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AXIONS AND COSMIC RAYS

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We investigate the propagation of a charged particle in a spatially constant but time-dependent pseudoscalar background. Physically, this pseudoscalar background could be provided by a relic axion density. The background leads to an explicit breaking of Lorentz invariance; processes such as $p \longrightarrow p\gamma$ or $e \longrightarrow e\gamma$ are consequently possible under some kinematic constraints. The phenomenon is described by the QED Lagrangian extended with a Chern–Simons term that contains a four-vector characterizing the breaking of Lorentz invariance induced by the time-dependent background. While the induced radiation (similar to the Cherenkov effect) is too small to influence the propagation of cosmic rays significantly, the hypothetical detection of the photons radiated by high-energy cosmic rays via this mechanism would provide an indirect way to verify the cosmological relevance of axions. We discuss the order of magnitude of the effect.

Keywords: axion, high-energy cosmic ray, galactic magnetic field

1. Axions

Cold relic axions resulting from vacuum misalignment [1], [2] in the early universe is a popular and so far viable candidate for dark matter. If we assume that cold axions are the only contributors to the matter density of the universe apart from ordinary baryonic matter, then its density must be [3]

$$\rho \simeq 10^{-30}\,\mathrm{g/cm^3} \simeq 10^{-46}\,\mathrm{GeV}^4.$$

Of course, dark matter is not uniformly distributed: its distribution traces that of visible matter (or rather the other way around). The galactic halo of dark matter (assumed to consist of axions) would correspond to a typical value for the density [4]

$$\rho_{\mathrm{a}} \simeq 10^{-24}\,\mathrm{g/cm^3} \simeq 10^{-40}\,\mathrm{GeV}^4,$$

extending over a distance of 30 to 100 kpc in a galaxy such as the Milky Way. The precise details of the density profile are not so important at this point. The axion background provides a very diffuse concentration of pseudoscalar particles interacting very weakly with photons and therefore indirectly with cosmic rays. What are the consequences of this diffuse axion background on high-energy cosmic ray propagation? Could this influence cosmic ray propagation similarly to the Greisen–Zatsepin–Kuz'min (GZK) cutoff [5]? This is the question that we answer here.

It is quite relevant that the axion is a pseudoscalar, being the pseudo-Goldstone boson of the broken Peccei–Quinn symmetry [6]. Its coupling to photons occurs through the anomaly term; the coefficient is hence easily calculable if the axion model is known:

$$\Delta\mathcal{L} = g_{\mathrm{a}\gamma\gamma}\frac{\alpha}{2\pi}\frac{a}{f_{\mathrm{a}}}\widetilde{F}F, \tag{1}$$

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Two popular axion models are the model in [7] and the model in [8], in both of which $g_{a\gamma\gamma} \simeq 1$. We provide some additional details of the theory in Sec. 3.

2. Cosmic rays

Cosmic rays consist of particles (such as electrons, protons, helium nuclei, and other nuclei) reaching the Earth from outside. Primary cosmic rays are those produced by astrophysical sources (e.g., supernovas), while secondary cosmic rays are particles produced by the interaction of primaries with interstellar gas. We study the effect of axions on the propagation of these cosmic rays. We separately consider proton and electron cosmic rays and ignore heavier nuclei because the effect on them is far less important, as becomes clear later (the axion-induced bremsstrahlung depends on the mass of the charged particle).

2.1. Cosmic ray energy spectrum. The number of protons in cosmic rays is an important characteristic. Experimental data indicate that the number of cosmic ray particles with a given energy depends on energy according to a power law $J(E) = N_i E^{-\gamma_i}$, where the spectral index γ_i takes different values in different regions of the spectrum [9]. For protons, we have

$$J_p(E) = \begin{cases} 5.87 \cdot 10^{19} E^{-2.68}\,\mathrm{eV^{-1}m^{-2}s^{-1}sr^{-1}}, & 10^9\,\mathrm{eV} \le E \le 4 \cdot 10^{15}\,\mathrm{eV}, \\ 6.57 \cdot 10^{28} E^{-3.26}\,\mathrm{eV^{-1}m^{-2}s^{-1}sr^{-1}}, & 4 \cdot 10^{15}\,\mathrm{eV} \le E \le 4 \cdot 10^{18}\,\mathrm{eV}, \\ 2.23 \cdot 10^{16} E^{-2.59}\,\mathrm{eV^{-1}m^{-2}s^{-1}sr^{-1}}, & 4 \cdot 10^{18}\,\mathrm{eV} \le E \le 2.9 \cdot 10^{19}\,\mathrm{eV}, \\ 4.22 \cdot 10^{49} E^{-4.3}\,\mathrm{eV^{-1}m^{-2}s^{-1}sr^{-1}}, & E \ge 2.9 \cdot 10^{19}\,\mathrm{eV}, \end{cases} \tag{2}$$

while the power law for electrons is [10]

$$J_e(E) = \begin{cases} 5.87 \cdot 10^{17} E^{-2.68}\,\mathrm{eV^{-1}m^{-2}s^{-1}sr^{-1}}, & E \le 5 \cdot 10^{10}\,\mathrm{eV}, \\ 4.16 \cdot 10^{21} E^{-3.04}\,\mathrm{eV^{-1}m^{-2}s^{-1}sr^{-1}}, & E \ge 5 \cdot 10^{10}\,\mathrm{eV}, \end{cases} \tag{3}$$

with the flux typically two orders of magnitude below the proton flux, although it is known less well. Our ignorance about electron cosmic rays is quite regrettable because it has a substantial impact in our estimate of the radiation yield.

We note that the above values measured locally in the inner solar system. It is known that the intensity of cosmic rays increases with distance from the sun because the modulation due to the solar wind makes it more difficult for them to reach us, particularly so for electrons. In addition, the hypothesis of homogeneity and isotropy holds for proton cosmic rays but not necessarily for electron cosmic rays. Indeed, because cosmic rays are deflected by magnetic fields they follow a nearly random trajectory within the Galaxy. We know that a hadronic cosmic ray spends an average of about 10^7 years in the galaxy before escaping into intergalactic space. This ensures the uniformity of the flux, at least for protons of galactic origin. In contrast, electron cosmic rays travel for an average of about 1 kpc before being slowed down. But because $l \sim t^{1/2}$ for a random walk, 1 kpc corresponds to a typical age of an electron cosmic ray of 10^5 yr [11]. In addition, the lifetime of an electron cosmic ray depends on the energy as

$$t(E) \simeq 5 \cdot 10^5 \left(\frac{1\,\mathrm{TeV}}{E}\right) \mathrm{yr} = \frac{T_0}{E}, \qquad T_0 \simeq 2.4 \cdot 10^{40}.$$

To complicate matters further, it has been argued that the local interstellar electron flux is not even representative of the Galactic flux and may reflect the electron debris from a nearby supernova 10^4 years ago [12].

2.2. The GZK cutoff. It was stated in [5] that the number of cosmic rays above a certain energy threshold (we call this the GZK limit) should be very small. Cosmic rays particles interact with photons from the cosmic microwave background (CMB) to produce pions $\gamma_{\text{CMB}} + p \longrightarrow p + \pi^0$ or $\gamma_{\text{CMB}} + p \longrightarrow n + \pi^+$. The energy threshold is about $10^{20}\,\text{eV}$. Because of the mean free path associated with these reactions, cosmic rays with energies above the threshold and traveling over distances larger than 50 Mpc should not be observed on the Earth. This is the reason for the rapid fall off of the proton cosmic ray spectrum above $10^{20}\,\text{eV}$ because there are very few nearby sources capable of providing such tremendous energies.

We note that the change in the slope of the spectrum at around $10^{18}\,\text{eV}$ is believed to be due to the appearance of extragalactic cosmic rays at that energy.

3. Solving QED in a cold axion background

In this section, in great detail, we describe the theoretical tools needed for understanding the interactions between the high-energy cosmic rays we have just described and the cold axion background described in Sec. 1.

The coupling of axions to photons is described by the following part of the Lagrangian:

$$\mathcal{L}_{\text{a}\gamma\gamma} = g_{\text{a}\gamma\gamma}\frac{\alpha}{2\pi}\frac{a}{f_{\text{a}}}F^{\mu\nu}\widetilde{F}_{\mu\nu}, \tag{4}$$

where

$$\widetilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

is the dual field strength tensor.

The axion field is originally in a nonequilibrium state, and in the process of its relaxation to the equilibrium configuration, coherent oscillations with $\mathbf{q} = 0$ are produced if the reheating temperature after inflation is below the Peccei–Quinn transition scale [6]. In late times, the axion field evolves according to $a(t) = a_0 \cos(m_{\text{a}}t)$, where the amplitude a_0 is related to the initial misalignment angle. Lagrangian (4) then becomes

$$\mathcal{L}_{\text{a}\gamma\gamma} = g_{\text{a}\gamma\gamma}\frac{\alpha}{2\pi}\frac{1}{f_{\text{a}}}a_0\cos(m_{\text{a}}t)F^{\mu\nu}\widetilde{F}_{\mu\nu} = g_{\text{a}\gamma\gamma}\frac{\alpha}{\pi f_{\text{a}}}a_0\cos(m_{\text{a}}t)\epsilon^{\mu\nu\alpha\beta}\partial_\mu A_\nu F_{\alpha\beta}.$$

Integrating by parts (dropping total derivatives) and taking into account that $\epsilon^{\mu\nu\alpha\beta}\partial_\mu F_{\alpha\beta} = 0$, we obtain

$$\mathcal{L}_{\text{a}\gamma\gamma} = g_{\text{a}\gamma\gamma}\frac{\alpha m_{\text{a}} a_0}{\pi f_{\text{a}}}\sin(m_{\text{a}}t)\epsilon^{ijk}A_i F_{jk}, \tag{5}$$

where the Latin indices run over only the spatial components.

A cosmic ray particle (which travels at almost the speed of light) sees regions with quasiconstant values of the axion background of a size depending on the axion mass but always many orders of magnitude larger than its wavelength. Therefore, we can approximate the sine in (5) by a constant (1/2, for example). It can then be written as

$$\mathcal{L}_{\text{a}\gamma\gamma} = \frac{1}{4}\eta_\mu A_\nu \widetilde{F}^{\mu\nu}, \qquad \eta^\mu = (\eta, 0, 0, 0), \qquad \eta = 4g_{\text{a}\gamma\gamma}\frac{\alpha m_{\text{a}} a_0}{\pi f_{\text{a}}}. \tag{6}$$

The “constant” η changes sign with a period $1/m_{\text{a}}$.

The oscillator has the energy density $\rho_{\text{a}} = \dot{a}^2_{\max}/2 = (m_{\text{a}}a_0)^2/2$, and hence $m_{\text{a}}a_0 = \sqrt{2\rho_{\text{a}}}$. The constant η is then

$$\eta = g_{\text{a}\gamma\gamma}\frac{4\alpha}{\pi}\frac{\sqrt{2\rho_{\text{a}}}}{f_{\text{a}}} \sim 10^{-20}\,\text{eV}$$

for $\rho_{\text{a}} = 10^{-4}\,\text{eV}$ and $f_{\text{a}} = 10^7\,\text{GeV} = 10^{16}\,\text{eV}$.

The extra term in (6) corresponds to the Maxwell–Chern–Simons electrodynamics. Although we can then in principle have any four-vector η^μ, the axion background provides a purely temporal vector. We assume that η^μ is constant within a time interval $1/m_{\text{a}}$.

3.1. Euler–Lagrange equations. In the presence of an axion background, the QED Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\,\not{\partial} - e\not{A} - m_e)\psi + \frac{1}{2}m_\gamma^2 A_\mu A^\mu + \frac{1}{4}\eta_\mu A_\nu \widetilde{F}^{\mu\nu}, \tag{7}$$

where $\not{A} = \gamma^\mu A_\mu$ and similarly for other slashed symbols. Here, we also introduce the effective photon with the mass m_γ (equivalent to a refractive index [2]). We can write the approximation $m_\gamma^2 \simeq 4\pi\alpha n_e/m_e$ for it. The electron density in the Universe is expected to be at most $n_e \simeq 10^{-7}\,\mathrm{cm}^3 \simeq 10^{-21}\,\mathrm{eV}^3$. This density corresponds to $m_\gamma \simeq 10^{-15}\,\mathrm{eV}$, but we here use the more conservative limit $m_\gamma = 10^{-18}\,\mathrm{eV}$ (compatible with [13]).

The second term in (7) gives the kinetic and mass term for the fermions and also their interaction with photons. Dropping it, we obtain the Lagrangian for (free) photons in the axion background (see [14] for further details):

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_\gamma A_\mu A^\mu + \frac{1}{4}\eta_\mu A_\nu \widetilde{F}^{\mu\nu} = \\ &= -\frac{1}{2}\partial_\mu A_\nu(\partial^\mu A^\nu - \partial^\nu A^\mu) + \frac{1}{2}m_\gamma^2 A_\mu A^\mu + \frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\eta_\mu A_\nu \partial_\alpha A_\beta.\end{aligned}$$

The Euler–Lagrange (EL) equations are

$$\partial_\sigma \frac{\partial \mathcal{L}}{\partial(\partial_\sigma A_\lambda)} - \frac{\partial \mathcal{L}}{\partial A_\lambda} = 0,$$

where

$$\begin{aligned}\partial_\sigma \frac{\partial \mathcal{L}}{\partial(\partial_\sigma A_\lambda)} &= \partial_\sigma \frac{\partial}{\partial(\partial_\sigma A_\lambda)}\left[-\frac{1}{2}\,\partial_\mu A_\nu g^{\alpha\mu}g^{\beta\nu}(\partial_\alpha A_\beta - \partial_\beta A_\alpha) + \frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\eta_\mu A_\nu \partial_\alpha A_\beta\right] = \\ &= \partial_\sigma\bigg\{-\frac{1}{2}[g^{\alpha\sigma}g^{\beta\lambda}(\partial_\alpha A_\beta - \partial_\beta A_\alpha) + \\ &\qquad + \partial_\mu A_\nu(g^{\sigma\mu}g^{\lambda\nu} - g^{\lambda\mu}g^{\sigma\nu})] + \frac{1}{4}\epsilon^{\mu\nu\sigma\lambda}\eta_\mu A_\nu\bigg\} = \\ &= \partial_\sigma\left[-(\partial^\sigma A^\lambda - \partial^\lambda A^\sigma) + \frac{1}{4}\epsilon^{\mu\nu\sigma\lambda}\eta_\mu A_\nu\right] = \\ &= -\partial_\sigma\partial^\sigma A^\lambda + \partial^\lambda\partial_\sigma A^\sigma + \frac{1}{4}\epsilon^{\mu\nu\sigma\lambda}\eta_\mu \partial_\sigma A_\nu, \\ \frac{\partial \mathcal{L}}{\partial A_\lambda} &= \frac{\partial}{\partial A_\lambda}\left(\frac{1}{2}m_\gamma^2 g^{\mu\nu}A_\mu A_\nu + \frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\eta_\mu A_\nu \partial_\alpha A_\beta\right) = \\ &= \frac{1}{2}m_\gamma^2(g^{\lambda\nu}A_\nu + g^{\mu\lambda}A_\mu) + \frac{1}{4}\epsilon^{\mu\lambda\alpha\beta}\eta_\mu\partial_\alpha A_\beta = m_\gamma^2 A^\lambda + \frac{1}{4}\epsilon^{\mu\lambda\alpha\beta}\eta_\mu \partial_\alpha A_\beta.\end{aligned}$$

Rearranging the indices, we bring the equations to the form

$$-\Box A^\lambda + \partial^\lambda\partial_\sigma A^\sigma - m^2 A^\lambda - \frac{1}{2}\epsilon^{\beta\lambda\mu\alpha}\eta_\mu\,\partial_\alpha A_\beta = 0.$$

If we choose the Lorenz gauge $\partial_\alpha A^\alpha = 0$, then the second term vanishes. The equations can also be written as

$$-g^{\beta\lambda}\Box A_\beta - g^{\lambda\beta}m_\gamma^2 A_\beta - \frac{1}{2}\epsilon^{\beta\lambda\mu\alpha}\eta_\mu\,\partial_\alpha A_\beta = 0.$$

We are interested in writing these equations in the momentum space. For this, we define the Fourier transform of the field:

$$A_\mu(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \tilde{A}_\mu(k).$$

The relevant derivatives are written as

$$\partial_\alpha A_\beta = \int \frac{d^4k}{(2\pi)^4} (-ik_\alpha) e^{-ikx} \tilde{A}_\beta(k), \qquad \Box A_\beta = \int \frac{d^4k}{(2\pi)^4} (-k^2) e^{-ikx} \tilde{A}_\beta(k).$$

The EL equations are then

$$\int \frac{d^4k}{(2\pi)^4} \left[g^{\beta\lambda}(k^2 - m_\gamma^2) + \frac{i}{2} \epsilon^{\beta\lambda\mu\alpha} \eta_\mu k_\alpha \right] e^{-ikx} \tilde{A}_\beta(k) = 0.$$

Therefore,

$$\left[g^{\beta\lambda}(k^2 - m_\gamma^2) + \frac{i}{2} \epsilon^{\beta\lambda\mu\alpha} \eta_\mu k_\alpha \right] \tilde{A}_\beta(k) = 0$$

or, equivalently,

$$K^{\mu\nu} \tilde{A}_\nu(k) = 0, \qquad K^{\mu\nu} = g^{\mu\nu}(k^2 - m_\gamma^2) + \frac{i}{2} \epsilon^{\mu\nu\alpha\beta} \eta_\alpha k_\beta. \tag{8}$$

3.2. Polarization vectors and dispersion relation. We now define

$$S^\nu_\lambda = \epsilon^{\mu\nu\alpha\beta} \eta_\alpha k_\beta \epsilon_{\mu\lambda\rho\sigma} \eta^\rho k^\sigma.$$

This can be more conveniently expressed using the contraction of two Levi-Civita symbols $\epsilon_{\mu\lambda\rho\sigma}\epsilon^{\mu\nu\alpha\beta} = -3!\delta^\nu_{[\lambda}\delta^\alpha_\rho\delta^\beta_{\sigma]}$ (the minus sign is here because $\epsilon_{0123} = -\epsilon^{0123}$ in Minkowski space):

$$S^{\mu\nu} = [(\eta \cdot k)^2 - \eta^2 k^2] g^{\mu\nu} - (\eta \cdot k)(\eta^\mu k^\nu + k^\mu \eta^\nu) + k^2 \eta^\mu \eta^\nu + \eta^2 k^\mu k^\nu.$$

It satisfies

$$S^\mu_\nu \eta^\nu = S^\mu_\nu k^\nu = 0, \qquad S = S^\mu_\mu = 2[(\eta \cdot k)^2 - \eta^2 k^2], \qquad S^{\mu\nu} S_{\nu\lambda} = \frac{S}{2} S^\mu_\lambda.$$

If $\eta^\mu = (\eta, 0, 0, 0)$, then we have $S = 2\eta^2 \vec{k}^2 > 0$.

We now introduce two projectors:

$$P^{\mu\nu}_\pm = \frac{S^{\mu\nu}}{S} \mp \frac{i}{\sqrt{2S}} \epsilon^{\mu\nu\alpha\beta} \eta_\alpha k_\beta.$$

These projectors have the properties

$$P^{\mu\nu}_\pm \eta_\nu = P^{\mu\nu}_\pm k_\nu = 0, \qquad g_{\mu\nu} P^{\mu\nu}_\pm = 1, \qquad (P^{\mu\nu}_\pm)^* = P^{\mu\nu}_\mp = P^{\nu\mu}_\pm,$$

$$P^{\mu\lambda}_\pm P_{\pm\lambda\nu} = P^\mu_{\pm\nu}, \qquad P^{\mu\lambda}_\pm P_{\mp\lambda\nu} = 0, \qquad P^{\mu\nu}_+ + P^{\mu\nu}_- = 2\frac{S^{\mu\nu}}{S}.$$

With these projectors, we can build a pair of polarization vectors to solve (8). We start from a spacelike unit vector, for example, $\epsilon = (0, 1, 1, 1)/\sqrt{3}$. We then project it: $\tilde{\varepsilon}^\mu = P^{\mu\nu}_\pm \epsilon_\nu$. To obtain a normalized vector, we need

$$(\tilde{\varepsilon}^\mu_\pm)^* \tilde{\varepsilon}_{\pm\mu} = P^{\nu\mu}_\pm \epsilon_\nu P_{\pm\mu\lambda} \epsilon^\lambda = P^\nu_{\pm\lambda} \epsilon_\nu \epsilon^\lambda = \frac{S^{\nu\lambda} \epsilon_\nu \epsilon_\lambda}{S} =$$

$$= \frac{S \epsilon^\mu \epsilon_\mu / 2 + \eta^2 (\epsilon \cdot k)^2}{S} = -\frac{1}{2} + \frac{(\epsilon \cdot k)^2}{2\vec{k}^2}.$$

(this of course is negative because ϵ is spacelike). The polarization vectors are then

$$\varepsilon^\mu_\pm = \frac{\tilde{\varepsilon}^\mu_\pm}{\sqrt{-\tilde{\varepsilon}^\nu_\pm \tilde{\varepsilon}^*_{\pm\nu}}} = \left[\frac{\vec{k}^2 - (\epsilon \cdot k)^2}{2\vec{k}^2}\right]^{-1/2} P^{\mu\nu}_\pm \epsilon_\nu .$$

These polarization vectors satisfy

$$g_{\mu\nu}\varepsilon^{\mu*}_\pm \varepsilon^\nu_\pm = -1, \qquad g_{\mu\nu}\varepsilon^{\mu*}_\pm \varepsilon^\nu_\mp = 0, \tag{9}$$

$$\varepsilon^{\mu*}_\pm \varepsilon^\nu_\pm + \varepsilon^\mu_\pm \varepsilon^{\nu*}_\pm = -2\frac{S^{\mu\nu}}{S} = -\frac{S^{\mu\nu}}{\eta^2 \vec{k}^2}. \tag{10}$$

Using the projectors, we can write the tensor in (8) as

$$K^{\mu\nu} = g^{\mu\nu}(k^2 - m_\gamma^2) + \sqrt{\frac{S}{2}}(P^{\mu\nu}_- - P^{\mu\nu}_+).$$

For $k = (\omega_\pm, \vec{k})$, we then have

$$K^\mu_\nu \varepsilon^\nu_\pm = \left[(k^2 - m_\gamma^2) \mp \sqrt{\frac{S}{2}}\right]\varepsilon^\nu_\pm = (k^2 - m_\gamma^2 \mp \eta|\vec{k}|)\varepsilon^\mu_\pm = \left(\omega^2_\pm - \vec{k}^2 - m^2_\gamma \mp \eta|\vec{k}|\right)\varepsilon^\mu_\pm .$$

Therefore, $\tilde{A}^\mu = \varepsilon^\mu_\pm$ is a solution of (8) iff

$$\omega_\pm(\vec{k}) = \sqrt{m^2_\gamma \pm \eta|\vec{k}| + \vec{k}^2}.$$

This is the new dispersion relation of photons in the cold axion background in the approximation where η is assumed to be piecewise constant.

4. The process $p \longrightarrow p\gamma$

4.1. Kinematic constraints. We now consider $p(p) \longrightarrow p(q)\gamma(k)$ or $e(p) \longrightarrow e(q)\gamma(k)$. This process is forbidden in normal QED by the energy conservation, but it is possible in this background (the cold axion background even allows the process $\gamma \longrightarrow e^+e^-$ [15]). Momentum conservation implies that $\vec{q} = \vec{p} - \vec{k}$. If m is interpreted as the mass of the charged particle (proton or electron), then conservation of energy leads to

$$\begin{aligned} &E(q) + \omega(k) = E(p), \qquad \sqrt{m^2 + (\vec{p} - \vec{k})^2} + \sqrt{m^2_\gamma \pm \eta|\vec{k}| + \vec{k}^2} = \sqrt{m^2 + \vec{p}^2}, \\ &\sqrt{E^2 + k^2 - 2pk\cos\theta} + \sqrt{m^2_\gamma \pm \eta k + k^2} - E = 0, \end{aligned} \tag{11}$$

where we use the expressions

$$E = E(p) = \sqrt{m^2 + \vec{p}^2}, \qquad p = |\vec{p}|, \qquad k = |\vec{k}|, \qquad pk\cos\theta = \vec{p}\cdot\vec{k}$$

in the last line. As becomes clear in what follows, if η is positive or negative, then the process is only possible for the respective negative or positive polarization. Therefore, $\pm\eta = -|\eta|$ in these cases. To take both of them into account, we use the minus sign and write η instead of $|\eta|$.

We square the last equality in (11) twice:

$$(4E^2 - 4p^2\cos^2\theta + 4p\eta\cos\theta - \eta^2)k^2 - 2(2E^2\eta + 2m^2_\gamma p\cos\theta - m^2_\gamma\eta)k + 4E^2m^2_\gamma - m^4_\gamma = 0.$$

Neglecting $-\eta^2 k^2$, $-m_\gamma^2 \eta k$, and $-m_\gamma^4$ in this equation, we obtain

$$(E^2 - p^2\cos^2\theta + p\eta\cos\theta)k^2 - (E^2\eta + m_\gamma^2 p\cos\theta)k + E^2 m_\gamma^2 = 0.$$

This equation has two solutions

$$k_\pm = \frac{E^2\eta + pm_\gamma^2\cos\theta \pm E\sqrt{E^2\eta^2 - 4E^2m_\gamma^2 + 4p^2m_\gamma^2\cos^2\theta - 2pm_\gamma^2\eta\cos\theta}}{2(E^2 - p^2\cos^2\theta + p\eta\cos\theta)}, \tag{12}$$

which make sense only if the discriminant Δ is positive. In the approximation $\cos\theta \simeq 1 - \sin^2\theta/2$, the condition $\Delta \geq 0$ becomes

$$\sin^2\theta \leq \frac{p^2\eta^2 - 2pm_\gamma^2\eta + m^2(\eta^2 - 4m_\gamma^2)}{4p^2m_\gamma^2(1 - \eta/4p)},$$

which can be rewritten as

$$\sin^2\theta \leq \frac{\eta^2}{4p^2m_\gamma^2}\frac{1}{1-\eta/4p}(p - p_+)(p - p_-), \tag{13}$$

where

$$p_\pm = \frac{m_\gamma^2}{\eta} \pm \frac{2mm_\gamma}{\eta}\sqrt{1 - \frac{\eta^2}{4m_\gamma^2}} \simeq \pm\frac{2mm_\gamma}{\eta}\sqrt{1 - \frac{\eta^2}{4m_\gamma^2}}.$$

It is clear that $p_+ > 0$ and $p_- < 0$. For $\sin^2\theta$ to be positive, the condition

$$p > p_+ = p_{\rm th} = \frac{2mm_\gamma}{\eta}\sqrt{1 - \frac{\eta^2}{4m_\gamma^2}}$$

must be satisfied. This is the threshold below which the process cannot occur kinematically. The energy threshold $E_{\rm th}^2 = m^2 + p_{\rm th}^2$ is written as

$$E_{\rm th} = \frac{2mm_\gamma}{\eta}.$$

As $\eta \to 0$, the threshold value goes to infinity (as expected, the process becomes impossible if η vanishes).

There is another relevant scale in the problem: m^2/η. It is many orders of magnitude above the GZK cutoff. Therefore, we always assume the limit $p \ll m^2/\eta$. The maximum angle of emission for a given momentum is given by (13). Its largest value is obtained when p is large ($p \gg p_{\rm th}$):

$$\sin^2\theta_{\rm max} = \frac{\eta^2}{4m_\gamma^2}.$$

Because this is a small number, photons are emitted in a narrow cone $\theta_{\rm max} = \eta/2m_\gamma$. This justifies the approximation for $\cos\theta$.

At $\theta_{\rm max}(p)$, the square root in (12) vanishes, and we have

$$k_+[\theta_{\rm max}(p)] = k_-[\theta_{\rm max}(p)] \xrightarrow{p_{\rm th} \ll p \ll m^2/\eta} \frac{2m_\gamma^2}{\eta}.$$

The minimum value for the angle is $\theta = 0$:

$$k_\pm(0) \simeq \frac{E^2\eta + pm_\gamma^2 \pm (E^2\eta - pm_\gamma^2 - 2m^2m_\gamma^2/\eta)}{2(m^2 + p\eta)}.$$

In the framework of the assumption $p_{\rm th} \ll p \ll m^2/\eta$, we hence obtain the maximum and minimum values of the photon momentum:

$$k_{\max} = k_+(0) = \frac{\eta E^2}{m^2}, \tag{14}$$

$$k_{\min} = k_-(0) = \frac{m_\gamma^2}{\eta}. \tag{15}$$

These two values coincide at the energy threshold. Here, we can see that the process is possible for negative or positive polarization only if respectively $\eta > 0$ or $\eta < 0$. Otherwise, the modulus of the photon momentum would be negative.

We note that the incoming cosmic ray wavelength fits perfectly within the $1/m_{\rm a}$ size, and the quantity η therefore turns out to be almost perfectly constant. Whether η is positive or negative, there is always a state with slightly less energy into which it is possible to decay with a loss of part of its energy (of the order $O(\eta)$), emitting a soft photon. Therefore, even if the process is rare, it does not average to zero. We will present an exact analysis of this situation in a future publication.

4.2. Amplitude. The next thing we need is to determine is the matrix element for the process. Using the standard Feynman rules, we obtain

$$i\mathcal{M} = \bar{u}(q) ie\gamma^\mu u(p)\varepsilon_\mu^*(k).$$

Its square is

$$|\mathcal{M}|^2 = \bar{u}(q) ie\gamma^\mu u(p)\varepsilon_\mu^*(k)[\bar{u}(q) ie\gamma^\nu u(p)\varepsilon_\nu^*(k)]^* =$$
$$= e^2\varepsilon_\mu^*(k)\varepsilon_\nu(k)\,\mathrm{tr}[u(q)\bar{u}(q)\gamma^\mu u(p)\bar{u}(p)\gamma^\nu].$$

We now must sum and average correspondingly over the initial and final proton helicities. We do not average over photon polarizations, because the process is possible only for one polarization. Taking the trace, we obtain

$$|\overline{\mathcal{M}}|^2 = \frac{1}{2}e^2\varepsilon_\mu^*(k)\varepsilon_\nu(k)\,\mathrm{tr}[(\not{q} + m)\gamma^\mu(\not{p} + m)\gamma^\nu] =$$
$$= \frac{1}{2}e^2\varepsilon_\mu^*(k)\varepsilon_\nu(k)\,\mathrm{tr}[\not{q}\gamma^\mu\not{p}\gamma^\nu + m^2\gamma^\mu\gamma^\nu] =$$
$$= \frac{1}{2}e^2\varepsilon_\mu^*(k)\varepsilon_\nu(k)[q^\mu p^\nu - q^\alpha p_\alpha g^{\mu\nu} + q^\nu p^\mu + m^2 g^{\mu\nu}].$$

Using four-momentum conservation, (9), and the fact that $p^\alpha p_\alpha = m^2$, we obtain

$$|\overline{\mathcal{M}}|^2 = 2e^2(-p^\alpha k_\alpha + 2\varepsilon_\mu^*\varepsilon_\nu p^\mu p^\nu) = 2e^2[-p^\alpha k_\alpha + (\varepsilon_\mu^*\varepsilon_\nu + \varepsilon_\mu\varepsilon_\nu^*)p^\mu p^\nu].$$

Now using (10), we obtain

$$(\varepsilon_\mu^*\varepsilon_\nu + \varepsilon_\mu\varepsilon_\nu^*)p^\mu p^\nu = -\frac{S^{\mu\nu}p_\mu p_\nu}{\eta^2 k^2} = p^2\sin^2\theta.$$

The average squared amplitude is then

$$|\overline{\mathcal{M}}|^2 = 2e^2(-p^\alpha k_\alpha + p^2\sin^2\theta). \tag{16}$$

The first term in the right-hand side is positive,

$$-p^{\alpha}k_{\alpha} = -E\omega + pk\cos\theta = -E\omega - pk\frac{m_{\gamma}^2 - \eta k - 2E\omega}{2pk} =$$

$$= \frac{1}{2}\eta\left(k - \frac{m_{\gamma}^2}{\eta}\right) = \frac{1}{2}(k - k_{\min}) > 0,$$

and $|\overline{\mathcal{M}}|^2$ is therefore also positive.

4.3. Differential decay width. The differential decay width is

$$d\Gamma = (2\pi)^4\delta^{(4)}(q + k - p)\frac{1}{2E}|\overline{\mathcal{M}}|^2\, dQ,$$

where the phase space element is

$$dQ = \frac{d^3q}{(2\pi)^3 2E(q)}\frac{d^3k}{(2\pi)^3 2\omega(k)}.$$

We can use $\delta^{(3)}(\vec{q}+\vec{k}-\vec{p})$ to eliminate d^3q. The remaining factor δ corresponds to the energy conservation law: $E(q) = E - \omega$. We next use a property of the Dirac delta function,

$$\delta[f(x)] = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|},$$

where x_i are the zeros of the function $f(x)$. In our case, we consider $E(q)$ a function of $\cos\theta$:

$$\delta[E(q) + \omega - E] = \delta(\sqrt{E^2 + k^2 - 2pk\cos\theta} + \omega - E) =$$

$$= \left|\frac{-2pk}{2\sqrt{E^2 + k^2 - 2pk\cos\theta}}\right|^{-1}\delta\left(\cos\theta - \frac{m_{\gamma}^2 - \eta k - 2E\omega}{-2pk}\right).$$

Further, we write $d^3k = k^2\, dk\, d(\cos\theta)\, d\varphi$, integrate over the angle φ (which gives the factor 2π), and use the delta function to eliminate $d(\cos\theta)$. This fixes the value of $\cos\theta$:

$$\cos\theta = \frac{m_{\gamma}^2 - \eta k - 2E\omega}{-2pk}. \tag{17}$$

Finally, the differential decay width is

$$d\Gamma = \frac{\alpha}{2}\frac{k}{Ep\omega}(-p^{\alpha}k_{\alpha} + p^2\sin^2\theta)\, dk,$$

where $\alpha = e^2/4\pi$ and $\sin\theta$ is given by (17). This decay width can be written more conveniently for future computations:

$$\frac{d\Gamma}{dk} = \frac{\alpha}{8}\frac{1}{k\omega}[A(k) + B(k)E^{-1} + C(k)E^{-2}]\Theta\left(\frac{E^2\eta}{m^2} - k\right),$$

where Θ is the step function,

$$A(k) = 4(\eta k - m_{\gamma}^2), \qquad B(k) = 4\omega(m_{\gamma}^2 - \eta k),$$

$$C(k) = -2m_{\gamma}^2k^2 + 2\eta k^3 - m_{\gamma}^4 - \eta^2k^2 + 2m_{\gamma}^2\eta k.$$

4.4. Effects on cosmic rays. We now compute the energy loss of protons in this background:

$$\frac{dE}{dx} = \frac{dt}{dx}\frac{dE}{dt} = \frac{1}{v}\left(-\int \omega \, d\Gamma\right).$$

Using the previous results and $v = p/E$, we obtain the energy loss (with the integration limits given by (14) and (15))

$$\begin{aligned}
\frac{dE}{dx} &= -\frac{\alpha}{2}\frac{1}{p^2}\int_{k_{\min}}^{k_{\max}} dk\, k\left[\frac{1}{2}(\eta k - m_\gamma^2) + p^2(1-\cos^2\theta)\right] = \\
&= -\frac{\alpha}{8p^2}\int_{k_{\min}}^{k_{\max}} dk \Bigg[2\eta k^2 - (4m^2 + 2m_\gamma^2 + \eta^2)k + 2\eta(2E^2 + m_\gamma^2) - \\
&\quad - m_\gamma^2(4E^2 + m_\gamma^2)\frac{1}{k} + 4E\eta\sqrt{m_\gamma^2 - \eta k + k^2} + 4Em_\gamma^2\frac{\sqrt{m_\gamma^2 - \eta k + k^2}}{k}\Bigg] = \\
&= -\frac{\alpha}{8p^2}\Bigg[\frac{2}{3}\eta\left(\frac{\eta^3E^6}{m^6} - \frac{m_\gamma^6}{\eta^3}\right) - \frac{1}{2}(4m^2 + 2m_\gamma^2 + \eta^2)\left(\frac{\eta^2E^4}{m^4} - \frac{m_\gamma^4}{\eta^2}\right) + \\
&\quad + 2\eta(2E^2 + m_\gamma^2)\left(\frac{\eta E^2}{m^2} - \frac{m_\gamma^2}{\eta}\right) - m_\gamma^2(4E^2 + m_\gamma^2)\log\left(\frac{\eta^2E^2}{m^2m_\gamma^2}\right) + \dots\Bigg].
\end{aligned}$$

The leading term is

$$\frac{dE}{dx} = -\frac{\alpha}{8p^2}\frac{2\eta^2E^4}{m^2} = -\frac{\alpha\eta^2E^2}{4m^2v^2} \simeq -\frac{\alpha\eta^2E^2}{4m^2}.$$

The energy as a function of the traveled distance is then

$$E(x) = \frac{E(0)}{1 + (\alpha\eta^2/4m^2)E(0)x}.$$

The fractional energy loss for a cosmic ray with the initial energy $E(0)$ traveling a distance x is

$$\frac{E(0) - E(x)}{E(0)} = \frac{(\alpha\eta^2/4m^2)E(0)x}{1 + (\alpha\eta^2/4m^2)E(0)x}.$$

This loss becomes more important as the cosmic ray energy increases. But $\alpha\eta^2/4m^2$ is a very small number. If we take $E(0) = 10^{20}\,\mathrm{eV}$ (the energy of the most energetic cosmic rays) and $x = 10^{26}\,\mathrm{cm}$ (about the distance to Andromeda, the nearest galaxy, and therefore larger than the galactic halo), then the energy loss is smaller than $1\,\mathrm{eV}$. For less energetic cosmic rays, the effect is even weaker.

As we have shown, the effect of the axion background on cosmic rays is quite negligible. Nevertheless, the emitted photons may be detectable. Using $m_\gamma = 10^{-18}\,\mathrm{eV}$ and $\eta = 10^{-20}\,\mathrm{eV}$ as characteristic values and having in mind the GZK cutoff for protons (and a similar one for electrons[1]), we find that the emitted photon momenta are in the range $10^{-16}\,\mathrm{eV} < k < 100\,\mathrm{eV}$ for primary protons and $10^{-16}\,\mathrm{eV} < k < 400\,\mathrm{MeV}$ for primary electrons.

The number of cosmic rays with a given energy crossing a surface element per unit time is $d^3N = J(E)\,dE\,dS\,dt_0$, where $J(E)$ is the cosmic ray flux. These cosmic rays radiate at a time t. The number of photons is given by

$$d^5N_\gamma = d^3N\frac{d\Gamma(E,k)}{dk}\,dk\,dt = J(E)\frac{d\Gamma(E,k)}{dk}\,dE\,dk\,dt_0\,dS\,dt.$$

[1] It is very doubtful that electrons could be accelerated to such energies, but this is unimportant for our discussion because the intensity is extremely small at these energies.

Assuming that the cosmic ray flux is independent of time, we integrate this equation over t_0 and obtain a factor $t(E)$: the age of the average cosmic ray with the energy E. Because we do not care about the energy of the primary cosmic ray (only that of the photon matters), we also integrate over E, starting from $E_{\min}(k)$, the minimum energy that the cosmic ray can have in order to produce a photon with momentum k given by (14). We obtain the photon flux

$$\frac{d^3 N_\gamma}{dk\, dS\, dt} = \int_{E_{\min}(k)}^{\infty} dE\, t(E) J(E) \frac{d\Gamma(E,k)}{dk}, \qquad E_{\text{th}} = 2\frac{m m_\gamma}{\eta}. \tag{18}$$

Further, we assume that $t(E)$ is approximately constant and take $t(E) \approx T_p = 10^7\,\text{yr}$ for protons and $t(E) \approx T_e = 5 \cdot 10^5\,\text{yr}$ for electrons. We know that this last approximation is incorrect because $t(E) \sim 1/E$, but we are now interested only in obtaining an order of magnitude estimate of the effect.

The photon energy flux is obtained by multiplying photon flux (18) by the energy of a photon with momentum k:

$$I(k) = \omega(k) \int_{E_{\min}(k) > E_{\text{th}}}^{\infty} dE\, t(E) J(E) \frac{d\Gamma}{dk} \approx$$
$$\approx \frac{\alpha T}{8k} \int_{E_{\min}(k)}^{\infty} dE\, N_i [A(k) E^{-\gamma_i} + B(k) E^{-(\gamma_i+1)} + C(k) E^{-(\gamma_i+2)}],$$

where $E_{\min}(k) = m\sqrt{k/\eta}$ (see (14). A numerical analysis shows that the only relevant term in the decay rate is $4\eta k$ from $A(k)$. The integral can then be approximated by

$$I(k) \simeq \frac{\alpha \eta T}{2} \frac{J[E_{\min}(k)] E_{\min}(k)}{\gamma_{\min} - 1} \propto k^{-(\gamma-1)/2}.$$

The value $\gamma_{\min}$ should be taken from (2) or (3) depending on the range where $E_{\min}(k)$ falls. Substituting the numerical values, we obtain the approximate expressions for $I_p(k)$ and $I_e(k)$:

$$I_p(k) = 6 \left(\frac{T_p}{10^7\,\text{yr}}\right) \left(\frac{\eta}{10^{-20}\,\text{eV}}\right)^{1.84} \left(\frac{k}{10^{-7}\,\text{eV}}\right)^{-0.84} \text{m}^{-2}\text{s}^{-1}\text{sr}^{-1},$$

$$I_e(k) = 200 \left(\frac{T_e}{5 \cdot 10^5\,\text{yr}}\right) \left(\frac{\eta}{10^{-20}\,\text{eV}}\right)^{2.02} \left(\frac{k}{10^{-7}\,\text{eV}}\right)^{-1.02} \text{m}^{-2}\text{s}^{-1}\text{sr}^{-1}.$$

As mentioned above, these expressions are only an estimate of the order of magnitude and assume constant average values for the age of a cosmic ray (either proton or electron). The interested reader can find a more detailed discussion in our recent paper [16], from which we take Fig. 1 describing the radiation yield.

5. Conclusions and outlook

We have investigated the effect on charged particles of a pseudoscalar background that is mildly time dependent (compared with the particle momentum). We considered both proton and electron cosmic rays. This effect is calculable because the axion background induces a modification of QED that turns out to be exactly solvable. This modification has several interesting features, such as the possibility of the photon emission process $p \to p\gamma$ and $e \to e\gamma$ (which we called the axion-induced bremsstrahlung processes). We obtained kinematic constraints on the process; in particular, we showed that the process is possible only for proton energies higher than a certain threshold. We computed the energy loss of protons in such a background. For protons with energies below the GZK cutoff, this loss is negligibly small.

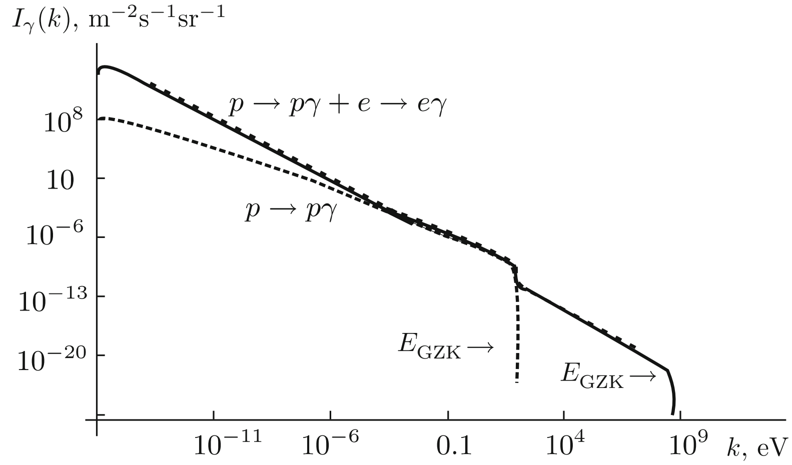

Fig. 1. Radiation yield using the exact formulas and a more appropriate parameterization of the electron cosmic ray average lifetime as a function of the energy (from [16]): We note that electrons in general dominate the effect at low energies.

Nevertheless, the radiated photons could still be detected. We have computed their flux and energy spectrum in some detail. Because the energy threshold depends on the mass of the charged particle, it is lower for lighter particles. Also, the energy loss is proportional to the inverse squared mass of the charged particle, and the effect is therefore more important for electrons. The value of $k_{\rm min}$ is independent of the charged particle mass, and the radiation spectra for electrons or protons differ very little (but the average lifetimes of electron and proton cosmic rays differ significantly, and this has observable effects on the radiation power spectrum).

The interested reader can refer to [16], where we describe this phenomenon in more detail and discuss the possibility of measuring this diffuse radiation. Below, we summarize the main conclusions in that work.

The dominant contribution to the radiation via the considered mechanism is from electron (and positron) cosmic rays. If we assume that the cosmic ray power spectrum is characterized by an exponent γ, then the produced radiation has an spectrum $k^{-(\gamma-1)/2}$ for proton primaries, which becomes $k^{-\gamma/2}$ for electron primaries. The dependence on the key parameter $\eta \sim \sqrt{\rho_*}/f_{\rm a}$ comes with the respective exponents $\eta^{(1+\gamma)/2}$ and $\eta^{(2+\gamma)/2}$ for protons and electrons. But for the regions where the radiation yield is largest, the electron contribution is determinative. We assumed that the flux of electron cosmic rays is uniform throughout the Galaxy and hence identical to the flux observed in our neighborhood, but relaxing this hypothesis could provide an enhancement of the effect by a relatively large factor. The effect for the lowest wavelengths, for which the atmosphere is transparent, and for values of η corresponding to the current experimental limit is of the order $O(10^{-1})$ mJy. This is at the sensitivity limit of antenna arrays that are already being deployed, and it there seems possible to conduct experimental observations of the discussed effect.

In the case of radiation originating from our galaxy, there is no answer to the main question relevant to our discussion: Is the flux of electron cosmic rays measured in our neighborhood representative of the whole galaxy? Because it seems possible to relate this flux to the galactic synchrotron radiation, this radiation could be deduced from measuring the flux. It appears that either the total number of electron cosmic rays is substantially larger than the number of rays measured in the Solar System or the galactic magnetic fields must be stronger than expected [17]. The quantitative analysis of this phenomenon requires further studies. There has also been no attempt to quantify the signal from possible extragalactic sources.

We note that the effect discussed here is a collective effect. This contrasts with the GZK effect discussed in Sec. 1: the CMB radiation is not coherent over large scales. For instance, no similar effect exists for hot axions. One more observation is that some of the scales that play a role in the present discussion are somewhat nonintuitive (e.g., the "crossover" scale m^2/η or the threshold scale $m_\gamma m/\eta$). This is related to the fact that this effect is not Lorentz invariant. It may seem surprising at first glance that an effect that has such a low probability can give a small, but not negligibly small, contribution. The reason is that the number of cosmic rays is huge. It is known that their contribution to the galactic energy density is comparable to the contribution of the galactic magnetic field [18].

There are several aspects of the present analysis that could be improved to make it more precise. In particular, the problem with a piecewise-constant oscillating axion background or even with a background with a serrated time profile can be easily solved exactly without using special functions (the sine profile involves Mathieu functions). This problem will be considered in subsequent publications, but the present analysis already suffices to obtain the order of magnitude of the effect. We hope that the considered mechanism will help to understand the possible relevance of cold axions as a candidate for the role of dark matter.

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