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On the limits of democracy

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Abstract In this article, we extend the analysis of Gersbach (2009) and explore the limits of democratic constitutions to achieve first-best outcomes. We establish the most general possibility result and we illustrate the efficiency gains of flexible majority rules by examples. We show that no first-best constitution exists if there is uncertainty regarding the size of losses and benefits from public projects.

1 Introduction

Gersbach (2009) has introduced democratic mechanisms and has shown how increasingly sophisticated treatment, agenda and decision rules can yield first-best allocations in a variety of circumstances. However, it remains unclear when we reach situations for which it will be impossible to design first-best democratic constitutions.[1] In this article, we extend the analysis of Gersbach (2009), to provide an answer to this question.

In particular, we explore the potential and the limitations of democratic constitutions to avoid the inefficiencies described above. A democratic constitution is a set of rules that specify (i) how the proposal maker is chosen and treated; (ii) restrictions on proposals that can be made and (iii) how the society decides on a proposal. The rules must satisfy the liberal-democracy constraint (see Gersbach (2009) and Section 2.3).

We establish the most general possibility result for achieving first-best allocations. First-best constitutions involve flexible majority rules. Under flexible majority rules, the size of the majority required for the adoption of a proposal depends on the proposal

[1] In this article, the term constitution is used rather than mechanisms. Both are synonyms.

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itself, e.g. on the share of the population who will have to pay taxes, or on the aggregate tax revenues generated by the proposal.[2]

Flexible majority rules can ensure that a winning majority supports the provision of a project proposal if and only if the public project is socially valuable. If such flexible majority rules are coupled with other constitutional principles which will be developed in the article (such as taxation restricted to majority winners, maximal taxation of the agenda-setter and costly agenda-setting), first-best allocation may be achieved, as such rules jointly ensure that efficient project proposals will be proposed and adopted and neither inefficient project proposals nor pure redistribution proposals are proposed and approved.

Then, we show that knowledge about negative utility realizations is an essential constraint for this result. We demonstrate that no first-best constitution exists if there is uncertainty regarding the size of losses and benefits from public projects at the constitutional stage. The reason is that proposals leading to the adoption of a public project do not generate information about negative utilities. Without such information, constitutional rules cannot discriminate between efficient and inefficient projects.

My article is a study in constructive constitutional economics, as outlined in the classic contribution by Buchanan and Tullock (1962). Under a veil of ignorance, individuals decide which rules should govern legislative decision-making. In a long tradition dating back to Rousseau (1762), Buchanan and Tullock (1962) have examined the costs and benefits of majority rules chosen by a society operating under a veil of ignorance. Aghion and Bolton (2003) have explicitly introduced contractual incompleteness for the design of optimal majority rules. They show how the simple or qualified majority rule can help to overcome ex-post vested interests.

The twin problem of societies—the risk of tyranny by the majority and the risk of legislation-blocking by the minority, as outlined in Aghion and Bolton (2003)—has been further examined in Aghion et al. (2004), who derive optimal supermajority governing rules that balance both of these dangers. Harstad (2005) develops a theory of majority rules based on the incentives of members of a club to invest in order to benefit from anticipated projects. Optimal majority rules balance two opposing forces. Large required majorities provide little incentive to invest because of hold-up problems, while the members of small majorities invest too much to become members of a majority coalition.

Our study is complementary to the articles mentioned above. We extend the analysis of Gersbach (2009) and explore the limitations of democratic constitutions. The main differences to Gersbach (2009) are threefold: First, at the legislative stage, valuations become common knowledge in this article. Second, we allow for more uncertainty regarding project parameters at the constitutional stage. In particular, we allow that the share of project winners, the size of benefits and the size of losses from projects may be unknown. This multidimensional uncertainty yields the impossibility theorem. Third, we will introduce two novel rules: maximal taxation of the agenda setter and

[2] Some homeowner associations in Irvine, California use a flexible majority rule: the higher the proposed increase of fees, the higher the required majority to adopt it. Thus, the size of the majority is dependent on the proposal. I am grateful to Ami Glazer for this information.

taxation of majority winners only. Those rules will help to achieve first-best allocations in the case when there is no uncertainty regarding the size of losses from a project.

The article is organized as follows: In the next section, we introduce the model. In Section 3, we present the most general result on first-best constitutions and we illustrate the working of first-best constitutions with a simple example. Section 4 contains the impossibility theorem and Section 5 concludes with a discussion. All proofs can be found in the Appendix.

2 Model and constitutional rules

2.1 Model

We consider a standard social-choice problem in public project provision and financing. Time is indexed by $t = 0, 1$. The first period $t = 0$ is the constitutional period in which a society of risk-neutral members decides upon the way in which public project provision and financing should be governed in the legislative period.

In the legislative period $t = 1$, each citizen is endowed with some private consumption good $e > 0$. The community can adopt a public project with costs $k > 0$ in units of the consumption good. Citizens are indexed by i or $j \in [0, 1]$. We use v_j to denote the utility for agent j from the provision of the public project. We assume that v_j can have two values (expressed in terms of the consumption good): $v_j = V_h > 0$ and $v_j = V_l < V_h$ with probability p and $1 - p$, respectively ($0 \leq p \leq 1$). Hence, by a suitable version of the law of large numbers, the fraction of project winners and project losers will be p and $(1 - p)$, respectively. It is sometimes convenient to arrange citizens in the following way: $v_j = V_h > 0$ if $j \in [0, p]$ (project winners); $v_j = V_l < V_h$ if $j \in (p, 1]$ (project losers). At $t = 0$, agents do not know whether they will be project winners or project losers in $t = 1$.

Public projects have to be financed by taxes. We assume that taxation is distortionary and thus there exists costs of redistribution. Let $\lambda > 0$ denote the shadow cost of public funds. This means that taxation uses $(1 + \lambda)$ of the tax-payers' resources to levy 1 for public projects or for transfers to citizens. Hence, the overall costs of the public project amount to $(1 + \lambda)k$, and we can represent the project data as a vector $\mathcal{P} = \Big(p, V_h, V_l, (1 + \lambda)k\Big)$.

We use t_j and s_j to denote the tax payment or subsidy of citizen j, respectively, and define the variable g that indicates whether the public project is provided ($g = 1$) or not ($g = 0$). The utility of citizen j in the legislative period is given by

$$U_j = e + gv_j - t_j + s_j.$$

Throughout the article, we assume that s_j and t_j interpreted as functions of j are integrable. We assume that e is sufficiently large such that the individuals can pay the taxes under any of the constitutions we will discuss.[3] Finally, the budget constraint on the society in the legislative period is given by

$$\int_0^1 t_j \, dj \geq (1+\lambda)\left[gk + \int_0^1 s_j \, dj\right].$$

2.2 Socially efficient solutions

The fact that citizens are risk-neutral implies that, from an ex ante point of view, it is socially efficient to provide the public project if and only if

$$V := pV_h + (1-p)V_l > k(1+\lambda).$$

We define the critical value p^* as

$$p^* = \begin{cases} 1, & \text{if } k(1+\lambda) > V_h \\ \frac{k(1+\lambda)-V_l}{V_h-V_l}, & \text{if } V_h \geq k(1+\lambda) \geq V_l \\ 0, & \text{if } k(1+\lambda) < V_l. \end{cases}$$

If $p > p^*$, the project is efficient, while it is inefficient if $p \leq p^*$. Moreover, taxes can be distributed across individuals in any fashion but money should only be raised to finance the public project. From an ex ante point of view, any redistribution activities are waste.

2.3 Democratic provision

In the following, we assume that complete social contracts cannot be written at the constitutional stage. As is common in the incomplete contracting literature, we assume that future nature states cannot be described precisely, therefore, a constitution can only specify rules for future social decision-making. To capture the notion of democratic processes, we use the *liberal-democracy constraint* as justified in Gersbach (2009). The liberal-democracy constraint consists of the following sub-constraints: Every agent has the same chance to make a proposal. Every individual has the right to vote. Only yes/no messages are allowed at the voting stages. Every individual is allowed to abstain from voting or proposal-making. The precise formalization of the liberal-democracy constraint is embodied in the game in the next subsection.

For the informational assumptions, we follow the incomplete social contract literature and assume that all entries of the project vector $\mathcal{P} = \big(p, V_h, V_l, (1+\lambda)k\big)$ are *observable* by all citizens in the legislative period and that the location j of each

[3] If v_j is a benefit that cannot be taxed, it will be sufficient to assume that e is larger than V_h. If v_j were taxable, it is sufficient to assume that e is larger than $\max[V_h, |V_l|]$

citizen is common knowledge [4] (see Aghion and Bolton 2003). Some entries of $\mathcal{P}$ may be verifiable in a constitutional court in the legislative period, and constitutional rules can then be formulated on the basis of those parameters. An equivalent alternative view is that, instead of being verifiable in the *legislative period*, these parameters of $\mathcal{P}$ are known in the *constitutional period* and hence can be written into the constitution directly. In the following, we describe our results in terms of the second view.

2.4 The game

We consider the sequence of constitutional and legislative decision-making. At the constitutional stage, the society decides which rules will govern the legislative processes. The sequence of events is given as follows:

Stage 1: In the constitutional period, the society decides unanimously about the constitutional principles governing legislative decision-making.[5] Some elements of $\mathcal{P}$ may be known.

Stage 2: At the start of the legislative period, the remaining project data of $\mathcal{P}$ will become known. Citizens observe their location j on the unit interval and the location of all other agents. Citizens decide simultaneously whether to apply for agenda-setting ($\psi_j = 1$) or not ($\psi_j = 0$) where ψ_j is the corresponding indicator variable.

Stage 3: Among all citizens who apply, one citizen $a \in [0, 1]$ is determined randomly to set the agenda. The agenda-setter proposes a project/financing package $\left(g, t_j, s_j\right)_{j\in[0,1]}$. Denote this choice by A_a.

Stage 4: Given A_a, citizens decide simultaneously whether to accept the proposal ($\delta_j(A_a) = 1$) or not ($\delta_j(A_a) = 0$) where δ_j is the corresponding indicator variable.

The game fulfils all four conditions constituting the liberal-democracy constraint. Note that if nobody applies for agenda-setting, the status quo will prevail. The status quo is characterized by $g = 0, t_j = s_j = 0, \forall j$. Hence, the utility of a citizen in this case is e. We further note that, at the voting stage, individuals know who will be taxed and who will receive subsidies if a proposal is accepted.

Given a constitution with a set of principles discussed in the next section, we now look at subgame perfect implementation in stages 2–4.[6] An equilibrium for the subgame consisting of stages 2–4 can be described as a set of strategies

$$\left\{\psi, A, \delta(\cdot)\right\},$$

[4] This assumption is plausible for a number of examples such as the construction of roads, labor market reforms or the scaling down of the defense industry where project winners and losers are given by their location or occupation.

[5] Strictly speaking, to make stage 1 part of the game, one has to specify a default, say simple majority voting, if unanimous agreement fails.

[6] A comprehensive overview of the theory of implementation can be found in Moore (1992) and Jackson (2001).

where $\psi = (\psi_j)_{j\in[0,1]}$, $A = (A_a)_{a\in\{j\in[0,1]:\psi_j=1\}}$, $\delta = (\delta_j)_{j\in[0,1]}$. Of course, $\delta_j = \delta_j(A_a)$ depends on the proposed agenda A_a.[7]

In deriving an equilibrium, we face the problem that, as we have a continuum of voters, an individual vote may have no influence on the outcome. As we will show, in some circumstances an individual has influence when constitutional rules are specified at an individual level. In other cases, individual voting is irrelevant. To describe the application and voting outcome in our model, we use weak dominance criteria that mimic the optimal voting and application behaviour of a society with a large but finite number of agents. In our model, voting is a simple binary decision, so individuals cannot gain anything from strategic voting. Hence, we assume sincere voting, i.e. agents vote for their most preferred alternative.

Clearly, sincere voting selects a unique voting outcome. Hence, we can use the weak dominance criterion for the decision whether to apply for agenda-setting (stage 2).

- **(EWSA)** *Agents eliminate weakly dominated strategies in stage 2.*

Since sincere voting selects a unique voting outcome, we can use $U_j(A_a)$ to define the utility level that an agent j will achieve if agent a has proposed the agenda A_a and voting has taken place. Accordingly, $U_a(A_a)$ is the utility level of the agenda setter a for his agenda A_a. Moreover, let the set of all possible constitutional agendas be denoted by $\mathcal{A}$. To simplify the exposition, we assume that the following tie-breaking rule is applied:

- If an agenda-setter is indifferent between an agenda that leads to $g = 1$ and another that yields $g = 0$, he will propose the former.

Note that $U_j(A_a)$ is a utility level evaluated at the optimal voting strategies of all agents. In what follows, we will always assume—without referring to the fact explicitly—that sincere voting, (EWSA) and the tie-breaking rule are applied. We are now ready to characterize the expected utility level a particular constitution can deliver. We say that a constitution $\mathcal{C}$ implements an expected utility U if all possible subgame perfect equilibria under constitution $\mathcal{C}$ yield the expected utility U.[8] We call a constitution *first-best* if it implements the expected utility $\bar{U}_{\text{opt}}$ induced by the socially efficient contract, i.e.

$$\bar{U}_{\text{opt}} = \begin{cases} e + V - (1+\lambda)k & \text{if } V - (1+\lambda)k > 0 \\ e & \text{else.} \end{cases}$$

To prove that the constitutions we propose are first-best, we show that

- equilibrium applying and voting strategies are unique;
- if $V - (1+\lambda)k > 0$, there exists $a \in [0, 1]$ with $\psi_a = 1$, and each agenda-setter makes a proposal that implements a socially efficient allocation;
- if $V - (1+\lambda)k \leq 0$, nobody applies for agenda-setting, i.e. $\psi_j = 0$ for all j.

[7] In principle, δ_j and A_a can depend on the entire history of the game which we omit to simplify the presentation.

[8] Non-uniqueness of equilibria only occurs in out-of-equilibrium strategies.

Finally, note that, in stage 1, the constitutional rules are decided by the unanimity rule. It is obvious that if a set of constitutional rules yields first-best, it will be approved unanimously in stage 1, since individuals are identical at this point.

2.5 Constitutional principles

The rules of the constitution now have to specify

1. whether there is to be a special treatment for the agenda-setter (**agenda-setter rules**);
2. restrictions on the agendas, i.e. definition of all constitutional agendas (**agenda rules**). An agenda consists of a project proposal and a financing package;
3. how the nation decides on a proposal (**decision rules**).

We assume open ballots. Therefore, individuals can be divided ex post into majority winners and the minority. In order to avoid ambiguous language, we distinguish between project winners (losers) and majority winners (losers), depending on whether V_h or V_l has been realized and whether an individual belongs to the majority or minority of voters, respectively. To formulate the rules we will be using in this article, we need the following notation:

Notation *Let A_a be an arbitrary agenda. We denote the fraction of citizens who have to pay positive taxes by n_T. An individual who has to pay a positive tax is called a taxed person or a tax-payer. Furthermore, we denote the maximal taxes*[9] *proposed for a citizen by t^{max} and the total tax payments proposed in A_a by $T = \int_0^1 t_j \, dj$.*

We will consider the following set of possibilities for designing constitutional rules. We will see that this set of rules allows us to construct the most general possibility theorem. Apart from the rules discussed in Gersbach (2009), we introduce two novel rules: maximal taxation of agenda setter and constraint taxation on majority winners which will be explained below. For the impossibility theorem, we will consider all conceivable constitutional rules.

Agenda-setter rules

- *Costs of agenda-setting [CA(b)]*
  The agenda-setter pays a fixed amount of $b > 0$ if his agenda does not lead to the provision of the public project.

Agenda rules

- *Maximal taxation of agenda setter [MTA]*
  The agenda setter pays the maximal tax rate proposed in his agenda.

9 The maximum tax may not be well-defined, as the tax rate for a particular individual can be set arbitrarily high. To avoid this difficulty, we use the essential supremum to define $t^{\max}$.

- *No subsidies [NS]*
  The agenda-setter is not allowed to propose any subsidies.
- *Constraint taxation on majority winners [CTW]*
  Only majority winners can be taxed.
- *Budget constraint [BC]*
  The financing package must satisfy the budget constraint.

Decision rules

- *m-majority rule [M(m)]*
  If a proposal to change the status quo receives a majority of m percent of the citizens, the proposal is adopted.
- *Flexible majority rule [FM(α, β)]*
  This rule divides the population into a part that—according to the proposal—pays positive taxes and the rest of the population. The fraction of taxed citizens is n_T. A proposal is adopted if it receives an α-majority in the taxed part of society and a β-majority in the rest of the society. The critical levels $\alpha = \alpha(n_T, T, t^{\max})$ and $\beta = \beta(n_T, T, t^{\max})$ may depend on the fraction n_T of tax-payers in the population, on the total taxes T, and on the maximal tax rate $t^{\max}$ proposed in the agenda. The following special cases of the flexible majority rule can be defined:
  – *Fixed participation rule:* $\alpha(\cdot) \equiv 1$ and $\beta(\cdot) \equiv \beta$ $(0 \leq \beta \leq 1)$.
  – *Threshold majority rule (fixed threshold) [TMf(q)]*
    Under this rule, β jumps from 1 to 0 when the proportion of tax-payers reaches the threshold level q.
    $$\alpha(\cdot) \equiv 1 \text{ and } \beta(n_T) := \begin{cases} 1 & \text{if } n_T \leq q \\ 0 & \text{otherwise.} \end{cases}$$
  – *Threshold majority rule (variable threshold) [TMv(q)]*
    $$\alpha(\cdot) \equiv 1 \text{ and } \beta(n_T, T, t^{\max}) := \begin{cases} 1 & \text{if } n_T \leq q(T, t^{\max}) \\ 0 & \text{otherwise.} \end{cases}$$

Flexible majority rules are defined with two arguments. In the special cases, however, one argument is sufficient. Note that the flexible majority rules [FM(α, β)] may depend on information generated by the proposal (n_T, $t^{\max}$ or T). Therefore, constitutions that use such rules produce a *feedback effect*: the actual rules governing the decision whether a proposal is constitutional depend on the proposal, so the proposals to be made will, in turn, depend on those rules. In contrast, rules [CA(b)] and [M(m)] do not depend on proposal information but may depend on project parameters. Finally, [NS] and [CTW] do not depend on any additional information.

It is important to note that constitutional verification occurs both at the proposal stage and at the voting stage. If a proposal or the majority voting outcome violates one of the agenda or decision rules, then the status quo prevails, since the constitution is violated and the proposal is void. This implies that the second and final constitutional check is only possible after the votes have been cast.

The constitutional violation of an agenda rule is of particular importance in the case of [CTW]. [CTW] means that an adopted proposal is only constitutional if agents who voted with the minority do not pay taxes. Clearly, [CTW] requires open ballots. [CTW] makes taxed individuals pivotal in the following sense: Suppose a proposal $g = 1$ has been made. If a taxed individual votes against the proposal, the status quo $g = 0$ prevails. If $g = 1$ is rejected, the prevalence of the status quo is obvious. If $g = 1$ is adopted, the tax-payer in question will necessarily belong to the minority, since he/she supports $g = 0$. Hence, the [CTW] rule is violated, and the outcome is unconstitutional, which implies that the status quo also prevails in this case. As a consequence of [CTW], taxed individuals will support a proposal as long as their net benefit is positive, since otherwise $g = 0$ will necessarily prevail.[10]

Here, some remarks about the role of the budget constraint are called for. If taxes exceed project costs and subsidies, we assume that excess revenues will be paid back uniformly to the citizens as lump sum transfers. We might also assume that excess revenues are destroyed. All constitutions in this article will ensure that no subsidies are paid and that taxes will never exceed project costs. A more delicate issue arises when a project/financing package is adopted that violates the budget constraint. Since all constitutions will contain [NS], such a proposal implies that taxes cannot cover project costs. In this case, we assume that the project cannot be realized and yields no benefits, while costs are sunk. For instance, if a public infrastructure project cannot be successfully completed, it is of no value to voters, while the costs are sunk. Since voters observe project costs at the legislative stage, they know whether costs will be covered when a project is undertaken. Hence, making a proposal that cannot cover project costs and does not include subsidies is always weakly dominated by proposing the status quo for any agenda-setter. In the following, we, therefore, neglect proposals with $g = 1$ when taxes cannot cover project costs.

We will call a proposal A_a by an agent a constitutional if the triple $\left(a, A_a, \delta^*(A_a)\right)$ does not violate the constitutional rules. $\delta^*(A_a)$ denotes the equilibrium voting strategies if A_a is proposed.

3 First-best constitutions

In this section, we explore the structure of first-best constitutions. Throughout this section, V_l may be positive or negative and constitutions are first-best for both cases. However, the proofs and arguments are only given for the more delicate case when V_l is negative.[11]

3.1 Uncertainty regarding p and $(1 + \lambda)k$

We start with the case when V_h and V_l are known at the constitutional stage, but not p and $(1 + \lambda)k$. This case will allow us to construct simple examples. We do not need

[10] Note that an agent can be pivotal when the condition [CTW] is evaluated, as this rule is defined at the individual level. In contrast, an individual vote has no impact on the voting outcome.

[11] If V_l were restricted to be positive, there exist simpler constitutions than the ones introduced in this section that are first-best as deterrence of project losers from agenda-setting is not crucial (see Gersbach 2005).

to make any specific assumptions about the statistical distribution of p and $(1+\lambda)k$ at the constitutional stage.

We consider the following constitution with a variable threshold for the flexible majority rule:

$$\mathcal{C}_1 := \left\{[\mathrm{CA}(\max\{-V_l, 0\})], [\mathrm{MTA}], [\mathrm{NS}], [\mathrm{CTW}], [\mathrm{TMv}(q^*(T))]\right\},$$

where[12]

$$q^*(T) = \begin{cases} 1, & \text{if } T > V_h \\ \frac{T-V_l}{V_h-V_l}, & \text{if } V_h \geq T \geq V_l \\ 0, & \text{if } V_l > T. \end{cases}$$

Proposition 1 *Suppose that the utility levels V_h and V_l are known at the constitutional stage. Then, constitution $\mathcal{C}_1$ is first-best.*

The intuition for the result is straightforward.[13] The rule [NS] ensures that no inefficient redistribution occurs. The rule [MTA] implies that an agenda setter who proposes the project will tax all project winners (and himself) by the same tax rate, which is given by $\frac{k(1+\lambda)}{p}$. Overall tax revenues T are given by $k(1+\lambda)$. The rule [CTW] ensures that only project winners are taxed. The rule [CA] ensures that project losers will never apply for agenda-setting. Project winners will apply for agenda-setting if the project is socially desirable. The threshold $q^*(T)$ is constructed in a way such that for $T = k(1+\lambda)$, the project, if proposed, will be adopted if the share of project winners is equal or larger than $q^*(T)$. In such cases, the project is efficient, as

$$q^*(T)V_h + (1-q^*(T))V_l = \frac{k(1+\lambda)-V_l}{V_h-V_l}V_h + \frac{V_h-k(1+\lambda)}{V_h-V_l}V_l = k(1+\lambda)$$

The important observation is that the critical threshold $q^*(T)$, above which the yes-votes of taxed individuals are sufficient for the adoption of the public project, itself depends on the aggregate tax revenues generated by a specific proposal. How the threshold $q^*(T)$ varies can be illustrated by comparing two projects with small and high costs, i.e. when $(1+\lambda)k$ is small or large. Small aggregate tax revenues correspond to small costs for the public project provision and thus the project is socially optimal for a smaller share of project winners. By construction, $q^*(T)$ is smaller. The opposite case occurs for the project with large costs.

3.2 Uncertainty regarding p, V_h and $(1+\lambda)k$

In this section, we assume that only V_l is known at the constitutional stage, but not p, V_h and $(1+\lambda)k$.

[12] Note that for $V_l < 0$ the case $V_l > T$ and thus $q^*(T) = 0$ cannot occur.

[13] A more detailed and formal proof is available upon request, see also Gersbach (2009).

As we shall see, this constellation will constitute the most general possibility theorem. We consider the following constitution involving a variable threshold q^*:

$$\mathcal{C}_2 := \left\{[\text{CA}(\max\{-V_l, 0\})], [\text{NS}], [\text{CTW}], [\text{TMv}(q^*(T, t^{\max}))]\right\},$$

where

$$q^*(T, t^{\max}) = \begin{cases} 1, & \text{if } T > t^{\max} \\ \frac{T-V_l}{t^{\max}-V_l}, & \text{if } t^{\max} \geq T \geq V_l \\ 0, & \text{if } V_l > T. \end{cases}$$

Proposition 2 *Suppose the utility level V_l is known at the constitutional stage. Then constitution $\mathcal{C}_2$ is first-best.*

The proof is given in the Appendix.[14] Note that the flexible majority rule above depends on the overall revenues and the maximal tax-rate proposed by the agenda-setter. The intuition for the result is as follows: If the public project is socially efficient, the agenda-setter can avoid the requirement of unanimous support by taxing a sufficiently high proportion of project winners and by imposing a sufficiently high maximal tax rate $t^{\max} \leq V_h$. In this case, the project will be adopted by the yes-votes of all project winners. Note that an agenda-setter will not propose a uniform tax rate $(1+\lambda)k/p$ for all project winners, as then $p < q^*$ and, therefore, $n_T \leq p < q^*$, which would imply that unanimous support for the proposal is required. Hence, $t^{\max}$ has to be higher than $(1 + \lambda)k/p$. Therefore, in order to avoid raising more overall taxes than are needed to finance the public project, the agenda-setter has to create at least two groups of project winners: one with a tax rate $t^{\max}$ and one with a lower tax rate. If, on the other hand, $p \leq q^*$, then the agenda-setter cannot avoid the requirement of unanimous support, since by [CTW] the maximal tax rate $t^{\max}$ cannot be higher than V_h.

Note also that [MTA] would be difficult to use when V_h is uncertain, since the incentives for project winners to apply for agenda-setting would not be clear anymore. If nobody applies, a single project winner has an incentive to apply if the project is socially valuable. But if another project winner applies, it is better for him to hold back, since not setting the agenda implies a positive probability of belonging to the low-tax group, while setting the agenda would always lead to the maximal tax rate.

3.3 Examples

In this section, we give two examples to illustrate the efficiency gains that can be achieved by flexible majority rules in comparison to simple majority rules. Suppose in the first example that only p and $(1 + \lambda)k$ are uncertain.

At the constitutional stage, suppose that p and $(1+\lambda)k$ are random variables. Both are uniformly distributed on [0, 1] and are stochastically independent. To give the

[14] A first version of this proof has been developed in Erlenmaier and Gersbach (2001).

simple majority rule the best chances of achieving utility gains, we supplement the simple majority rules by the same constitutional principles and compare

$$\mathcal{C}_1 := \left\{[\text{CA}(\max\{-V_l, 0\})], [\text{MTA}],[\text{NS}],[\text{CTW}],[\text{TMv}(q^*(T))]\right\}$$

with

$$\hat{\mathcal{C}}_1 := \{[\text{CA}(\max\{-V_l, 0\})], [\text{MTA}],[\text{NS}],[\text{CTW}],[\text{M}(m)]\}\,.$$

We know that constitution $\mathcal{C}_1$ yields first-best allocation. The expected utility is denoted by EU^{FB}. The expected utility for the constitution $\hat{\mathcal{C}}_1$ when we choose the fixed majority rule m is denoted by EU^m. We denote by m^* the fixed majority rule that maximizes EU^m.

We calculate the relative efficiency loss associated with the optimal fixed majority rule, denoted by $\Delta\widehat{EU}$ which yields:[15]

$$\Delta\widehat{EU} := \frac{EU^{m^*} - EU^{FB}}{EU^{FB}} = \frac{-\frac{1}{12}(1+\lambda)^2}{V_h^2 - V_h(1+\lambda) + \frac{1}{3}(1+\lambda)^2}$$

The relative efficiency loss depends on parameters. Suppose first that $\lambda = 0.3$ and $V_h = 1$. Then, $\Delta\widehat{EU} = -0.53$, and applying an optimal fixed majority rule implies a utility loss of 53%. If $\lambda = 0.3$ and $V_h = 2$, the utility loss would amount to 7%. The example illustrates that the relative efficiency loss of optimal super-majority rules varies strongly with parameters.

The second example is identical to the first example, except that the value for V_h is either 1 or 2, and each value occurs with probability $\frac{1}{2}$. Then, using the constitution in Proposition 2, we can perform the same exercise. As the appropriate flexible majority rules is still first-best, the utility loss of using fixed majority rules is more than the average of the previous losses in the first example.

4 The impossibility case

Our most general possibility result depends on the fact that V_l is known. In this section, we show that this is indeed an essential constraint. We establish an impossibility theorem for the case where V_h and V_l are uncertain.[16]

In particular, we assume, at the constitutional stage, that V_h and V_l are stochastically independent and distributed according to some density functions with $V_h \in [\underline{V}_h, \overline{V}_h]$ and $V_l \in [\underline{V}_l, \overline{V}_l]$. $\underline{V}_h$ and $\underline{V}_l$ are the lowest possible realizations. $\overline{V}_h$ and $\overline{V}_l$ are the

[15] Details of the calculations are available upon request.

[16] We note that if V_h is known, but not V_l, it is possible to construct constitutions that are approximately first-best. This can be achieved by the majority requirement $m = p + \epsilon$, where ϵ is a small but positive real number, and by allowing a maximal amount of subsidies of $\epsilon\gamma$ ($\gamma > 0$). The constant γ is set such that $pV_h + (1-p)(-\gamma) = (1+\lambda)(k+\epsilon\gamma)$. Thus, if and only if $V_l \geq -\gamma$, any agenda-setter will compensate a measure ϵ of losers by the amount γ for each, and the necessary majority will accept. Thus, the decision is first-best. Subsidies are costly, however, but this loss vanishes by letting $\epsilon \to 0$.

upper bounds. We assume $\underline{V}_l < \overline{V}_l < 0 < \underline{V}_h < \overline{V}_h$. We use $V_l^*(V_h)$ to denote the critical benefit level that characterizes the socially efficient public good provision:

$$V_l^*(V_h) := \min \{V_l \mid pV_h + (1-p)V_l \geq (1+\lambda)k\} \tag{1}$$

To simplify the presentation of our arguments, we assume that V_h and V_l are uniformly distributed[17] and that $\underline{V}_l < V_l^*(\underline{V}_h) < \overline{V}_l$. Hence, $1 > Pr\{V_l \geq V_l^*(\underline{V}_h)\} > 0$ where $Pr\{\cdot\}$ denotes the probability function. Finally, we assume $p\underline{V}_h < k(1+\lambda) < p\overline{V}_h$. We obtain:

Proposition 3 *There does not exist a constitution that is first-best.*

The proof of Proposition 3 is given in the Appendix. The main point of Proposition 3 is that a proposal leading to the adoption of the public project does not generate information about V_l. Hence if V_l is uncertain, constitutional rules cannot discriminate between efficient and inefficient projects when V_l varies.

Two remarks are in order: First, the impossibility theorem in this article is based on incomplete information and is quite different to the famous Arrow theorem on the impossibility collective rationality which requires at least three alternatives.[18] Second, there are other types of democracies that do not fit into our framework and for which it is open whether our impossibility theorem can be applied to. For instance, procedural rules often allow amendments and decisions may be taken by a smaller group representing a larger population of citizens.[19]

5 Discussion and conclusion

In this article, we have explored the potential and limits of democratic constitutions using flexible majority rules. Several remarks are useful to put our results into perspective. First, flexible majority rules are essential ingredients of first-best constitutions. As illustrated in the examples, the use of simple or super-majority rules in the derived constitutions cannot yield first-best allocations. Moreover, with the logic of the proof of Proposition 2, one can show that there exists no constitution with a fixed supermajority rule that yields first-best allocations if p and $(1+\lambda)k$ are uncertain. Second, Propositions 1 and 2 show that the constitutions $\mathcal{C}_1$ and $\mathcal{C}_2$ are first-best and thus yield socially optimal project decisions for any realization of project parameters. Thus, in terms of aggregate utility, they dominate the corresponding constitutions in which the flexible majority rule is replaced by the optimally chosen super-majority rule, ex ante and at the stage when project parameters are known. However, $\mathcal{C}_1$ and $\mathcal{C}_2$ do not necessarily Pareto-dominate constitutions with super-majority rules ex post, when agents

[17] The proof highlights the fact that impossibility results can be obtained for other distributional assumptions.

[18] For a recent unified characterization of the relationship between collective rationality and permissible collective choice rules that satisfy the four axioms unrestricted domain, strong Pareto, anonymity and neutrality, see Cato and Hirata (2010).

[19] A famous historical example is the Athenian democracy in which representatives are selected by lot (see Tangian (2008) for an interesting model of this type of democracy).

have learned whether they will be winners or losers. Losers will not be compensated under socially optimal constitutions and thus they may prefer inefficient voting rules ex post to make project adoption impossible.

Third, applying for agenda-setting may be costly for citizens. This introduces new considerations, as agents may want to free-ride, and may renounce applying for agenda-setting. Then, the constitutions in Propositions 1 and 2 do not guarantee first-best allocations anymore. The problem can be dealt with by modifying the rule [MTA] in the following way: the agenda setter has to pay the highest tax rate minus the cost of applying for agenda-setting. This would eliminate the incentive for free-riding. Fourth, an important avenue for future research is to consider divisible public goods. In such circumstances, the risk of under- or overprovision of public goods becomes more pronounced. In such cases, flexible majority rules, in which the required majority depends on the aggregate tax burden, may provide a tool allowing democracies to steer the level of public goods towards a socially desirable level. A thorough exploration of this conjecture is left for future research.

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Appendix A: First-best constitution

In this Appendix, we give the proof for Proposition 2 for $V_l < 0$. The following observation will be needed.

Lemma 1 *Suppose that a constitution involves the principles [CA*$(\max\{-V_l, 0\})$*], [NS], and [CTW] and that* $V_l < 0$. *Then in any equilibrum* $\psi_j^* = 0$ *for all* $j \in (p, 1]$.

Proof First note that [CTW] means that the utility of a citizen j who does not apply for agenda-setting is never smaller than $e + V_l$. The reason is as follows: Suppose that an agenda setter has proposed an agenda A_a. Suppose that $e + gv_j + s_j - t_j < e + V_l$ for some agent j. But then $t_j > 0$, so agent j will reject the proposal, implying that A_a is unconstitutional according to [CTW] and $U_j(A_a) = e$.

Suppose now that a project loser a sets the agenda. Suppose that his proposal A_a is either unconstitutional, will not be adopted, or involves $g = 0$. Then, according to [CA($\max\{-V_l, 0\}$)], the agenda setter will have to pay $(-V_l)$ and because of [NS] will receive no subsidies implying that $U_a(A_a) = e + V_l$.

If A_a involves $g = 1$, we again have $U_a(A_a) = e + V_l$ because of the negative utility and the ban on subsidies. Hence,

$$\sup_{a \in (p,1],\, A_a \in \mathcal{A}} U_a(A_a) = e + V_l$$

and the Lemma follows by weak dominance. □

Proof of Proposition 2

Step 1: Suppose $V > (1+\lambda)k$, and thus for a given realization of V_h and $(1+\lambda)k$, we have $p > p^*(V_h, (1+\lambda)k)$, where p^*, viewed as a function of V_h and $(1+\lambda)k$, has been defined in Section 2.2 Suppose also that a project winner determines the agenda. He can achieve the utility level $e + V_h$ by proposing an agenda of the following type: He chooses $t_a = 0$, and taxes the remaining project winners as follows: A fraction w of project winners pays $t_{w1} := V_h$, and the other fraction $(1-w)$ has to pay for the rest of the project costs, i.e. their tax rate is

$$t_{w2} := \frac{(1+\lambda)k}{(1-w)p} - \frac{wV_h}{1-w}.$$

The fraction w has to be small enough to ensure that $t_{w2} > 0$. Hence in this case, $n_T = p$ and $t^{\max} = V_h$, which implies that $q^* = p^* < n_T$, so that the proposal only needs the unanimous support of the project winners. But since $t^{\max} = V_h$, all project winners have a non-negative net benefit from the proposal and will vote for it.

We next observe that because of [NS], the agenda setter cannot achieve a higher utility level than $e + V_h$, and that all agendas leading to the same utility level $e + V_h$ for the agenda setter necessarily involve $g = 1$, overall taxes $T = (1+\lambda)k$, and no subsidies.

Step 2: Suppose again $V > (1+\lambda)k$. From step 1, we conclude that $\psi_j = 0$ is a weakly dominated strategy for project winners, since, $\psi_j = 1$ never leads to a lower utility than $\psi_j = 0$, but, given that nobody applies, $\psi_j = 1$ is strictly better than $\psi_j = 0$. Using Lemma 1, we, therefore, find that project losers never apply for agenda-setting while project winners apply. Hence, the constitution yields the socially efficient solution. Moreover, applying strategies are unique.

Step 3: Suppose $V \le (1+\lambda)k$. Suppose that a project winner determines the agenda and wants to achieve a higher utility than e. This can only be achieved by a constitutional agenda with $g = 1$ that will be adopted. Consider such an attempt. [NS] implies that $T = (1+\lambda)k$, and [CTW] requires that $n_T \le p$ and $t^{\max} \le V_h$. Hence,

$$q^* \ge p^*.$$

Therefore, if $p \le p^*$, unanimous support for the proposal is required. As project losers cannot be subsidized, they would reject such a proposal. Hence, the utility of the agenda setter is $e + V_l < e$. The rule [CTW] implies that project winners, who do not apply for proposal making, can always ensure that their utility is at least e. Together with Lemma 1 this implies that nobody will apply for agenda-setting, i.e. $\psi_j^* = 0$ for all j. Hence, the status quo prevails which completes the proof.

Appendix B: Impossibility theorem

Proof of Proposition 3

The proof proceeds in several steps. Suppose there exists a constitution $\hat{\mathcal{C}}$ that is first-best.

Step 1: A first-best allocation is never associated with positive subsidies. We assume that $\hat{\mathcal{C}}$ contains [NS].[20]

Step 2: $\hat{\mathcal{C}}$ must contain [CA(x)] with $x \geq -V_l^*(\overline{V}_h)$. Otherwise, project losers would apply for agenda-setting when $x < -V_l < -V_l^*(\overline{V}_h)$. In such circumstances, the project may be efficient (if $V_l > V_l^*(V_h)$), but project losers apply for agenda-setting, and as [NS] holds, they would propose $g = 0$ when they can set the agenda.

Step 3: For efficient projects, the required majority to adopt $g = 1$ cannot be larger than p. As project losers will reject any proposal $g = 1$, this point is obvious.

Step 4: Any constitutional rule can depend on the known project parameters p and $(1+\lambda)k$ and on the proposal, i.e. on g and on the tax scheme $(t_j)_{j\in[0,1]}$. Suppose that a constitution $\hat{\mathcal{C}}$ contains a set of such rules, consider two constellations $(V_l^1, \underline{V}_h)$ and $(V_l^2, \underline{V}_h)$, where $V_l^*(\overline{V}_h) < V_l^1 < V_l^*(\underline{V}_h) < V_l^2$. The second project is efficient, the first is not. According to step 2, no project loser will apply for agenda-setting in either case.
Since $\hat{\mathcal{C}}$ is assured of being first-best, project winners must have an incentive to apply for agenda-setting in the case $(V_l^2, \underline{V}_h)$ and want to make a proposal $g = 1$. Otherwise $\hat{\mathcal{C}}$ is not first-best. Suppose that there exists such a proposal with a tax scheme, say, $(\hat{t}_j)_{j\in[0,1]}$ that is adopted. We note that such a proposal will be rejected by all project losers.

Step 5: Suppose case $(V_l^1, \underline{V}_h)$ occurs. If a project winner applies for agenda-setting and makes a proposal $g = 1$ and $(\hat{t}_j)_{j\in[0,1]}$, this proposal will be adopted under $\hat{\mathcal{C}}$. The reason is as follows: The proposal is the same as in step 4 and thus generates the same information as with $(V_l^2, \underline{V}_h)$. Hence, the same rules apply. Project losers will reject $g = 1$. Net benefits for project winners are the same, so they will vote the same way as in step 4. Hence, since the proposal, the voting outcome, and the rules are the same as in step 4, this proposal will be adopted. Moreover, if it is profitable for project winners to apply for agenda-setting with $(V_l^2, \underline{V}_h)$, it is also profitable with $(V_l^1, \underline{V}_h)$. Hence, project winners will apply for agenda-setting, and $g = 1$ will be adopted for $(V_l^1, \underline{V}_h)$. Therefore, $\hat{\mathcal{C}}$ is not first-best.

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[20] Note that a first-best constitution $\hat{\mathcal{C}}$ may not contain [NS]. In this case, as there is only one round of proposals, we can add [NS] to the constitution without destroying its efficiency properties.

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