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Viewing the future through a warped lens: Why uncertainty generates hyperbolic discounting

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Abstract A large body of experimental research has demonstrated that, on average, people violate the axioms of expected utility theory as well as of discounted utility theory. In particular, aggregate behavior is best characterized by probability distortions and hyperbolic discounting. But is it the same people who are prone to these behaviors? Based on an experiment with salient monetary incentives we demonstrate that there is a strong and significant relationship between greater departures from linear probability weighting and the degree of decreasing discount rates at the level of individual behavior. We argue that this relationship can be rationalized by the uncertainty inherent in any future event, linking discounting behavior directly to risk preferences. Consequently, decreasing discount rates may be generated by people's proneness to probability distortions.

Keywords Time preferences · Risk preferences · Hyperbolic discounting · Probability weighting · Institutionally generated uncertainty

JEL Classification D01 · D81 · D91

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"Future income is always subject to some uncertainty, and this uncertainty must naturally have an influence on the rate of time preference, or degree of impatience, of its possessor."

Fisher (1930)

It has long been recognized by practitioners and theorists alike that the domains of choice under risk and over time are intimately related. In the realm of economic theory, the dimensions of risk and time are treated as largely independent attributes, modeled in an equivalent way (Prelec and Loewenstein 1991): The classical models of choice, expected utility theory (EUT) and discounted utility theory (DUT), view decision makers as maximizing a weighted sum of utilities with the weights representing either probabilities or exponentially declining discount weights, respectively.

A large body of empirical evidence has challenged the validity of EUT and DUT as descriptive models of choice, however. In the domain of risk, one of the best documented phenomena concerns the *common ratio effect*: Often, people's preference for a smaller more probable outcome over a larger less probable one changes in favor of the larger outcome when both outcome probabilities are scaled down by a common factor. This pattern of behavior constitutes a violation of the independence axiom of EUT (Kahneman and Tversky 1979; Starmer and Sugden 1989).[1] The stationarity axiom of DUT, according to which preferences should depend on the absolute time interval between the delivery of the objects, has met a similar fate. The *common difference effect* describes the empirical regularity that indifference between a smaller earlier payoff and a larger later payoff shifts to preference for the larger later payoff when both payoff dates are pushed into the more remote future by a common delay (Thaler 1981; Benzion et al. 1989).

Researchers have reacted to these anomalies by relaxing the assumptions on the corresponding decision weights while leaving the overall separable structure of the models intact. Violations of independence can be captured by a suitable nonlinear transformation of the probabilities (Quiggin 1982; Tversky and Kahneman 1992). The crucial characteristic of probability weights that produces common ratio violations is *subproportionality* (Kahneman and Tversky 1979). Subproportionality means that, for a fixed ratio of probabilities, the ratio of the corresponding probability weights is closer to unity when the probabilities are low than when they are high. Intuitively speaking, scaling down the original probabilities makes them less distinguishable from each other, thus favoring preference reversals of the common-ratio type. Violations of stationarity, on the other hand, are accounted for by giving up the requirement of constant discount rates and allowing decreasing ones, which has become known as *hyperbolic discounting* (Ainslie 1991; Laibson 1997; Prelec 2004). These generalizations seem to perform much better at explaining

[1] Prominent special cases are the Allais paradox (Allais 1953) and the Bergen paradox (Hagen 1972).

aggregate choices than do EUT and DUT (Rachlin et al. 1991; Harless and Camerer 1994; Hey and Orme 1994; Myerson and Green 1995; Kirby 1997). At the individual level, however, there is vast heterogeneity in observed behavior in both decision domains (Hey and Orme 1994; Chesson and Viscusi 2000; Abdellaoui et al. 2010; Bruhin et al. 2010), and it is an open question whether the superior fit of the generalized models is actually a manifestation of common regularities of individual behavior, as argued for example by Prelec and Loewenstein (1991). In their opinion, violations of independence and stationarity are not coincidental but reflect certain fundamental principles of prospect evaluation which govern people's sensitivities to probability and delay. If there is such a common driving force underlying risk taking and discounting behavior, we should observe a significant correlation between subproportionality of probability weights and hyperbolicity of discount weights at the level of individual behavior.

In the extant empirical literature, the relationship between individuals' attitudes towards risk and delay has been examined from various different angles. One strand of the literature focuses on people's risk tolerance measured independently from their degree of impatience. These studies find that more risk averse people tend to discount the future more heavily (Leigh 1986; Anderhub et al. 2001; Eckel et al. 2004). Discount rates are inferred directly from choices over dated monetary amounts and, therefore, their measurement is confounded by the curvature of the utility function. Andersen et al. (2008) correct for utility curvature and still find a positive, but much reduced, correlation in their predicted degrees of risk aversion and impatience. None of the studies so far have accounted for probability weighting and, therefore, they cannot address the question of whether departures from linear probability weighting are systematically related to departures from exponential discounting. The psychological literature has dealt with comparisons of highly reduced forms of discounting functions for delay and probability, ignoring utility curvature, and finds a positive correlation between both types of discounting (Rachlin et al. 1991; Myerson et al. 2003).

Another strand of the literature investigates people's choices when both risk and delay are present (Keren and Roelofsma 1995; Ahlbrecht and Weber 1997; Weber and Chapman 2005; Noussair and Wu 2006; Anderson and Stafford 2009; Baucells and Heukamp 2010; Coble and Lusk 2010). These studies generally conclude that there are interaction effects between time and risk, such as risk tolerance increasing with delay, which are not easily justifiable within the frameworks of EUT and DUT. Again, probability weighting does not feature in any of these papers. A notable exception is the contribution by Abdellaoui et al. (2011) who estimate individual probability weights over varying delays, but do not elicit discount functions for guaranteed payoffs.

Finally, some recent papers examine the effects of risk in the payment date, rather than in outcome magnitude. Parallel to the findings on delayed guaranteed outcomes, Chesson and Viscusi (2000) report discount rates to decline with time horizon. Moreover, Chesson and Viscusi (2003) show that aversion to timing risk is positively related to ambiguity aversion, suggesting that uncertainty may be processed similarly in both the dimensions of time and risk. In a follow-up study Onay and Öncüler (2007) argue that the prevalence of timing risk aversion, which runs against the predictions of EUT, can be accommodated within a rank-dependent model involving probability weighting. They do not test their conjecture empirically, however.

This brief review of the literature shows that, to the best of our knowledge, there is no previous study that investigates the link between individuals' probability weights and discount weights. While evidence of hyperbolic discounting is occasionally reported, simultaneous estimates of individual probability weights are usually not provided. This lack may be due to the fact that a comparatively rich data set, and for that matter also a fairly sophisticated estimation strategy, is needed to be able to disentangle utility curvature and probability weighting.

In order to address the issue of a relationship between subproportionality and hyperbolicity, we conducted an experiment with salient monetary incentives, which exhibits a number of distinguishing features: First, the experiment generated data rich enough to be able to estimate individual probability weighting functions and relate them to the same subjects' revealed discount rates. Second, in contrast to many previous discounting experiments, every single subject got paid for her intertemporal choices, involving substantial payoffs, in an incentive compatible manner. Third, we kept transaction costs equal across different payment dates in order to preclude confounding effects. Finally, we controlled for utility curvature.

We present the following experimental results. First, we show that the degree of subproportionality is highly significantly associated with the strength of decreasing discount rates. The curvature of the utility function, however, seems not to be directly related to their decline. Second, estimation results are robust to controlling for socioeconomic characteristics, such as gender, age, experience with investment decisions and cognitive abilities. In fact, the only variable associated with decreasing discount rates turns out to be the degree of subproportionality of probability weights, which explains a—by any standard—large percentage of the variation in the extent of the decline. Moreover, all our results are insensitive to model specification.

Our findings demonstrate that, consistent with the hypothesis of a common underlying factor, departures from linear probability weighting are significantly correlated with departures from exponential discounting. However, the relationship between risk taking and time discounting behavior cannot be attributed to one of the candidate factors discussed in the literature: Cognitive skills measured by the *Cognitive Reflection Test* (Frederick 2005) do not contribute to explaining the correlation between subproportionality and hyperbolicity.

We favor an alternative hypothesis, namely, that the correlation is driven by a natural link between the domains of time and risk (Halevy 2008; Saito 2011): Arguably, only immediate consequences can be totally certain whereas delayed ones are uncertain by their very nature. For instance, a promised reward may, due to unforeseen circumstances, materialize later or turn out to be smaller than expected, or sudden illness or death may keep the decision maker from collecting her reward. For these reasons, future consequences are inextricably associated with uncertainty, implying that the decision maker's valuation of delayed outcomes not only depends on her *pure* time preference, i.e. her preference for immediate utility over delayed utility, but also on her perception of uncertainty and, consequently, on her risk preferences. If the probability of receiving a promised reward gets transformed like any other probability, the reward gets devalued by an additional factor which equals this subjectively weighted probability. Moreover, if the decision maker is prone to common ratio violations, i.e. if her probability weights are subproportional, this additional discount weight declines at a decreasing rate: Any further delay by another unit of time corresponds to a scaling down of the probability of receiving the reward to which the decision maker becomes progressively insensitive. Hence, total discount weights decline at a decreasing rate, i.e. hyperbolically, even if the pure rate of time preference is constant, and the effect is more pronounced for comparatively more subproportional probability weights. Figuratively speaking, hyperbolic discounting is driven by viewing the uncertain future through a warped lens, produced by systematic distortions of probabilities.

This theoretical framework not only organizes our experimental findings but also accounts for previous evidence of interactions of time and risk. A number of studies detected preference reversals when either risk is added to temporal prospects (Baucells and Heukamp 2010) or delay is added to simple risky prospects (Keren and Roelofsma 1995; Weber and Chapman 2005).

Our analysis suggests that institutionally generated uncertainties, such as lack of contract enforcement and weak property rights, may induce extreme short-run impatience even if people's pure rate of time preference is constant and relatively low. This insight is important because it implies that revealed behavior may be predominantly driven by environmental factors rather than by the underlying preferences themselves and, consequently, may be amenable to economic policy. While uncertainty may be an important channel through which hyperbolicity of discount rates is generated there may be other sources of hyperbolic discounting behavior as well. For instance, pure time preferences may be hyperbolic *per se*, as could be argued for addictive behavior. And when visceral motives, such as hunger or lust, come into play, uncertainty may not be the dominant dimension decision makers are concerned about. An excessive preference for the present may then be driven by factors other than potential disappearance of the object of desire.

The remainder of the paper is structured as follows: In Section 1 we describe the experimental design and procedures. Section 2 outlines our approach to estimation. Section 3 presents our results. Section 4 discusses our hypothesis on the role of risk preferences in time discounting. Section 5 concludes.

1 Experimental design

The experiment took place at the Institute for Empirical Research in Economics (IERE), University of Zurich, in May 2006. Participants were recruited from the IERE subject pool, which consists of students from all fields offered at the University of Zurich and the ETH Zurich. In total, we analyzed 112 subjects' responses.[2] The experiment consisted of two main parts, one dedicated to eliciting certainty equivalents for non-delayed risky prospects,[3] the other one to eliciting future equivalents and their corresponding imputed discount rates for temporal prospects involving guaranteed payments.[4]

We used similar procedures to elicit certainty equivalents and discount rates in order to economize on subjects' cognitive effort. For both types of tasks we implemented choice menus containing a list of 20 varying alternatives which had to be judged against a fixed option. To familiarize subjects with the nature of the procedure the instructions contained examples and trial problems. Besides a show up fee of Swiss Francs (CHF) 10 (CHF 1 $\approx$ USD 0.8 at the time of the experiment), each subject was paid according to one of her risky choices and one of her temporal choices selected randomly at the very end of the experiment. Subjects received their compensation for the risky choices and the show-up fee in cash immediately after completion of all the tasks. The compensation for their intertemporal choices was paid out to them at the respective dates when they cashed in vouchers issued to them at the end of the experiment. Payment modalities are described in detail below. Subjects could work at their own speed. On average, it took them 1.25 hour to complete the experiment, including a socioeconomic questionnaire presented after the choice tasks.

1.1 Elicitation of certainty equivalents

Since the objective of the risk task was to obtain data on the basis of which individual probability weights could be estimated, a fairly large number of observations per person was needed. To elicit individual lottery evaluations, subjects were presented with 20 choice menus, each one involving a specific binary lottery $\mathcal{L} = (x_1, p; x_2)$, with $x_1 > x_2 \geq 0$, labeled *Option A* in Fig. 1. *Option B* in the choice menu represented the guaranteed alternatives, ranging

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[2]We omitted six subjects' responses from our analysis. Four subjects reported that they would not be able to cash in their delayed payments at the respective payment dates. Three of them would have been on vacation then, the fourth person had planned a long visit abroad. Hence, their choices were not informative of their time preferences. Concerning the remaining two subjects we could not disentangle utility effects from probability weighting effects. Nonetheless, our results do not change when we include these two individuals in the data set.

[3]The risk data was also used in Bruhin et al. (2010).

[4]Instructions are available upon request. The experiment was programmed and conducted with the software z-Tree (Fischbacher 2007).

	Option A	Your Choice		Option B *(guaranteed reward)*
1	Gain of CHF 50 with a probability of 75% *and* Gain of CHF 10 with a probability of 25%	A ○	● B	CHF 50
2		A ○	● B	CHF 48
3		A ○	● B	CHF 46
4		A ○	● B	CHF 44
5		A ○	● B	CHF 42
6		A ○	● B	CHF 40
7		A ○	● B	CHF 38
8		A ●	○ B	CHF 36
9		A ●	○ B	CHF 34
10		A ●	○ B	CHF 32
11		A ●	○ B	CHF 30
12		A ●	○ B	CHF 28
13		A ●	○ B	CHF 26
14		A ●	○ B	CHF 24
15		A ●	○ B	CHF 22
16		A ●	○ B	CHF 20
17		A ●	○ B	CHF 18
18		A ●	○ B	CHF 16
19		A ●	○ B	CHF 14
20		A ●	○ B	CHF 12

Fig. 1 Choice menu—risk

from the higher lottery outcome x_1 to the lower outcome x_2. Every subject had to choose her preferred option in each row of the choice menu. In Fig. 1, a hypothetical subject prefers all guaranteed payments larger than CHF 36 to the stated lottery, and prefers the lottery in the remaining rows. This subject's valuation of the lottery, her certainty equivalent CE, is calculated as the arithmetic mean of the two amounts next to her indifference point, amounting to CHF 37 in the example here. The set of lotteries, listed in Table 1, included a wide range of outcomes and probabilities. Every subject was confronted with this set of lotteries once. The choice menus appeared in an individualized random order.

At the end of the experiment, after the subject had completed all the tasks, one row of one choice menu was randomly selected for payment. If the subject had opted for the lottery there, her decision was played out for real. If the subject had opted for the guaranteed payoff, the respective amount was paid out to her. On average, subjects earned CHF 37.22 in cash for the risk task, including the show-up fee of CHF 10, to be paid out immediately. Cash payments were considerably higher than the local student assistant's hourly wage.

Table 1 Risky prospects $(x_1, p; x_2)$

p	x_1	x_2	p	x_1	x_2
0.1	20	10	0.25	50	20
0.5	20	10	0.5	50	20
0.9	20	10	0.75	50	20
0.05	40	10	0.95	50	20
0.25	40	10	0.05	150	50
0.5	40	10	0.5	10	0
0.75	40	10	0.5	20	0
0.95	40	10	0.05	40	0
0.05	50	20	0.95	50	0
0.1	150	0	0.25	40	0

Payoffs x_1 and x_2 are stated in Swiss Francs (CHF)
p denotes the probability of x_1 materializing

1.2 Elicitation of discount rates

Using a format similar to the risk task, we elicited individual discount rates for temporal prospects $\mathcal{T} = (x, t)$, with $x > 0$, over payments x delayed by t months. The choice menus, designed as in Fig. 2, contained 20 binary choices each.[5] Subjects had to choose between a guaranteed payment PE of CHF 60 the next day (*Option A*) and a guaranteed later payment x (*Option B*), delayed by two months or four months, respectively. The varying alternatives x were sorted in descending order from the highest amount to the lowest amount, incorporating an interest payment at a simple annualized rate of $\delta_t \in [0\%, 95\%]$ over the corresponding time interval $[0, t]$.[6] These rates were exhibited in the right-most column of the choice menu. The present amount of CHF 60 and the range of interest rates were chosen to provide salient incentives, so that deferring the reward was actually worthwhile. The arithmetic mean of the two monetary amounts next to the indifference point on the choice menu provided the imputed discount rate δ_t. The hypothetical subject in Fig. 2, for instance, is indifferent between CHF 60 and CHF 70.50, implying a discount rate of 52.5% *per annum*.

We applied a similar random payment method in the time task as in the risk task: One of each subjects' choices was paid out for real at the corresponding payment date. Average payoffs for the time task amounted to CHF 64.34. Therefore, total average payments for both risk and time tasks summed to more than four times students' opportunity costs, measured by the student assistants' hourly wages.

[5] A similar design was proposed by Coller and Williams (1999).

[6] For convenience of notation we use "0" to denote the day after the experiment. Consequently, PE is labeled "present equivalent".

	Option A *payment tomorrow*	Your Choice				Option B *payment in 4 months + 1 day*	
1		A	○	●	B	CHF 79	95%
2		A	○	●	B	CHF 78	90%
3		A	○	●	B	CHF 77	85%
4		A	○	●	B	CHF 76	80%
5		A	○	●	B	CHF 75	75%
6		A	○	●	B	CHF 74	70%
7		A	○	●	B	CHF 73	65%
8		A	○	●	B	CHF 72	60%
9		A	○	●	B	CHF 71	55%
10	Amount	A	●	○	B	CHF 70	50%
11	of CHF 60	A	●	○	B	CHF 69	45%
12		A	●	○	B	CHF 68	40%
13		A	●	○	B	CHF 67	35%
14		A	●	○	B	CHF 66	30%
15		A	●	○	B	CHF 65	25%
16		A	●	○	B	CHF 64	20%
17		A	●	○	B	CHF 63	15%
18		A	●	○	B	CHF 62	10%
19		A	●	○	B	CHF 61	5%
20		A	●	○	B	CHF 60	0%

Fig. 2 Choice menu—time

Since our objective was to elicit discount rates over guaranteed payments, we took special care with the payment procedure: First, every single subject got paid for one of her intertemporal choices, all of which entailed a payment of the same order of magnitude. Not paying off everyone may render the stochastic nature of the experimental earnings salient and interfere with the objective of eliciting discount rates over guaranteed amounts of money. The second issue concerns the credibility of payment. In order to control for uncertainty arising from subjects' doubts about experimenter reliability, an official voucher of the Swiss Federal Institute of Technology was issued to them. This payment method was explained in detail in the instructions, and a specimen of the voucher was included in the instruction set. A third possibly confounding factor are transaction costs. Transaction costs should be the same regardless of the payment date in order to avoid inducing present bias resulting from immediately available cash payments. Therefore, every subject had to make a trip to the cash desk to collect her earnings for the discounting task.[7]

[7] People entitled to payoffs the next day were issued vouchers immediately after the experiment. All the others received official certificates of indebtedness after the experiment. The vouchers themselves were sent to them by registered mail several days before they could cash them in, so they did not have to worry about forgetting encashment or misplacing their vouchers.

2 Econometric specification

The data elicited in the experiment provide two types of main variables: certainty equivalents CE for risky prospects, and discount rates δ_t imputed from observed present equivalents PE for temporal prospects. We first discuss our econometric approach to risky choice and, subsequently, describe the method employed to test for a link between subproportionality and hyperbolicity.

2.1 Behavior under risk

Modeling decisions under risk encompasses two components, a model of behavior on the one hand, and assumptions regarding decision errors on the other hand. Risk taking behavior is modeled by rank dependent utility theory (RDU).

According to RDU, an individual values a two-outcome lottery $\mathcal{L} = (x_1, p; x_2)$, where $x_1 > x_2 \geq 0$, by $w(p)u(x_1) + (1 - w(p))u(x_2)$. The function $u(x)$, with $u(0) = 0$ and $u'(x) > 0$, describes how monetary outcomes x are subjectively valued. The function $w(p)$ assigns a subjective weight to every outcome probability p, with $w(0) = 0$, $w(1) = 1$, and $w'(p) > 0$. The decision maker's predicted certainty equivalent $\hat{CE}$ for this lottery can then be written as

$$\hat{CE} = u^{-1}\left[w(p)u(x_1) + (1 - w(p))u(x_2)\right]. \tag{1}$$

In order to make RDU operational, we have to assume specific functional forms for the utility function $u(x)$ and the probability weighting function $w(p)$. Given our objective of describing individual behavior, we choose flexible functional forms for u as well as for w. A natural candidate for utility u is a power function. In its extensive form, as discussed by Wakker (2008), u is modeled as[8]

$$u(x) = \begin{cases} x^\eta & \text{if } \eta > 0, \\ \ln(x) & \text{if } \eta = 0, \\ -x^\eta & \text{if } \eta < 0. \end{cases} \tag{2}$$

A variety of parameterizations of probability weighting functions $w(p)$ have been proposed in the literature (Karmarkar 1979; Lattimore et al. 1992; Tversky and Kahneman 1992). Since our primary interest lies in common ratio violations we focus on the characteristic of subproportionality. Expressed formally (Prelec 1998), *subproportionality* holds if $1 \geq p > q > 0$ and $0 < \lambda < 1$ implies the inequality

$$\frac{w(p)}{w(q)} > \frac{w(\lambda p)}{w(\lambda q)}. \tag{3}$$

[8] Note that $\ln(x)$ is not defined for $x = 0$. Therefore, estimation is carried out after shifting all outcomes by one unit of money (cmp. Wakker (2008), p.1335).

As we rely on subproportionality as the crucial characteristic of the probability weighting function, to be estimated for each single individual, we adopt the flexible and empirically well-founded two-parameter specification suggested by Prelec (1998),

$$w(p) = e^{-\beta(-\ln p)^{\alpha}}. \tag{4}$$

For $\alpha < 1$, the function is subproportional everywhere, with the parameter α measuring the degree of subproportionality.[9] A smaller value of α reflects a more subproportional curve, departing more strongly from linear weighting. Therefore, this specification enables us to rank individuals according to their proneness to common ratio violations. The second parameter, $\beta > 0$, is a net index of convexity in that increasing β increases the convexity of the function without affecting subproportionality (Prelec (1998), p.505). Linear weighting is characterized by $\alpha = \beta = 1$.

With regard to error specification we have to reconsider our measurement procedure. In the course of the experiment, an individual's risk taking behavior was captured by her certainty equivalents CE_l for a set of 20 different lotteries $\mathcal{L}_l = (x_{1l}, p_l; x_{2l}),\ \ l \in \{1, \ldots, 20\}$. Since RDU explains *deterministic* choice, actual certainty equivalents CE_l are likely to deviate from the predicted certainty equivalents $\hat{CE}_l$ by a stochastic error ϵ_l, which has to be taken account of. Therefore, we assume that the observed certainty equivalents CE_l can be expressed as $CE_l = \hat{CE}_l + \epsilon_l$, with ϵ_l being normally distributed with zero mean.[10]

Concerning the error variance, we need to account for heteroskedasticity: For each lottery subjects had to consider 20 guaranteed outcomes, equally spaced throughout the lottery's outcome range $x_{1l} - x_{2l}$. Since the observed certainty equivalent CE_l is calculated as the arithmetic mean of the smallest guaranteed amount preferred to the lottery and the subsequent guaranteed amount, the error is proportional to the outcome range. Therefore, the standard deviation ν_l of the error term distribution has to be normalized by the outcome range, yielding $\nu_l = \nu(x_{1l} - x_{2l})$, where ν denotes an additional parameter to be estimated. In total, therefore, four parameters per subject were estimated by maximum likelihood: the curvature of the utility function η, subproportionality and convexity of the probability weighting function α and β, as well as the normalized standard deviation of the decision error parameter ν.

[9]Prelec (2000) even uses the term “Allais paradox index” (p.78).

[10]Since CE is calculated as the arithmetic mean of two neighboring amounts in the choice menu it possibly contains some measurement error. As CE is the dependent variable in the model a measurement error does not pose a problem other than potentially increasing noise.

2.2 Behavior over time

Subjects' responses to the intertemporal choice tasks in the experiment provided us with measurements of discount rates δ_2 and δ_4, imputed from the intertemporal tradeoffs between present equivalents PE and payments x delayed by two and four months, respectively. However, the true underlying discount weights $D(t)$ are defined in terms of utilities, not payoffs.[11] For a temporal prospect $\mathcal{T} = (x, t)$, true discount rates are inferred from the indifference relation $u(PE) = D(t)u(x)$. Measured discount rates, therefore, deviate from the underlying true rates unless u is linear. While in our specification utility curvature affects the level of discount rates but cannot, by itself, induce their decline, it may have a confounding effect on the magnitude of the change in the measured discount rates $\Delta\delta = \delta_2 - \delta_4$: In the presence of nonlinear probability weighting, $\Delta\delta$ gets amplified by the concavity of the power utility function (see Appendix A.4). Specifically, the more concave u, the larger the measured difference in the discount rates $\Delta\delta$ if $w(p)$ is not linear. Therefore, we control for the degree of concavity η in the regression model.

2.3 Regression model

We investigate the hypothesized relationship between probability weighting and changing discount rates by regressing the difference between the imputed discount rates δ_2 and δ_4, $\Delta\delta$, on a vector of regressors c. In the base model, Model 1, the vector c consists of a constant and the individuals' estimated risk preference parameters: η captures concavity of the utility function, α captures subproportionality of the probability weighting function, and β its convexity. If there is a link between subproportionality and hyperbolicity, we expect to find a negative correlation between subproportionality α and the extent of the decline in discount rates $\Delta\delta$. Additionally, we estimate an extended version of the base model, Model 2, by controlling for a set of individual characteristics. In particular, these controls comprise gender (labeled *Female*), age (*Age*), the logarithm of disposable income per month (*Log-Income*), a binary variable indicating whether the subject is familiar with investment decisions (*Experience*) as well as the test score for the *Cognitive Reflection Test* (*CRT*) (Frederick 2005).[12] This three-question test measures specific aspects of cognitive ability which were found to be strongly correlated with risk taking and discounting behavior.

[11] We assume that the utility of money is a general cardinal function which applies to risky as well as to delayed payoffs. A theoretical justification for this assumption is provided by Wakker (1994), empirical support by Abdellaoui et al. (2010).

[12] Summary statistics of the controls are included in Appendix E, Table 4.

Unlike the exemplary choice pattern displayed in Fig. 2, a decision maker may have opted for the same option in all rows of the choice menu, which results in a censored observation. In particular, she may have always preferred the smaller sooner option, indicating that her discount rate may lie beyond the maximum value of 95%.[13] As a consequence, the difference between the observed discount rates δ_2 and δ_4 is affected by censoring as well. As ordinary least square (OLS) may yield biased estimates in this case, we account for this issue by a censored regression model, described in detail in Appendix B. The model has the following form:

$$\Delta\delta_i^* = c_i \underbrace{(\gamma_2 - \gamma_4)}_{\Delta\gamma} + \underbrace{e_{2,i} - e_{4,i}}_{\Delta e_i}, \tag{5}$$

where $\Delta\delta_i^*$ specifies the true, but potentially unobserved, difference between δ_2 and δ_4 for individual i, $i \in \{1, \ldots, 112\}$. The error term Δe_i is normally distributed with mean zero and variance σ^2. The interpretation of the regression coefficients $\Delta\gamma$ is equivalent to those of OLS regression, also displayed in the regression output (see Table 3).

3 Results

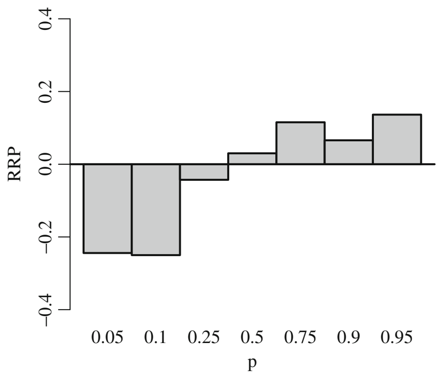
In the following section we analyze the raw data on risk taking behavior and time discounting, and present the estimates for subjects' probability weights. Finally, we examine the relationship between subjects' sensitivities with respect to changes in probability and delay.

3.1 Descriptive analysis

For the domain of risk taking, Fig. 3 summarizes observed behavior by the median relative risk premia $RRP = (EV - CE)/EV$, where EV denotes the expected value of a lottery's payoff and CE stands for the observed certainty equivalent. $RRP > 0$ indicates risk aversion, $RRP < 0$ risk seeking, and $RRP = 0$ risk neutrality. The median relative risk premia, sorted by the probability p of the higher gain, show a systematic relationship between aggregated risk attitudes and lottery probabilities: Subjects' choices display the familiar pattern, i.e. they are risk averse for high-probability gains, but risk seeking for low-probability gains, supporting the existence of probability distortions.

[13] A decision maker may also always prefer the later larger option. In this case, we assume a discount rate of 0%. The number of observations at the boundary of the choice menu are listed in Table 5 of Appendix E.

Fig. 3 Median relative risk premia *(RRP)*. The bars depict the dependence of observed relative risk premia (RRP) on level of probability. $RRP = (EV - CE)/EV$, where $RRP > 0$ indicates risk aversion, $RRP < 0$ risk seeking and $RRP = 0$ risk neutrality



As far as intertemporal choices are concerned, people's average behavior exhibits decreasing discount rates, i.e. subjects discount less remote outcomes more strongly than more remote ones: The first column in Table 2 reveals that the discount rates imputed from subject's choices decline on average by 7 percentage points *per annum* when the time horizon is extended from two months to four months. The average data veil heterogeneity as well as the extent of decreasing discount rates, however. Whereas the majority of approximately 54% of all subjects exhibit decreasing discount rates over time, $\Delta\delta > 0$ (second column), about 29% exhibit constant discount rates (third column), and the residual group reveals increasing discount rates (fourth column). Average discount rates of subjects with decreasing discount rates amount to $\delta_2 = 47\%$ *p.a.* and $\delta_4 = 31\%$ *p.a.*, respectively, reflecting a much greater change than do the overall averages.[14]

3.2 Risk preference parameters

Whereas one of our central variables, change in discount rates $\Delta\delta$, is directly observable, the other one, departure from linear probability weighting, has to be estimated from our data on certainty equivalents.

Individual risk preference parameters η, α and β were estimated on the basis of the econometric model discussed in Section 2.1. As Table 2 reveals, the average values of the curvature parameter η of the utility function reflect slight concavity or linearity. The average subproportionality index α amounts to 0.505, indicating a pronounced departure from linear probability weighting in line with previous findings (Tversky and Kahneman 1992; Gonzalez and Wu 1999; Abdellaoui 2000). The average estimates for β lie in the vicinity of one, implying that the respective curves intersect the diagonal at about $p = 1/e$. [15]

[14]The distributions of the observed discount rates are shown in Appendix C.

[15]Histograms of the parameter distributions are included in Appendix D.

Table 2 Average discount rates and risk parameters

	All	Subjects with		
	100%	$\Delta\delta > 0$ 53.9%	$\Delta\delta = 0$ 29.2%	$\Delta\delta < 0$ 16.9%
δ_2	0.368	0.465	0.213	0.328
	(0.023)	(0.029)	(0.045)	(0.058)
δ_4	0.299	0.307	0.213	0.418
	(0.020)	(0.025)	(0.045)	(0.068)
$\Delta\delta$	0.070	0.157	0	−0.090
	(0.012)	(0.015)		(0.019)
η	0.873	0.808	0.948	0.953
	(0.032)	(0.046)	(0.074)	(0.072)
α	0.505	0.426	0.574	0.634
	(0.021)	(0.027)	(0.040)	(0.063)
β	0.974	0.936	1.064	0.940
	(0.026)	(0.036)	(0.068)	(0.045)
Observations	89	48	26	15

The table lists average observed discount rates δ_2 and δ_2 and estimated risk parameters η, α and β (standard errors in parentheses). δ_2 (δ_4) denotes the imputed discount rate for the two (four) month delay, and $\Delta\delta = \delta_2 - \delta_4$. η is the (power) utility parameter (see Eq. 2). α is an index for subproportionality of the Prelec probability weighting function and β an index for its convexity. 23 subjects were excluded from the descriptive statistics due to censoring

The overall picture revealed by our data is consistent with the typical empirical findings: On average, subjects systematically violate linear probability weighting and constant-rate discounting. But the central question, namely whether the degree of subproportionality of probability weighting is associated with hyperbolicity of discounting behavior at the level of the *individual* has yet to be answered.

3.3 Relationship between probability weights and hyperbolic discounting

A first indication of a systematic relationship between probability weighting and discounting can be found in Table 2. The average estimated subproportionality index α varies substantially across discounting types and exhibits a systematic pattern: α is lowest for the group with decreasing discount rates and highest for the group with increasing discount rates.

This finding is confirmed by the estimates of the regression models. Table 3 displays the results derived by OLS as well as by the censored regression method. Inspection of the coefficients indicates that censoring seems not to be an important problem: After omitting the 23 censored observations, OLS yields coefficients very close to the estimates of the censored regression model. Furthermore, both specifications (Models 1 and 2) lead to the same conclusion: Subproportionality of probability weighting is significantly associated with decreasing discount rates $\Delta\delta$. Table 3 shows that, in line with our conjecture, the estimated coefficient of α is negative, amounting to approximately −0.2.

Table 3 Regression results

Dependent variable: $\Delta\delta$ ($\Delta\delta^*$)	OLS		Censored	
	Model 1	Model 2	Model 1	Model 2
Intercept	0.226***	0.279	0.247***	0.321
	(0.063)	(0.228)	(0.057)	(0.225)
η	0.018	0.002	−0.006	−0.022
	(0.042)	(0.043)	(0.039)	(0.041)
α	−0.205***	−0.220***	−0.185***	−0.203***
	(0.066)	(0.074)	(0.062)	(0.075)
β	−0.070	−0.040	−0.074	−0.045
	(0.067)	(0.068)	(0.060)	(0.063)
Female		−0.012		−0.011
		(0.031)		(0.032)
Age		−0.001		−0.002
		(0.007)		(0.007)
Log-Income		−0.013		−0.012
		(0.024)		(0.023)
Experience		0.015		0.020
		(0.032)		(0.033)
CRT		0.021		0.021
		(0.017)		(0.017)
$\hat{\sigma}$	0.123	0.124	0.084	0.082
R^2 or (LogLik)	0.137	0.170	(48.693)	(51.123)
Observations	89	89	112	112
Parameters	4	9	9	19

The table presents estimation results for the ordinary least square model (OLS; censored observations were omitted) and the censored regression model. It shows whether and to what extent risk preference parameters η, α, β and personal characteristics explain the difference in imputed discount rates $\Delta\delta = \delta_2 - \delta_4$. CRT denotes the test score for the Cognitive Reflection Test (Frederick 2005), a three-item test measuring specific aspects of cognitive ability. Standard errors (in parentheses) are obtained by the bootstrap method with 10,000 replications. Bootstrapping accounts for the fact that the regressors α, β and η are estimated quantities

All the respective estimates are significant at the 1%-level and remain robust to inclusion of additional controls. The effect is not only highly significant, it is also quite substantial: A decrease in the subproportionality index α by 0.1 is associated with an increase in $\Delta\delta$ by 2 percentage points *per annum*. In particular, the decline in discount rates is only related to the degree of subproportionality, but not to the index of convexity β. We obtained the same order of magnitude for the coefficient of α when we restricted β to be equal to one. Regression coefficients and their standard errors also remain stable when either η or β are deleted from the list of regressors.[16]

[16]Moreover, it can be shown that estimates are totally robust to alternative parameterizations of the probability weighting curve as well. Results are available upon request.

The coefficients of the utility parameter η are not statistically different from zero, either.[17] This result is consistent with our hypothesis that utility curvature *per se* does not impact the extent of decreasing discount rates. Furthermore, none of the other individual characteristics show a significant effect.[18]

An F-test comparing the OLS Model 1 with Model 2 renders a p-value of 0.670, favoring the more parsimonious Model 1, as the controls do not substantially contribute to explaining the variance in $\Delta\delta$.[19] Furthermore, the regression models explain a rather large fraction of total variance: Model 2, for instance, yields an R-squared value of 17%.[20] These findings present conclusive evidence that comparatively more subproportional probability weighting is associated with a stronger decline in discount rates.

4 Discussion

The strong and significant correlation between subproportionality of probability weighting and extent of hyperbolic discounting begs the question of whether this relationship can be explained in causal terms. In principle, there are three pathways through which correlation could be generated. First, the tendency towards hyperbolic discounting could cause distortions in probability weights. Since, in experiments, estimates of probability weights are generally based on atemporal choices, i.e. when there is practically no time delay between choice and payment, this possibility can be effectively ruled out. Second, the direction of causality could work the other way round, with proneness to probability distortions inducing hyperbolic discounting. Finally, there could be a third factor driving both types of departures from the standard model predictions. We will discuss the latter possibility first and then turn to the second alternative.

Since the common ratio effect and the common difference effect pertain to diminishing sensitivity towards probability and delay, respectively, similar cognitive processes may govern the evaluation of risky and delayed outcomes. A natural candidate for a common factor driving both processes is cognitive abilities. Several papers have looked into the relationship between cognitive abilities and risk tolerance on the one hand, and between cognitive abilities and patience on the other hand (Frederick 2005; Benjamin et al. 2006; Dohmen et al. 2007). Generally, they conclude that better cognitive abilities tend to be associated with higher risk tolerance as well as higher patience. For

[17]Nor does an interaction term $\alpha \times \eta$ contribute to explaining variation in $\Delta\delta$.

[18]While not significantly different from zero, coefficients exhibit the expected signs: Females have a slightly more subproportional weighting curve, consistent with previous findings (Fehr-Duda et al. 2006). Both experience with investment decisions and high *CRT* scores are associated with smaller departures from linearity.

[19]The same is the case when the two censored models are compared. A likelihood ratio test of Model 2 against Model 1 renders a p-value of 0.9.

[20]When regressing $\Delta\delta$ exclusively on the socioeconomic variables, R-squared amounts to 3.9%.

instance, Frederick (2005) finds that students with high *Cognitive Reflection Test* scores gamble significantly more often that do the low *CRT* group, and exhibit lower imputed discount rates, albeit not for choices involving longer time horizons. These previous findings seem to conflict with the insignificant coefficient of *CRT* in Table 3. Since *CRT* is not correlated with α,[21] the lack of correlation between *CRT* and $\Delta\delta$ indeed suggests that *CRT* scores cannot explain the variance in the hyperbolicity of discount rates. However, we are concerned with sensitivities towards changes in probability and delay and not with measures of average risk aversion and impatience, the focus of previous research. Of course, there could be other factors than cognitive abilities, or aspects of cognitive ability not captured by *CRT* scores, that drive the correlation between subproportionality and hyperbolic discounting. Clearly, this possibility cannot be ruled out and needs further exploration.

Finally, we discuss the last one of our options, direct impact of subproportionality on hyperbolicity of discounting. Many authors have noted before that "[a]nything that is delayed is almost by definition uncertain" (Prelec and Loewenstein 1991, p.784). For instance, a promised reward may, due to unforeseen circumstances, materialize later or turn out to be smaller than expected, or death may keep the decision maker from collecting her reward at all. For these reasons, future consequences are inextricably associated with uncertainty, implying that the decision maker's valuation of temporal prospects not only depends on her *pure* time preference, i.e. her preference for immediate utility over delayed utility, but also on her perception of uncertainty and, consequently, on her risk preferences. In other words, uncertainty drives a wedge between pure time preferences and time discounting.

If this account is an accurate description of intertemporal choice it has far reaching implications for observed discounting behavior, the most obvious one being that behaviorally revealed discount rates will be higher than the rate of *pure* time preference as they include a risk premium. Not surprisingly then, uncertainty has been identified to be an important confound in the measurement of time preferences, which may, at least partly, explain the notoriously high discount rates found in empirical studies (Frederick et al. 2002). The story does not stop here, however. If risk preferences influence time discounting, then people's proneness to probability weighting has to be taken into account as well. Recent contributions have examined the impact of nonlinear probability weighting on discounting behavior theoretically (Halevy 2008; Saito 2011). Halevy, motivated by the interaction effects between time and risk found by Keren and Roelofsma (1995) and Weber and Chapman (2005), is concerned with convex probability transformations that can accommodate the certainty effect inherent in the classical Allais paradox. We focus on the more general case of common ratio violations which can be modeled by subproportional probability weights. Subproportionality is not confined to convex functions but may also be present in inverse S-shaped probability transformations, which

[21] The Pearson correlation coefficient amounts to −0.0049 (p-value 0.964).

organize a large part of the empirical evidence. In the following, we show that the degree of subproportionality of probability weights indeed predicts the extent of decreasing discount rates. Furthermore, we derive comparative static results with respect to degree of uncertainty and exemplify the model predictions by graphical illustrations.

4.1 A model of discounting: the warped lens

If the future is perceived as uncertain, an allegedly guaranteed delayed outcome $\mathcal{T} = (x, t)$ is effectively evaluated as a risky prospect. Suppose that any future payment is perceived to materialize with a constant per-period probability of contract survival s, $0 < s \leq 1$. Consequently, $\mathcal{T}$ is evaluated as $\mathcal{L} = (x, s^t)$, rendering x with probability s^t and zero otherwise.

As far as the rate of pure time preference is concerned, we adopt the conventional assumption: the rate of pure time preference is characterized by a constant per-period rate $r \geq 0$, resulting in a pure time discount factor ρ equal to e^{-r}.

These assumptions imply that the present equivalent PE of the future payment x, such that the decision maker is indifferent between PE and x, is defined by

$$u(PE) = w(s^t)\rho^t u(x). \tag{6}$$

The *effective* discount weight $D(t)$ at delay t equals the weight attached to $u(x)$, i.e.

$$D(t) = w(s^t)\rho^t, \tag{7}$$

which depends not only on the *pure* rate of time preference r, but also on the probability of contract survival s as well as on the shape of the probability weighting function w. Clearly, if w is linear, $D(t)$ declines exponentially irrespective of the magnitude of s. If $0 < s < 1$, the resulting discount weight is lower than for $s = 1$, implying that uncertainty *per se* increases the absolute level of discount rates, but cannot account for discount rates declining over time. In fact, due to uncertainty, discounting would be observed even for a zero rate of pure time preference. If, however, w is nonlinear and $0 < s < 1$, the component $w(s^t)$ distorts the discount weight: As shown formally in Appendix A.1, subproportionality of w generates hyperbolicity of $w(s^t)$ in t and, consequently, decreasing discount rates if the future is perceived as uncertain. Metaphorically speaking, the decision maker, when looking into the future, perceives delayed events through the warped lens of probability distortions. A natural extension of this insight is that higher degrees of subproportionality induce more strongly declining discount rates (see Appendix A.2). The effective discount weight $D(t)$ also depends on the level of uncertainty s. Higher uncertainty implies more strongly declining discount rates as well. A formal proof appears in Appendix A.3.

In order to illustrate the predictions of our model, which hold for any subproportional probability weighting function, we demonstrate the comparative static effects of subproportionality α and uncertainty s graphically, using Prelec's specification. Figure 4 shows the comparative static effects of varying α, Fig. 5 is dedicated to the effects of varying s.

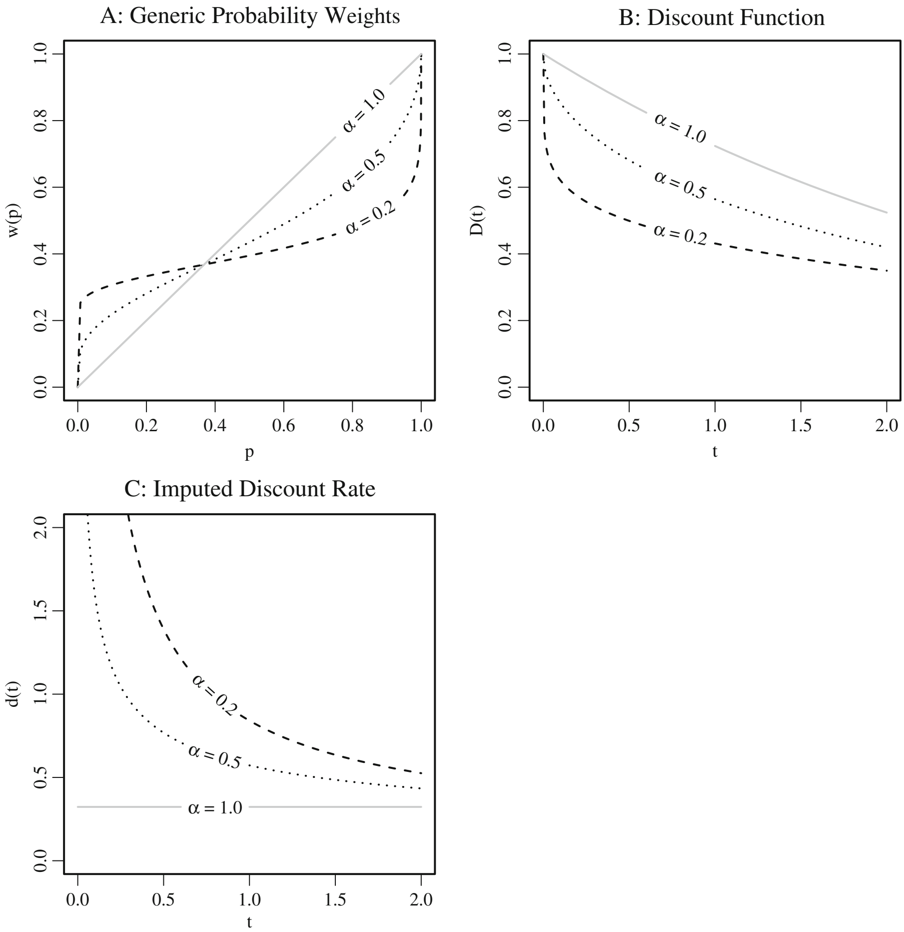


Fig. 4 Effects of subproportionality on discounting. The three panels illustrate the effect of varying degrees of subproportionality α. Panel **a** shows Prelec probability weighting curves for different values of $\alpha \in \{0.2, 0.5, 1\}$: The lower is α the greater is subproportionality and the greater is the departure from linearity. Panel **b** shows, for each of the three cases of probability weighting, the effective discount function resulting from the model in Eq. 6: exponential discounting for $\alpha = 1.0$, hyperbolic discounting otherwise. The lower is α, the more pronounced is hyperbolicity. Panel **c** displays discount rates d_t inferred from $D(t) = e^{-d_t t}$: constant discounting for $\alpha = 1.0$, decreasing rates otherwise. The less subproportional are probability weights, the more quickly discount rates converge to the base constant rate. Note that the pure rate of time preference $r = 0.1$ and the probability of contract survival $s = 0.8$ for all three panels

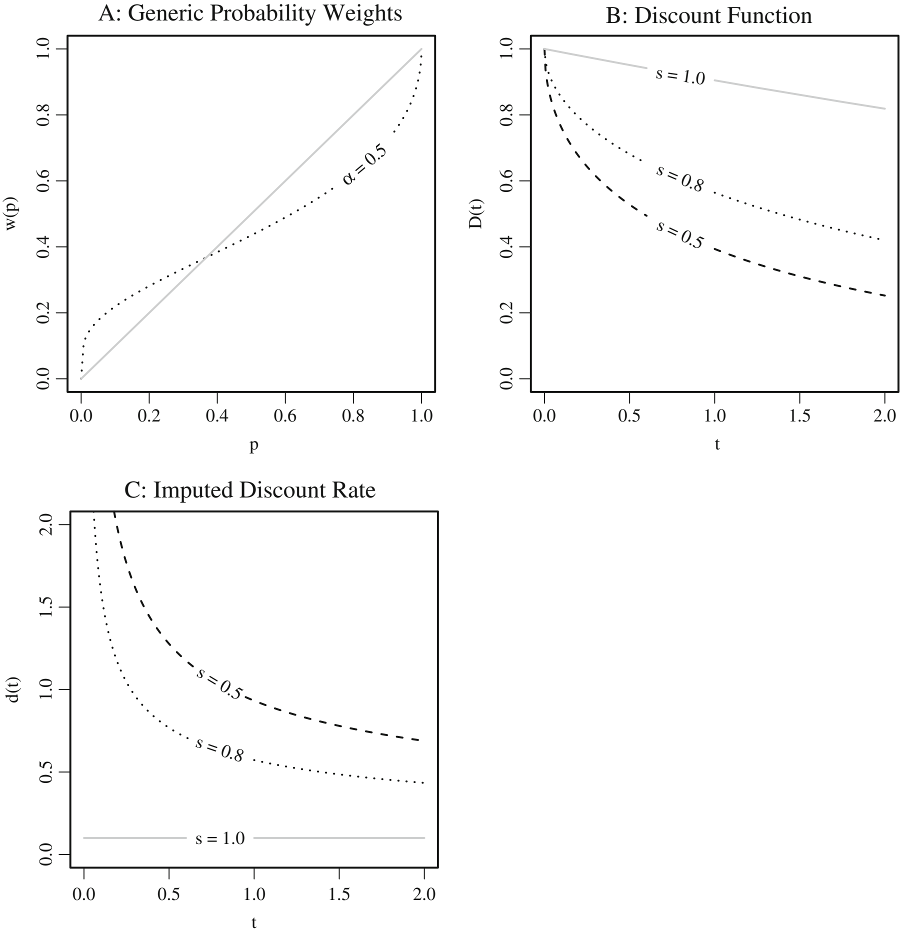


Fig. 5 Effects of inherent uncertainty on discounting. Panel **a** depicts probability weights for $\alpha = 0.5$. For these probability distortions, Panel **b** shows the effect of varying probabilities of contract survival s on the discount function defined in Eq. 6. The higher inherent uncertainty is (the lower is s), the more strongly the discount function declines in time horizon. Panel **c** shows imputed discount rates for the discount functions presented in Panel **b**. For $s = 1$, discount rates are constant and equal to the rate of pure time preference r. The greater inherent uncertainty is, the more excessive is discounting. r is held fixed at 0.1 in all three panels

Panel A of Fig. 4 depicts the probability weighting curves for three distinct parameter values of α, with $\beta = 1$: a medium-sized departure from linearity ($\alpha = 0.5$), as exhibited on average by our experimental subjects, a strong departure from linearity ($\alpha = 0.2$), as well as the limiting case of linear probability weighting ($\alpha = 1$). Panel B of Fig. 4 shows, for each of the three cases of probability weighting, the effective discount weights resulting from Eq. 7 as they evolve over time.[22] For a decision maker with linear probability weighting the discount function, represented by the solid gray curve, is exponential. In contrast, the dotted discount function of a typical decision maker with $\alpha = 0.5$ departs from exponentiality. By comparison, the decision maker characterized by the most strongly S-shaped probability weighting curve underweights (overweights) large (small) probabilities more strongly than does the decision maker with $\alpha = 0.5$, which leads to an even more pronounced departure from exponential discounting (dashed curve).

Finally, Panel *C* of Fig. 4 displays the imputed discount rates d_t inferred from $D(t) = e^{-d_t t}$. The solid gray line corresponds to linear probability weighting. Since this decision maker is not prone to probability distortions, her discount rate is independent of time delay and, consequently, constant over time. In contrast to this decision maker, the discount rates of the decision makers with nonlinear probability weights start out at very high levels and then decline sharply. As is evident from comparing the dashed curve with the dotted one, the more subproportional probability weighting function generates a larger decline in discount rates between period 2 and period 1, i.e. the difference $d_2 - d_1$ is greater for higher degrees of subproportionality α. For this prediction to hold the probability of contract survival s needs to be smaller than one. Since people vary in their perceptions of uncertainty our framework predicts a correlation between subproportionality and decreasing discount rates. This is exactly what we find in our data.

Another important insight from our approach concerns the direct impact of uncertainty on discounting behavior. Hyperbolicity of discount rates is crucially influenced by people's perceptions of uncertainty: Increasing uncertainty not only raises the level of discount rates but also exacerbates revealed short-term impatience. Figure 5 illustrates this effect for $\alpha = 0.5$, the average index of subproportionality in our data, and $r = 0.1$. When the survival probability s declines from 0.8 to 0.5, the resulting discount function departs more strongly from exponentiality as Panel *B* shows. Hence, the decrease in discount rates associated with higher uncertainty is more pronounced as well (Panel *C*).

All the effects described so far operate solely via probability weighting, with inherent uncertainty providing the link between the domains of time and risk, while the utility of money is measured on the same scale in both decision domains. What the model cannot handle is the *magnitude effect*, another anomaly in discounting behavior: Typically, large payoffs are discounted less heavily than small payoffs (Thaler 1981; Benzion et al. 1989). In our model, this effect would have to manifest itself in the characteristics of the utility function. However, risk aversion has been found to increase with payoff magnitude (Kachelmeier and Shehata 1992; Holt and Laury 2002), which implies higher, rather than lower, discounting of large payoffs. Prelec and Loewenstein (1991), confronted with the same problem of incompatibility when attempting to

[22]For illustrative purposes, in Fig. 4 r is fixed at 0.1 and s is assumed to be 0.8, which means that 80% of all contracts are anticipated to survive at least one period.

integrate time and risk in a single model, speculate that this discrepancy is caused by interaction effects that are different for risky and intertemporal choice: Magnifying a risky payoff renders the probability dimension more important because, due to fear of disappointment, the decision maker's concern with obtaining the more desirable outcome increases. Evidence for such an interaction effect has been recently provided by Fehr-Duda et al. (2010) who show that probability weights decrease with stake size, i.e. large-payoff gambles are evaluated comparatively more pessimistically. On the other hand, magnifying a delayed payoff presumably does not have a strong impact on the importance of time delay or even increases utility due to savoring. In any case, the existence of interaction effects challenges the assumption of separability and calls for systematic future research.

4.2 Perceived uncertainty and the pure rate of time preference

The model presented in the previous section provides a theoretical underpinning of our empirical finding that the degree of subproportionality α predicts the extent of hyperbolic discounting $\Delta\delta$. Our theoretical framework implies such a relationship *ceteris paribus*, holding constant the other model parameters, specifically the subjective probability of contract survival s and the pure rate of time preference r, both of which are not observable. In our experimental setting with decisions over a short time horizon, the subjective probability of contract survival s should lie very close to unity since mortality risk is very low in our age group of subjects and we took great care to communicate experimenter reliability. One way of checking the plausibility of the theoretical model is to investigate whether, on average, actual choices are indeed consistent with this conjecture, i.e. whether values of s implied by our data lie in the vicinity of one *for a wide range of plausible values of the pure rate of time preference* r.

For this purpose, we examine the combinations of s and r that are consistent with the observed average intertemporal tradeoffs between present equivalents PE and delayed payments x. We solve for all feasible combinations of $\hat{s}$ and $\hat{r}$ that are compatible with the observed choices by inserting the estimates for subjects' average behavioral parameters η, α and β into Eq. 6. As is clear from Eq. 6, a higher probability of contract survival needs to be compensated by a higher pure rate of time preference, *ceteris paribus*, to keep individuals indifferent between more immediate and more remote rewards.

As Fig. 6 shows, the feasible $(\hat{s}, \hat{r})$-combinations indeed exhibit a rising profile, with $\hat{s}$ starting out at below 97% *p.a.* and converging to 100% *p.a.*, when the pure rate of time preference increases from 0% to 15% *p.a.* and beyond. For instance, $s = 99\%$ is compatible with $r \simeq 8.5\%$ *p.a.* What this exercise shows is that the data, interpreted within our framework, is consistent with a very high subjective probability that contracts survive at least one year, in accordance with our conjecture. Furthermore, accounting for inherent uncertainty implies rates of pure time preference in a plausible range lying considerably below the observed average discount rates of more than 30% *p.a.*

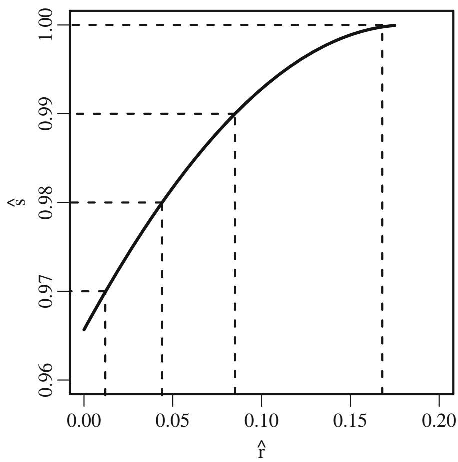


Fig. 6 Feasible ($\hat{s}$, $\hat{r}$)-combinations. The solid line represents combinations of $\hat{s}$ (probability of contract survival) and $\hat{r}$ (rate of pure time preference) that are feasible with our data and model estimates when interpreted within the framework of Eq. 6. It suggests that observed discounting behavior is compatible with quite reasonable values for the probability of contract survival and the rate of pure time preference

This suggests that even allegedly guaranteed future outcomes are viewed as slightly uncertain, in line with direct questionnaire evidence provided in Patak and Reynolds (2007). The authors asked respondents to rate their certainties for the same rewards, delayed by 1, 2, 30, 180, and 365 days, respectively, which they had encountered during a preceding choice experiment. The respondents reported ratings that clearly decline with the length of delay. Moreover, using a similar method, Takahashi et al. (2007) found that such subjective probabilities of obtaining delayed rewards decay in a hyperbolic-like manner, consistent with probability weights $w(s^t)$ declining hyperbolically with delay t.

5 Conclusion

For several decades, decision research has been dominated by the quest for better descriptive theories of behavior under risk and over time, triggered by a large body of experimental evidence challenging the classical models of choice, expected utility theory and discounted utility theory. Alternative models, accounting for nonlinear probability weighting and hyperbolic discounting, describe behavior much more accurately than do the classical models, at least at the aggregate level. In this paper we address the question of whether the better fit of the generalized models is actually a consequence of the same subjects' anomalous behaviors. We present the first evidence that more pronounced systematic departures from linear probability weighting are indeed associated with more strongly declining discount rates at the level of the individual decision makers. This result is robust to inclusion of additional controls as well as model specification. In fact, the only variable explaining a substantial fraction of heterogeneity in individual discounting patterns turns out to be the degree of subproportionality of probability weights.

Several authors have proposed that the existence of matching violations of the classical axioms is not coincidental, but rather reflects the close relationship between risk and delay (e.g. Prelec and Loewenstein 1991). Some researchers have even argued that the two attributes are virtually the same, but there is no consensus as to which one is the more fundamental of the two. We favor the view that, if there is a hierarchical relationship between them at all, risk is the more likely candidate. To bolster this view, we provide a theoretical model predicting the observed link between probability distortions and decreasing discount rates. For hyperbolic discounting to emerge two factors need to interact: probability distortions and future uncertainty.

Arguably, the future is uncertain by definition. Uncertainty may stem from different sources, either tied to the individual herself, such as lifetime expectancy, or to environmental factors. Lack of contract enforcement and weak property rights, for instance, may make people skeptical that promises will be actually kept. Therefore, institutionally generated uncertainties may induce extreme short-run impatience even if people's pure rate of time preference is low and constant. This insight is important because it implies that revealed behavior may be predominantly driven by environmental factors rather than by the underlying preferences themselves and, consequently, may be amenable to economic policy.

The channel through which uncertainty generates hyperbolic discounting is nonlinear probability weighting, a robust regularity of risk taking behavior. If probability weighting plays such an important role in risk taking and discounting behavior, the obvious question concerning the source of these probability distortions arises. Unfortunately, little is known empirically about the forces driving probability distortions. A number of theoretical contributions have invoked emotions to explain probability weighting (Wu 1999; Caplin and Leahy 2001). Walther (2003, 2010), for instance, rationalizes nonlinear probability weighting by generalizing expected utility theory: He assumes that, in addition to monetary outcomes, the decision maker cares about emotions triggered by the resolution of uncertainty. His approach predicts that, if the decision maker anticipates experiencing elation or disappointment when the actual outcome lies above or below some normal level, she will distort outcome probabilities according to an S-shaped pattern. The more emotional a person expects to be, the stronger will be her departure from linear probability weighting and, consequently, the more pronounced hyperbolic discounting will be. Preliminary evidence for the role of affect in probability weighting is provided by Rottenstreich and Hsee (2001) who show that probability weights depart more strongly from linearity for emotion-laden targets, such as a kiss from one's favorite movie star, than for comparatively pallid monetary outcomes. A systematic test of the affect hypothesis is still lacking, and we have to leave it to future research to investigate whether anticipated emotions or some other factors are the primary drivers of probability weighting and whether, indeed, there is a causal link from probability weighting to hyperbolic discounting.

Acknowledgements We thank Ernst Fehr, Drew Fudenberg, David Laibson, David Levine, Drazen Prelec, Peter Wakker and Herbert Walther for helpful comments. The usual disclaimer applies. This research is supported by the Swiss National Science Foundation (Grants 100012-109907 and 100014-120705). An earlier version of this paper, "Uncertainty Breeds Decreasing Impatience: The Role of Risk Preferences in Time Discounting", is available at http://www.iew.uzh.ch/wp/iewwp412.pdf.

Appendix A: Formal proofs

A.1 Hyperbolicity

In the framework proposed here, the discount weight $D(t)$ equals

$$D(t) = w(s^t)\rho^t, \tag{8}$$

with ρ defined as e^{-r}. In order to establish that subproportional probability weights are sufficient[23] for discount rates to decrease, we define decreasing impatience at t in the following way (Prelec 2004): Let (x, t) be a temporal prospect paying off x at t with certainty. A preference relation $\succeq$ exhibits *decreasing impatience* if for any $t > 0$, $0 < x < y$, $(x, v) \sim (y, z)$ implies $(y, z + t) \succeq (x, v + t)$.

According to our framework the temporal prospects $(x, 0) \sim (y, 1)$ are evaluated as $u(x)w(s^0)\rho^0 = u(y)w(s^1)\rho^1$. As subproportionality of w implies that $w(s) < w(s^{t+1})/w(s^t)$, deferring the prospects by t periods renders

$$1 = \frac{u(y)w(s)\rho}{u(x)} < \frac{u(y)w(s^{t+1})\rho^{t+1}}{u(x)w(s^t)\rho^t} \tag{9}$$

and, therefore, $(y, t+1) \succ (x, t)$, meeting the requirement for decreasing impatience if $s < 1$. ■

In the intertemporal tradeoff between the present and the subsequent period the discount weight equals $w(s)\rho$. At time t, $u(x)$ is discounted by $w(s^t)\rho^t$. Compounding by the initial one-period discount weight $w(s)\rho$ would render $w(s)w(s^t)\rho^{t+1}$ at $t+1$, but the discount weight effectively amounts to $w(s^{t+1})\rho^{t+1}$ then. Therefore, $w(s^{t+1})/(w(s)w(s^t))$, the wedge between the relative discount weights $D(0)/D(1)$ and $D(t)/D(t+1)$, provides a measure for the extent of departure from stationarity at t.

[23] Note that subproportionality, aside from $s < 1$, is also necessary (Saito 2011).

A.2 Comparative hyperbolicity

The previous proof shows that, provided that $s < 1$, subproportionality of w engenders hyperbolic discounting. As will become clear shortly, a decision maker with a comparatively more subproportional probability weighting function will also tend to exhibit more strongly decreasing discount rates:

A preference relation $\succeq_2$ exhibits *more strongly decreasing impatience* than $\succeq_1$ if for any intervals $0 \leq v < z, t, \Delta t$ and outcomes $0 < x < y$, $0 < x' < y'$, $(x, v) \sim_1 (y, z)$, $(x, v + t) \sim_1 (y, z + t + \Delta t)$, and $(x', v) \sim_2 (y', z)$ imply $(x', v + t) \preceq_2 (y', z + t + \Delta t)$ (Prelec 2004).

In order to examine the effect of the degree of subproportionality on hyperbolicity suppose that the probability weighting function w_2 is comparatively more subproportional than w_1, as defined in Prelec (1998), and that the following indifference relations hold for two decision makers 1 and 2 at periods $v = 0$ and $z = 1$:

$$u_1(x) = u_1(y)w_1(s)\rho \text{ for } 0 < x < y,$$

$$u_2(x') = u_2(y')w_2(s)\rho \text{ for } 0 < x' < y'.$$

Due to subproportionality, the following relation holds for decision maker 1 in period t:

$$1 = \frac{u_1(y)w_1(s)\rho}{u_1(x)} < \frac{u_1(y)w_1(s^{t+1})\rho^{t+1}}{u_1(x)w_1(s^t)\rho^t}. \tag{10}$$

Therefore, the subjective probability of contract survival has to be reduced by compounding s over an additional time period Δt to re-establish indifference:

$$u_1(x)w_1(s^t)\rho^t = u_1(y)w_1(s^{t+1+\Delta t})\rho^{t+1}. \tag{11}$$

It follows from the definition of comparative subproportionality that this adjustment of the survival probability by Δt is not sufficient to re-establish indifference with respect to w_2, i.e.

$$u_2(x')w_2(s^t)\rho^t < u_2(y')w_2(s^{t+1+\Delta t})\rho^{t+1}. \tag{12}$$

Therefore, $(x', t) \prec (y', t + 1 + \Delta t)$. ∎

A.3 Uncertainty and hyperbolicity

In order to derive the effect of increasing uncertainty on hyperbolicity we examine the sensitivity of the departure from stationarity at t, measured by $w(s^{t+1})/(w(s)w(s^t))$, with respect to changing s:

$$\begin{aligned}
&\frac{\partial}{\partial s}\left[\frac{w(s^{t+1})}{w(s)w(s^t)}\right] \\
&\quad = \frac{1}{[w(s)w(s^t)]^2}\big[(1+t)s^t w(s)w(s^t)w'(s^{t+1}) \\
&\qquad\qquad - ts^{t-1}w(s)w(s^{t+1})w'(s^t) - w(s^t)w(s^{t+1})w'(s)\big] \\
&\quad = \frac{1}{s[w(s)w(s^t)]^2}\big[(1+t)s^{t+1} w(s)w(s^t)w'(s^{t+1}) \\
&\qquad\qquad - ts^{t}w(s)w(s^{t+1})w'(s^t) - sw(s^t)w(s^{t+1})w'(s)\big] \\
&\quad = \frac{w(s^{t+1})}{sw(s)w(s^t)}\left[\frac{(1+t)s^{t+1}w'(s^{t+1})}{w(s^{t+1})} - \frac{ts^t w'(s^t)}{w(s^t)} - \frac{sw'(s)}{w(s)}\right] \\
&\quad = \frac{w(s^{t+1})}{sw(s)w(s^t)}\left[(1+t)\varepsilon(s^{t+1}) - t\varepsilon(s^t) - \varepsilon(s)\right] \\
&\quad < 0
\end{aligned}$$

with $\varepsilon(p) = pw'(p)/w(p)$ defined as the elasticity of the probability weighting function w. According to Segal (1987), p. 148, subproportionality holds iff $\varepsilon(p)$ is increasing. As $s^{t+1} < s^t < s$, $\varepsilon(s^{t+1}) < \varepsilon(s^t) < \varepsilon(s)$ and, hence, the sum of the elasticities in the final line of the derivation is negative. Therefore, increasing uncertainty, i.e. decreasing s, entails a greater departure from stationarity and, consequently, a higher degree of hyperbolicity. ■

A.4 Effect of concavity

In the course of the experiment we cannot observe discount weights at delay t, $D(t)$, directly but infer $\tilde{D}(t)$ from the intertemporal tradeoffs between payments at different dates, i.e. $PE = \tilde{D}(t)x_t$. According to our assumptions, utility is modeled by a power function $u(x) = x^\eta$, $\eta > 0$, which renders $\tilde{D}(t) = D(t)^{\frac{1}{\eta}}$. It follows that

$$\frac{\tilde{D}(0)/\tilde{D}(1)}{\tilde{D}(t)/\tilde{D}(t+1)} = \left(\frac{D(0)/D(1)}{D(t)/D(t+1)}\right)^{\frac{1}{\eta}} \qquad (13)$$

and therefore the observed decrease in discount rates resulting from nonlinear probability weighting gets amplified by $\eta < 1$ and, hence, concavity has to be controlled for in the regression model.

Appendix B: Censored regression model

This appendix discusses the way we model the difference in the censored observed discount rates, $\Delta\delta = \delta_2 - \delta_4$, and link it to individual risk preferences.

To relate time discounting to risk preferences, the model assumes the following linear relationship between the discount rate $\delta^*_{t,i}$ of individual $i \in \{1, \ldots, N\}$ over delay $t \in$ {two months, four months} and a vector of regressors c_i, containing a constant, the parameters of risk preferences, η_i, α_i and β_i, as well as some socioeconomic characteristics:

$$\delta^*_{t,i} = c_i \gamma_t + e_{t,i}, \tag{14}$$

where γ_t denotes a vector of slope parameters and $e_{t,i}$ stands for a normally distributed error term with mean zero and variance $\frac{1}{2}\sigma^2$. Under the assumption of nonnegative discounting, the choice menu, depicted in Fig. 2, allows us to directly elicit individual discount rates that lie between 0 and 92.5%. However, if individual i always opts for being paid out at the earlier point in time (*Option A*), we do not necessarily observe her true discount rate $\delta^*_{t,i}$ as we only know that it amounts to at least 95%. Thus, the elicited discount rates, $\delta_{2,i}$ and/or $\delta_{4,i}$, are censored from above at $b = 0.95$. In the data we observe

$$\delta_{t,i} = \begin{cases} \delta^*_{t,i} \text{ if } \delta^*_{t,i} < b\,, \\ b \quad \text{otherwise.} \end{cases} \tag{15}$$

This immediately yields the difference in the discount rates over two and four months,

$$\Delta\delta^*_i = c_i \underbrace{(\gamma_2 - \gamma_4)}_{\Delta\gamma} + \underbrace{e_{2,i} - e_{4,i}}_{\Delta e_i}\,, \tag{16}$$

where Δe_i is normally distributed with mean zero and variance σ^2. Consequently, this difference $\Delta\delta^*_i$ is affected by censoring, too, and only observed if both $\delta_{2,i} < b$ and $\delta_{4,i} < b$.

In order to avoid biased estimators for γ_2, γ_4, and σ, the model needs to take the censored nature of the data into account. Therefore, its log likelihood takes on the following form:

$$\ln L\left(\gamma_2, \gamma_4, \sigma; c, \delta_2, \delta_4\right) = \sum_{i:\,\delta_{2,i}=b\,,\,\delta_{4,i}=b} P\left(\delta_{2,i} = b\,,\ \delta_{4,i} = b \mid c, \delta_2, \delta_4\right)$$

$$+ \sum_{i:\,\delta_{2,i}<b\,,\,\delta_{4,i}=b} P\left(\delta_{2,i} < b\,,\ \delta_{4,i} = b \mid c, \delta_2, \delta_4\right)$$

$$+ \sum_{i:\,\delta_{2,i}=b\,,\,\delta_{4,i}<b} P\left(\delta_{2,i} = b\,,\ \delta_{4,i} < b \mid c, \delta_2, \delta_4\right)$$

$$+ \sum_{i:\,\delta_{2,i}<b\,,\,\delta_{4,i}<b} \frac{1}{\sigma}\phi\left(\frac{\Delta\delta_i - c_i\left(\gamma_2 - \gamma_4\right)}{\sigma}\right), \qquad (17)$$

where ϕ represents the standard normal distribution's density and the probabilities P, accounting for the different ways by which an observation may be censored, are given by

$$P\left(\delta_{2,i} = b\,,\ \delta_{4,i} = b \mid c, \delta_2, \delta_4\right)$$

$$= \left[1 - \Phi\left(\frac{b - c_i\gamma_2}{\sigma}\right)\right]\left[1 - \Phi\left(\frac{b - c_i\gamma_4}{\sigma}\right)\right],$$

$$P\left(\delta_{2,i} < b\,,\ \delta_{4,i} = b \mid c, \delta_2, \delta_4\right)$$

$$= \Phi\left(\frac{b - c_i\gamma_2}{\sigma}\right)\left[1 - \Phi\left(\frac{b - c_i\gamma_4}{\sigma}\right)\right],$$

$$P\left(\delta_{2,i} = b\,,\ \delta_{4,i} < b \mid c, \delta_2, \delta_4\right)$$

$$= \left[1 - \Phi\left(\frac{b - c_i\gamma_2}{\sigma}\right)\right]\Phi\left(\frac{b - c_i\gamma_4}{\sigma}\right),$$

with Φ denoting the cumulative density function of the standard normal distribution. Numerical maximization of $\ln L\left(\gamma_2, \gamma_4, \sigma; c, \delta_2, \delta_4\right)$ yields the maximum likelihood estimates of $\hat{\gamma}_2$, $\hat{\gamma}_4$, and $\hat{\sigma}$. To obtain the maximum likelihood estimate of $\Delta\hat{\gamma}$ we utilize the invariance property.

Appendix C: Observed discount rates

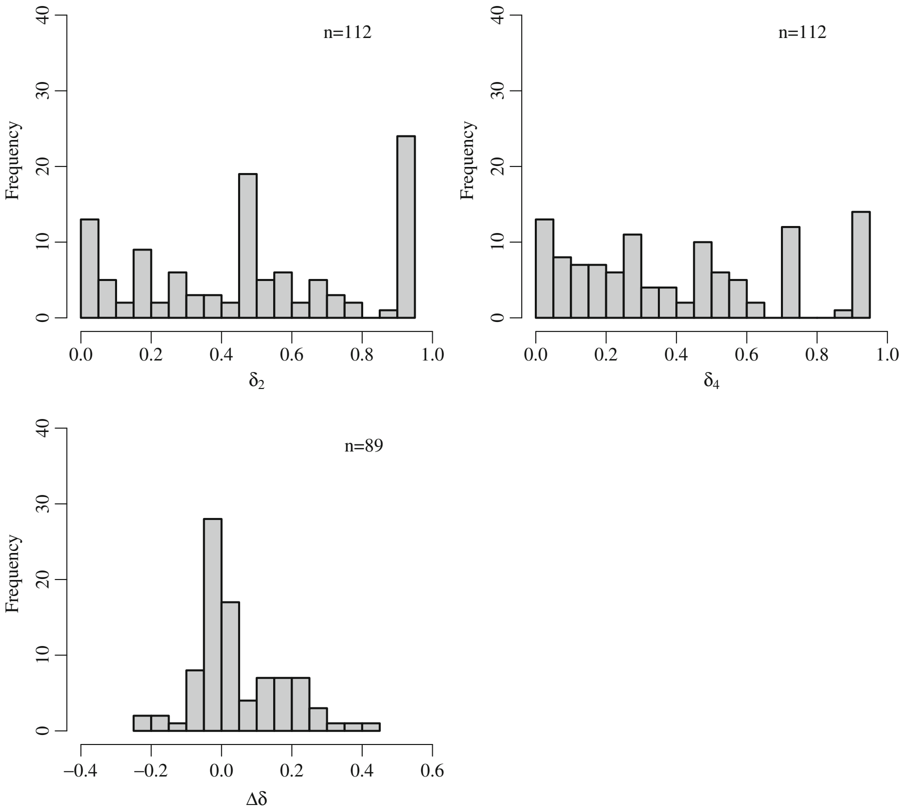


Fig. 7 Distributions of discount rates δ_2 and δ_4 and their differences

Appendix D: Estimated risk parameters

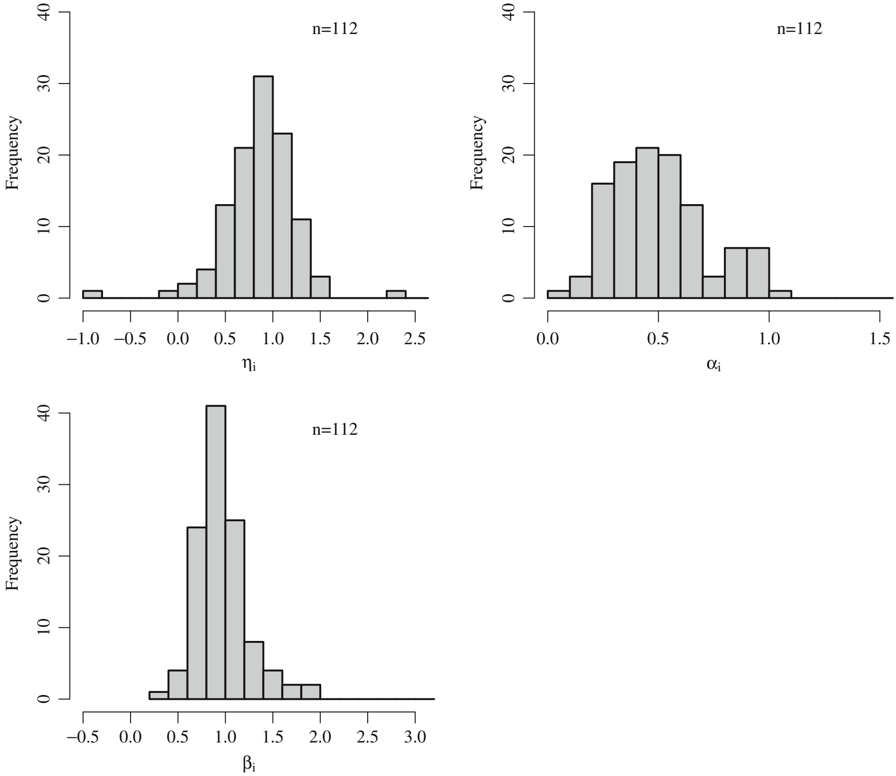


Fig. 8 Distributions of η, α and β

Appendix E: Controls

Table 4 Summary statistics ($n = 112$)

	Type	Mean	Std.Err.
Female	Binary	0.446	0.047
Age	Numeric	22.625	0.209
Log-income	Numeric	6.380	0.067
Experience	Binary	0.304	0.044
CRT	Numeric	2.214	0.082

CRT is the test score for the Cognitive Reflection Test (Frederick 2005), a three-item test measuring specific aspects of cognitive ability. *Experience* is a binary variable equal to one if experience with investment decisions was reported, zero otherwise

Table 5 Number of observations at the bounds ($n = 112$)

	δ_2	δ_4
$\geq$ 95%	23	14
0%	2	0

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