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Eating Goldstone Bosons in a Phase Transition: A Critical Review of Lyre's Analysis of the Higgs Mechanism

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Abstract In this note, I briefly review Lyre's (2008) analysis and interpretation of the Higgs mechanism. Contrary to Lyre, I maintain that, on the proper understanding of the term, the Higgs mechanism refers to a physical process in the course of which gauge bosons acquire a mass. Since also Lyre's worries about imaginary masses can be dismissed, a realistic interpretation of the Higgs mechanism seems viable. While it may remain an open empirical question whether the Higgs mechanism did actually occur in the early history of the universe and what the details of the mechanism are, I claim that the term can certainly refer to a physical process.

1 Does the Higgs Mechanism Exist?

"Does the Higgs mechanism exist?" asks Lyre (2008), and comes to the conclusion that "it certainly does not describe any dynamical process in the world" (130). He examines "the technical derivation" (119) and finds that "the whole story about the 'mechanism' is just a story about ways of representing the theory and fixing the gauge" (130). I disagree, and argue that Lyre's conclusions are based on an inadequate understanding of the concept. While it may remain an open empirical question whether the Higgs mechanism did actually occur in the early history of the universe and what the details of the mechanism are, I claim that the concept can certainly refer to a physical process. I present the Higgs mechanism in a way that emphasizes that it is not only about "reshuffling degrees of freedom" (119) but about the transition between two distinct physical systems.

That the Higgs mechanism is only about "reshuffling degrees of freedom" is Lyre's first of "three observations" which make him particularly skeptical about its reality (125–126). The second observation reminds the reader that every Lagrangian which is involved in the

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description of the Higgs mechanism is, as a matter of fact, invariant under the gauge transformations; only the individual ground states of the system are not. Therefore, Lyre recommends not to speak of a *broken* symmetry but rather of a *hidden* symmetry. In several respects, this point about terminology (which also other authors have made) is well taken, and I will not discuss it in the remainder of this article.

I will, however, touch on Lyre's third observation which states that "[one of the Lagrangians] does not allow for any quick, literal interpretation, since here we are facing the obscure case of a ϕ-field with imaginary mass μ" (126). I will briefly describe why and under what conditions one can "read off" the mass of the particles described by a Lagrangian from one of its coefficients. The Lagrangian which, for Lyre, suggests an imaginary mass does not meet the necessary conditions, and I agree with Lyre that, therefore, the "quick, literal interpretation" cannot be applied. However, this does not mean that the mass which would result from a correct calculation using this Lagrangian would be imaginary.

2 Scalar Electrodynamics

For the present purposes of reviewing Lyre's analysis concerning the ontology of the Higgs mechanism, I will not go into the details of the complete model of spontaneous symmetry breaking of the electroweak interaction. The simpler model of the electrodynamics of charged spinless particles also exhibits the relevant features. I, like Lyre for the main part of his analysis, will therefore restrict myself to this model. For the following exposition of this model, I use the lecture notes by (Wiese 2010, chapter 2) as my basis; (Peskin and Schroeder 1995, 690–692), for instance, give a similar exposition.

The simplest version of the model is one in which the particles are free. The model contains a complex scalar field $\Phi = \Phi_1 + i\Phi_2, \Phi_1, \Phi_2 \in R$ and the dynamics of this field and its quanta are described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi^*\partial^\mu\Phi - V(\Phi), \tag{1}$$

where

$$V(\Phi) = \frac{m^2}{2}|\Phi|^2. \tag{2}$$

From the Lagrangian of this most simple of systems we can derive the corresponding Euler–Lagrange equations

$$\partial_\mu \frac{\delta\mathcal{L}}{\delta(\partial_\mu\Phi_i)} - \frac{\delta\mathcal{L}}{\delta\Phi_i} = 0 \qquad i = 1, 2, \tag{3}$$

which coincide, in this case, with the familiar Klein–Gordon equations for two free, spinless, charged fields with quanta of mass m:

$$\partial_\mu\partial^\mu\Phi_i + m^2\Phi_i = 0. \tag{4}$$

In the next complex version of the model, the scalar particles interact directly among themselves. The interaction is described by a power of 4 in the field's absolute value which is added to the potential V such that it now reads

$$V(\Phi) = \frac{m^2}{2}|\Phi|^2 + \frac{\lambda}{4!}|\Phi|^4, \tag{5}$$

see, for instance, (Peskin and Schroeder 1995, 348–350). The parameter λ measures the strength of the interaction and the factor 4! is introduced for convenience in the perturbative solution of the dynamical equations of the model.

For the model to be interpretable physically, the potential V must be bounded from below. Otherwise the energy spectrum would also not be bounded from below and, accordingly, there would be no ground state of the system, which clearly cannot be the case for any real system. For the purely quadratic potential of Eq. (2) there is thus no other choice than $m^2 > 0$. For the potential describing the self-interaction of the field Φ (see Eq. 5), however, $m^2 < 0$ is possible also.

3 Spontaneous Breakdown of a Global Symmetry

The model defined by Eqs. (1) and (2) is globally symmetric with respect to $U(1)$ transformations $\Phi' = e^{iq\phi}\Phi$, where $\phi \in R$ is the parameter of the transformation and the factor q is introduced for more convenient identification of the charge of the particles. With $m^2 > 0$, the Lagrangian from Eqs. (1) and (5) describes a system of particles of *approximately* the mass m. The mass of the particles is, in this version of the model, not exactly equal to m because of the interactions among the particles. A more precise treatment of the mass has to employ renormalization techniques, which, however, are not relevant for the present discussion. However, this reminds us that the coefficient of the quadratic term in the Lagrangian is equal to (half the square of) the mass of the particles only as long as the potential is approximately quadratic (like Eq. 2 or, more generally, like the potential of a harmonic oscillator). Only then do the Euler-Lagrange equations approximately coincide with the Klein–Gordon equation, on which our identification of the coefficient with the mass of the particle was based, see Sect. 2.

For $m^2 < 0$, the global $U(1)$ symmetry of the model is *spontaneously broken*. This means that the symmetry of the Lagrangian is still intact but the field configurations which lead to a minimal value of V are not invariant under the $U(1)$ transformations any longer. Before, in the case of $m^2 > 0$, the field configuration with minimal V was simply $\Phi = 0$. Now, with $m^2 < 0$, there is no unique configuration which minimizes V. A whole class of field configurations,

$$\Phi = \sqrt{-\frac{6m^2}{\lambda}}e^{i\chi}, \tag{6}$$

yield a minimum value for V. χ is the real parameter which characterizes a particular member of the class. For convenience, I will abbreviate $\sqrt{-\frac{6m^2}{\lambda}}$ by v such that the minimal configurations read $ve^{i\chi}$.

In order to estimate the masses of the quanta of the interacting fields from the coefficients in the Lagrangian, we have to restrict ourselves to only small fluctuations around the field configurations for which the potential V is at its minimal value, in other words, we have to perform a series expansion of the Lagrangian around one of the points of minimal value of V. Only then can we approximately equate the actual potential of Eq. (5) with the potential of Eq. (2), which is more readily interpretable in terms of a Klein–Gordon equation as discussed in Sect. 2. Because the Lagrangian is invariant under (global) $U(1)$

transformations, our results will not depend on which particular member of the class of minimal configurations we choose for our expansion around it. A choice in which the expansion takes a particularly simple form is $\Phi_0 = v$, that is we set $\chi = 0$. We can expand around that particular configuration of minimal V by substituting $v + \sigma(x) + i\pi(x)$ for $\Phi(x)$, where $\sigma(x)$ and $\pi(x)$ are two real fields of which we only consider the infinitesimal excitations. In terms of the newly introduced σ and π fields the Lagrangian takes the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi - \frac{1}{2}(-2m^2)\sigma^2 + \cdots, \tag{7}$$

where I left out higher order terms in the fields, which can be neglected for the present purposes. This form of the Lagrangian, valid for small absolute values of σ and π, allows us to read-off the approximate masses of the quanta of this system of self-interacting fields: zero for the quanta of the π field, $\sqrt{-2m^2}$ for the quanta of the σ field.[1]

We now also see that Lyre's worries, based on his "third observation" (Lyre 2008, 126), about the imaginary mass that would result from the identification of the coefficient of $|\Phi|^2$ as (half the square of) the particles' mass are unjustified. The identification can only be made if small fluctuations are considered of a field configuration for which V takes its minimal value. This is not the case for the Lagrangian which we get from Eqs. (1) and (5) with $m^2 < 0$. The fact that the parameter m^2 is negative in that Lagrangian does, therefore, not mean that the quanta described by it have imaginary mass.

4 Spontaneous Breakdown of a Local Symmetry

For reasons not to be discussed here, one prefers models which exhibit even a local symmetry, instead of a merely global one. In order to promote the global $U(1)$ symmetry, discussed above, to a *local* symmetry, one has to introduce a *gauge* field and a covariant derivative. The gauge field will eventually describe an interaction between the fields whereas, in the case of the global symmetry, the interaction between the particles was direct and immediate.

The Lagrangian that describes a locally symmetric model of spinless charged particles which interact through a gauge field is

$$\mathcal{L} = \frac{1}{2}(D_\mu\Phi)^*D^\mu\Phi - V(\Phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{8}$$

where $D_\mu = \partial_\mu - iqA_\mu$ is the covariant derivative, A_μ the gauge field, q the strength of the coupling of the scalar field to the gauge field (in other words: the charge of the scalar field) and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the field strength tensor associated with the gauge field. The combination $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ describes the kinetic energy of the gauge field. For the purposes of our simplified model of the electroweak interactions, A_μ is the electromagnetic field and its quanta the photons. $V(\Phi)$ reads, as in the case of global symmetry, $\frac{m^2}{2}|\Phi|^2 + \frac{\lambda}{4!}|\Phi|^4$.

As in the case of global symmetry, the Lagrangian describes either the symmetric phase (if $m^2 > 0$) or the broken phase (if $m^2 < 0$). In the symmetric phase, the field configuration which minimizes V is again just $\Phi = 0$. The masses are approximately given by the coefficients of the quadratic terms in the Lagrangian. There are two fields, Φ_1 and Φ_2,

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[1] Remember $m^2 < 0$. Therefore, $-2m^2 > 0$ and $\sqrt{-2m^2}$ positive and real.

Table 1 Approximate masses of the quanta which exist in the symmetric or broken model of interacting spinless charged particles, in the case of global or local symmetry

	Global	Local
Symmetric	$m_\sigma = m_\pi = m$	$m_\pi = m_\sigma = m,\ m_A = 0$
		$\downarrow *$
Broken	$m_\sigma = \sqrt{2}|m|,\ m_\pi = 0$	$m_\sigma = \sqrt{2}|m|,\ m_A = qv$

"m_A" denotes the masses of the quanta of the gauge field, the photons. The transition, denoted by the starred arrow, from the symmetric to the broken phase, in the case of the local symmetry, is the Higgs mechanism

which both have quanta of mass m. Because there is no quadratic term of the gauge field, its quanta (the photons) are massless.

In the broken phase, i. e. when $m^2 < 0$, we have to do again the series expansion around one of the field configurations which minimize V, i. e. around $ve^{i\chi}$ for some $\chi \in R$. Again, since the Lagrangian is $U(1)$ symmetric, we can set $\chi = 0$ and perform the expansion around $\Phi_0 = v$ and substitute $\Phi(x)$ by $v + \sigma(x) + i\pi(x)$. However, now the Lagrangian is even invariant under *local* $U(1)$ transformations, and $v + \sigma(x)$ and $v + \sigma(x) + i\pi(x)$ differ, in a first order approximation, just by such a local $U(1)$ transformation, albeit an infinitesimal one: $e^{i\pi(x)/v} \approx 1 + i\pi(x)/v$. Therefore, because of the symmetry of the Lagrangian, any conclusion we draw from the Lagrangian will not depend on this difference, and we can set $\pi(x) = 0$ and substitute $\Phi(x)$ just by $v + \sigma(x)$.

Apart from higher order terms, we then obtain

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}(-2m^2)\sigma^2 + \frac{1}{2}q^2v^2A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \cdots \tag{9}$$

In this form, we see that the Lagrangian describes massive quanta of the σ field and massive quanta of the gauge field. This is indicated by the quadratic terms in these fields; the other terms describe the kinetic energy of the fields. Contrary to the case of the spontaneous breakdown of the *global* symmetry, we see that here, in the case of *local* symmetry, we have a massive photon instead of a massless Goldstone boson. Compared to the locally *symmetric* phase, the difference is that we have a massive, instead of massless, photon and only one massive scalar field, instead of two, see Table 1.

5 The Higgs Mechanism

Using Table 1 we can see that the number of physical degrees of freedom is unaffected by the spontaneous breakdown of the symmetry, either local or global. In the case of the global symmetry, the number of degrees of freedom is two, before and after the spontaneous breakdown of the symmetry, because each scalar particle has one degree of freedom, irrespective of its mass. In the case of local symmetry, it might seem, at first sight, that one degree of freedom is somehow lost in the course of the spontaneous breakdown of the symmetry, because in the symmetric phase there is a quantum of the π field while in the broken phase there is none. However, the number of degrees of freedom of the gauge field depends on whether it is massive or not. In the symmetric phase, the photons are massless and thus have two physical degrees of freedom only. In the broken phase, the photon has a mass and thus has three physical degrees of freedom.

The *Higgs mechanism* is the transition from the symmetric to the broken phase in the case of a local symmetry, see Table 1.[2] This is the transition from a state in which there are two massive scalar fields, σ and π, and a massless gauge field, A_μ, to a state in which there is only one scalar field, σ, with massive quanta, and a massive gauge field. The Goldstone boson, the massless quanta of π, which appears in the broken phase of a global symmetry, does not appear in the broken phase of a local symmetry. In a metaphorical manner of speaking, one therefore often says that the Goldstone boson, which would appear if the symmetry were global, is "eaten" by the photon which thus becomes massive.[3]

At the same time, this metaphor of eating might be responsible for the confusion behind Lyre's claim that the Higgs mechanism is nothing but a reshuffling of degrees of freedom and as such cannot possibly refer to a physical process. Such a claim can only be maintained if one means by "Higgs mechanism" the transition from the system described by the Lagrangian of Eq. (8) (with $\Phi(x) = v + \sigma(x) + i\pi(x)$) to the Lagrangian of Eq. (9). However, this is clearly not a transition between two physically distinct systems, as Lyre correctly points out, but a mere transition from one description of the system to another equivalent description. One might be tempted to apply the eating metaphor to this transition, too, because in the first description the π field appears in the Lagrangian while in the second description it does not. However, because of the local $U(1)$ symmetry of the Lagrangian, it is the same physical system, without π quanta, that is described in both cases.[4] The only difference between the two cases is that one form of the description (Eq. 9) clearly shows that, in fact, there are no π quanta, while the other form of the description (Eq. 8) is less directly interpretable.

None of Lyre's worries, therefore, gives us reason to doubt that the Higgs mechanism can have the same ontological status as any other mechanism of spontaneous symmetry breaking, which we observe, for instance, in ferromagnets or superconductors. Lyre's analysis concerns the transition between two equivalent descriptions of the same physical system which should and, in fact, usually is not called the Higgs mechanism.[5] The proper understanding of the term is that of a transition from a symmetric phase of a physical system to an asymmetric (or broken) phase. In the course of this transition, one type of massive charged spinless particle disappears and the gauge field, the quanta of which are massless in the symmetric phase, becomes massive. Such a process might or might not have happened in the cooling of the early universe[6], but in any case, whether it happened or

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[2] For some purposes, this statement may be over-simplified. The relation $m_A = qv$ (see Table 1) shows how the mass of the gauge boson depends on the strength of the coupling q of the scalar field to the gauge field. The second row of Table 1 shows how, in the broken phase, the introduction of a gauge field and the requirement of a *local* symmetry, instead of only a global one, leads to the disappearance of the (massless) Goldstone boson π. These observations are emphasized in Higgs (1964) and Anderson (1963), for instance. Accordingly, in a more complete characterization, the Higgs mechanism should be regarded as the combination of the two processes of coupling the scalar field to the gauge field (going from left to right in the second row of Table 1) *and* the transition from the symmetric to the broken phase of a local symmetry (going from top to bottom in the second column of Table 1).

[3] To my knowledge, the metaphor goes back to (Coleman 1985, 123).

[4] See also (Weinberg 1996, 296), for instance.

[5] Textbooks are often not completely clear about what they call the "Higgs mechanism". However, this has mainly to do with the fact that a complete characterization of the mechanism should also include the transition from a system with zero coupling of the scalar and the gauge field to a system with non-zero coupling, see footnote 2. The clearest expositions I have found are (Peskin and Schroeder 1995, chapter 20), (Ryder 1996, chapter 8) and (Weinberg 1996, chapter 21), which, as far as I can tell, never share Lyre's understanding of the "Higgs mechanism".

[6] Cf., e. g., (Linde 1990, 17) and references therein such as Weinberg (1974), ref. 20 in Linde.

not is a meaningful empirical question and is not answered to the negative by Lyre's conceptual argument.

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