Heat Mass Transfer (2008) 45:127–129
DOI 10.1007/s00231-008-0425-1

TECHNICAL NOTE

Cases of mutual compensation of the magnetic and buoyancy forces in mixed convection past a moving vertical surface

E. Magyari

Received: 13 December 2007 / Accepted: 10 July 2008 / Published online: 29 July 2008

Abstract It is shown in this Note that under certain conditions, in the hydromagnetic mixed convection flow over a stretching vertical surface, between the magnetic and the buoyancy forces a mutual compensation effect can occur, such that the mixed convection problem reduces to a simple forced convection problem.

1 Introduction

In a very recent paper of Ishak et al. [1], the steady hydromagnetic mixed convection boundary layer flow over a stretching vertical sheet was investigated (see also the comprehensive list of references in [1]).

The aim of the present note is to show that in this class of heat and fluid flow phenomena, under certain conditions, between the magnetic and the buoyancy forces a mutual compensation effect can occur. The consequence of this effect is that in this case, the hydromagnetic mixed convection problem considered in [1] reduces to a simple forced convection problem.

2 Governing equations

In [1] the continuity, momentum and energy equations,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \upsilon\,\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u + s\,g\beta(T - T_\infty) \tag{2}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial^2 T}{\partial y^2} \tag{3}$$

along with the boundary conditions

$$\begin{aligned} &u|_{y=0} = U_w(x), \quad v|_{y=0} = 0, \quad T|_{y=0} = T_w(x), \\ &u|_{y\to\infty} = 0, \quad T|_{y\to\infty} = T_\infty \end{aligned} \tag{4}$$

were considered. These equations describe the steady velocity and temperature boundary layers of an electrically conducting incompressible fluid past a vertical plane wall of temperature distribution $T_w(x)$, when the wall is stretching with velocity $U_w(x)$ and when an external magnetic field $B = B(x)$ is applied in the horizontal direction. In the Boussinesq term of the momentum equation (2) the sign function s takes the value $+1$ and -1 when the wall velocity is directed vertically upward and downward, respectively (see Fig. 1 of [1]). Concerning the dependence of the wall velocity, of the wall temperature distribution and of the applied field on the wall coordinate x, it has been assumed that all of them are power law functions of the form [1]

$$U_w(x) = a\,x^m, \quad T_w(x) = T_\infty + b\,x^{2m-1}, \quad B(x) = B_0\,x^{\frac{m-1}{2}} \tag{5}$$

such that the problem (1)–(4) admits similarity solutions. Here $a > 0$, $b > 0$, B_0 and m are constants. Under these assumptions, the velocity and temperature fields are, [1],

$$\begin{aligned} &u = U_w(x) f'(\eta), \quad \eta = \sqrt{\frac{a}{\upsilon}}\,x^{\frac{m-1}{2}}\,y, \\ &v = -\sqrt{\upsilon a}\,x^{\frac{m-1}{2}}\left[\frac{m+1}{2}f(\eta) + \frac{m-1}{2}\eta f'(\eta)\right], \\ &T = T_\infty + b\,x^{2m-1}\theta(\eta) \end{aligned} \tag{6}$$

where the similar stream function $f(\eta)$ and the similar temperature field $\theta(\eta)$ are obtained as solutions of the boundary value problem

E. Magyari (✉)
Institut für Hochbautechnik, ETH Zürich,
Wolfgang-Pauli Str.1, 8093 Zürich, Switzerland
e-mail: magyari@hbt.arch.ethz.ch; magyari@bluewin.ch

$$f''' + \frac{m+1}{2} f f'' - m f'^2 - M^2 f' + \lambda\,\theta = 0 \tag{7}$$

$$\frac{\theta''}{Pr} + \frac{m+1}{2} f\,\theta' - (2m-1) f'\theta = 0 \tag{8}$$

$$f(0) = f_0, \quad f'(0) = 1, \quad f'(\infty) = 0, \quad \theta(0) = 1, \quad \theta(\infty) = 0 \tag{9}$$

In the above equations the primes denote differentiations with respect to the similarity variable η, and M, λ and Pr are the magnetic parameter, the mixed convection parameter and the Prandtl number respectively,

$$M^2 = \frac{\sigma B_0^2}{\rho a}, \quad \lambda = \frac{Gr_x}{Re_x^2} = s\frac{g\beta b}{a^2}, \quad Pr = \frac{\upsilon}{\alpha} \tag{10}$$

In the expression of λ, the signs $s = +1$ and $s = -1$ are associated with the aiding and opposing flow regimes, respectively.

3 Compensation effect in the similarity case

A simple inspection of Eqs. (7) and (8) shows that in the special case

$$Pr = 1, \quad m = 1, \quad \lambda = M^2 \tag{11}$$

equations (7) and (8) reduce by the substitution

$$\theta(\eta) = f'(\eta) \tag{12}$$

to one and the same (ordinary) differential equation

$$f''' + f f'' - f'^2 = 0 \tag{13}$$

At the same time, the boundary conditions (9) reduce to

$$f(0) = f_0, \quad f'(0) = 1, \quad f'(\infty) = 0 \tag{14}$$

In other words, under the conditions (11), the mixed convection boundary value problem (7)–(9) involving two *coupled* differential equations reduces to the problem (13), (14) involving a single differential equation. The latter problem coincides precisely with the forced convection problem for a boundary layer flow driven by a continuous stretching surface in a quiescent isothermal fluid.

The forced convection problem (13), (14) admits the exact analytical solutioñ, [2],

$$f(\eta) = 1 - e^{-\eta} \tag{15}$$

On account on Eq. (12), the corresponding similar temperature is (see also [3])

$$\theta(\eta) = e^{-\eta}, \quad \theta'(0) = -1 \tag{16}$$

Accordingly, the corresponding dimensional velocity and temperature fields are obtained as

$$u = a\,x\,e^{-\eta}, \quad v = -\sqrt{\upsilon a}\,(1 - e^{-\eta}), \quad T = T_\infty + b\,x\,e^{-\eta}, \quad \eta = \sqrt{(a/\upsilon)}\,y \tag{17}$$

It is important to notice that the third condition (11),

$$s\frac{g\beta b}{a^2} = \frac{\sigma B_0^2}{\rho a} \tag{18}$$

can be satisfied only in the aiding flow regime ($s = +1$). In this case the driving shear forces due to the upward moving wall and the buoyancy forces due to the hot wall are directed upward. The magnetic force, on the other hand always points in the opposite direction of motion [see Eq. (2)]. Therefore, when the conditions (11) are satisfied, the (opposite) buoyancy and the magnetic forces compensate each other exactly. As a consequence, the observer sees in this case a pure forced convection boundary layer flow induced by the upward moving surface. The magnetic and buoyancy effects are simply eliminated. Equations (5), (11) and (17) show explicitly that this compensation effect occurs for electrically conducting fluids with $Pr = 1$, when the transversal magnetic field is uniform, $B(x) = B_0$ and when, at the same time, both the surface velocity $U_w(x)$ and the surface temperature $T_w(x)$ are linearly increasing functions of the wall coordinate x. In the opposing flow regime ($s = -1$) due to the downward moving wall, the magnetic and the buoyancy forces are of the same (upward) direction, such that in this case their mutual compensation is physically impossible, in full agreement with the mathematical message of Eq. (18) for $s = -1$.

4 Compensation effect in the non-similarity case

The existence of the self-similar solutions is related in the present case to the conditions (5) which imply in turn quite strong restrictions. Accordingly, the self-similar flows correspond only to a "small" subset of the full solution space of the mixed convection problem (1)–(4). The aim of the present section is to show that in the boundary value problem (1)–(4), the mutual compensation of magnetic and buoyancy forces can take place also under conditions which are much less restrictive than in the case of self-similar flows described above.

Indeed, Eqs. (2) and (3) show that for a uniform magnetic field, $B(x) = \text{const.} = B_0$ and a fluid of $Pr = 1$, these equations reduce by the substitution

$$T(x,y) = T_\infty + s\frac{\sigma B_0^2}{\rho g \beta} u(x,y) \tag{19}$$

to one and the same (partial) differential equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2} \tag{20}$$

At the same time, the boundary conditions (4) reduce to

$$u|_{y=0}=U_w(x),\quad v|_{y=0}=0,\quad u|_{y\to\infty}=0 \tag{21}$$

For a given stretching velocity distribution $u(x,0)=U_w(x)$, Eq. (19) requires the wall temperature distribution

$$T_w(x)=T_\infty+s\frac{\sigma B_0^2}{\rho g\beta}U_w(x) \tag{22}$$

The first essential difference compared to the similarity case discussed in Sect. 3 is that $U_w(x)$ is an *arbitrary* function of the wall coordinate x which may, but must not necessarily coincide with the power law function (5). Nevertheless, with the aid of the substitution (19), the reduction of the mixed convection problem (1)–(4) to the general forced convection problem (20), (21) is in the case $B(x)=B_0$ and $Pr=1$ still possible.

Compared to the *forward* boundary layer flows examined in [1], however, there occurs in Eq. (22) a further essential difference. A *forward* or usual boundary layer flow results when the stretching velocity $U_w(x)$ is directed from the definite edge of the surface at $x = 0$ to $x = +\infty$. In this case the definite edge $x = 0$ is the *leading edge* of the flow. Such a flow can be realized on an upward projecting *hot* plate (discussed in [1]), or on a downward projecting *cold* plate (not discussed in [1]). Both of these forward boundary flows are *aiding* flows corresponding in Eq. (22) to $s=+1$ and $s=-1$, respectively. When however, $U_w(x)$ is directed from $x=+\infty$. to the definite edge $x = 0$ of the plate, i.e. $U_w(x)<0$ for all $x>0$, one is faced with *backward* boundary layer flows. In this case the definite edge $x=0$ becomes the *trailing edge* of the flow, while the leading edge is receded to an indefinite station far upstream. On this reason, the forward and backward boundary layer flows are essentially different phenomena, from physical and mathematical point of view as well. In this sense, Eq. (22) predicts that the mutual compensation of the buoyancy and magnetic forces can also be realized in further two *aiding* flow regimes. These correspond to backward boundary layer flows on an upward projecting cold plate, $s=+1$, $U_w(x)<0$, $T_w(x)<T_\infty$, and on a downward projecting hot plate, $s=-1$, $U_w(x)<0$, $T_w(x)>T_\infty$, respectively.

5 Summary and Conclusion

The MHD mixed convection flow of an electrically conducting fluid of $Pr=1$, driven simultaneously by (i) the shear forces due to a moving vertical wall, (ii) the thermal buoyancy and (iii) an applied horizontal magnetic field was examined. The main result of this short report is that in a uniform magnetic field, in the flow over the heated or cooled surface, between the buoyancy and magnetic forces a mutual compensation effect can occur. The result of this effect is that several types of aiding, mixed convection, forward and backward boundary layer flows become physically undistinguishable from some simpler forced convection flows, induced by a stretching surface in a quiescent, electrically insulating isothermal fluid. Our present result illustrates in fact a wide and useful applicability of the Reynolds analogy.

References

1. Ishak A, Nazar R, Pop I (2008) Hydromagnetic flow and heat transfer adjacent to a stretching vertical sheet. Heat Mass Transf 44:921–927
2. Crane LE (1970) Flow past a stretching plane. Z Angew Math Phys 21:645–647
3. Grubka LJ, Bobba KM (1985) Heat transfer characteristics of a continuous stretching surface with variable temperature. ASME J Heat Transf 107:248–250