ASYMPTOTIC SAFETY OF GRAVITY AND THE HIGGS-BOSON MASS

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If gravity is asymptotically safe, then the ultimate theory might be just the standard model (minimally supplemented by a few light particles to accommodate neutrino masses and oscillations, dark matter, and the baryon asymmetry of the Universe) plus gravity. If this is indeed the case, then the Higgs-boson mass can be predicted ($m_H = m_{min} \simeq 130\,\text{GeV}$ with an uncertainty of only a few GeV) or constrained to be in the interval $m_{min} < m_H < m_{max} \simeq 174\,\text{GeV}$.

Keywords: asymptotic safety, Higgs boson

1. Introduction

The most minimalistic approach to quantum gravity is associated with asymptotic safety [1]. Although general relativity is nonrenormalizable by perturbative methods, it can exist as a nonperturbative field theory with a nontrivial ultraviolet fixed point (see [2] for a review). A very economical description of all interactions in Nature may be possible in this setting. We can assume that there is no new physics at an intermediate energy scale between the Fermi scale and the Planck scale $M_P = 2.44 \cdot 10^{18}\,\text{GeV}$. All confirmed observational signals in favor of physics beyond the standard model (SM) such as neutrino masses and oscillations, dark matter and dark energy, the baryon asymmetry of the Universe, and inflation can be associated with new physics below the electroweak scale (see [3], [4] and the references therein). The minimal model νMSM contains three relatively light singlet Majorana fermions and the dilaton in addition to the SM particles. These fermions could be responsible for neutrino masses, dark matter, and the baryon asymmetry of the Universe. The dilaton can lead to dynamical dark energy [5], [6] and realizes a spontaneously broken scale invariance that either emerges from the cosmological approach to a fixed point [5], [7] or is an exact quantum symmetry [8], [9]. Inflation can occur as a result of either the presence of the Higgs particle in the SM [10] or the asymptotically safe character of gravity [11]. One more part of new physics, for example, including the strong CP problem or the flavor problem, can be associated with the Planck scale.

But this standpoint encounters an obstacle, which is related to the Landau-pole problem for several couplings in the SM (or the νMSM). Namely, the $U(1)$-gauge coupling $g' \equiv g_1$, the Higgs self-coupling λ, and Yukawa couplings (most notably, that of the top quark, y_t) are not asymptotically free. This makes it impossible to formulate the *fundamental* SM, leaving it the role of an *effective* field theory applicable only below some energy scale.

Here, based on [12], we discuss a scenario that can overcome the indicated difficulty and allows predicting the Higgs mass, which can be tested at the LHC. In Sec. 2, we briefly review asymptotic safety. In Sec. 3, we discuss how the asymptotically safe SM can emerge as a result of combining the SM with asymptotically safe gravity, and we predict the Higgs mass. We present conclusions in Sec. 4.

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2. Asymptotic safety

A "good" quantum field theory can be sought as follows:

1. We take some specified set of quantum fields and write the most general Lagrangian respecting chosen symmetries and including operators of arbitrary dimension.

2. We compute all scattering amplitudes in all orders of the perturbation theory.

3. We require that the theory be unitary, Lorentz invariant, and causal, which leads to an infinite number of conditions for an infinite number of coupling constants defining the theory.

4. We solve these consistency equations, hoping that the theory is characterized by a finite number of essential parameters (coupling constants), making predictions possible.

Of course, it is very difficult, if not impossible, to realize this program. One approach is based on the renormalization group (RG) [1]. We introduce dimensionless coupling constants g_i for all terms in the action: $g_i = \mu^{D_i} G_i$, where D_i is the canonical dimension of the coupling constant G_i and μ is an arbitrary parameter with the dimension of mass. The RG equations are derived from the requirement that physical amplitudes be independent of μ. This leads to the running of couplings $g = \{g_i\}$ as

$$\mu \frac{\partial g_i}{\partial \mu} = \beta_i(g)$$

and fixes the β-functions.

The renormalizable asymptotically free theories correspond to Gaussian ultraviolet (UV) fixed points: essential couplings $g_i(\mu) \to 0$ as $\mu \to 0$. The number of these couplings is finite: only operators with a dimension not exceeding four are allowed. Well-known examples of asymptotically free theories include quantum chromodynamics, certain grand unified theories, and renormalizable theories in two- and three-dimensional space–time.

The asymptotically safe theories are associated with non-Gaussian UV fixed points $g^* \neq 0$: $\beta_i(g^*) = 0$. Although they are nonrenormalizable, they are predictive if the dimensionality of the critical surface in the space of coupling constants (points are attracted to g^* as $\mu \to \infty$) is finite. Known examples include the scalar field theory in three dimensions at the Wilson–Fischer fixed point (the critical surface is two-dimensional), the nonlinear σ model [13], and gravity in $2+\epsilon$ dimensions [1], [14]–[16].

Determining whether some theory is asymptotically safe is complicated because the standard perturbative expansion fails. Common methods include the ϵ-expansion [17], lattice simulations [18], [19], and the functional RG [20], [21]. Weinberg's original conjecture that gravity might be asymptotically safe was based on the ϵ-expansion. The extensive studies of the functional RG for gravity were initiated in [22] and continued in [23], [24], where further evidence for it was presented. In what follows, we assume that gravity is indeed asymptotically safe.

3. Asymptotically safe SM and the Higgs-boson mass

The standalone SM is neither asymptotically free nor asymptotically safe. It suffers from Landau-pole behavior of the $U(1)$-gauge constant, the Yukawa terms, and the Higgs self-coupling. But it is not excluded that a combination of the SM with asymptotically safe gravity can change the situation and lead to a consistent theory. In what follows, we discuss how this can happen. We focus on the evolution of the SM gauge coupling constants g_1, $g \equiv g_2$, and g_3 corresponding to the $U(1)$, $SU(2)$, and $SU(3)$ groups and also on the Higgs self-coupling λ and Yukawa couplings y_t for top quarks. We fix the values of the gauge

couplings according to their experimental values at small energies but leave λ and y_t undetermined for the time being.

The RG equations for the matter self-couplings contain a contribution from the gravity sector [1], [25], [26]. In the general case, the RG equations for these couplings with gravity corrections incorporated have the form

$$\frac{dh}{dt} = \beta_h^{\text{SM}} + \beta_h^{\text{grav}}, \tag{1}$$

where $t = \log \mu$; h is any of the couplings g_i, λ, or y_t; β_h^{SM} is the SM contribution; and β_h^{grav} are the gravity corrections. In the one-loop approximation,

$$\beta_1^{\text{SM}} = \frac{1}{16\pi^2}\frac{41}{6}g_1^3, \qquad \beta_2^{\text{SM}} = -\frac{1}{16\pi^2}\frac{19}{6}g_2^3, \qquad \beta_3^{\text{SM}} = -\frac{1}{16\pi^2}7g_3^3,$$

$$\beta_y^{\text{SM}} = \frac{1}{16\pi^2}\left[\frac{9}{2}y_t^3 - 8g_3^2 y_t - \frac{9}{4}g_2^2 y_t - \frac{17}{12}g_1^2 y_t\right],$$

$$\beta_\lambda^{\text{SM}} = \frac{1}{16\pi^2}\left[24\lambda^2 + 12\lambda y_t^2 - 9\lambda\left(g_2^2 + \frac{1}{3}g_1^2\right) - 6y_t^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2\right].$$

The structure of gravity corrections can be deduced from a dimensional analysis:

$$\beta_h^{\text{grav}} = \frac{a_h}{8\pi}\frac{\mu^2}{M_{\text{P}}^2(\mu)}h,$$

where a_1, a_2, a_3, a_y, and a_λ are some constants (anomalous dimensions) corresponding to g_1, g_2, g_3, y_t, and λ and $M_{\text{P}}^2(\mu)$ is the running Planck mass. From studies of the functional RG, we infer a characteristic scale-dependence of the gravitational constant or Planck mass,

$$M_{\text{P}}^2(\mu) = M_{\text{P}}^2 + 2\xi_0\mu^2, \tag{2}$$

where ξ_0 is a number whose exact value is inessential for our considerations. The value $\xi_0 \approx 0.024$ was found from a numerical solution of the functional RG equations [22], [25], [27]. Hence, for large momentum transfer $q^2 \gg M_{\text{P}}^2$, the effective gravitational constant $G_N(q^2)$ scales as $1/16\pi\xi_0 q^2$, ensuring the regular behavior of high-energy scattering amplitudes. Different anomalous dimensions were computed explicitly in [25]–[32]. But we note that there is no agreement between different authors on the magnitude and even the signs of the coefficients a_i. Moreover, the definitions of the matter couplings used in different papers are not the same. The coefficients a_i found in different papers depend on the gauge used and on the form of truncation of the functional RG equations. We assume that some gauge-invariant definition of these couplings will be possible eventually. It will most probably be based on gauge-invariant high-energy scattering amplitudes, as suggested in [1]. We stress that this definition of couplings does not coincide with that based on the minimal subtraction scheme (cf. [33]).

The running of different couplings in the SM can be divided into two regimes. Up to the scales $\mu^2 \sim M_{\text{P}}^2$, the gravitational corrections to the beta functions of the SM are suppressed by the factor μ^2/M_{P}^2 and are therefore small. The couplings run logarithmically up to $\mu^2 \sim M_{\text{P}}^2$. For $\mu^2 \gtrsim M_{\text{P}}^2$, the corrections coming from gravity become important. If the gravitational part of the β-functions dominates and $\mu^2 \gtrsim M_{\text{P}}^2/2\xi_0$, then the running is a power law,

$$h \propto \mu^{a_h/16\pi\xi_0}. \tag{3}$$

Clearly, the signs of the anomalous dimensions a_h play a crucial role for the validity of the SM at any energy scale.

We first consider the gauge sector. For simplicity, we assume that $a_1 = a_2 = a_3 = a$, which holds for one-loop computations performed up to now because the gravitational interactions are universal. All gauge constants are then asymptotically free if $a < a^{\mathrm{crit}} \simeq -0.013$ (the value a^{crit} corresponds to the fixed point $g_1^* \simeq 0.5$ in the $U(1)$ one-loop coupling running if it starts from the experimental value at low energies). If this is the case, then the Landau-pole problem for the $U(1)$ coupling is solved by the gravity contribution to the RG running. And the computations in [28], [26] indeed give a negative sign for a with $|a| \sim 1$. In what follows, we assume that

$$a < a^{\mathrm{crit}}. \tag{4}$$

In this case, the gauge coupling constants cannot be predicted. If $a = a^{\mathrm{crit}}$, then the value of the $U(1)$ coupling is predictable.

We now consider the Yukawa coupling for top quarks. Setting $a = -1$, for example, and taking the (central) experimental value $m_{\mathrm{t}} = 173.1\,\mathrm{GeV}$ for the top-quark mass [34], we find that the behavior of y_{t} is asymptotically free for $a_y < a_y^{\mathrm{crit}} \simeq -0.005$ (it corresponds to a non-Gaussian fixed point with $y_{\mathrm{t}}^* \simeq 0.38$ at $a_y = a_y^{\mathrm{crit}}$), and we obtain the Landau-pole behavior for $a_y > a_y^{\mathrm{crit}}$. The critical value of a_y is only weakly sensitive to a. For example, for $a = -0.02$, we obtain $a_y^{\mathrm{crit}} \simeq -0.002$ and $y_{\mathrm{t}}^* \simeq 0.25$. For smaller values of the top-quark mass, the corresponding values of a_y^{crit} are even closer to zero, while larger m_{t} move a_y^{crit} further from zero.

We suppose that $a_y > a_y^{\mathrm{crit}}$. Then to have a consistent theory for all energy scales, we must set $y_{\mathrm{t}} = 0$. This corresponds to the massless top quark and disagrees with the experimental data. In other words, if this happens to be the case, then we should reject the assumption that there is no new physics between the Fermi and Planck scales and modify the pattern of the y_{t} RG running. Therefore, the hypothesis that the SM or νMSM is fundamental can only be true if $a_y \leq a_y^{\mathrm{crit}}$. Unfortunately, we were unable to extract a reliable value and sign of a_y from the existing literature. For example, it was shown in [35] that gravity contributions make the Yukawa coupling asymptotically free in the quantum R^2 gravity with matter. The gravitational running of the Yukawa couplings f was studied in [32] in the functional RG approach for the Einstein–Hilbert type of truncation, and different signs were found for a_y in different gauges. Because of this lack of agreement, we simply assume that $a_y < a_y^{\mathrm{crit}}$ in what follows. As in the case of the $U(1)$ coupling, the special case $a_y = a_y^{\mathrm{crit}}$ would allow predicting m_{t}.

We turn to the behavior of the scalar self-coupling λ. The gravitational corrections can only promote the SM to the rank of a fundamental theory if the running of λ does not lead to any pathologies up to the Planck scale. In other words, the Landau pole must be absent for $k \lesssim M_{\mathrm{P}}$ [36]–[39], and λ must be positive for all momenta up to M_{P} [40]–[42], ensuring the stability of the electroweak vacuum. There is a large parameter space on the plane $(m_{\mathrm{H}}, m_{\mathrm{t}})$ where both conditions are satisfied; close to the experimental value of the top mass, it is described by

$$m_{\min} < m_{\mathrm{H}} < m_{\max}. \tag{5}$$

Here,

$$m_{\min} = 129.5 + \frac{m_{\mathrm{t}} - 173.1}{2.1} \cdot 4.1 - \frac{\alpha_{\mathrm{s}} - 0.1183}{0.002} \cdot 1.5, \tag{6}$$

$$m_{\max} = 174.0 + \frac{m_{\mathrm{t}} - 173.1}{2.1} \cdot 0.6 - \frac{\alpha_{\mathrm{s}} - 0.1183}{0.002} \cdot 0.1 \tag{7}$$

(in GeV), where α_{s} is the strong coupling at the scale of the Z-boson mass; the theoretical uncertainty in $m_{\min}$ equal to $\pm 2.2\,\mathrm{GeV}$. These numbers are taken from the recent two-loop analysis [43] (also see [44], [45] and earlier computations in [46]–[49]). The value of $m_{\max}$ corresponds to the (somewhat arbitrary) criterion
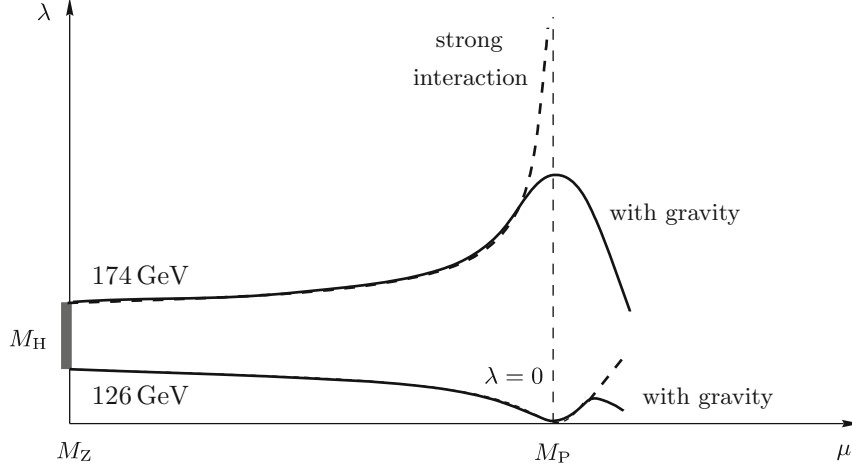
Fig. 1. Evolution of the Higgs self-coupling λ in the SM and asymptotically safe gravity in the case of a negative anomalous dimension a_λ.

$\lambda(M_P) < 6$. The allowed region also contains very small top-quark and Higgs-boson masses, which are excluded according to the experimental data.

We first suppose that a_λ is negative and has a sufficiently large magnitude,

$$a_\lambda < a_\lambda^{\mathrm{crit}} \simeq -\frac{24\xi_0\lambda(M_P)}{\pi}$$

($a_\lambda^{\mathrm{crit}} \simeq -1$ if $\lambda(M_P) \simeq 6$). Then the Higgs coupling is asymptotically free in all the region of the parameter space bounded by (5). The gravity contribution removes the Landau-pole behavior at energies exceeding the Planck mass (see Fig. 1). Setting $m_t = 173.1\,\mathrm{GeV}$ (we recall that the top-quark mass cannot be predicted if $a_y < a_y^{\mathrm{crit}}$) and neglecting uncertainties in the theoretical computations and in α_s, we find that the Higgs mass must lie in the interval [129.5, 174.0] GeV. The upper limit on the Higgs mass decreases if the actual value of $a_\lambda^{\mathrm{crit}}$ is less than one.

The most interesting situation is realized if a_λ is positive, which leads to a specific prediction for the Higgs-boson and top-quark masses. Evidence that this is the case comes from computations in [25], [27], according to which $a_\lambda \simeq 3.1$. A contribution with the same sign and a similar magnitude was previously found in [50]. We elucidate the structure of the solution of the RG equation for λ in this case. Because a_λ has a positive sign, the general solution of (1) diverges as $\mu \to \infty$, which leads to an inconsistent theory. But there can exist a particular solution leading to $\lambda \to 0$ (or, in a special case, $\lambda \to \mathrm{const} \neq 0$) in the UV. It is easy to see that the required behavior is only possible if the top-quark contribution appearing together with a negative sign of $\beta_\lambda^{\mathrm{SM}}$ dominates the gauge contribution as $t \to \infty$, leading to the constraint
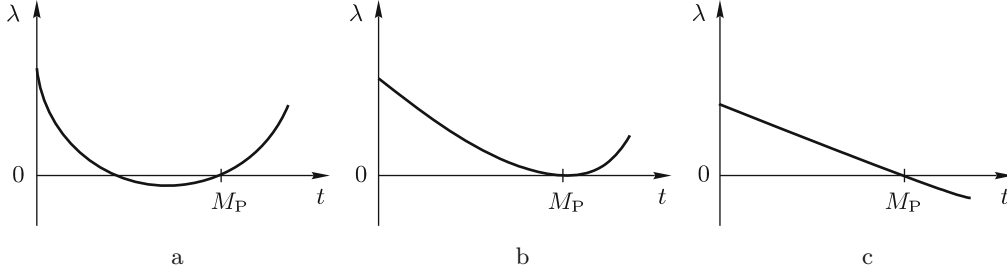
$$a \leq a_y \leq a_y^{\mathrm{crit}}. \tag{8}$$

If $a < a_y < a_y^{\mathrm{crit}}$, then the UV asymptotic behavior for λ is

$$\lambda(\mu) \approx \frac{6y_t^4(\mu)\xi_0}{\pi a_\lambda}.$$

At $a_y = a$, we have $\lambda(\mu) \propto \mu^{a/2\pi\xi_0}$, while at $a_y = a_y^{\mathrm{crit}}$, there is a non-Gaussian fixed point for λ^*, which satisfies the equation

$$24\lambda^{*2} + 12\lambda^* y_t^{*2} - 6y_t^{*4} + \frac{\pi a_\lambda \lambda^*}{\xi_0} = 0.$$

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Fig. 2. Evolution of the Higgs self-coupling λ in the SM with the boundary condition $\lambda(M_P) = 0$.

For $a_\lambda \geq 0$, the scalar self-coupling amplitude does not exceed $\lambda^* < 0.3y_t^{*2}$.

In summary, for any given set of anomalous dimensions a_h satisfying conditions (4) and (8) and for any $a_\lambda \geq 0$, there is a unique value of the low-energy scalar self-coupling that leads to a consistent theory. This means that the Higgs-boson mass can be predicted. To find m_H, we should solve the RG equations fixing the initial values (e.g., at the Z-boson mass M_Z) for the gauge and Yukawa couplings and adjust λ such that it tends to zero as $\mu \to \infty$ or approaches the fixed point λ^*. Moreover, only RG trajectories with positive λ are acceptable.

The following consideration allows localizing the value of the Higgs-boson mass and leads to a consistent theory. The RG equation for λ satisfying the asymptotic safety requirement can be rewritten as an integral equation

$$\lambda(\mu) = -\int_\mu^\infty \frac{d\mu'}{\mu'} \left(\frac{1 + 2\xi_0\mu^2/M_P^2}{1 + 2\xi_0(\mu')^2/M_P^2} \right)^{a_\lambda/32\pi\xi_0} \beta_\lambda^{SM}\big(h(\mu')\big).$$

Assuming that all couplings tend to zero as in (3), we obtain the boundary condition at $\mu = M_P$:

$$\lambda(M_P) = -C\beta_\lambda^{SM}\big(h(M_P)\big),$$

where C is positive and is of the order of unity. Because $\beta_\lambda^{SM} \ll \lambda$ at the point $k = M_P$, this can be replaced with

$$\lambda(M_P) \approx 0. \tag{9}$$

Therefore, the running of λ in the SM must bring it close to zero at the Planck scale.

This is not all the story. In a consistent theory, $\lambda(\mu)$ must be positive at *all* energy scales. To find the consequences of this requirement, we consider the SM evolution of λ for $\mu < M_P$ with boundary condition (9). Three different possibilities are shown in Fig. 2. The case in Fig. 2a, where λ hits zero before the Planck scale, is excluded: the SM breaks down below M_P in such a case. The case in Fig. 2c is potentially dangerous: the negative value of β_λ^{SM} at $k = M_P$ by continuity pushes λ to negative values above the Planck scale. In other words, not only the scalar self-coupling must be close to zero, but also its SM β-function should be small at $k = M_P$:

$$\beta_\lambda^{SM}(M_P) \approx 0. \tag{10}$$

How accurately Eqs. (9) and (10) should be satisfied depends on the specific values of the anomalous dimensions a_h and requires a numerical solution of the RG equations. It is important that there are two conditions instead of one: this allows fixing (or at least constraining) the Higgs-boson and top-quark masses *simultaneously*.

For better accuracy in the numerical computations, we used the two-loop RG equations. The low-energy coupling constants were expressed in terms of the physical parameters in the one-loop approximation (see [44], [51] and also [43]). Below, we describe the most essential features of our findings.

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The requirement that λ be positive for all energy scales leads to strong bounds on the top-quark mass. The lower bound $m_{\mathrm{t}} \gtrsim 170\,\mathrm{GeV}$ is practically independent of the anomalous dimensions a_h. Essentially, if $m_{\mathrm{t}} < 170\,\mathrm{GeV}$, then we obtain the behavior in Fig. 2a, leading to an unstable vacuum. Larger values of m_{t} correspond to the pattern shown in Fig. 2c. If the magnitudes of a and a_y are sufficiently large, then the constants g_i and y_{t} rapidly tend to zero for $k > M_{\mathrm{P}}$, leading to a small value of $\beta_\lambda^{\mathrm{SM}}$ above the Planck scale and thus to healthy behavior of λ. If the magnitudes of a and a_y are smaller, then the absolute value of $\beta_\lambda^{\mathrm{SM}}$ just above the Planck scale increases and forces λ into the negative region. Therefore, the upper limit on the mass of the top quark derived from the positivity considerations depends substantially on a and a_y. For example, for $a = a_y = -1$ and $a_\lambda = 3$, admissible RG trajectories exist for a large variety of top masses: $m_{\mathrm{t}} = 171.3\,\mathrm{GeV}$ leads to $m_{\mathrm{H}} \simeq 126\,\mathrm{GeV}$, while $m_{\mathrm{t}} = 230\,\mathrm{GeV}$ requires $m_{\mathrm{H}} \simeq 227\,\mathrm{GeV}$. The choice of $a = a_y = -0.25$ and $a_\lambda = 3$ leads to an upper bound $m_{\mathrm{t}} \lesssim 174\,\mathrm{GeV}$, which is very close to the lower limit. The fact that the experimental value of the top-quark mass is amazingly close to the lower limit (and to the upper limit for sufficiently small a_y) can be regarded as a support for the ideas presented here.

We now choose the experimental value for the top-quark mass and determine the Higgs-boson mass. The prediction is quite insensitive to the specific values of a, a_y, and a_λ and is

$$m_{\mathrm{H}} = m_{\min}, \qquad (11)$$

where $m_{\min}$ is given by (6). It is easy to understand why this occurs. The SM behavior of λ corresponding to $m_{\mathrm{H}} = m_{\min}$ and $m_{\mathrm{t}} = 173.1\,\mathrm{GeV}$ is exactly what is shown in Fig. 2b. Decreasing m_{H} moves us to $\lambda(t)$ in Fig. 2a, which is excluded. Increasing m_{H} makes $\lambda(M_{\mathrm{P}})$ positive and drives it to infinity above the Planck scale for $a_\lambda > 0$, which is also excluded. The latter behavior can only be modified if the Yukawa coupling for the top quark has a non-Gaussian fixed point $a_y = a_y^{\mathrm{crit}}$, which leads to the existence of a nontrivial fixed point in λ. Taking $a = -1$ and $a_y \simeq -0.005$ as an example, we find that $\lambda^* < 0.043$, which increases the predicted Higgs-boson mass by not more than $8\,\mathrm{GeV}$. Taking a smaller a reduces this increase. But this situation requires some fine tuning and therefore seems unlikely.

Our prediction (11) (or (5) if a_λ is in fact negative) can be verified at the LHC. Given that the accuracy in the Higgs-boson mass measurements at the LHC can reach $200\,\mathrm{MeV}$, the reduction of theoretical uncertainty and of experimental errors in the determination of the top-quark mass and of the strong coupling constant are highly desirable. As discussed in [43], the theoretical error can decrease from $2.2\,\mathrm{GeV}$ to $0.4\,\mathrm{GeV}$ if the one-loop pole matching at the electroweak scale and two-loop running up to the Planck scale are upgraded to the two-loop matching and three-loop running. We note that three-loop beta-functions for the SM are not yet known and that the two-loop pole matching has never been performed.

The prediction $m_{\mathrm{H}} \approx m_{\min}$ holds not only under the hypothesis that the SM plus gravity describes all the physics relevant for the running of couplings. It generalizes to many extensions of the SM and gravity, possibly even including theories with extra spatial dimensions. Of course, the precision of the prediction is weakened if a much larger class of models is considered. Nevertheless, only two crucial ingredients are necessary for predicting $m_{\mathrm{H}} \approx m_{\min}$. First, above a transition scale k_{tr}, the running should drive the quartic scalar coupling rapidly to an approximate fixed point at $\lambda = 0$, only perturbed by small contributions to β_λ from Yukawa and gauge couplings. This is generally the case for a sufficiently large anomalous dimension $a_\lambda > 0$. Second, around k_{tr}, there should be a transition to the SM running in the low-energy regime. This transition may actually involve a certain splitting of scales as "threshold effects," for example, by extending the SM to a grand unified theory at a scale near k_{tr}. It suffices that these threshold effects do not lead to a rapid increase of λ in the threshold region. This will be the case if the λ-independent contributions to β_λ only involve perturbatively small couplings in a threshold region extending over only a few orders of magnitude.

We make a few comments.

1. The amazing fact that the SM scalar self-coupling is equal to zero *together* with its β-function at the Planck scale for the particular values of the top-quark and Higgs-boson masses (to the best of our knowledge) was first noted in [52], where the hypothesis of a "multiple point principle" was advanced, stating that the effective potential for the Higgs field must have two minimums: one corresponding to our vacuum and the other at the Planck scale. Our reasoning is completely different. Although the meaning of the "multiple point principle" remains unclear to us, we note that the prediction of the Higgs-boson mass from it coincides with ours (the specific numbers in [52] differ because they were based on a one-loop computation).

2. The values of the Higgs-boson mass that we found are consistent with a possibility of inflation due to the SM Higgs boson [10]. The Higgs inflation requires the consistency of the SM up to an energy scale lower than M_P, $k \sim M_\mathrm{P}/\xi$, where $\xi = 700$ to 10^5 is the value of the nonminimal coupling of the Higgs field to the Ricci curvature scalar [43], [53] (also see [54], [55]). Smaller ξ correspond to smaller Higgs-boson masses.

3. Here, we implicitly assumed that the Fermi scale is fixed at its experimental value. It was found in [25], [27] that in a scalar–gravity system, the anomalous dimension of the scalar mass is negative, making it the relevant (and hence unpredictable) coupling. If this is indeed the case for the SM, then the smallness of the Fermi scale compared with the Planck scale remains a puzzle. If, on the contrary, this anomalous dimension happens to be positive for the SM, then the consistency of the theory will require setting the Fermi scale to zero in the asymptotic region. If true, then this may eventually shed light on the huge difference between the electroweak and the Planck scales.

4. Conclusion

We have discussed the possibility that the SM supplemented with an asymptotically safe gravity plays the role of a fundamental, not just an effective, field theory. We showed that this is possible if the gravity contributions to the running of the Yukawa and Higgs coupling have the appropriate signs. The mass of the Higgs scalar is predicted to be $m_\mathrm{H} = m_\mathrm{min} \simeq 130\,\mathrm{GeV}$ with a few GeV uncertainty if all the SM coupling constants except the Higgs self-coupling λ are asymptotically free, while λ is strongly attracted to an approximate fixed point $\lambda = 0$ (in the limit of vanishing Yukawa and gauge couplings) by the flow in the high-energy regime. This can be achieved by a positive gravity-induced anomalous dimension for the running of λ. A similar prediction holds for extensions of the SM to grand unified theories if the split between the unification and Planck scales remains moderate and all relevant couplings are perturbatively small in the transition region. Detecting the Higgs scalar with a mass around $130\,\mathrm{GeV}$ at the LHC could strongly suggest that there is no new physics influencing the running of the SM couplings between the Fermi and Planck/unification scales.

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