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On the O'Brien–Jarrett–Marchi law

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Abstract The relationship between the total water volume entering a lagoon during a characteristic tidal cycle (i.e., the prism) and the size of its inlet is well established empirically since the classic work of O'Brien and Jarrett widely cited in the geomorphic and hydrodynamic literature. Less known is a rather deep theoretical explanation proposed by Marchi. This paper reviews the empirical and theoretical evidence on which the relation is based, setting the various theoretical approaches so far pursued within the general framework ensured by Marchi's theoretical treatment of the problem. We conclude that the depth of the empirical and theoretical validations and the breadth and the importance of its implications suggest that the O'Brien–Jarrett–Marchi law relating the minimum inlet cross-sectional area and the tidal prism flowing through it may be referred to thereinafter.

Keywords Lagoons · Inlets · Morphodynamics · Hydrodynamics · Venice Lagoon

1 Introduction

The study of the mechanisms controlling long-term exchange of sediments between enclosed tidal basins and adjacent seas requires focus on control sections, typically tidal inlets, where cross-sectional forms adjust to hydrodynamic and sediment transport conditions. The mobile nature of the bed material and the self-tuning of channel forms under sediment production and transport, however, shape the morphology of tidal basins

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anywhere. In this context, the possible validity of geomorphic relationships synthesizing form and function, in particular relating land-forming flow rates to key morphological features, is of utmost importance both for practical and theoretical reasons.

Empirical synthesis of complex dynamic processes is commonplace in geomorphology. In fluvial basins, for instance, it is usually assumed that total contributing area, say A, is proportional to landscape-forming discharges, say Q, i.e., $Q \propto A^{\beta}$ with $\beta \leq 1$ (e.g., Leopold et al. 1964). Surrogating Q by drainage area A in landscape evolution theories much simplifies matters without eliminating the type of complexity that is central to network self-organization (Rinaldo et al. 1995; Rodrìguez-Iturbe and Rinaldo 1997). In tidal networks, however, analogous assumptions require some elaboration because of the need to define a dynamically meaningful framework for computing both landforming discharges and watershed area (Leopold et al. 1993; Myrick and Leopold 1963; Friedrichs 1995; Fagherazzi et al. 1999; Rinaldo et al. 1999a, 1999b). Note that a wide literature exists addressing the hydrodynamics of tidal inlets, channels and creeks (e.g., Boon 1975; Friedrichs and Aubrey 1988; Lanzoni and Seminara 1998; Tambroni and Seminara 2006), the consequences of tidal currents and asymmetries on sediment dynamics and other morphological characteristics of tidal channels (e.g., Boon and Byrne 1981; Friedrichs 1995; Lanzoni and Seminara 2002), morphometric analyses of tidal networks (Myrick and Leopold 1963; Leopold et al. 1993; Fagherazzi et al. 1999; Rinaldo et al. 1999a, 1999b; Fagherazzi and Furbish 2001; Marani et al. 2002, 2003; D'Alpaos et al. 2005, 2006, 2007b; Tambroni and Seminara 2006), sedimentation and accretion patterns within salt marshes (e.g., Christiansen et al. 2000), ecological dynamics, and patterns in salt marshes (e.g., Marani et al. 2004, 2006). Moreover, simplified models have also been proposed to simulate the morphological behavior of tidal basins (e.g., van Dongeren and de Vriend 1994) and to describe the vertical movement of a marsh platform relative to a datum (zero-dimensional model) (e.g., Allen 2000; French 1993; Marani et al. 2007), or such movement combined with the growth of the vertical sequence of underlying sediments (e.g., Allen 2000).

The basic relationship employed for coupling tidal hydrodynamic and morphodynamic processes is an originally empirical linkage of cross-sectional area of inlets, say Ω, with spring (i.e., maximum astronomical) tidal prism, P or the landforming discharge Q, i.e.,

$$\Omega \propto P^{\alpha} \sim Q^{\beta} \qquad (1)$$

where α, β are scaling coefficients typically assumed to lie in the range $0.85-1.10$ (e.g., Myrick and Leopold 1963; Jarrett 1976; Marchi 1990; Hughes 2002).

The validity of Eq. 1 for sheltered channels (those not exposed to littoral transport or open sea) has been questioned (Friedrichs 1995), whereas empirical relationship have confirmed its overall validity for whole tidal basins (Rinaldo et al. 1999b; D'Alpaos et al. 2009). Complex and site-specific feedbacks between tidal channel morphology and tidal flow properties occur both in inlet and sheltered channel sections (e.g., Bruun 1978; O'Brien 1969; Jarrett 1976; Friedrichs 1995; D'Alpaos et al. 2009), and this has found theoretical explanations (Marchi 1990; Di Silvio and Dal Monte 2003; Tambroni and Seminara 2006) on which we shall return later in this paper. Moreover, complex tidal network structures generated through a simplified model of channel development (D'Alpaos et al. 2005) were obtained under the assumption implied by relations of the type (1) that channel cross-sectional areas are in dynamic equilibrium with the flowing tidal prism. The validity of such an assumption, however, was assessed only indirectly (D'Alpaos et al. 2005, 2007a, 2007b), by observing that the synthetically generated networks meet distinctive real network statistics (e.g., Marani et al. 2003). Man-made interventions are also key geomorphic agents in many lagoonal environments. For instance, dredging of tidal channels essential to navigation in many tidal environments may cause accelerated deposition, as well as reductions of the tidal prisms, by infilling or dyking marshes and/or lagoons. These modifications introduce time-dependent scales of influence on flow and erosion processes. Short-term, rapid hydrodynamic adjustments may thus occur (Byrne et al. 1981). Longer-term adjustments, e.g., due to subsidence and eustasy affecting the tidal prism, may also be important.

Various attempts have so far been proposed in order to find a theoretical explanation of Eq. 1. Comparisons with river regime equations led Mason (1973) to conclude that inlet channels are in an equilibrium state in which, similarly to regime flow conditions, erosion balances deposition. In the presence of negligible littoral drift and wave effects, Krishnamurthy (1977) obtained a simplified relationship of the form (1) with an exponent $\alpha = 1$. This result was derived by assuming a logarithmic velocity profile over the depth, integrating across a rectangular section and assuming that, at equilibrium, maximum bed shear stresses were at the most equal to the threshold shear stress for bed erosion. This latter postulate is also set at the basis of the theory proposed by Marchi (1990). In this case, the problem is solved without resorting to any particular form of the velocity profile, but simply considering the one-dimensional (i.e., cross-sectionally averaged) equations governing the flow field within an inlet channel connecting a given lagoon basin to the sea. As a result, the inlet cross-sectional area turns out to be related to the tidal prism by a power law with exponent $\alpha = 6/7$. A similar relationship, but with an exponent $\alpha = 8/9$, was recently obtained by Hughes (2002) by considering a power law velocity profile and assuming that the maximum discharge per unit width through the inlet is at equilibrium with every depth across the minimum cross section.

In the present contribution, we show how the various theoretical treatments of the problem can be simply cast within a common framework following the reasoning put forth by Marchi (1990).

The paper is organized as follows: Sects. 2, 3 and 4 recall in detail the reference framework from the literature, in particular focusing on the most relevant theoretical support for the empirical law (1). In Sect. 5, we show the generality embodied by Marchi's (1990) approach. Finally, Sect. 6 proposes our conclusions on the generality of the results presented.

2 O'Brien (1969) and Jarrett (1976)

The first attempts to find an empirical relationship between the cross-sectional area of a tidal channel and the tidal prism flowing through it go back to O'Brien (1931, 1969). This approach was motivated by navigation purposes and, therefore, focused on the morphological characteristics of tidal inlets. On the basis of data referring to sandy inlets in sedimentary equilibrium under a semi-diurnal tidal period, O'Brien (1969) thus proposed an empirical relationship of the form $\Omega = kP^{\alpha}$ where Ω is the minimum cross-sectional area (gorge) of the inlet channel, i.e., below mean water level, P is the tidal prism based on the spring tidal range, $\alpha = 0.85$, and $k = 4.69 \times 10^{-4}$, provided that Ω and P are expressed in [ft^2] and in [ft^3], respectively.

Since then, many attempts have been made to confirm the validity of the above relationship despite the notable discrepancies inevitably associated with the empirical data used to derive it, as typically observed also for river regime formulas (e.g., Leopold et al. 1964). The most comprehensive work on this subject was by Jarrett (1976), who reanalyzed Eq. 1 considering a large number of tidal inlets located in North America, and determining the coefficients k and α through a regression analysis (Fig. 1). In particular, Jarrett (1976) distinguished between various groups of inlets, including inlets with no jetties, one jetty and two jetties, and depending on the location (Atlantic, Pacific or Gulf of Mexico) along the North American coast. Table 1, derived from Bruun (1978), reports the values of the coefficients k and α resulting from this analysis. As noted by Jarrett (1976), the interpretation of these data must take into account the sources of errors that are implicit in the procedures adopted for estimating the geometry of the section and, above all, the tidal prism. Nevertheless, a similar trend also emerged from the model experiments of Mayor-Mora (1973) and Seabergh et al. (2001) that were carried out under controlled conditions including wave actions.

One must note, however, that the sediment transport conditions in the models differed considerably from those typically occurring in the field. Figure 1 shows a collection of

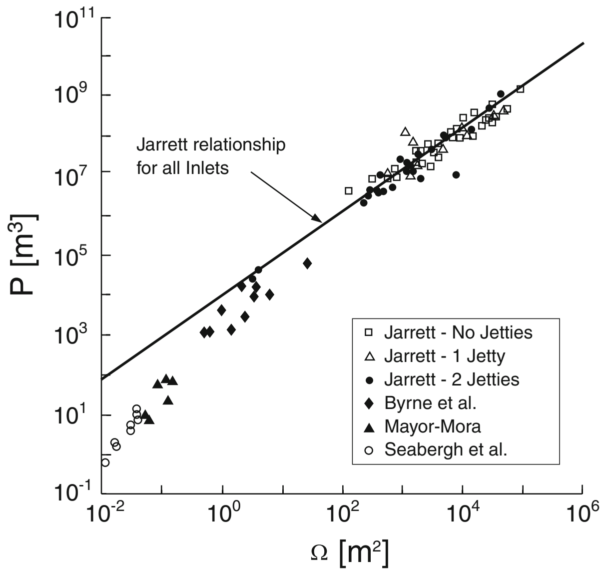

Fig. 1 Equilibrium cross-sectional area, Ω, versus tidal prism, P, for field and laboratory data collected after the seminal work of O'Brien (1969) and Jarrett (1976) (after Hughes 2002)

Table 1 Values of the coefficients α and k of the relationship $\Omega = kP^{\alpha}$ where Ω is the minimum cross-sectional area (gorge) of the inlet channel below mean water level and P is the tidal prism based on the spring tidal range

Location	No. of jetties	k	α
Atlantic	0,1,2	7.75×10^{-6}	1.05
	0,1	5.37×10^{-6}	1.07
	2	5.77×10^{-5}	0.95
Pacific	0,1,2	1.19×10^{-4}	0.91
	0,1	1.91×10^{-6}	1.03
	2	5.28×10^{-4}	0.85
Gulf of Mexico	0,2	5.02×10^{-4}	0.84
	0	3.51×10^{-4}	0.86
All data	0,1,2	5.74×10^{-5}	0.95
	0,1	1.04×10^{-5}	1.03
	2	3.76×10^{-4}	0.86

Ω and P are expressed in [ft^2] and in [ft^3], respectively (after Bruun 1978) observational values of equilibrium cross-sectional areas versus tidal prism values for field and laboratory data (Hughes 2002). In any case, it is remarkable that when the inlet is protected from the littoral drift (i.e., dual jetty inlets in the Atlantic and Pacific) or is subject to a limited drift (inlets in the Gulf of Mexico), the values attained by the exponent α (as well as those of the coefficient k) tend to be nearly similar. This result can be explained by the fact that in these types of inlets, shoals and bars are nearly absent, the tidal entrance gets better organized, and the gorge section can adjust itself to the combined action of tidal currents and waves (Bruun 1978).

3 Krishnamurthy (1977) and Hughes (2002)

The relationships between the tidal prism passing through an inlet and the size of the inlet throat derived by Krishnamurthy (1977) and Hughes (2002) are essentially based on the assumption of a given velocity profile along any vertical, which is subsequently integrated across the inlet cross section to obtain the flow discharge flowing through it and, eventually, the tidal prism. In both cases, the equilibrium condition is considered with reference to the concept of equilibrium depth associated with maximum discharge per unit width, i.e., the discharge leading to a bed shear stress, at the most equal to the critical shear stress for incipient sediment motion, τ_c.

In particular, Krishnamurthy (1977) calculates the tidal prism as:

$$P = \int_0^{T/2} BU(t)D(t)\mathrm{d}t \tag{2}$$

where B is the width of the rectangular cross section approximating the inlet cross section, U is the local depth averaged velocity, and D is the instantaneous flow depth at the inlet caused by a sinusoidal tidal forcing with amplitude a and period T. Further assuming that, as often occurs, a is much smaller than D, Krishnamurthy (1977) ends up with the relationship

$$\Omega = kP \quad k = \left[1.25u_{*c}T\left(1 + \frac{2a}{\pi D_0}\right)\ln\frac{10.93D_0}{e_s}\right]^{-1} \tag{3}$$

where Ω is the inlet area below mean sea level, u_{*c} is the friction velocity corresponding to the critical shear stress (i.e., $u_{*c} = (\tau_c/\rho)^{1/2}$), D_0 is the mean flow depth at mean sea level, and e_s is a coefficient characterizing the bed roughness (proportional to the sediment grain size d_s in the case of a plane sediment bed).

On the other hand, Hughes (2002) assumes that the velocity profile along a given vertical is described by a power law with exponent 1/8. The maximum discharge per unit width, q, determining a bed shear stress critical for sediment motion then results:

$$q = c_\tau u_{*c} d_s^{-1/8} D^{9/8} \tag{4}$$

where d_s is the sediment median grain size, D is the water depth at maximum discharge, and c_τ is a boundary layer shape factor that includes the unknown relationship between d_s and bed roughness. Hughes (2002) further observes that the tidal prism can be approximated as $P = \varphi 2aS$ (with φ an empirical factor accounting for the effects of non-sinusoidal tides) and takes advantage of the mass balance applied to the entire tidal basin,

$$Q = S\frac{\mathrm{d}h}{\mathrm{d}t} \tag{5}$$

(with Q the discharge flowing through the inlet, S the lagoon surface area, and h the sinusoidal tide elevation), to finally obtain:

$$\Omega = kP^{8/9} \quad k = \varphi\left[\frac{\pi}{c_\tau}\frac{B^{1/8}d_s^{1/8}}{u_{*c}T}\right]^{8/9}. \tag{6}$$

The quantities c_τ, u_{*c}, and φ appearing in this latter relationship are determined by Hughes (2002) by comparing the values of q computed through Eq. 4 and those resulting from velocity measurements carried out at two dual jetty tidal inlets, and applying Eq. 6 to field and laboratory data available in literature.

4 Marchi (1990)

Marchi (1990) considers a wide, rectangular inlet channel where ebb flows develop in the positive x direction (Fig. 2).

The inlet channel connects a lagoon basin of surface area S with the sea, where a sinusoidal landscape-forming tidal oscillation of amplitude a_1, period T and frequency ω (= $2\pi/T$) is imposed. Marchi (1990) further assumes that the inlet channel length, L, is much smaller than the characteristic length of the tidal wave (cT, where c is the celerity of propagation of the tidal wave) and, therefore, neglects along channel gradients in the cross-sectionally averaged velocity, U. A sinusoidal forcing of the type $h_1(t) = h_0 + a_1\sin\omega t$ is assumed, where h_0 is the mean sea level with respect to the datum.

Under such stipulations, if $h_1(t)$ and $h_2(t)$ are the sea-bound and lagoon elevations, respectively (Fig. 2), where kinetic head is negligible, one obtains through 1-D momentum balance:

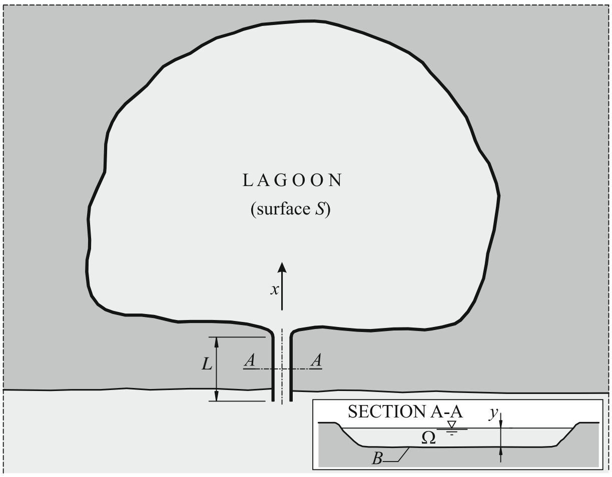

Fig. 2 Sketch of the modelled system constituted by the sea where the forcing tide $h_1(t)$ occurs, the inlet channel of length L, the inner lagoon where the delayed and damped tidal oscillation $h_2(t)$ takes place (after Marchi 1990)

$$h_1(t) - h_2(t) = \frac{L}{g}\frac{\mathrm{d}U}{\mathrm{d}t} + \left(\frac{2gL_e}{k_s^2 R^{4/3}}\right)\frac{U|U|}{2g} \tag{7}$$

where $U(t)$ is the cross-sectionally averaged velocity, L_e is an effective length that accounts for the effects of localized and distributed energy losses, k_S is Strickler's flow resistance parameter $[L^{1/3}T^{-1}]$ and R is the hydraulic radius of the inlet.

Momentum balance is complemented by mass balance, written as

$$\Omega U = \frac{\mathrm{d}V}{\mathrm{d}t} = \varphi S \frac{\mathrm{d}h_2}{\mathrm{d}t} \tag{8}$$

where Ω is the inlet cross-sectional area, V is the tidal volume entering or leaving the lagoon, and φ is a reduction coefficient whose departure from the unit value defines the differences from a static propagation scheme, that is, accounting for the instantaneous differences in the free surface elevations anywhere within the lagoon.

Stability of the cross section implies that, at any time, the maximum shear stress produced by the flow, say $\tau_{\max}$, does not exceed the threshold τ_c for the incipient motion of the bed sediment, i.e.,

$$\tau_{\max} = \frac{\rho g}{k_S^2 R^{-1/3}} U_{\max}^2 \le \tau_c \tag{9}$$

where $U_{\max}$ is the highest value reached by the velocity in the tidal exchange through the inlet within a landforming tidal cycle, T, usually assumed to be the widest spring oscillation (the characteristic spring tide).

Clearly, the nonlinear nature of the problem prevents the water surface oscillation at the lagoon entrance $h_2(t)$ to be described by a single frequency, ω (e.g., Dronkers 1964; Bruun 1978). Cleverly, however, Marchi (1990) proposed to simplify the original problem by observing that $h_2(t) = h_0 + a_2 \sin\omega(t - \theta)$ occurs with a delay θ, and that the instantaneous difference $h_1(t)$—$h_2(t)$ corresponding to $U_{\max}$ is the maximum [because the inertial term $\mathrm{d}U/\mathrm{d}t$ in Eq. 7 is negligible when $U \approx U_{\max}$ (Fig. 3)]. As a result, with acceptable approximation, one has $|h_1 - h_2|_{U\max} \sim a_1 \sin\omega\theta$. Moreover, assuming θ to be nearly constant throughout the tidal cycle, one obtains $h_1(t) \sim h_2(t + \theta)$ (Fig. 3). It then results that:

$$U_{\max} = \Omega^{-1}\varphi\omega a_1 \cos(\omega\vartheta) S \tag{10}$$

and, recalling that the tidal prism, defined as the total water volume entering the lagoonal basin within each tidal cycle, is $P = \varphi 2 a_2 S$, where (Figs. 2, 3) a_2 is the tidal amplitude at the end of the considered tidal channel, yields:

$$P = 2\omega^{-1} U_{\max}\Omega, \tag{11}$$

Marchi (1990) proceeded to compute the relationship $U_{\max} = f(\Omega)$ and its intersection with the curve given by Eq. 9 when the shear stress under maximum tidal velocity matches the threshold stress, τ_c. It is interesting to note that typically two intersections occur when equalizing the dynamic equation to the stability condition of the bed material. Of these, only one corresponds to a stable condition where a reduction of the inlet corresponds to an increase in the velocity tending to recast the original cross section. This behavior also characterizes the relation linking the cross-sectional area, Ω, to the surface area of the tidal basin, S:

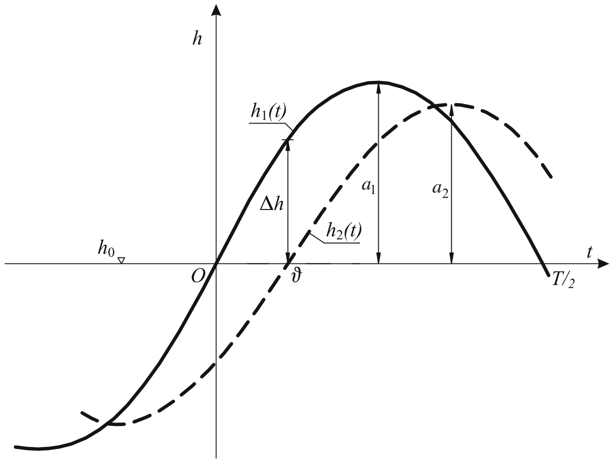

Fig. 3 Sketch of the relevant tidal oscillations at the sea- and lagoon-bound ends, $h_1(t)$ and $h_2(t)$ respectively, of the inlet (after Marchi 1990)

$$S = \frac{k_s}{\sqrt{g}} \frac{u_{*c}}{\varphi a_1 \omega B^{1/6}} \frac{1}{\sqrt{1 - [u_{*c}^2 BL_e/(ga_1\Omega)]^2}} \Omega^{7/6}. \tag{12}$$

Indeed, the curve resulting from this functional dependence (shown in Fig. 4 in terms of dimensionless quantities and with reference to the three inlets of the Venice Lagoon)

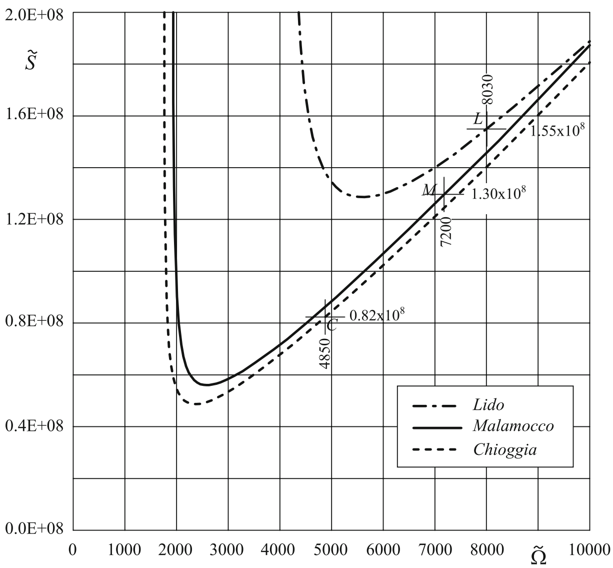

Fig. 4 The dimensionless curves, $\tilde{S}(\tilde{\Omega})$, for the three inlets of the Lagoon of Venice. Note that, for a given value of the dimensionless lagoon basin area $\tilde{S}(= S/(2a_1)^2)$, only for larger values of the dimensionless cross section, $\tilde{\Omega}(= \Omega/(2a_1)^2)$ implies a stable condition. The labels *L*, *M* and *C* indicate, respectively, the data for the inlets of Lido, Malamocco, and Chioggia of the Lagoon of Venice (after Marchi 1990). The adopted scaling is: $\tilde{S} = S/(2a_1)^2$, $\tilde{\Omega} = \Omega/(2a_1)^2$, with a_1 the amplitude of the sinusoidal forcing tide

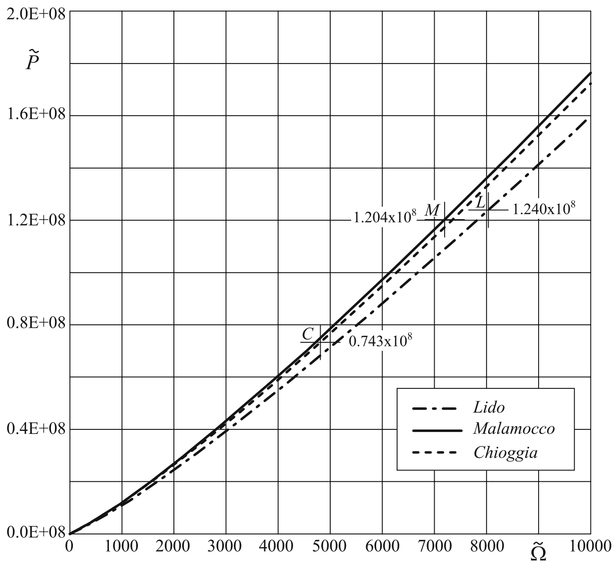

Fig. 5 The dimensionless curves $\tilde{P}(\tilde{\Omega})$ are plotted for the three inlets of the Lagoon of Venice. Note that the relationship does not exhibit unstable conditions. Tidal prisms, however, are not independent variables on which the processes can be affected, being a function of the lagoonal surface and of the forcing amplitude. The labels *L*, *M* and *C* indicate, respectively, the data for the inlets of Lido, Malamocco and Chioggia of the Lagoon of Venice (after Marchi 1990). The adopted scaling is: $\tilde{P} = P/(2a_1)^3$, $\tilde{\Omega} = \Omega/(2a_1)^2$, with a_1 the amplitude of the sinusoidal forcing tide

exhibits two branches of which only the right one is associated with equilibrium conditions. Furthermore, at larger sections, one observes a nearly linear relationship with the dimensionless lagoonal surface, i.e., $\tilde{S} \sim \tilde{\Omega}^{7/6}$.

Finally, through the same procedure, Marchi (1990) determines the relationship between the cross-sectional area of the inlet, *Ω*, and the ensuing tidal prism, *P*, namely

$$\Omega = kP^{6/7} \quad k = \left[\pi\sqrt{g}\frac{B^{1/6}}{u_{*c}Tk_s}\right]^{6/7} \tag{13}$$

shown in Fig. 5 in terms of dimensionless quantities and with reference to the three inlets of the Venice Lagoon. This equation, which has been largely confirmed by relaxing a number of simplifying assumptions and solving numerically the resulting problem (Tambroni and Seminara 2006), agrees very well with the empirical law $\Omega \sim P^{0.85}$ proposed by O'Brien (1969) and further verified by Jarrett (1976). Moreover, the structure of the proportionality coefficient *k* shows some similarities with the analogous coefficients appearing in relationships (3) and (6).

5 Generalization of Marchi's (1990) formulation

The three theoretical approaches described in Sects. 3 and 4 exhibit some common interesting features, which are worthwhile to recall here. All of the proposed relationships (3), (6), and (13) point at a power law of the form (1), although with different values of the exponent α, and indicate that the proportionality coefficient, *k*, cannot be universal. Indeed, in all cases the equilibrium cross-sectional area, *Ω*, tends to decrease for increasing values of the tidal period, *T*, and of the critical friction velocity for sediment erosion, u_{*c}. Moreover, Eqs. 6 and 13 suggest that *Ω* decreases with decreasing flow resistance (i.e., for increasingly small values of d_s and increasingly high values of k_s) and decreasing channel width, *B*. This latter behavior does not clearly appear from Eq. 3, since the coefficient *k* embeds also an implicit dependence on *Ω* through the quantity $D = \Omega/B$.

The similarities emerging from the various relationships can be set within a common comprehensive framework by resorting to Marchi's (1990) physical approach. We first observe that imposing that the bed shear stress under maximum tidal velocity is equal to the critical shear stress for incipient sediment motion τ_c yields $U_{max} = Cu_{*c}$, with $u_{*c} = (\tau_c/\rho)^{1/2}$. Hence, recalling Eq. 11, we end up with the relationship:

$$P = \frac{2u_{*c}}{\omega} C\Omega \tag{14}$$

indicating that the exponent α appearing in (1), as well as the proportionality coefficient, k, strictly depends on the relationship used to estimate the flow conductance, C. In particular, the power law dependence of Ω on P obtained by Krishnamurthy (1977) (Eq. 3), Marchi (1990) (Eq. 13) and Hughes (2002) (Eq. 6) are recovered by expressing C through the classical flow resistance relationships proposed by Keulegan (1938), Strickler (1923) and Engelund and Hansen (1967), which read as

$$C = 2.25 \ln\left(11 \frac{\Omega}{Be_s}\right); \quad C = \frac{k_s}{\sqrt{g}} \left(\frac{\Omega}{B}\right)^{1/6}; \quad C = 9.45 \left(\frac{\Omega}{2.5Bd_s}\right)^{1/8}, \tag{15}$$

respectively. On the other hand, Eq. 14 clearly supports the observation that the proportionality constant k cannot be universal. Indeed, k increases with ω and decreases with u_{*c}, at a rate that depends on the flow resistance formula used to estimate C. In any case, k proves independent of the tidal forcing amplitude, under the assumption of linear propagation of the tide through the inlet.

In practice, various causes can lead to a departure from the above described ideal equilibrium conditions, and therefore a certain amount of sediment transport can occur through the different phases of the tidal cycle. In fact, in many lagoons, a quasi-equilibrium condition can be attained according to which basin vertical growth, resulting from the interplay of erosional and depositional processes, nearly balances the rate of relative sea level rise. In this case, the assumption of maximum bottom shear stress always lower or equal than its critical value is not strictly met. Note also that even for a constant value of relative mean sea level, inlet cross-sectional areas can tend to be at equilibrium only asymptotically. Other causes responsible for a departure from the theoretical relationships (15) are related to the effects of along shore currents, waves, changes in the external forcings and human interventions.

6 Conclusions

The thorough review of literature reported in the present paper defines a comprehensive framework for geomorphic relationships linking the minimum cross-sectional area of a tidal inlet Ω to the water volume (the tidal prism) entering its embedded lagoonal expansion, P. The relationship is rooted in wide and diversified empirical observations and theory. Empirical evidence, gathered from a large number of tidal inlets of the Atlantic, Gulf and Pacific coasts of the USA and European coasts, emphasizes the existence of the power law linking Ω to P, with an exponent α in the range 0.85–1.10 and a proportionality coefficient k, which in general depends on the hydrodynamic and sedimentologic conditions of the specific site, as well as on the possible presence of jetties.

The theoretical treatment of the problem proposed by Marchi (1990) not only explains such an empirical evidence, but also provides a comprehensive theoretical framework in which the various analytical models can be rationally set. In particular, the exponent α and the coefficient k depend on the particular formula used to estimate the flow conductance. Moreover, k is related to the tidal period and to the friction velocity critical for sediment erosion.

The quality and nature of the empirical and theoretical validations thus suggest that the O'Brien–Jarrett–Marchi law may be referred to thereinafter.

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