Transport in Porous Media **51:** 227–230, 2003.

The Opposing Effect of Viscous Dissipation Allows for a Parallel Free Convection Boundary-Layer Flow Along a Cold Vertical Flat Plate

E. MAGYARI⋆ and B. KELLER
Chair of Physics of Buildings, Institute of Building Technology, Swiss Federal Institute of Technology (ETH) Zürich, Wolfgand-Pauli-Str. 1, HIL E, CH-8093 Zürich, Switzerland

(Received: 15 February 2002)

Abstract. External free convection boundary-layer flows are usually treated by neglecting the effect of viscous dissipation. This assumption always results in a non-parallel flow, besides a strong parallel component also a weak transversal component of the (steady) velocity field occurs. The present paper shows, however, that the weak opposing effect of the buoyancy forces due to heat release by viscous dissipation, can give rise along a cold vertical plate adjacent to a fluid saturated porous medium to a strictly parallel steady free convection flow. This boundary-layer flow is described by an algebraically decaying exact analytical solution of the basic balance equations.

Key words: free convection, viscous dissipation, boundary-layer, Gebhart number, exact solution, algebraic decay.

1. Introduction

In recent years much effort has been directed on the effect of viscous dissipation in porous media. The familiar u^2-model of this effect for Darcy flows (Ene and Sanchez-Palencia, 1982; Bejan, 1995), as a counterpart of the $(\partial u/\partial y)^2$)-model of clear fluids, has been extended by Murthy and Singh (1997) to the case of non-Darcy free convection flows. A paradox occurring in the approach of Murthy and Singh (1997) has been resolved by Nield (2000). An attempt to extend the method of Murthy and Singh (1997) to the case of mixed convection flows, was undertaken recently by Tashtoush (2000) and Murthy (2001). The corresponding transformation of energy equation in a pseudo-similar form, as well as the transferability of the boundary condition $T(x, y \to \infty) = \text{const.} = T_\infty$, which in the case of a mixed convection contradicts the energy equation, were discussed by Magyari *et al.* (2002a). The excess temperature due to heat release by viscous dissipation in a quasi-parallel free convection flow over a hot vertical plate has been calculated recently by Magyari and Keller (2002b).

The aim of the present paper is to further contribute to this open research field by describing a surprising effect of viscous dissipation, namely, as summarized

⋆Author for correspondence: e-mail: magyai@hbt.arch.ethz.ch

already in the title, its ability to give rise along a cold vertical plate to a strictly parallel free convection boundary-layer flow.

2. Basic Equations and Solution

Following Nield and Bejan (1999), we write the mass, momentum and energy conservation equations (of a Darcy-Boussinesq free convection boundary-layer flow) in the form:

$$u_x + v_y = 0, \tag{1}$$

$$u_y = -\frac{g\beta K}{\upsilon} T_y, \tag{2}$$

$$uT_x + vT_y = \alpha T_{yy} + \frac{\upsilon}{K c_p} u^2. \tag{3}$$

Here x and y are the Cartesian coordinates along and normal to the plate, respectively, u and v are the velocity components along x and y axes, T is the fluid temperature, K is the permeability of the porous medium, g is the acceleration due to gravity, c_p is the specific heat at constant pressure, α, β and $\upsilon = \mu/\rho$ are the effective thermal diffusivity, thermal expansion coefficient and kinematic viscosity, respectively, and the subscripts x and y indicate partial derivatives. The positive x-axis, with its origin on the leading edge, points vertically downwards in the direction of g. The second term on the right-hand side of Equation (3) is proportional to the volumetric heat generation rate $q''' \equiv \mu u^2/K$ by viscous dissipation. The aim of the present paper is to examine the possibility of a parallel free convection boundary-layer flow over a vertical flat plate of constant temperature Tw when the effect of viscous dissipation is taken into account. The plate is assumed to be 'cold', that is, $T_w < T_\infty$, where T_∞ denotes the ambient temperature of the fluid. The minus sign on the right-hand side of Equation (2), as well as the choice of the coordinate system are correlated with this assumption. Under 'parallel' we mean (as usual) a plane boundary-layer flow with identically vanishing transversal velocity, that is,

$$(u, v) = (u, 0). \tag{4}$$

Hence, our boundary conditions accompanying Equations (1)–(3) read:

$$\text{On } y = 0: \qquad T = T_w, \tag{5a}$$

$$\text{As } y \to \infty: \qquad u = 0, \qquad T = T_\infty > T_w. \tag{5b}$$

Equation (2) and the boundary condition (5b) imply immediately:

$$u = \frac{g\beta K}{\upsilon}(T_\infty - T). \tag{6}$$

As a consequence of assumption (4), all the physical quantities will depend only on the coordinate y. Thus, having in mind (3) and (6), the problem reduces to solve equation

$$\frac{\mathrm{d}^2}{\mathrm{d}y^2}\left(\frac{T_\infty - T}{T_\infty - T_w}\right) = \frac{\mathrm{RaGe}}{L^2}\left(\frac{T_\infty - T}{T_\infty - T_w}\right)^2 \tag{7}$$

along with the boundary conditions (5). In Equation (7), L denotes a reference length and the Rayleigh and Gebhart numbers have been defined as follows:

$$\mathrm{Ra} = \frac{g\beta K|T_w - T_\infty|L}{\upsilon\alpha}, \qquad \mathrm{Ge} = \frac{g\beta L}{c_p}. \tag{8}$$

The effect of viscous dissipation is controlled by the Gebhart number. For $\mathrm{Ge} = 0$, Equation (7) does not admit solutions satisfying all the boundary conditions (5). Thus, we recover the well-known result that in the absence of viscous dissipation no parallel boundary-layer solution exists. If, however, $Ge \neq 0$, the problem admits the exact solution:

$$\begin{aligned} T(y) &= T_\infty - \frac{T_\infty - T_w}{(1 + \sqrt{(\mathrm{RaGe}/6)}(y/L))^2}, \\ u(y) &= \frac{\alpha}{L}\frac{\mathrm{Ra}}{(1 + \sqrt{(\mathrm{RaGe}/6)}(y/L))^2}. \end{aligned} \tag{9a, b}$$

The corresponding wall heat flux and Nusselt number are given by:

$$q_w = -\lambda\left.\frac{\partial T}{\partial y}\right|_{y=0} = \frac{\lambda(T_\infty - T_w)}{L}\mathrm{Nu}, \qquad \mathrm{Nu} = -\sqrt{\frac{2\mathrm{RaGe}}{3}}. \tag{10}$$

Both the temperature and velocity profiles (9) are quadratically decreasing functions of the distance y from the plate. The Nusselt number depends only on the product of the Rayleigh and Gebhart numbers and it is negative. Therefore, in full agreement with physical expectation, heat is transferred in this case everywhere from the fluid to the surface. With increasing value of the Gebhart number, the amount of heat transferred increases as $\mathrm{Ge}^{1/2}$.

3. Discussion and Conclusions

The main result of the present paper (as already summarized in its title) is the prediction that owing to viscous dissipation, in certain free convection boundary-layer flows a ‘selfparallelization’ effect of the velocity field can occur. The necessary condition of this phenomenon is that the buoyancy forces induced by viscous dissipation are opposing to the ‘main’ buoyancy forces sustained by the wall temperature gradient. Having in mind that the former forces always are directed vertically upwards, the parallel flow described in this paper is only possible along a cold plate, that is, as a descending free convection flow. Along a hot vertical plate, the heat release by viscous dissipation does assist the ascending flow induced by the wall temperature gradient, amplifying both of its parallel and transversal velocity components. Thus, in the latter case no self-parallelization of the velocity field is possible.

We may conclude, therefore, that, although the buoyancy forces induced by heat release by viscous dissipation in general are much too weak to exert a substantial influence on the strong parallel component, they can throughout be able to prevent the occurrence of a weak transversal velocity component of a descending free convection boundary layer flow.

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