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ORIGINAL RESEARCH PAPER

Equalization reserves for natural catastrophes and shareholder value: a simulation study

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Abstract This paper investigates the effects on the company value for shareholders of keeping equalization reserves for catastrophic risk in an insurance company. We perform an extensive simulation study to compare the performance of the company with and without equalization reserves for several standard profitability measures. Equalization reserves turn out to be beneficial for shareholders in terms of the resulting expected Sharpe ratio and also with respect to the value of the call option on assets at some reasonably large maturity time. Moreover, the expected total discounted tax payments are not smaller when using equalization reserves. The results are robust with respect to model parameters such as interest rate, time horizon, cost of raising capital and business cycle dynamics.

1 Introduction

Over the last years, there have been many debates to what extent the new regulatory rules and accounting standards for insurance companies are appropriate. Many of these rules are inspired by regulations and accounting procedures designed for banks and it is a challenging question whether the insurance industry indeed should be regulated in a similar way. In this paper, we would like to contribute to this discussion for the case of equalization reserves for natural catastrophe (CAT) risks.

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Equalization reserves (also called fluctuation reserves) provide a buffer against large claim amount fluctuations over the years (in addition to the usual solvency margin). When the claim amount is below its expectation in one year, the difference is saved in order to be available for excessive losses in other years (instead of being paid out as an (arguably) 'false' profit to the shareholders). This procedure is particularly relevant in lines of business with rather heavy-tailed claim amount distributions (such as CAT risks), as it equalizes insurance business over time.

Equalization reserves have been used by insurers in many countries for more than half a century to dampen the effects of natural catastrophes on their balance sheet, putting aside reserves during good years for future possibly bad years (as the probability of occurrence is low, often substantial reserves can be built up before a large claim happens). For a general survey, see Pentikäinen [7] and the references therein. In particular when empirical evidence shows that geographical diversification does not suffice to smoothen the large fluctuations, it seems to be a natural approach to diversify this risk over time (and some countries particularly exposed to catastrophic risk—like Japan—even require their insurance companies to hold equalization reserves). The element of time diversification is also at the heart of the concept of ruin probabilities (cf. [2] for an overview). In an attempt to harmonize accounting principles in different countries, the International Accounting Standard Board (IASB) has developed International Financial Reporting Standards (IFRS), which are nowadays binding for listed companies in many countries (including all countries of the European Union). The IFRS rules were largely inspired by the United States-Generally Accepted Accounting Principles (US-GAAP), see for instance [5] and [9]. According to US-GAAP and new IFRS rules, insurance companies are not allowed to carry over reserves for future business, i.e. if no loss has occurred during the year, then the reserves must be released as profits. So—in contrast to previous insurance practice—equalization reserves are not permitted anymore. Apart from the intention to harmonize accounting standards, the purpose of the IFRS rules is to protect the shareholders and to bring more transparency into the value creation of the firm, thus to restore the investors' confidence in the insurance industry. By diminishing the amount of free cash-flows at the disposal of managers, the potential of misuse ('agency risk') is reduced. Also, tax authorities are concerned that equalization reserves are 'artificial' reserves which reduce the taxable profit of a company. While these arguments should be considered seriously, one also has to keep in mind that insurance companies and banks are quite different with respect to risk (and the topic of reserving is crucial for insurance; historically, about two thirds of the occurred insurance insolvencies were caused by insufficient reserves, cf. [1]). There is only a small part of risk taken by banks because their main activities are related to other services than risk management. On the other hand insurers and reinsurers carry more risk on their balance sheet and use their own capital to face it (see [4] for a detailed discussion). In addition, after all, the rules set up for banks have not been so successful to avoid recent crises.

For catastrophic risks, most of the time the claims will be *below* the expectation and in years where a catastrophe occurs, the expected claim size can easily be exceeded by so much that the yearly premium will not suffice to cover the liabilities. There is a common argument that capital should be used instead of reserves to cover large claims, and capital should only be raised at the time when it is needed. However, if an insurance company is at distress in view of large claims to pay, the cost of raising capital will be very high (and certainly much more expensive than keeping some profit of previous years on the balance sheet) and there may also be less cash available in general. In addition, the risk for a bankruptcy is much higher. Uncertainty in the results is penalized by investors, as they will require higher reward for their investments. This, in turn, increases the cost of insurance policies. While the extra cushion provided by time diversification is clearly beneficial for the policyholders, for the shareholders the short-term profits can be expected to be bigger if the reserves are released as profits at the end of each year. However, under a longer term perspective, the volatility of the profits will be lower in the presence of equalization reserves, and the probability of bankruptcy can be considerably lower, so that the invested notional amount is less likely to be lost. Finally, for the tax authorities, tax payments may be lower in the first years, but can be expected to continue over a longer time horizon and are hence equalized. So there is a trade-off that needs to be studied quantitatively.

In this paper, we illustrate for a simple, yet insightful model that equalization reserves can indeed be beneficial for all involved parties, in particular for a long-term investor. Under several performance measures for the cash-flows resulting from the initial investment of the shareholders we assess whether it is preferable to allow for equalization reserves in catastrophe insurance or not. Concretely, we build a stochastic model for the cash-flows of two companies, one carrying over reserves for future business (the "time diversified company") and the other one applying the new accounting rules and distributing all profits to its shareholders by the end of each year (the "US-GAAP company"). Both companies write the same catastrophe risks (distributed according to some heavy-tailed distribution) against the same initial risk-adjusted capital which is determined according to the Value-at-Risk. Simulating the actual insurance losses over a time span of 30 years, we determine the profits of the firm and the resulting dividend payments to shareholders. In a second step, we use the internal rate of return, the profitability index, the Sharpe ratio and the call option value of Merton type to compare which of the companies performs better. We then analyze the sensitivity of the results on certain model parameters (interest rate, time horizon, cost of raising capital and business cycle dynamics). Finally, we compare the resulting tax payments of the two companies. In order to focus on the effects of equalization reserves, we choose a number of simplifying assumptions in the model [e.g. all investment gains are according to a risk-free interest rate and we do not consider other types of claim reserves (such as IBNR)].

In Sect. 2 we present the model and its dynamics. After specifying the insurance loss model in Sect. 2.1, the calculation of premiums and the implementation of the business cycle is described in Sect. 2.2. The concrete accounting procedure is given in Sect. 2.3, whereas Sect. 2.4 gives the modifications when equalization reserves are employed. In Sect. 2.5, the four performance measures used later on are specified. As the model is intended to capture a number of stylized facts from catastrophe insurance practice, it is too complex to allow for an analytical expression of the expected value of these performance measures. We hence use a Monte Carlo simulation algorithm. In Sect. 3, the concrete implementation and subsequently the simulation results are given and discussed. In particular, the sensitivity of the results with respect to the involved parameters and the effects of equalization reserves on the total amount of paid taxes is studied. Section 4 concludes.

2 The model

Consider two insurance companies which write the same (heavy-tailed) CAT risk, both for an initial capital of $C(0)$. They use the same principle for premium calculation (specified below) and face the same losses over a period of τ years.

The investment universe of this model consists of two options to invest money, namely either in risk-free investments or in the risky CAT-company with the capital amount $C(0)$. However, the central difference in investment behavior within the risky investment, which is the main topic of this paper, is the possibility to either "keep" a part of risk-free investments "in" the company (the "time-diversified company") or to manage "all remaining" risk-free investments outside the company and to keep the balance sheet of the company "lean" (the "US-GAAP" company).

So, one of the two companies (the "US-GAAP" company) applies the IFRS rules (i.e. covers the annual losses with the annual premium received for the risk and, if not sufficient, with the capital) and the second company (the "time-diversified company") is allowed to carry over reserves for future business (i.e. covers the losses with the premium received for the risk plus the equalization reserves and, only if that is not sufficient, with the capital). If the premium is sufficient to pay the claims, the US-GAAP company pays the remaining difference as profit (dividends) to the shareholders which will be taxed. If the actual size of the claims is below its expectation, then, on the other hand, the time-diversified company takes the difference between the expectation and the claim size aside as equalization reserve. The part of the premium that exceeds the expected value of the claim size is also paid out as profit. When the premium (plus equalization reserves in the time-diversified case) is not sufficient to pay the claims, capital has to be (partially) used, after which it is rebuilt for the next year's business. We assume that the acquired wealth of the shareholder is used for rebuilding capital, up to the original value of $C(0)$, which can be quite expensive (*cost of raising capital*). If the capital can not be fully rebuilt back to the level $C(0)$, then in the next period the company is only allowed to write risk commensurate with the remaining capital (*reducing the exposure*). If the whole available capital is needed to settle the annual claim, the company is bankrupt and can no longer write new business. For the determination of the premium, we also include business cycles over the years. In the following we describe the model ingredients in more detail.

2.1 The insurance loss model

Assume that $X(t)$ is the aggregate claim amount of year t to be paid at time t, where $(X(t))_{t=1,2,\ldots}$ is a sequence of independent and identically distributed random variables with generic random variable X, which is calibrated in such a way that the resulting risk-adjusted capital (RAC) is $\rho(X) = C(0)$.[1] As a risk measure, we use for both companies the Value-at-Risk (VaR)

$$\rho(X) = \mathrm{VaR}[X;\theta] = F_X^{-1}(\theta) \tag{2.1}$$

where F_X^{-1} is the inverse of the cumulative distribution function of X and θ is the quantile at which the risk is covered. If the insurance company cannot afford to hold the capital of $C(0)$ in a certain year, it will only write the fraction of the corresponding policies, which leads to a RAC that is affordable. In this paper we will consider two types of loss distributions for the (heavy-tailed) CAT risk:

- **Lognormal losses**: $X \sim \mathrm{LN}(\mu,\sigma^2)$ with density

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma x}\exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right), \quad x \geq 0. \tag{2.2}$$

The expectation and standard deviation are given by

$$E[X] = e^{(\mu+\frac{\sigma^2}{2})}, \quad \mathrm{std}[X] = \sqrt{(e^{\sigma^2}-1)(e^{2\mu+\sigma^2})}, \tag{2.3}$$

and the Value-at-Risk is given by the simple formula

$$\mathrm{VaR}[X;\theta] = \exp(\mu + \sigma\Phi^{-1}(\theta)), \tag{2.4}$$

where $\Phi^{-1}(\theta)$ is the inverse cumulative distribution function of a standard normal distribution.
As we fix the initial capital $\rho(X) = C(0)$ and want to vary the coefficient of variation $\mathrm{CoV}(X) = \mathrm{std}[X]/E[X]$ to examine different degrees of heaviness, it is convenient to express the parameters μ and σ through those two quantities:

$$\sigma = \sqrt{\ln(1+[\mathrm{CoV}(X)]^2)}, \quad \mu = \ln\rho(X) - \sigma\cdot\Phi^{-1}(\theta). \tag{2.5}$$

- **Fréchet losses**: Here the cumulative distribution function of X is defined by

$$F_X(x) = \begin{cases} 0 & \text{if} \quad x \leq 0 \\ \exp(-(\frac{x}{s})^{-\alpha}) & \text{if} \quad x > 0, \end{cases} \tag{2.6}$$

where $\alpha > 0$ is a shape parameter (tail index), and $s > 0$ a scale parameter. Its expected value is given by

$$E[X] = s\cdot\Gamma\left(1-\frac{1}{\alpha}\right) \tag{2.7}$$

and the Value-at-Risk can be expressed as

$$\rho(X) = \mathrm{VaR}[X,\theta] = s\cdot(-\ln(\theta))^{(-1/\alpha)}. \tag{2.8}$$

Since we fix $\rho(X)$ and want to vary the scale parameter α, s is computed by means of

[1] For the way in which the collected premiums enter in the calculations, see Sects. 2.2 and 3.1.

$$s = \frac{\rho(X)}{(-\ln(\theta))^{-1/\alpha}}. \tag{2.9}$$

This distribution has a much heavier tail than the Lognormal distribution. By a series expansion at ∞ one observes that the tail behavior of the Fréchet distribution is asymptotically equivalent to the one of a Pareto distribution with tail parameter α. In the implementations in Sect. 3, we will use values of α less than 2, in which case the variance of X is infinite. Altogether, this will allow to compare the case of heavy-tailed X of Lognormal type (where still all moments exist) with very heavy-tailed power-type tails of Fréchet type.

2.2 Premiums and business cycles

The technical premium $P(t)$ for year t (collected at time $t-1$) should cover the expected claims costs $E[X(t)]$ plus cost of capital plus expenses and operational costs. For year 1 we define the technical premium (to be collected at time 0) as follows:

$$P(1) = \frac{E[X(1)] + \kappa \cdot \rho(X(1)) + e(1)}{1+r}, \tag{2.10}$$

where κ is the cost of capital rate before tax which can also be interpreted as the performance required by the shareholders in order to invest in the company, r is the risk-free interest rate and $e(1)$ represents the internal expenses and operational costs for the first year.[2]

In the re/insurance market premiums are seldom at the technical level. The price at which the cover is sold is governed by the market. Indeed, premiums are either below (soft market) or above (hard market) the technical premium. In order to take this business cycle into account in our model, consider for year t the loss ratio

$$\mathrm{LR}(t) = \frac{X(t)}{P(t)}. \tag{2.11}$$

The business cycle is then modeled through a recursion formula for the premiums (we assume that the relative loss experience is similar for the entire market). We compute the market premium, as opposed to the technical premium, for year t ($t \geq 2$) as

$$P(t) = P(t-1) \cdot \begin{cases} (1-s) & \text{if} \quad \mathrm{LR}(t-1) < \mathrm{LR}_s & \text{(softening)} \\ 1 & \text{if} \quad \mathrm{LR}_s \leq \mathrm{LR}(t-1) < \mathrm{LR}_h & s,h>0 \\ (1+h) & \text{if} \quad \mathrm{LR}(t-1) \geq \mathrm{LR}_h & \text{(hardening)} \end{cases} \tag{2.12}$$

where LR_s and LR_h are two fixed thresholds for the loss ratio above (below) which a softening (hardening) of the premium by the factor s (h) takes place. Putting s and h to zero will suppress the cycle. In addition, we impose the premium to be at least as large as the expected loss.

If at the end of a year t, the available capital can not be raised back to the level $C(0)$, but only to a smaller level $C(t)$, then the company can only write a

[2] In Sect. 3.1, a modified formula, where the premium income at the beginning of the year reduces the required risk-adjusted capital amount will also be considered.

corresponding fraction of the business for the next year.[3] By the homogeneity property of the value-at-risk, we correspondingly have the exposure rate for year t as

$$\varepsilon(t) = \frac{C(t-1)}{\rho(X(1))}. \tag{2.13}$$

Clearly, $\varepsilon(1) = 1$. The actual written premium, the incurred loss and the incurred expenses for year t are then

$$W(t) = P(t) \cdot \varepsilon(t), \quad L(t) = X(t) \cdot \varepsilon(t) \quad \text{and } I(t) = e(t) \cdot \varepsilon(t), \tag{2.14}$$

where $e(t)$ are the internal expenses for year t.

2.3 Accounting procedure: profit and loss and booking variables

We assume that the premiums are received by the company at the beginning of the year while the claims and expenses are settled at the end of the year. For the case with time diversification, the new reserves are also set at the end of the year, when the actual size of aggregate claim payment is known. From (2.14), the underwriting result $U(t)$ at the end of year t is given by

$$U(t) = W(t) - L(t) - I(t). \tag{2.15}$$

Furthermore, the company has two additional sources of income: the interest on capital and the interest on the premiums (we choose again a risk-free interest rate r). This leads to the operating result, $R(t)$:

$$R(t) = U(t) + \rho(X(t)) \cdot r + W(t) \cdot r. \tag{2.16}$$

We assume here a limited liability company, i.e. if $R(t) < -\rho(X(t))$, then the company goes bankrupt. In that case, the invested capital is lost (technically, we will assume it to remain on level zero for the rest of time) and the company is not allowed to enter new business anymore. Nevertheless, the shareholders continue to receive interest on their cumulated earnings until maturity τ.

If $-\rho_X(t) < R(t) < 0$, then (part of) the capital has to be used for the settlement of claims, and the company has to raise capital again to regain the original capital amount $C(0)$ if possible. For that purpose, it can use the accumulated dividends and interest of the previous years (if this amount is not sufficient, then the company has to reduce the exposure ε as discussed above). If the needed amount for the capital increase is $N(t)$, then the actual pre-tax cost, $Ex(t)$ of forced increase in capital is

$$Ex(t) = N(t) \cdot c \tag{2.17}$$

where c is the cost of increasing capital.[4] All other costs are neglected.

If $R(t) > 0$ and $\varepsilon(t) < 1$ (i.e. current exposure is less than 100 %), the profit of the running year is used to rebuild capital (increasing exposure) again for the next

[3] In our model the capital never exceeds $C(0)$, as additional profits are paid out as dividends.

[4] An increase of capital happens through intermediaries (investment banks), who will ask for money for their services, which we refer to as the *cost of raising capital*. It stems from the fact that the company is in distress and thus the price of their shares goes down. Correspondingly c is to be distinguished from κ.

year (up to the original amount of $C(0)$). The profit before taxes at the end of year t is hence the operating result $R(t)$ minus the cost of increasing capital $C(t)$:

$$\Pi(t) = R(t) - Ex(t). \tag{2.18}$$

The amount of taxes that the company has to pay is then

$$T(t) = \gamma \cdot \Pi(t) - D(t-1) \tag{2.19}$$

where γ is the tax rate and $D(t-1)$ are the deferred taxes from the previous year ($D(0) = 0$). Indeed, if there is a negative profit in a certain year, this amount can be subtracted from taxable profit in the following years, i.e.

$$D(t) = D(t-1) - \gamma \cdot \Pi(t) \tag{2.20}$$

(in other words, $D(t)$ increases in case of negative profit and decreases in case of positive profit). The profit after tax is finally

$$\widehat{\Pi}(t) = \Pi(t) - T(t). \tag{2.21}$$

If $\varepsilon(t) < 1$ (i.e. the capital could not be raised back to the original level $C(0)$ for the time period $(t-1, t)$, even when using earlier profits), this profit is first used to rebuild capital. The remaining profit is then paid as dividends $\delta(t)$ to shareholders. At the end of the first year, the amount of dividends is $\delta(1) = \widehat{\Pi}(t)_+$ (because $\varepsilon(1) = 1$), where $a_+ = \max(a, 0)$. For $t \geq 2$, we correspondingly have

$$\delta(t) = \left[\widehat{\Pi}(t) - (\rho(X(0) - \rho(X(t))\right]_+. \tag{2.22}$$

The 'shareholder account' balance $A(t)$ at the end of year t is

$$A(t) = [(1+r)A(t-1) + \delta(t) - N(t)]_+, \quad t \geq 1,$$

which is the previous amount plus interest (according to the risk-free interest rate r) plus new dividends minus possibly needed capital $N(t)$, when built up from earlier profits. Note that $A(0) = 0$. The wealth of the shareholders at time t finally is

$$M(t) = A(t) + \rho(X(t+1)) \cdot 1_{\{\text{no bankruptcy up to time } t\}}, \tag{2.23}$$

where $1_{\{F\}}$ is the indicator function of the event F. The annual profit $Z(t)$ paid to the shareholders at the end of year t ($t \geq 1$) is given by

$$Z(t) = \max(rA(t-1) + \delta(t) - N(t), -A(t-1)),$$

which includes dividend gains and interest on previous payments minus possibly needed capital $N(t)$. In particular, $Z(t)$ can be negative in a year where capital needs to be rebuilt.

2.4 Equalization reserves

Whereas the US-GAAP company pays out all the profits as dividends (in the way described above), the time-diversified company is allowed to build up equalization reserves (we assume up to an upper limit of $C(0)$). Concretely, whenever the actual incurred loss $L_T(t)$ is below the expected claim size[5], the difference between the two is added to the equalization reserves. Let $\Re_T(t)$ be the value of the equalization reserves at time t. By definition, $\Re_T(0) = 0$. We then have

$$\Re_T(t) = \min\left(\Re_T(t-1) + (\varepsilon(t)\cdot E[X] - L_T(t))_+ - V_T(t), C(0)\right) \tag{2.24}$$

where $V_T(t) = \min\left((L_T(t) + I_T(t) - W_T(t))_+, \Re_T(t-1)\right)$ are the reserves that are released in case there is a negative underwriting result to neutralize. The underwriting result at time t for the time-diversified company is given by

$$U_T(t) = W_T(t) - L_T(t) - I_T(t) - (\varepsilon(t)\cdot E[X] - L_T(t))_+ + V_T(t). \tag{2.25}$$

The operating result reads

$$R_T(t) = U_T(t) + r(\rho(X(t)) + W_T(t) + \Re_T(t-1))$$

and its loss is bounded by $\rho(X(t)) + \Re_T(t-1)$. From here on, the procedures for using capital to deal with the case $R_T(t) < 0$, the event of bankruptcy for $R_T(t) < -\rho(X(t))$ as well as the distribution of dividends (and corresponding tax payments) in the case of $R_T(t) > 0$ are identical to those of the US-GAAP company.

We would like to emphasize at this point the fundamental difference between reserves and capital, as it is sometimes argued that capital should cover the entire risk. In all actuarial practice, the reserves are invested at the risk-free rate while the capital needs to cover the cost of capital (see Eq. 2.10), which is much higher than the risk-free rate. For being able to reward the capital, re/insurers take risks. On the reserves, the companies are not allowed to take more risk. They are here to cover losses. That is why we only take risk on the capital as shown in (2.13) and not on the equalization reserves.

2.5 Performance measures

In order to compare whether equalization reserves can be an advantage for the shareholders of the company, we need to settle the criteria according to which we measure the performance of the two companies. In the following we discuss four possibilities of performance measures.

2.5.1 Profitability index

Consider first a very simple measure related to the cash-flows generated by the company. The net present value NPV of the cash-flows $Z(t)$ (the shareholders' earnings) of each year can be discounted to time zero by a risk-free interest rate r (see e.g. [3]). Then we obtain

$$\mathrm{NPV} = \sum_{t=0}^{\tau} \frac{Z(t)}{(1+r)^t}, \tag{2.26}$$

where τ is the number of years considered in the analysis. The profitability index is then defined as

[5] We use the index T to denote the respective quantity for the time-diversified company.

$$\mathrm{PI} = \frac{\mathrm{NPV}}{\rho(X(1))}, \tag{2.27}$$

which allows to quantify the amount of value created per unit of investment. A value bigger than 1 means that the investment produces value for the shareholders, whereas a value lower than 1 means that the investment is not profitable. This measure is sometimes used to rank different investment opportunities of a firm. If the company is still solvent at time τ, the capital $C(t)$ will be part of the last cash-flow ($C(\tau) = C(0)$ if $\varepsilon(\tau) = 1$). Apart from that, the PI does not account for the involved risk.

2.5.2 Internal rate of return

Another common way to look at the performance of an investment is the so-called internal rate of return (IRR), cf. [3]. The IRR is the rate that makes the net present value of all cash-flows earned from an initial investment equal to zero:

$$\sum_{t=1}^{\tau} \frac{Z(t)}{(1 + IRR)^t} - \rho_0(X) = 0 \tag{2.28}$$

Thus, the higher the IRR (for the same initial investment), the more desirable and valuable this investment opportunity. Again, the final cash-flow at time τ will include the still available invested capital $C(\tau)$ in case the company is not bankrupt. The IRR is quite popular because of its strong intuitive appeal. Note that it may not always be possible to find a positive rate IRR for which (2.28) holds (in the simulations below, those trajectories will then not be used for estimating the average value of IRR).

2.5.3 Sharpe ratio

Another possible performance measure for the shareholder is the Sharpe ratio [8]. It is widely used among investors and is risk-adjusted, giving the excess of return over the risk-free rate per unit of risk taken. Define the yearly return by

$$\mathrm{Re}(t) = \frac{M(t) - M(t-1)}{M(t-1)}$$

for each $t = 1,\ldots, \tau$, where $M(t)$ is the total wealth of the shareholders at time t. Then the Sharpe ratio is calculated by

$$SR = \frac{\frac{1}{\tau}\sum_{t=1}^{\tau} \mathrm{Re(t)} - \mathrm{r}}{v}, \tag{2.29}$$

where ν is the empirical standard deviation of the excess returns ($Re(t) - r$) over the τ years.

2.5.4 Call option value based on the Merton model

Using the concept of real options for the valuation of a firm's equity (cf. Myers [6]), one can define a performance measure in terms of the present value of a call option on the assets of the firm at the maturity date τ, where the exercise price is the initial amount of invested capital $C(0) = \rho(X(1))$.

The value of this call option can be written as

$$\mathrm{MM} = \frac{E[M(\tau) - \rho(X(1))]_+}{(1+r)^\tau}, \tag{2.30}$$

where $M(\tau)$ is the shareholder's wealth at maturity and $\rho(X(1))$ is the initial investment. Here the expectation is assumed with respect to the physical probability measure.[6] Note that this approach implicitly assumes that the investment has no value for the investor, if the final wealth is less than the amount of the initial investment. We will use Monte Carlo simulation to calculate the value of the option, which may be interpreted as a risk-corrected measure of the equity value of the firm. A shareholder will want to hold shares of the company with the highest equity value.

3 Simulation results

In this section we simulate the expected value of the above performance measures for both the US-GAAP and the time diversification company. Based on 50,000 independent replications of the dynamics of the wealth of the insurance company over the time period of τ years, we give a Monte Carlo estimate for each of these performance measures. In Table 1, we define a standard set of parameters on which the simulations are based. These numbers are motivated from the magnitudes one could typically have in insurance practice.[7] In subsequent subsections we will vary the parameters one at a time to assess the sensitivity with respect to that parameter, when the other parameters remain at the standard value.

In Tables 2 and 3, we give the simulation results together with the 95% asymptotic confidence interval for the expected values of the performance measures based on the standard set of parameters. One sees that quite consistently the PI and the IRR measure favor the US-GAAP company, whereas time diversification is preferable for the Sharpe ratio (SR) and for the Merton call option value (MM).[8] One should note that both PI and IRR do not treat the risk component of the cash-flows. Because of discounting, they favor shorter term projects with earlier cash-inflows, and as there is no penalization of bankruptcy beyond the loss of the initial capital $C(0)$, PI and IRR of the US-GAAP can then outperform the one of the time-diversified company. Note also that the results for the IRR are slightly biased as it can happen that in case of bankruptcies there exists no positive IRR rate to match (2.28). These trajectories are then not considered for the estimation of the expected

[6] Alternatively, the shareholder's wealth at maturity could also be compared with the initial capital amount invested at the risk-free rate, i.e. $E[M(\tau) - (1+r)^\tau \rho(X(1))]_+/(1+r)^\tau$. The numerical results turn out to be similar and we therefore restrict the present analysis to (2.30).

[7] We use the VaR at the level $\theta = 0.99$ for illustrative purposes (the results do not change in any significant way if the value $\theta = 0.995$, employed in Solvency II, is used). The choice of $C(0)$ is without loss of generality.

[8] It takes about 3 minutes on a usual PC to obtain all the estimates of one such table.

Table 1 Standard set of parameters

Standard parameters	
Risk-free rate r	3 %
Cost of raising capital rate c	5 %
Cost of capital rate κ	15 %
Hardening h	200 %
Softening s	20 %
Threshold LR_h	150 %
Threshold LR_s	60 %
Tax rate γ	25 %
Time horizon τ	30 years
Risk quantile θ	0.99
Initial capital level $C(0)$	100,000

Table 2 Expected performance index, internal rate of return, Sharpe ratio and the Merton call option value for lognormal losses for the US-GAAP and the time diversification company as a function of coefficient of variation

CoV:	0.1	1	10	20
E(PI)				
US-GAAP	3.3174 ± 0.0035	3.4221 ± 0.0126	1.6783 ± 0.0076	1.6565 ± 0.0072
Time div.	3.1628 ± 0.0031	3.3719 ± 0.0122	1.522 ± 0.0077	1.4681 ± 0.0073
E(IRR)				
US-GAAP	16.282 ± 0.016	16.176 ± 0.045	11.275 ± 0.034	11.327 ± 0.033
Time div.	14.504 ± 0.010	14.317 ± 0.042	9.161 ± 0.034	8.897 ± 0.033
E(SR)				
US-GAAP	1.1033 ± 0.0016	0.4314 ± 0.0014	0.4227 ± 0.0023	0.4485 ± 0.0024
Time div.	1.4303 ± 0.0015	0.6085 ± 0.0021	0.4752 ± 0.0024	0.4781 ± 0.0024
MM				
US-GAAP	262,598 ± 236	269,520 ± 874	144,094 ± 543	142,008 ± 514
Time div.	262,690 ± 235	279,778 ± 834	146,039 ± 557	143,033 ± 536

IRR. On the other hand, SR and MM are risk-adjusted measures of the performance. For these two measures, the time diversification company outperforms the US-GAAP company for all simulated degrees of heaviness of the loss distribution.

It is interesting to note that the average value of SR decreases for higher values of the coefficient of variation (Table 2) and increases again for very heavy tails (Table 3).[9]

A measure that does not consider profitability, but safety only, is to merely compare the number of resulting bankruptcies for the US-GAAP and the time

[9] This is further underlined by the fact that additional simulations show that for CoV = 200 in Table 2, one would have E(SR) = 0.6194 (US-GAAP) and E(SR) = 0.7642 (Time div.), whereas for much lighter Fréchet tails ($\alpha = 10$ in Table 3) one would have E(SR) = 1.0066 (US-GAAP) and E(SR) = 1.3547 (Time div.).

Table 3 Expected performance index, internal rate of return, Sharpe ratio and the Merton call option value for Fréchet losses for the US-GAAP and the time diversification company as a function of α

α:	1.9	1.5	1.3	1.1
E(PI)				
US-GAAP	1.9577 ± 0.0096	1.6933 ± 0.0083	1.6637 ± 0.0076	2.5183 ± 0.0081
Time div.	1.6442 ± 0.0095	1.3212 ± 0.0081	1.2259 ± 0.0073	1.9735 ± 0.0075
E(IRR)				
US-GAAP	13.187 ± 0.042	12.512 ± 0.038	12.651 ± 0.036	16.658 ± 0.036
Time div.	11.459 ± 0.041	10.445 ± 0.038	10.063 ± 0.036	11.905 ± 0.035
E(SR)				
US-GAAP	0.4111 ± 0.0018	0.4305 ± 0.0021	0.4675 ± 0.0023	0.6451 ± 0.0027
Time div.	0.538 ± 0.0023	0.543 ± 0.0025	0.5738 ± 0.0026	0.9372 ± 0.0038
MM				
US-GAAP	196,819 ± 744	175,514 ± 659	171,869 ± 618	229,758 ± 690
Time div.	203,002 ± 738	180,206 ± 664	175,551 ± 630	235,373 ± 693

diversification company among the 50,000 simulated 30 year periods (cf. Table 4). In general, the heavier the tail, the more defaults we observe, and the differences between the two types of companies are substantial. These differences are even more pronounced when the tail of the loss distribution is not very heavy, as in particular in those cases the equalization reserve can be the decisive difference to survive a big loss (for very heavy tails, the equalization reserves can typically be built up slightly quicker, but are then still less often sufficient to avoid bankruptcies, as very large claims are more likely). Altogether, it is clear that keeping equalization reserves in order to limit the future losses has a good influence on the well-being of the company.

The average equalization reserve level over time for all 50'000 simulations is depicted in Figure 1. One sees that typically the heavier the tail of the loss distribution, the higher the equalization reserve level that is built up (recall that it is built up by the positive difference of expected and actual claim size, and bounded from above by $C(0) = 100'000$). At the same time, the differences are not strongly pronounced, except for the very heavy Fréchet loss distribution ($\alpha = 1.1$).

Table 4 Number of bankruptcies for the lognormal (left) and Fréchet (right) distribution

Lognormal distribution			Fréchet distribution		
CoV	US-GAAP	Time div.	α	US-GAAP	Time div.
0.1	0	0	1.9	1,717	1,067
1	1,030	399	1.5	1,987	1,288
10	2,243	1,517	1.3	2,123	1,364
20	2,305	1,594	1.1	2,154	1,324

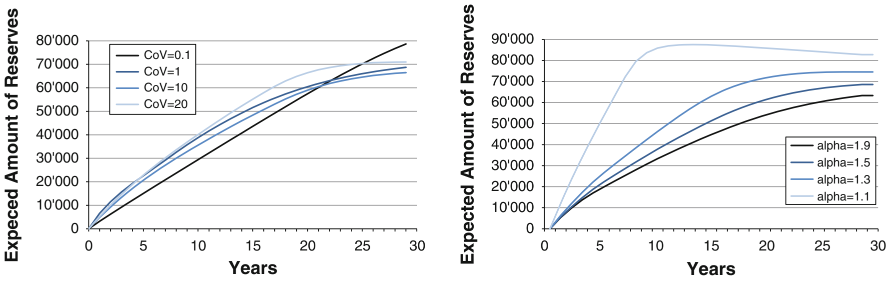

Fig. 1 Expected equalization reserve level over time for Lognormal (*left*) and Fréchet (*right*) losses

3.1 Sensitivity with respect to the RAC calculation

Instead of using (2.10) for the technical premium, one could also argue as follows. As the premium $P(t)$ is already collected at the beginning of the year, only the additional capital $\rho(X(t) - P(t))$ is actually needed. By translation invariance of VaR this leads to

$$P(t) = \frac{E[X(t)] + \kappa \cdot (\rho(X(t)) - P(t))}{1 + r},$$

which results in

$$P(t) = \frac{E[X(t)] + \kappa \cdot \rho(X(t))}{(1 + r)(1 + \kappa)} + e(t), \qquad (3.1)$$

and is smaller than the previous premium (because the RAC is lower). Here the expenses $e(t)$ are added at the end, as they do not contribute to the risk bearing. The simulated values under this alternative premium rule are depicted in Tables 5 and 6. One sees that this modification does not affect the relative differences between the US-GAAP and the time diversification companies. Yet, the absolute values of the resulting performance measures are all slightly lower (and for CoV = 0.1 of the Lognormal case substantially lower), which may be explained by the fact that for lower RAC more risk can be accepted for the same capital amount $C(0)$, resulting in a more volatile insurance business.

3.2 Impact of time horizon

It is natural to look at the sensitivity of the results with respect to the time horizon under consideration. In Tables 7 and 8 below we give the adapted values of Tables 5 and 6 when the time horizon is reduced to 15 years. The US-GAAP company is still preferable w.r.t. the PI and IRR measure, and now also for heavier tails w.r.t. the Sharpe ratio, whereas w.r.t. the Merton call option value time diversification remains preferable throughout. Note that only the (absolute) values of the expected Sharpe ratio increase when halving the time horizon.

We now study a couple of further sensitivities by varying one parameter and leaving the others at their standard value (cf. Table 1), i.e. we study the deviations

Table 5 Expected performance index, internal rate of return, Sharpe ratio and the Merton call option value for lognormal losses for the US-GAAP and the time diversification company as a function of coefficient of variation, premium rule (3.1)

CoV:	0.1	1	10	20
E(PI)				
US-GAAP	0.6242 ± 0.0031	3.276 ± 0.0125	1.5462 ± 0.0075	1.5216 ± 0.007
Time div.	0.5684 ± 0.0027	3.2298 ± 0.0121	1.3914 ± 0.0075	1.3364 ± 0.0071
E(IRR)				
US-GAAP	5.858 ± 0.014	15.186 ± 0.043	10.329 ± 0.034	10.356 ± 0.033
Time div.	5.349 ± 0.011	13.515 ± 0.041	8.370 ± 0.034	8.100 ± 0.033
E(SR)				
US-GAAP	0.1991 ± 0.0015	0.4138 ± 0.0014	0.4194 ± 0.0025	0.4544 ± 0.0027
Time div.	0.4112 ± 0.0041	0.5721 ± 0.002	0.4525 ± 0.0025	0.4557 ± 0.0025
MM				
US-GAAP	81,577 ± 200	261,325 ± 867	138,962 ± 534	134,754 ± 502
Time div.	82,618 ± 198	271,579 ± 828	138,962 ± 547	135,822 ± 522

Table 6 Expected Performance index, Internal Rate of Return, Sharpe ratio and Merton Call option value for Fréchet losses for the US-GAAP and the time diversification company as a function of α, premium rule (3.1)

α:	1.9	1.5	1.3	1.1
E(PI)				
US-GAAP	1.8331 ± 0.0094	1.5636 ± 0.0082	1.5280 ± 0.0074	2.3908 ± 0.0081
Time div.	1.5207 ± 0.0093	1.194 ± 0.008	1.0913 ± 0.0071	1.8451 ± 0.0075
E(IRR)				
US-GAAP	12.293 ± 0.041	11.539 ± 0.038	11.647 ± 0.036	15.530 ± 0.035
Time div.	10.708 ± 0.041	9.656 ± 0.038	9.267 ± 0.036	11.125 ± 0.035
E(SR)				
US-GAAP	0.3954 ± 0.0018	0.4198 ± 0.0022	0.4663 ± 0.0025	0.6853 ± 0.0032
Time div.	0.5075 ± 0.0022	0.5147 ± 0.0025	0.5485 ± 0.0026	0.9193 ± 0.0038
MM				
US-GAAP	190,145 ± 735	167,875 ± 650	164,074 ± 604	222,753 ± 680
Time div.	196,385 ± 727	172,317 ± 654	167,794 ± 613	228,181 ± 683

from the values in Tables 2 and 3. For brevity, we depict the corresponding simulation results graphically only. Also, we restrict this sensitivity analysis to the Sharpe ratio and the Merton call option values, as those two performance measures are of particular interest in the present context.

3.3 Impact of risk-free rate

Figure 2 shows the expected Sharpe ratio for Fréchet losses when the risk-free rate r is varied from its standard value of 3 %. The advantage of the time diversification

Table 7 Performance index, internal rate of return, Sharpe ratio and Merton call option value for lognormal losses for the US-GAAP and the time diversification company as a function of coefficient of variation (premium rule (3.1), time horizon 15 years)

CoV:	0.1	1	10	20
E(PI)				
US-GAAP	0.2461 ± 0.0021	1.5104 ± 0.0073	0.7863 ± 0.0048	0.7831 ± 0.0045
Time div.	0.2264 ± 0.0019	1.4848 ± 0.0072	0.7268 ± 0.0049	0.7116 ± 0.0046
E(IRR)				
US-GAAP	5.018 ± 0.017	13.904 ± 0.053	9.245 ± 0.05	9.341 ± 0.049
Time div.	4.676 ± 0.014	12.526 ± 0.05	7.733 ± 0.053	7.588 ± 0.052
E(SR)				
US-GAAP	0.2225 ± 0.0023	0.4838 ± 0.0018	0.6065 ± 0.0033	0.66 ± 0.0035
Time div.	0.3706 ± 0.0052	0.5654 ± 0.002	0.4801 ± 0.0026	0.4544 ± 0.0024
MM				
US-GAAP	31,769 ± 107	99,585 ± 391	59,210 ± 242	58,975 ± 227
Time div.	32,230 ± 106	102,021 ± 383	59,952 ± 241	59,613 ± 227

Table 8 Performance index, internal rate of return, Sharpe ratio and Merton call option value for Fréchet losses for the US-GAAP and the time diversification company as a function of α (premium rule (3.1), time horizon 15 years)

α:	1.9	1.5	1.3	1.1
E(PI)				
US-GAAP	1.0661 ± 0.0062	0.9476 ± 0.0055	0.9506 ± 0.0052	1.4187 ± 0.0054
Time div.	1.0277 ± 0.0063	0.8936 ± 0.0056	0.8761 ± 0.0053	1.2735 ± 0.0055
E(IRR)				
US-GAAP	10.951 ± 0.054	10.275 ± 0.053	10.548 ± 0.051	14.548 ± 0.048
Time div.	9.675 ± 0.055	8.766 ± 0.055	8.512 ± 0.054	10.284 ± 0.054
E(SR)				
US-GAAP	0.4987 ± 0.0024	0.5578 ± 0.0029	0.6373 ± 0.0033	0.9451 ± 0.0043
Time div.	0.5096 ± 0.0023	0.5011 ± 0.0025	0.4734 ± 0.0024	0.6026 ± 0.0023
MM				
US-GAAP	117,408 ± 509	107,116 ± 445	106,320 ± 412	143,927 ± 438
Time div.	119,641 ± 504	108,951 ± 442	107,858 ± 410	145,830 ± 437

company over the US-GAAP company becomes larger for larger values of r. The Merton call option value becomes lower, the higher the value of r is (cf. Fig. 3). This is quite intuitive, as the expected payoff at maturity τ is discounted with r. Whereas time diversification is then still preferable, the degree of outperformance becomes smaller. It turns out that for lognormal claims the behavior is very similar. On the other hand, the IRR and PI scale up as a linear function of r.

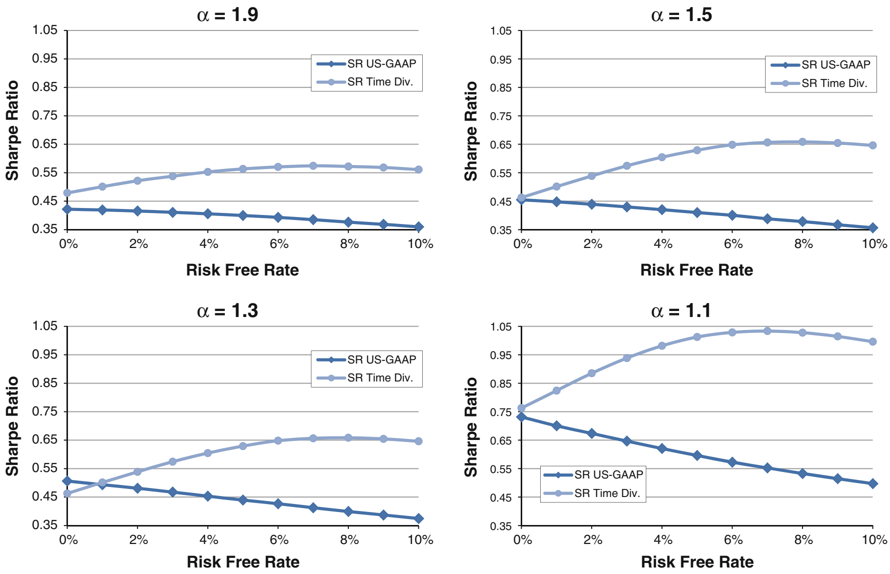

Fig. 2 Expected Sharpe ratios with Fréchet losses as a function of risk-free rate

3.4 Impact of cost of raising capital

Figure 4 shows the expected Sharpe ratio for Lognormal losses, when the cost of raising capital rate c is varied between 0 to 80 %. Whereas this value decreases with increasing c, it decreases less for the time diversification company, increasing the advantage of the latter (this effect becomes less pronounced for heavier tails, as then both companies are more likely to experience a claim that is too large to survive, even with equalization reserves).

3.5 Impact of market conditions

Whereas an increase of the hardening constant h does not have much effect on the expected Sharpe ratio in the lognormal loss case (in particular for low values of the CoV, see Fig. 5), a larger value of h leads to a substantial (and almost linear) increase of the Merton call option values for Fréchet losses (Fig. 6). In each of these cases, keeping equalization reserves remains preferable. Likewise, a more substantial softening of the market leads to a lower performance (in particular for more heavy-tailed losses). The difference in performance decreases with increasing value of s (cf. Figs. 7, 8), but is always in favor of time diversification. In the case of Lognormal claims with CoV = 0.1, the results are robust with respect to the choices of h and s, as there is very little fluctuation of the losses, so the actual values of s and h are only seldomly applied.

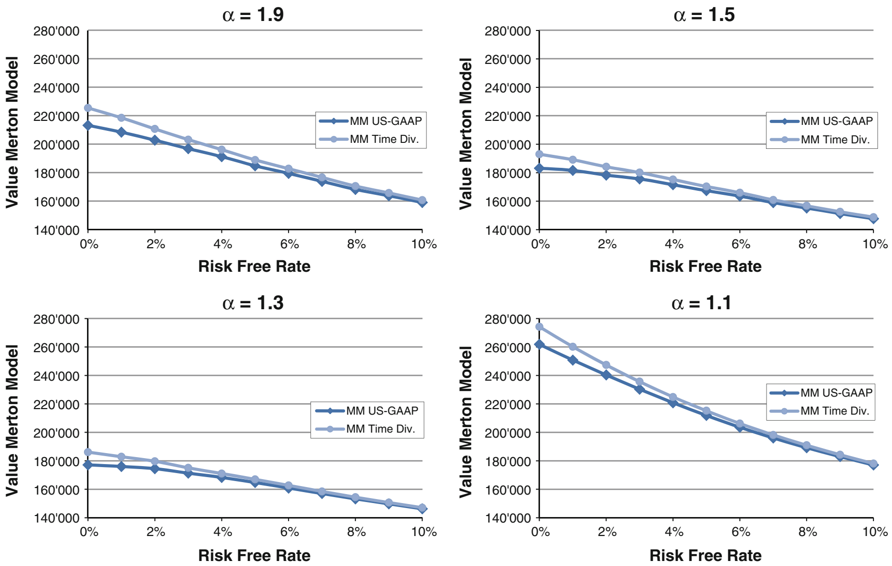

Fig. 3 Merton call option values with Fréchet losses as a function of risk-free rate

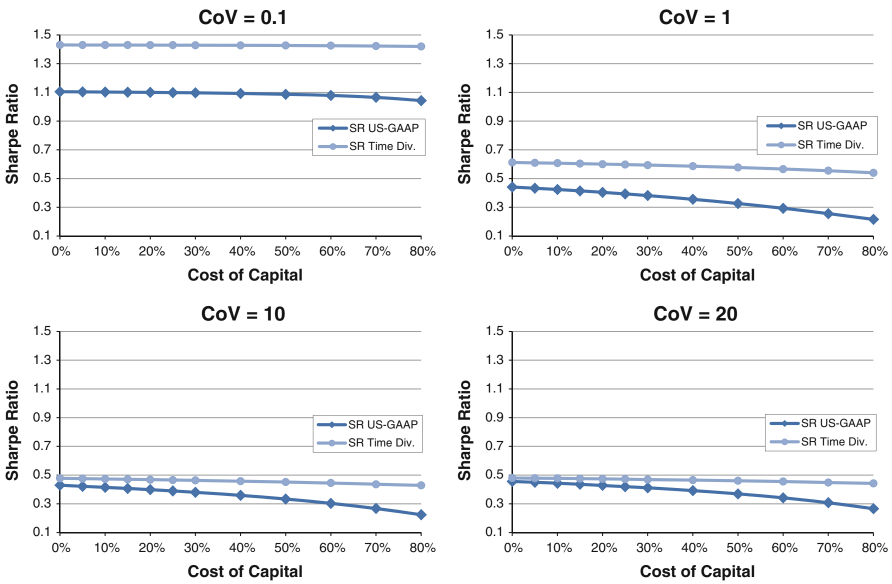

Fig. 4 Expected Sharpe ratio with lognormal losses as a function of cost of raising capital rate

3.6 Taxation

Finally, we would like to address the question whether equalization reserves reduce tax income for the authorities, as is claimed in some discussions. Figure 9 depicts

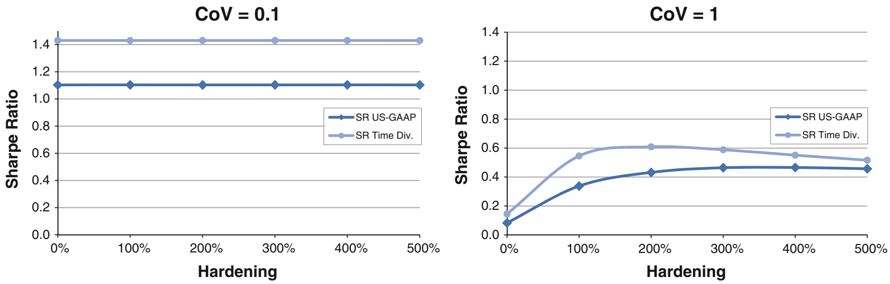

Fig. 5 Expected Sharpe ratios under a hardening of the market cycle for lognormal losses

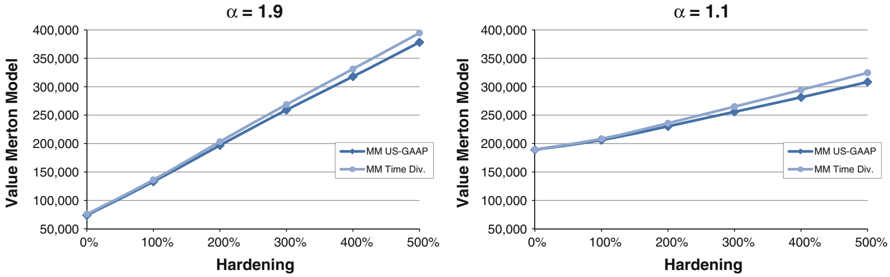

Fig. 6 Value of the call option under a hardening of the market cycle for Fréchet losses

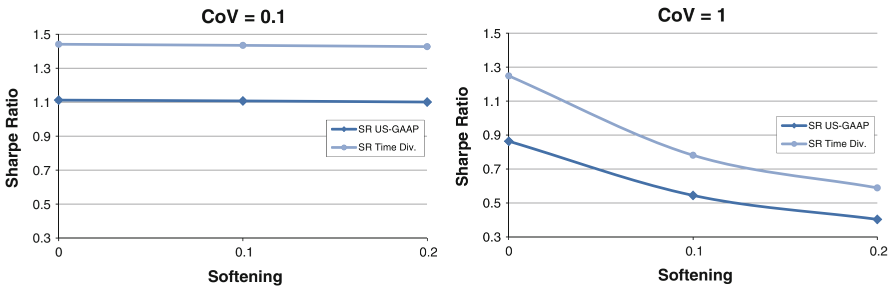

Fig. 7 Value of the call option under a softening of the market cycle for lognormal losses

the total amount of taxes up to maturity τ, and Fig. 10 gives the same plots, but now all tax payments are discounted to time 0 at the risk-free rate r. One can see that even when including discounting, there is no advantage of the US-GAAP company over the time diversification company when considered over the entire time horizon. Larger early tax payments of the US-GAAP company are later compensated by continuing tax payments of the time diversification company, in particular when bankruptcy can be avoided and after τ years the released equalization reserves are taxed. In the absence of discounting, this latter effect even dominates and the total tax payments are on average larger under time diversification.

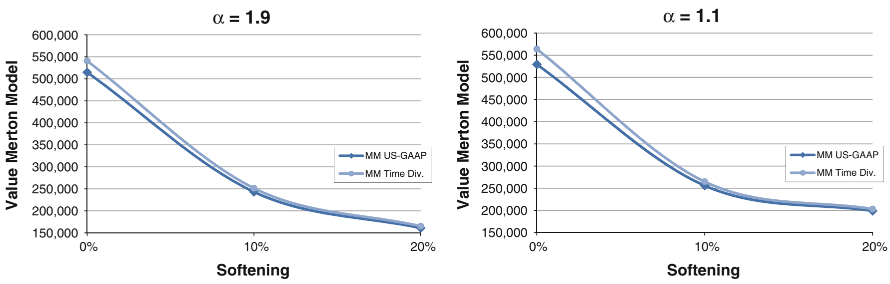

Fig. 8 Expected Sharpe ratios under a softening of the market cycle for Fréchet losses

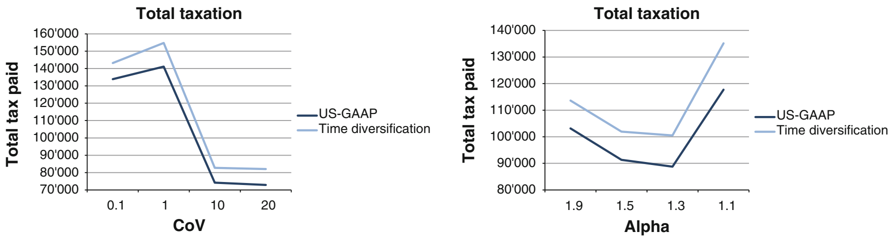

Fig. 9 Total non-discounted tax payments for lognormal (*left*) and Fréchet (*right*) claims

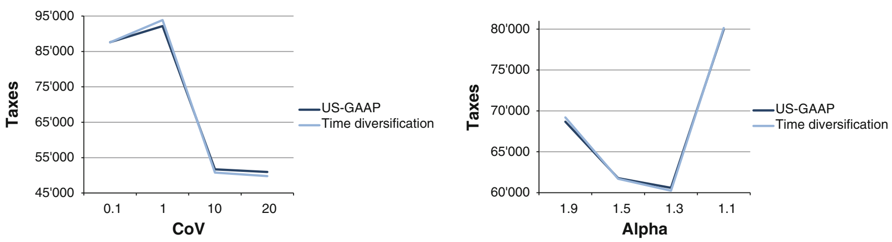

Fig. 10 Total discounted tax payments for lognormal (*left*) and Fréchet (*right*) claims

4 Conclusion

The time horizon of insurers goes much beyond the one year requested by Solvency II for the quantitative risk assessment. When new accounting rules are introduced, one needs to make sure that the need for long-term thinking of the management of insurance is sufficiently considered. Equalization reserves are a time-honored concept for the insurance of heavy-tailed risks, which lead to a more balanced view of the insurer's long-term profitability. Major strategic advantages for a time-diversified company are

- less need for expensive recapitalization in case of losses
- lower probability to experience bankruptcy with the company (and then not being able to reinvest again)

- (as a consequence) higher chance to profit from business cycles

On the down-side, a disadvantage for a time-diversified company is

- risk to lose more capital if the loss is bigger than the initial capital $C(0)$.

In this paper, we illustrate by implementing a CAT risk insurance model that equalization reserves can be beneficial for shareholders, even when classical performance measures such as the Sharpe ratio and the call option value of Merton type are used. Although not allowed in the US-GAAP and IFRS accounting rules, they seem to represent a viable and cheap alternative to reinsurance when facing large claim fluctuations. At the same time, the resulting total discounted tax payments are not smaller, they are just more equalized. The results are remarkably robust when varying model parameters.

All in all, using this model we indicate that the main objections against equalization reserves—diminished shareholder value and scheme to evade taxes—can not be claimed in general. Introducing a transparent rule on how to build equalization reserves (as attempted in this paper) could be a way to satisfy all stakeholders and help reduce the price of catastrophe covers for policyholders by reducing the capital requirements through increased diversification.

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