PCO 2010, KUCHING, SARAWAK
2 – 4 DEC 2010

Dynamic Modeling of a Double-Pendulum Gantry Crane System Incorporating Payload (C17)

Authors:
R. M. T. Raja Ismail, M. A. Ahmad, M. S. Ramli, R. Ishak and M. A. Zawawi
Faculty of Electrical & Electronics Engineering, Universiti Malaysia Pahang, Malaysia

ABSTRACT

- This paper presents dynamic modelling of a double-pendulum gantry crane system based on closed-form equations of motion.
- A dynamic model of the system incorporating payload is developed and the effects of payload on the response of the system are discussed.
- Extensive results that validate the theoretical derivation are presented in the time and frequency domains.

PROBLEM STATEMENT

- Purpose of controlling a gantry crane:
  - To transport the load at short period of time (fast) without causing any excessive swing at the final position.
- Problems that arise:
  - Gantry crane results in a swing motion when the payload stops suddenly after a fast rope movement.
  - It requires more time (larger settling time) to minimize the swing motion (swing angle).
  - The needs for skillful operators to manually control and stop the swing at the right position*

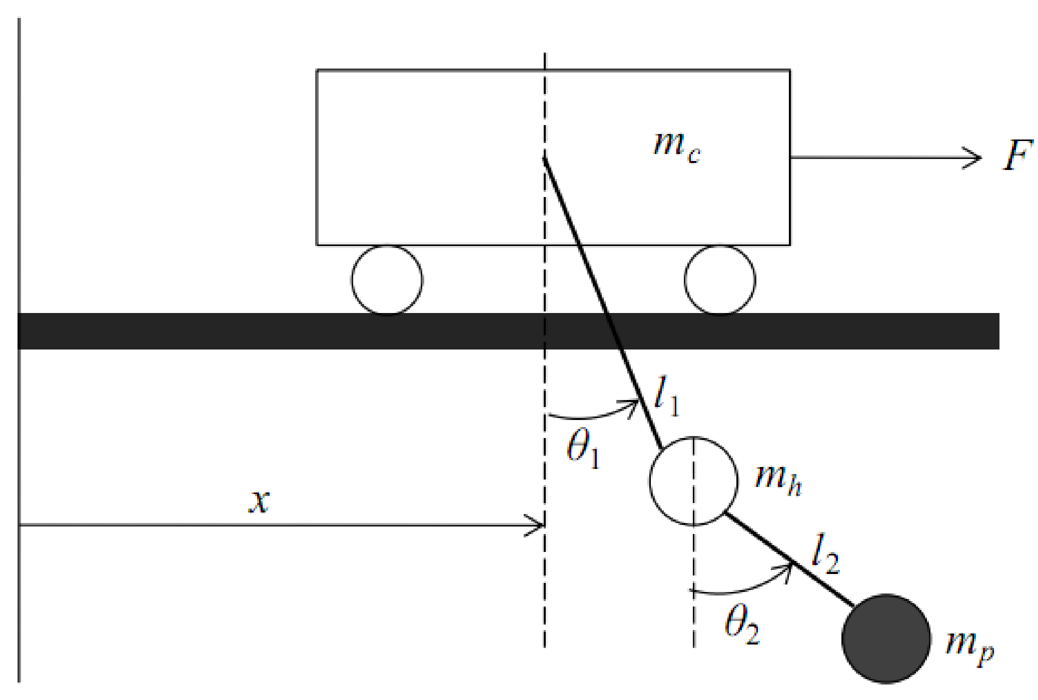
* Failure to control the crane might cause accident and may harm people and surrounding.

OBJECTIVES

- To study the dynamic modelling of a double-pendulum gantry crane system based on closed-form equations of motion.
- To investigate the effects of payload on the dynamic behaviour of a double pendulum gantry crane system.

BRIEFING ON Gantry Crane Sys : Model structure

- The double-pendulum gantry crane system with its hook and load considered in this work is shown below.
- Where x is the cart position, m_c is the cart mass, m_h is the hook mass and m_p is the payload mass.
- Meanwhile, θ_1 is the hook swing angle, θ_2 is the load swing angle, l_1 and l_2 are the cable length of the hook and load, respectively, and F is the cart drive force.



BRIEFING ON Gantry Crane Sys : System parameter values

Symbol	Parameter	Value
m_c	Cart mass	5 kg
m_h	Hook mass	2 kg
m_p	Payload mass	1-10 kg
l_1	Hook pendulum length	2 m
l_2	Load pendulum length	1 m
g	Gravity acceleration	9.8 m-s^{-2}
F	Bang-bang input	10 N (amplitude) / 1 s (width)

BRIEFING ON Gantry Crane Sys : System's variable concerned

Symbol	Variable	The importance
x (m)	Cart position	To achieve steady state position with minimum error
θ_1 (rad)	Hook swing angle	To avoid excessive swing at hook
θ_2 (rad)	Load swing angle	To avoid excessive swing at load
$PSD\ of\ \theta_1$ (dB)	Power spectral density of the hook swing angle	To minimize the vibration at hook due to rope movement
$PSD\ of\ \theta_2$ (dB)	Power spectral density of the load swing angle	To minimize the vibration at load due to rope movement

BRIEFING ON Gantry Crane Sys : Other parameters assumption

1) Cart friction force is ignored.
2) The tension force that may cause the hook and load cables elongate is also ignored.
3) The cart (translational) and the payload (rotational) are assumed to move in two dimensional only (2D – movements)

BRIEFING ON Gantry Crane Sys : Mathematical model

1) The dynamic model of the double-pendulum gantry crane system is expressed as :

$$\mathrm{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathrm{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathrm{G}(\mathbf{q}) = \mathbf{F}$$

Where:

Inertia → $$\mathrm{M}(\mathbf{q}) = \begin{bmatrix} m_c + m_h + m_p & (m_h + m_p)l_1 \cos\theta_1 & m_p l_2 \cos\theta_2 \\ (m_h + m_p)l_1 \cos\theta_1 & (m_h + m_p)l_1^2 & m_p l_1 l_2 \cos(\theta_1 - \theta_2) \\ m_p l_2 \cos\theta_2 & m_p l_1 l_2 \cos(\theta_1 - \theta_2) & m_p l_2^2 \end{bmatrix}$$

Centrifugal coriolis → $$\mathrm{C}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} 0 & -(m_h + m_p)l_1 \sin\theta_1\dot{\theta}_1 & -m_p l_2 \sin\theta_2\dot{\theta}_2 \\ 0 & 0 & m_p l_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_1 \\ 0 & -m_p l_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_1 & 0 \end{bmatrix}$$

Gravity → $$\mathrm{G}(\mathbf{q}) = \begin{bmatrix} 0 \\ (m_h + m_p) g l_1 \sin\theta_1 \\ m_p g l_2 \sin\theta_2 \end{bmatrix}$$

SIMULATION RESULTS …(1)

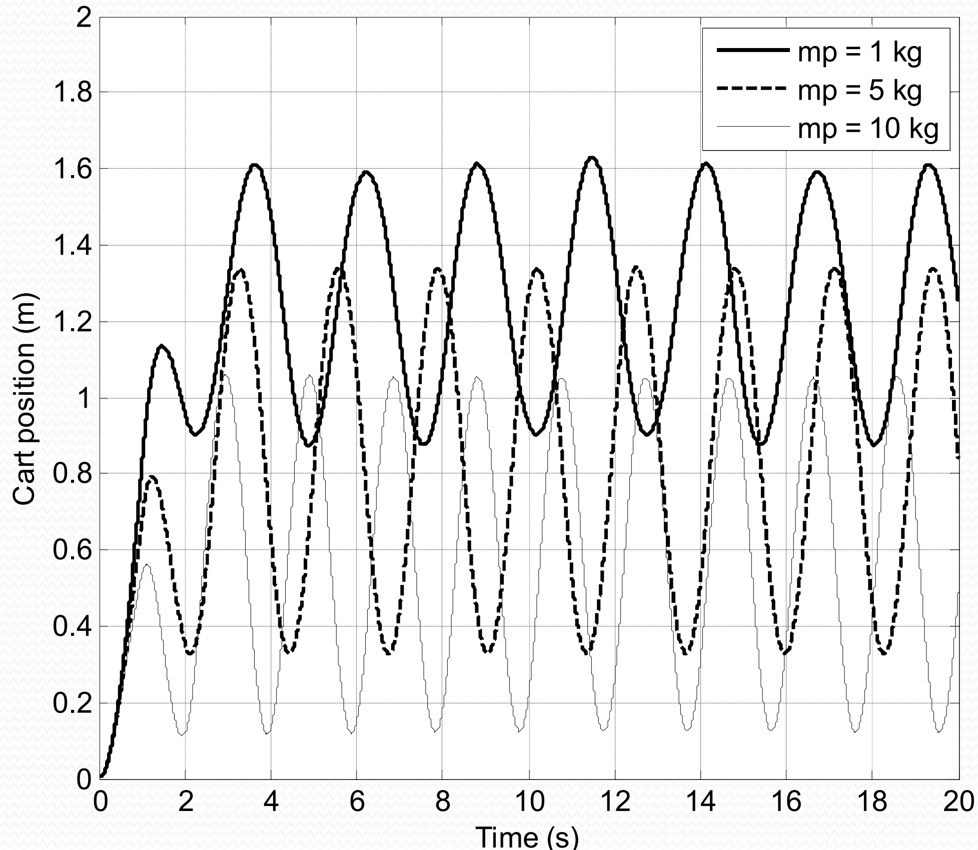


Fig 1: Response of the cart position (m_p = 1, 3, & 5 kg)

- It is noted that the average final position of the cart decreases and the chattering of the final position increases with increasing payloads.

SIMULATION RESULTS …(2)

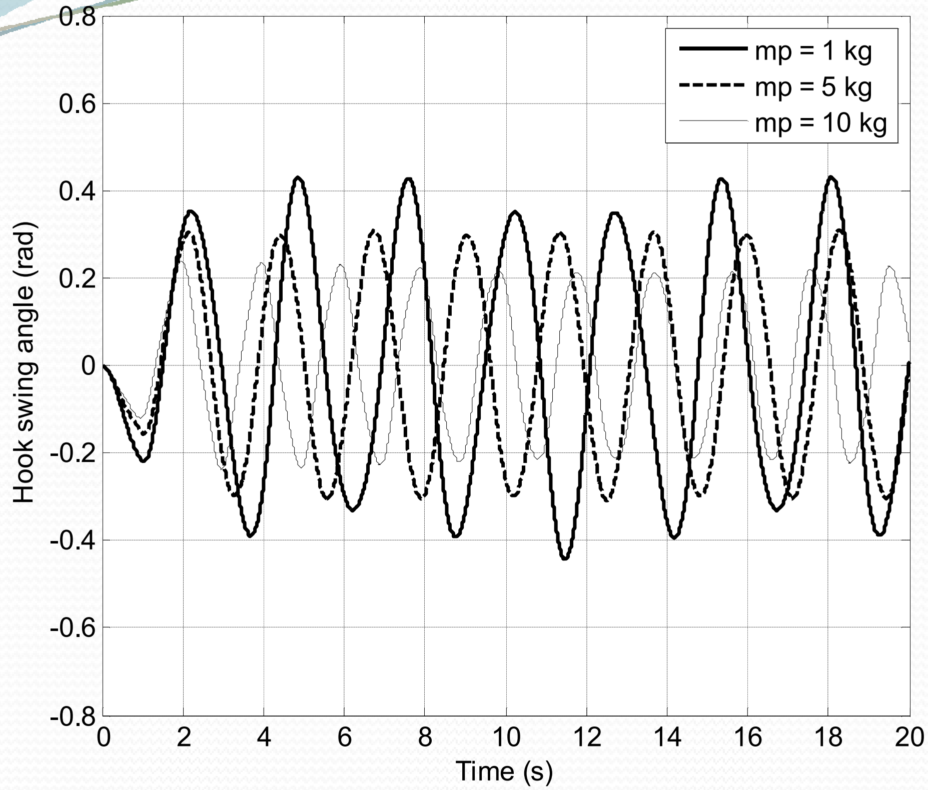


Fig 2(a): Response of the hook swing angle (m_p = 1, 3, & 5 kg)

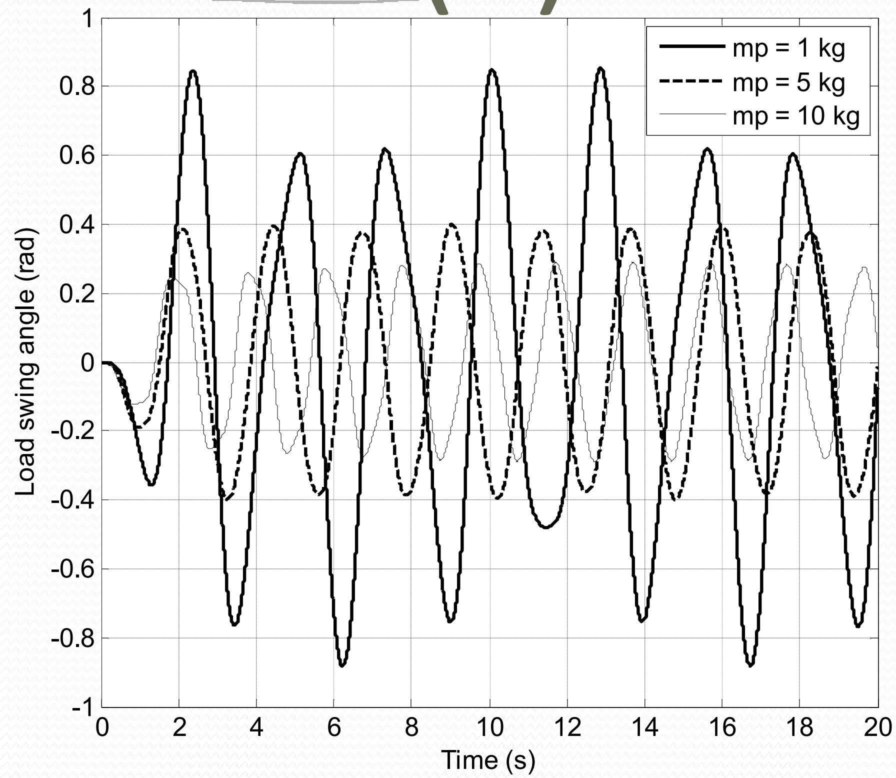


Fig 2(b): Response of the load swing angle (m_p = 1, 3, & 5 kg)

- It is shown that, the hook swing angle and load swing angle responses for various payloads requires more than 20 sec. to settle down.
- Besides that, it can be seen the oscillations of the hook swing angle and the load swing angle decrease with increasing payloads.

SIMULATION RESULTS …(3)

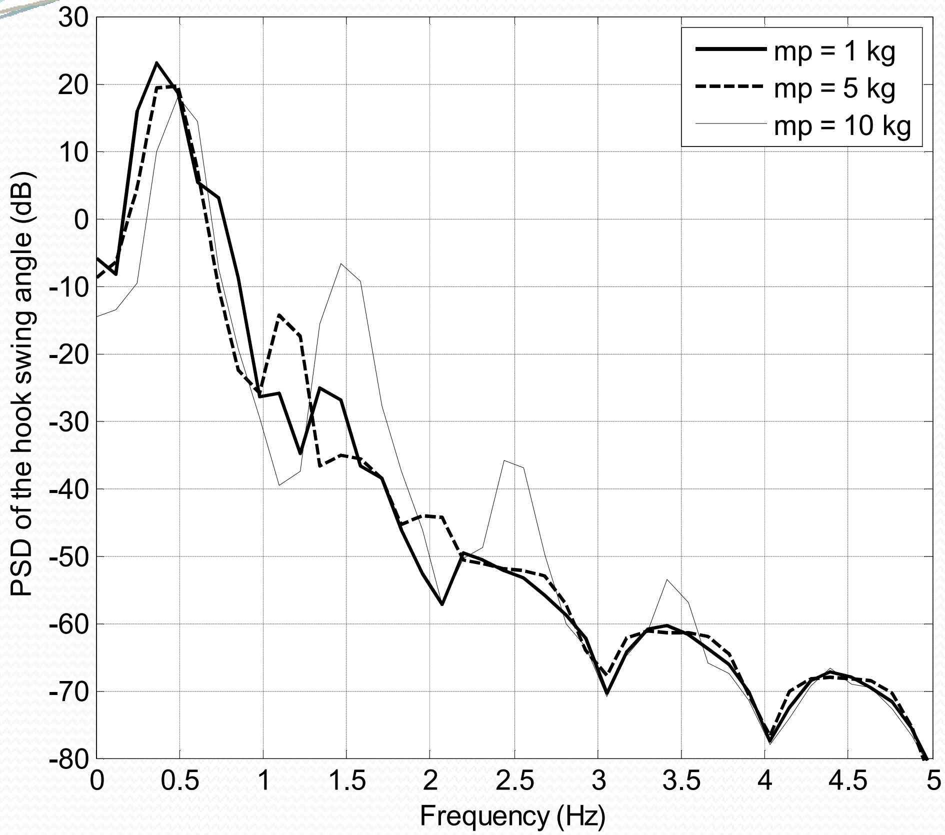


Fig 3(a): PSD of the hook swing angle (m_p = 1, 3, & 5 kg)

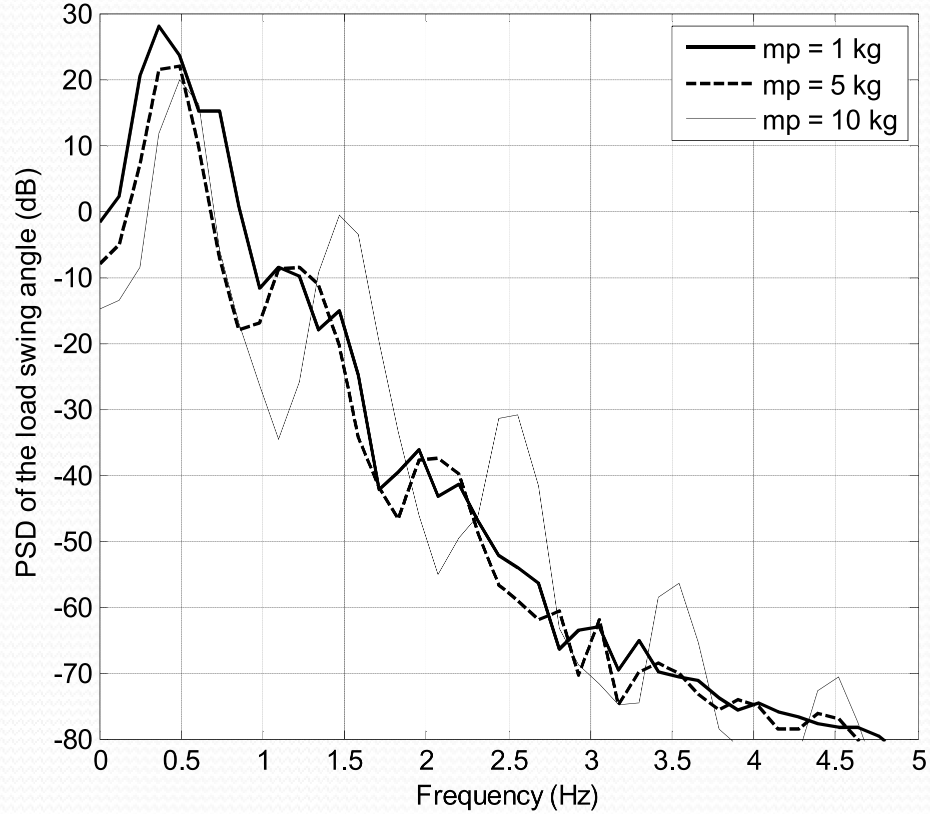


Fig 3(b): PSD of the load swing angle (m_p = 1, 3, & 5 kg)

- Fig. 3 (a),(b) demonstrates that the resonance modes of vibration of the system shift to higher frequencies with increasing payloads.

ANALYSIS AND DISCUSSION

Table 1: Payload vs. Cart position responses

Payload (kg)	Average cart position (m)	Oscillation (m)
0	-	-
1	1.9920	±0.3630
2	1.9151	±0.3831
3	1.9145	±0.4228
4	1.8521	±0.4929
5	1.8419	±0.5049
6	1.7219	±0.5057
7	1.6940	±0.5075
8	1.6063	±0.5091
9	1.5472	±0.5100
10	1.5154	±0.5102

Table 2: Payload vs. Hook & load swing angles

Payload (kg)	Hook swing angle (°)	Load swing angle (°)
0	-	-
1	±0.4132	±0.8826
2	±0.4063	±0.7418
3	±0.3770	±0.6140
4	±0.3493	±0.5319
5	±0.3080	±0.3982
6	±0.3049	±0.3791
7	±0.2919	±0.3431
8	±0.2813	±0.3244
9	±0.2333	±0.3007
10	±0.2305	±0.2902

- From table 1, the average cart position decreases but the oscillation itself increases for heavier loads.
- Meanwhile, both hook and load swing angle decrease with the load increments (Refer table 2)

ANALYSIS AND DISCUSSION

Table 3: Payload vs. Hook & load swing angles resonanse freq. (Hz)

Payload (kg)	Resonance frequency (Hz)			
	Hook swing angle		Load swing angle	
	Mode 1	Mode 2	Mode 1	Mode 2
0	-	-	-	-
1	0.3662	1.343	0.3662	1.099
2	0.3662	1.587	0.3662	1.221
3	0.3662	1.709	0.3662	1.221
4	0.3662	1.709	0.3662	1.221
5	0.4883	1.099	0.4883	1.221
6	0.4883	1.221	0.4883	1.221
7	0.4883	1.343	0.4883	1.343
8	0.4883	1.343	0.4883	1.343
9	0.4883	1.465	0.4883	1.465
10	0.4883	1.465	0.4883	1.465

- From table 3, it shows that both hook and load swing angles have the same resonance frequencies of mode 1.
- It is due to the sway of the payload is always follow the oscillation of the hook.

ANALYSIS AND DISCUSSION

- Besides, the system has the same resonance frequencies of mode 1 that is 0.3662 Hz, when the payload is varied from 1 kg to 4 kg and has the same frequency of 0.4883 Hz when the payload is varied from 5 kg to 10 kg.
- This shows that, in order to decrease the oscillation of the system, a same control design can be used for several systems although they have different payloads.
- Besides, the hook and the load swing angles have different resonance frequencies of mode 2. However, these resonance frequencies do not affect much on the system since the mode 1 frequency is the dominant mode to the system

FUTURE RECOMMENDATION

- Comparative studies on the cart position, hook & load swing angle as well as their respective PSD for various rope length (l_1 & l_2) and input forces (F).
- Implementation of experimental studies by using a different type of crane, (e.g. rotary crane).

CONCLUSION

- Investigation into the development of a dynamic model of a double-pendulum gantry crane system incorporating payload has been presented
- The dynamic model has been simulated with bang-bang force input.
- The cart position, hook swing angle and load swing angle responses of the gantry system have been obtained and analysed in time and frequency domains.
- Moreover, the effects of payload on the dynamic characteristic of the system have been studied and discussed.

REFERENCES

[1] D. T. Liu, W. P. Guo and J. Q. Yi and D. B. Zhao, Double-pendulum-type Overhead Crane Dynamics and its Adaptive Sliding Mode Fuzzy Control, Proc. Of the Third International Conference on Machine Learning and Cybernetics, 2004, pp. 423-428.

[2] D. T. Liu, W. P. Guo and J. Q. Yi, GA-based Composite Sliding Mode Fuzzy Control for Double-pendulum-type Overhead Crane, Lecture Notes in Computer Science, Springer Berlin, 2005, pp. 792-801.

[3] D. T. Liu, W. P. Guo and J. Q. Yi, Dynamics and GA-based Stable Control for a Class of Underactuated Mechanical Systems, International Journal of Control, Automation, and System, Vol. 6, No. 1, 2008, pp. 35-43.

[4] D. T. Liu, W. P. Guo and J. Q. Yi, Dynamics and Stable Control for a Class of Underactuated Mechanical Systems, Acta Automatica Sinica, Vol. 32, No. 3, 2006, pp. 422-427.

[5] D. Kim and W. Singhose, Reduction of Double-Pendulum Bridge Crane Oscillations, The 8th International Conference on Motion and Vibration Control, 2006, pp. 300-305.

REFERENCES (cont…)

[6] M. Kenison and W. Singhose, Input Shaper Design for Double-pendulum Planar Gantry Cranes, 1999 IEEE Conference on Control Applications, 1999.

[7] M. A. Ahmad, Z. Mohamed and N. Hambali, Dynamic Modelling of a Two-link Flexible Manipulator System Incorporating Payload, 3rd IEEE Conference on Industrial Electronics and Applications, 2008, pp. 96-101.

[8] M. W. Spong, S. Hutchinson and M. Vidyasagar, Robot Modeling and Control. New Jersey: John Wiley, 2006.

[9] M. W. Spong, Underactuated Mechanical Systems, Control Problems in Robotics and Automation. London: Springer

THANK YOU