THEIR SOLUTION ON DIFFERENTIATION QUESTION ($n=113$)

Question no.	Mathematical problem	Score				
		1	2	3	4	5
1	$y=x^3$	34	5	1	0	73
2	$y=\frac{1}{2}x^4$	20	6	0	0	87
3	$y=x^2-3x+4$	26	3	3	5	76
4	$y=(x+1)(x-6)$	22	1	11	8	71
5	$y=x(x+3)^2$	24	8	13	13	55
6	$y=x^2+\frac{3}{x}$	32	5	3	6	67
7	$y=x^2+\frac{5}{x}+\frac{6}{x^2}$	35	1	3	3	71

As shown in Fig. 4, when an equation involves product of different function; polynomial, exponential or trigonometry, student should apply product rule. However, in this problem, student can choose either to expand the expression first before performing the derivative or to use product rule. However, this student believed that this kind of function can be solved directly using $y=ax^n$ formula by assuming the outside x as a constant.

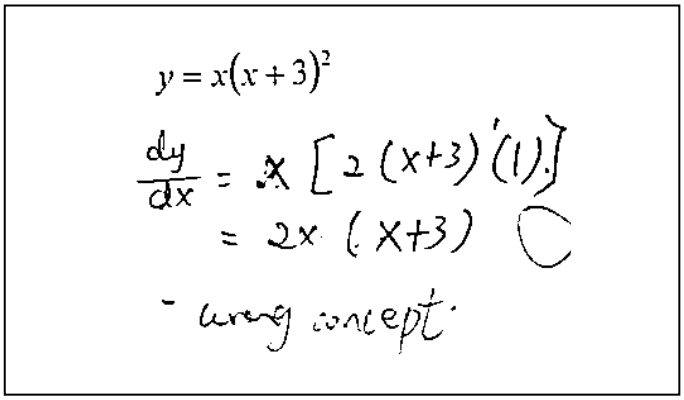

Figure 4. Student Score 2 due to resulted in wrong solution

On the other hand, the solution in Fig. 5 explained that this student chooses the right technique by expand the equation first, so that it is become easy to differentiate. However, the error encounter at the mathematical operation of performing the first derivative for the new equation. The student still has a problem in understanding the principle of first derivative of the function using $y'=nax^{n-1}$ formula. He knew that the degree n of the independent variable must decrease by one but failed to multiply that n with constant coefficient.

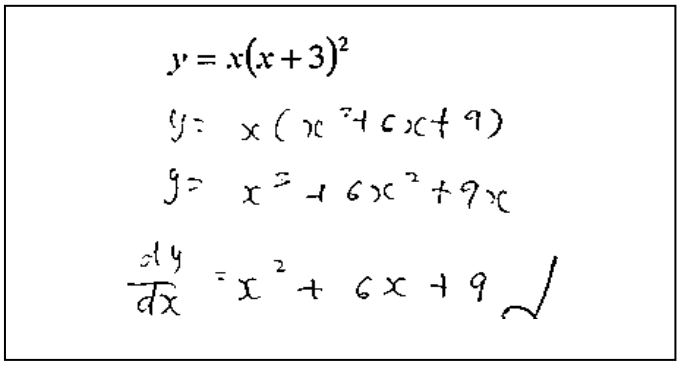

Figure 5. Student Score 3 in solving problem on differentiation.

C. Students performance based on basic application of differentiation

Table VI shows that students' performances on application of differentiation are low about (50%) compare to performance on solving algebraic and differentiation problem.

TABLE VI. STUDENTS SCORE PRIOR TO THEIR SOLUTION ON DIFFERENTIATION QUESTION ($n=113$)

Question no.	Mathematical problem	Score				
		1	2	3	4	5
1	$y=5x^{\frac{2}{3}}-2x^{\frac{1}{4}}$	46	4	3	5	55
2	$s=3p^2-p+6$	38	6	2	6	61
3	$r=-2t^2+\frac{4}{t}-1$	42	3	5	5	58

In this section, students were tested on differentiation of any variable, not only on common variable x and y. This is due to the distinct future study area will use different variables that represent their problem study. For instance, in the real world problem in electrical engineering that involve electrical circuit, current flow, i is normally discussed with respect to time, t.

Figure 6 gave an overview that this student lacks of strong knowledge on differentiation. He performed an error at mathematical procedure part on degree of x. He just put the negative sign in front of the power of x since he knew that the degree should be decreased after the differentiation process.

He just applied his own differential rules without fully comprehend the concept underlying them.

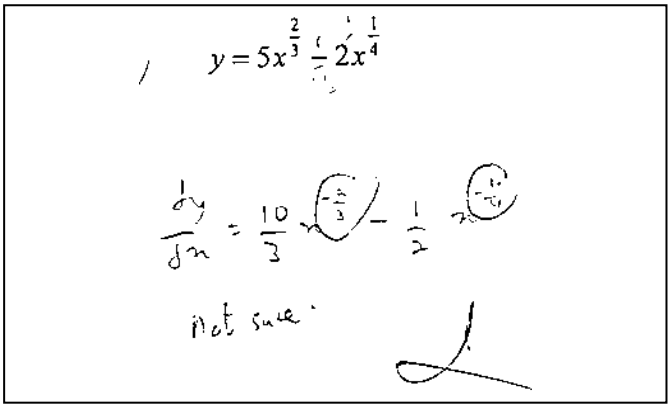

Figure 6. Student Score 3 because of misinterpretations obstruct the valid solution

Besides that, Figure 7 demonstrated the error occurred at the procedure of performing product rule method for the function which is not product of different functions. Student also showed a low understanding on derivative when the function involves independent variable as a denominator for the given function. At the same time, he did not perform derivative for the right dependent and independent variable since the final answer was given in term of $\frac{dy}{dx}$ not $\frac{dr}{dt}$. This shows that this student was unable to perform the appropriate derivative algorithm.

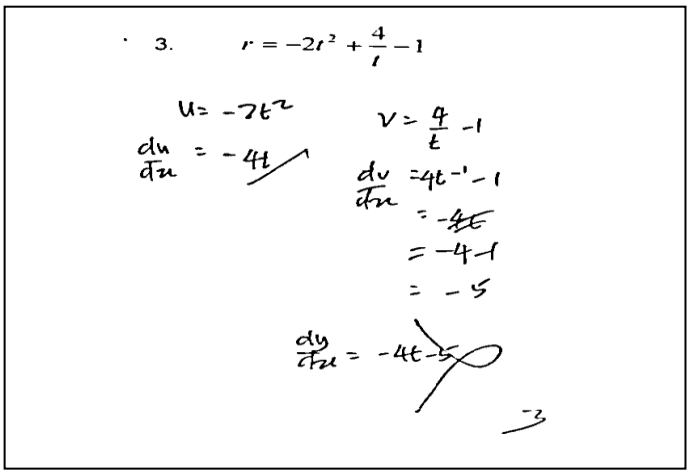

Figure 7. Student Score 2 in solving problem on differentiation of any variable

IV. CONCLUSION

The main conclusions of the current study indicate that students' performance on basic algebra is better than students' performance on differentiation. Thus, mathematical teacher and lecturers beliefs on their student having difficulties in solving differentiation problems are true. These finding also indicated several attribute on students' performance in solving mathematical problems of algebraic and differentiation. Students tend to merely memorize what they had learned without understand the real concepts. A majority of the students try to memorize and apply mathematical rules and definition without fully understand the underlying concept [9]. Students faced difficulties in solving algebraic problems that involved manipulation technique of dependent and independent variables. Students showed a good insight in derivatives that involve addition or subtraction of algebraic terms. However, misconceptions on algebraic expansions let the student fail to differentiate the functions correctly. Due to this, it is suggested that educator to revise the algebraic concept before further explanation on differentiation part in order to fostering students' ability in solving differentiation problem. Besides that, when the students were given unfamiliar functions with different variables, they were unable to solve them. In order to overcome the students' problems, it is highly recommended that the educators should give more emphasis and explanation on differentiation topics in their classes. If this is not done, the students are likely to become progressively more confused and in the long run they may not survive in their mathematics problem solving experiences [7]. Reflection on this study, we tried to design appropriate teaching approach in order to facilitate our students on differentiation topic at early school level as an extension of this study.

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