ORIGINAL ARTICLE

A new method for the characterisation of rounded cutting edges

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Abstract The influence of the cutting edge micro geometry on cutting process and on tool performance is subject of several research projects. Recently published papers focus on optimising the cutting edge rounding. The results are partly inconsistent. Unfortunately, no international standard yet exists to properly describe the cutting edge micro geometry. This is seen as the root cause for detected discrepancies. To develop a common understanding for the influence of rounded cutting edges, it is indispensable to use the same basis to characterise the edge profile. This paper gives a review on existing characterisation methods, analyses the difficulties in their application and discusses different modelling ideas to describe the cutting edge profile. Based hereon, a new algorithm and geometrical parameterisation of the cutting edge is proposed, which reduces uncertainties and difficulties in the application of currently available methods. The proposed method considers measurement uncertainties and is robust against form errors and creates thus the basis required for the study of the influence of rounded cutting edges.

Keywords Cutting tool · Cutting edge · Micro geometry · Characterisation

Greek symbols

α	clearance
β	wedge angle
γ	tool rake
γ_b	chamfer angle
γ_{eff}	tool effective rake
δ	blasting angle
θ	turning angle
ρ	polar angle
ρ	orientation angle of ellipse
φ	included angle between profile apex and wedge bisector
ψ	apex angle
Δr	flattening of cutting edge profile
Δr_{mz}	minimum zone between inner and outer circle with common centre point
K	asymmetry value

Latin symbols and acronyms

a	half length of elliptical major axis
b	half length of elliptical minor axis
b_n	chamfer width
c	circle centre point position
d	point distance
h	uncut chip thickness
n	position of cutting edge tip
n	number of points
p	distance between profile tip and flank fitting line
p_i	point position
p_c	fitting lines intersection point
p_{int}	intersection of wedge bisector and cutting edge profile
p_α	flank intersection point of circle and edge profile
p_γ	rake intersection point of circle and edge profile
q	distance between profile tip and face fitting line
r_i	radius
r_n	rounded cutting edge radius
u	standard uncertainty
$\bar{r}$	observed mean radius
$\widehat{r}$	predicted radius

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B_f	apex width
D_α	flank asymmetry distance
D_γ	rake asymmetry distance
S	asymmetry value
R^2	coefficient of determination
S_γ	distance of rake separation point to ideal tool tip
S_α	distance of flank separation point to ideal tool tip
S_c	asymmetry value
SS_reg	estimated sum of squares
SS_tot	total sum of squares
U	measurement uncertainty

Indices

i	continuous index
n	normal to surface
ls	least square
max	maximum
min	minimum
mz	minimum zone
x	in x direction
y	in y direction

1 Introduction

The micro geometry of a cutting tool describes the actual shape of the cutting edge, which is the intersection of a tool's flank and face. It is well-known that the micro geometry significantly influences the cutting tool's machining performance. Among other effects, influences on cutting forces [1–5], wear development [2, 5–11], surface properties [2, 5, 7, 8, 11, 12] or chip formation [6, 9, 10] have been reported. All papers have in common that the profile of the cutting edge has somehow been characterised by ideal geometrical elements such as a radius of a circle or the angle and the width of a chamfer. The influence of asymmetrically rounded cutting edges has also been studied [13, 14].

The published results, however, are partly contradictory. While Denkena et al. [4] reports on decreasing forces with increasing cutting edge radii r_n, the opposite has for example been document by Albrecht [1] or Cortés Rodríguez [15].

The root cause for the inconsistent results is seen to lie in the characterisation method used to describe the cutting edge shape. The general approach in characterising cutting edges is to first generate a set of data points representing the cutting edge profile. This is typically acquired by optical or tactile measurement. Methods that are principally able of measuring surface textures and can thus be used for edge detection are described in [16]. Optical systems that are widely spread for the acquisition of cutting edge profiles are focus variation-based systems and fringe projection-based systems. Due to difficulties that arise from the steepness of flank and face when measuring the cutting wedge, interferometry is less commonly used. Optical systems generate a 3D data set of the cutting edge. To characterise the profile of a cutting edge, either different profiles along the edge are extracted from the 3D file, or the data are summarized in an average cutting edge profile. In the next step, the resulting data points representing the edge profile are used for the characterisation. The characterisation by a rounded cutting edge radius r_n is not unique. This is also pointed out in a recent survey [17] which compares the cutting edge radius measurement of different institutions and detects significant deviations in the determined radii. Depending on user, measurement uncertainty and fitting algorithm used, the same cutting edge profile is described by different radii. The survey and the number of published papers treating the effect of rounded cutting edges underline the necessity to develop a new characterisation algorithm which reduces the uncertainties of existing methods. While the characterisation of the cutting tool's macro geometry is internationally standardised [18], no standard yet exists to characterise the profile of a cutting edge itself.

According to [19], which corresponds with [18] but is more detailed, only a distinction between a rounded, chamfered or sharp cutting edge is drawn, see Fig. 1. A rounded cutting edge is a cutting edge which is formed by a rounded transition between the face and the flank. The nominal radius of a rounded cutting edge measured in the cutting edge normal plane is r_n. A chamfered cutting edge is a cutting edge which has an angulated straight transition between the flank and the face. Cutting edges can be designed with one or multiple chamfers. A sharp cutting edge is neither round nor chamfered. No further differentiation is given. Combinations of the different shapes are not dealt with in this standard.

A more detailed differentiation of ideal reference profiles is given in [20]: asymmetrically rounded cutting edges as well as combinations of chamfers and roundings are taken into consideration. The different types are shown in Fig. 2. The top row shows variations of rounded cutting edges, while the bottom row represents different chamfered versions. Chamfered cutting edges with rounded transitions are available as well. The parameters S_α and S_γ already
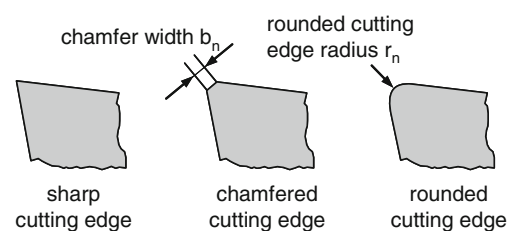
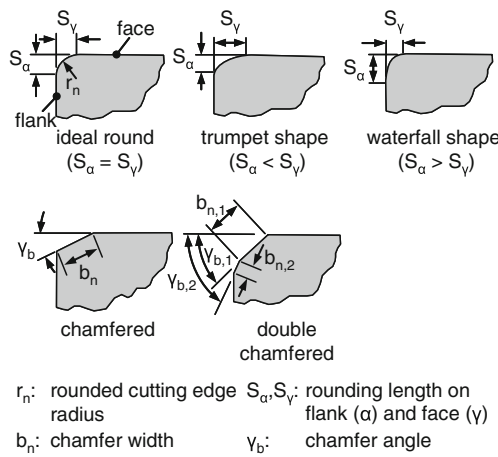**Fig. 1** Possible shapes of a cutting edge according to [19]

Fig. 2 Classification of possible cutting edge geometries after [20]

give a first indication of the geometry. The values will be dealt with in section 2.

Recently, attempts have been made to introduce new characterisation methods [4, 15, 20–22] which aim at describing asymmetrically rounded cutting edges. However, the published approaches are often limited in their applicability. The significance of the parameters used to describe the cutting edges is influenced by the user, by measurement uncertainty and by geometrical variations of the real cutting edge profile.

For a better understanding of the impact of the cutting edge geometry on the process performance, it is important to accurately know how the micro geometry looks like. Therefore, significant parameters are needed which can be used to link influences of the edge geometry to the machining performance. In this paper, currently available characterisation methods are analysed regarding applicability, uncertainty and comprehensibility. Based on the results, a new method for the cutting edge characterisation of rounded edges is proposed. This method uses a new algorithm which reduces the inadequateness and arbitrariness of already existing methods, it minimises the influence of measurement uncertainties, it is easy to apply, significant and robust and describes the main features of a cutting edge in few parameters.

2 Description and analysis of currently available characterisation methods

Different characterisation methods have been recently proposed. Their significance is dependent on the measurement uncertainty, i.e. the quality of the raw data used, form errors on the cutting edge to be characterised, in some cases by the user and the characterisation procedure itself. This section gives an overview of existing methods to describe rounded cutting edges and points out influences that impairs a reliable characterisation result.

2.1 Rounded cutting edge radius r_n

The probably most widespread and simplest way to characterise a rounded cutting edge is by fitting a circle into the cutting edge profile and to subsequently determine the radius r_n of that circle. The circle radius is a common parameter. The method is easy to handle, especially if no further information is given about how the fitting is to be done. If the curvature of a cutting edge is given by a set of points, mathematically at least three points along this curvature have to be chosen for the circle fitting. The radius r_n of a circle that is determined by three points with positions (x_i/y_i) can be calculated by the following mathematical equation:

$$(x_i - x_0)^2 + (y_i - y_0)^2 = r_n^2 \tag{1}$$

The solution is unique. The position (x_0/y_0) and the radius r_n of the inscribed circle are unequivocally solved, see Fig. 3. The circle runs exactly through the three points considered. If the result is not visually pleasing in terms of representing the cutting edge rounding, the procedure is generally repeated with three different points.

Thus, the chosen points on a curvature are strongly dependent on the individual user. As a result, the centre and the radius of the fitted circle will differ—the same cutting edge is characterised differently. As indicated by the inscribed circles in Fig. 3 (right), this user influence increases with increasing profile deviation of the edge profile in comparison to a perfect circle.

2.1.1 Circle fitting algorithms for overdetermined systems

To improve the circle fitting, a larger amount of points or all points along a curvature can be considered respectively. Typically, software programmes ask for the beginning and the end of the fitting area. This means that these two values are still individually chosen by the user and as such the fitting is influenced.

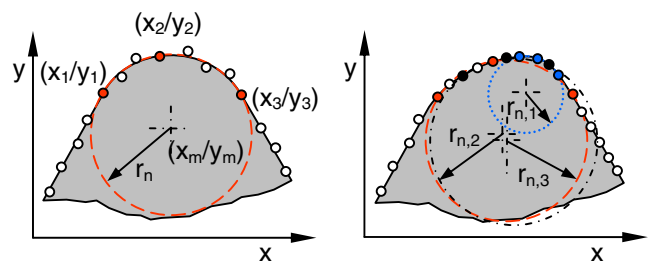In the case of fitting with more than three data points, the mathematical system of equations is overde-

Fig. 3 Circle described by three points (*left*); possible variation of cutting edge radius r_n when choosing different points along a curvature for the circle radius determination (*right*)

termined. Thus, a criterion for approximating the curvature by a circle is necessary. Dependent on the criterion chosen, the fitting results, i.e. the radii and the centre point coordinates, change. The commonly used fitting criterions are [23]:

- Gaussian condition or least square circle
- Tschebyscheff condition or minimum zone circle
- Minimum circumscribed circle
- Maximum inscribed circle

The algorithms are illustrated in Fig. 4.

The Gaussian condition is based on minimising the sum of all squared distances between fitting element and data points.

$$\sum_i d_i^2 \stackrel{!}{=} \min \qquad (2)$$

In the present case, the fitting element is a circle described by:

$$(x - x_0)^2 + (y - y_0)^2 = r^2 \qquad (3)$$

The distance between a data point (x_i, y_i) and a circle with the centre (x_0, y_0) and the radius r is given by:

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} - r \quad \text{with } i = 1 \ldots n \qquad (4)$$

To determine the values of x_0, y_0 and r, Eq. (3) has to be differentiated with respect to these parameters. If the resulting partial derivatives are identified by zero, the mathematical equations behind a Gaussian circle fitting correspond to:

$$\frac{\partial \sum_i d_i^2}{\partial x_0} = 0, \quad \frac{\partial \sum_i d_i^2}{\partial y_0} = 0, \quad \frac{\partial \sum_i d_i^2}{\partial r} = 0 \qquad (5)$$

$$x_0 = \frac{1}{n} \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \frac{r \cdot (x_i - x_0)}{\sqrt{(x_i - x_o)^2 + (y_i - y_o)^2}} \qquad (6)$$

$$y_0 = \frac{1}{n} \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \frac{r \cdot (y_i - y_0)}{\sqrt{(x_i - x_o)^2 + (y_i - y_o)^2}} \qquad (7)$$

$$r = \frac{1}{n} \sum_{i=1}^{n} \sqrt{(x_i - x_o)^2 + (y_i - y_o)^2} \qquad (8)$$

This nonlinear system of equations can be solved iteratively. The least square condition is the commonly applied method when approximating geometrical elements. For a given set of data points, the least square circle is unique. Further advantages of the least square circle fitting are its high repeatability and its outlier insensitivity. The more points are used, the more stable the fitting generally becomes.

The Tschebyscheff condition reduces the maximum distance between a point given and the fitted circle [23]:

$$\max_i |d_i| \stackrel{!}{=} \min \qquad (9)$$

It is commonly used to calculate form deviations of geometrical elements. According to [24], the roundness deviation of a profile corresponds to the minimum distance between two concentric circles which include all measurement points. The location of the centres of these so called minimum zone reference circles and the value of their radii shall be chosen so that the difference in radii between the two concentric circles is the least possible value [25]. The mean minimum zone reference circle is the arithmetic mean circle of the inner and outer circle. The Tschebyscheff circle is strongly influenced by outliers. Its repeatability is inferior to the Gauss condition and its solution is not always unique.

Further fitting algorithms are the maximum inscribed reference circle and the minimum circumscribed reference circle. The maximum inscribed circle is the largest possible circle that can be fitted within a set of data points:

$$r \stackrel{!}{=} \max \quad \text{with} \quad d_i \leq 0 \qquad (10)$$

The minimum circumscribed reference circle is the smallest circle that can be fitted around a set of data points.

$$r \stackrel{!}{=} \min \quad \text{with} \quad d_i \geq 0 \qquad (11)$$

The signed distance d_i is negative if the measurement point lies within the circle, while it is positive when it is outside the circle [23].

Typically, these conditions are used to evaluate fits. As for the Tschebyscheff algorithm, the solution for the maximum inscribed circle is not in every case unique. Generally, the algorithms are sensitive regarding outliers and have a low repeatability.

Despite the characteristics that the different fitting algorithms have, all circle fitting algorithms suffer that in
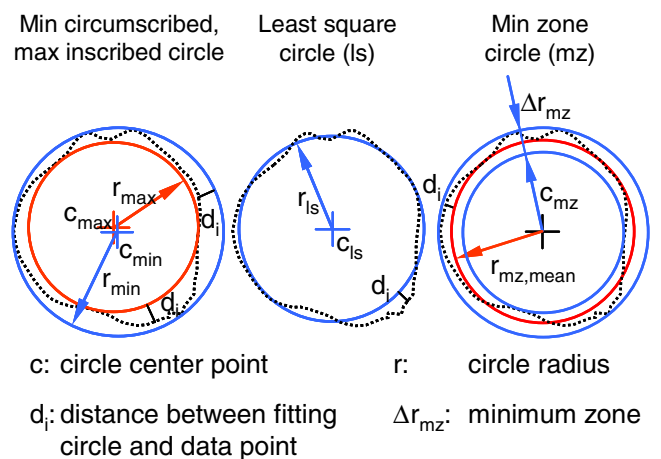

Fig. 4 Circle fitting algorithms for overdetermined systems

characterising cutting edge radii the fitting area is generally chosen manually (by the operator) and thus, as mentioned above, individually influenced by the user. No guideline is available for choosing this area.

In the research area of cutting edge characterisation, it has been criticised that different shapes might be characterised by the same radius [13]. The circumstance is caused by the choice of fitting points and is illustrated in Fig. 5.

Furthermore, a general drawback of the cutting edge radius is a lack of information about asymmetry.

2.1.2 Effect of point uncertainty and number of points on radius uncertainty when using least square circle fitting

Despite the characteristics that the different fitting algorithms have, the circle fitting is susceptible to point uncertainties. Every measured point is affected by an uncertainty. According to [26], the uncertainty of measurement describes a region about an observed value of a physical quantity which is likely to enclose the true value of that quantity.

The uncertainty of measurement is influenced by the measuring device, the user, the environment and the workpiece. This uncertainty has a direct impact on the accuracy of a characterisation.

In the above discussion, it has been assumed that the data points given represent the real shape of the cutting edge, but in real measurements this is not the case. The influence of measurement uncertainty on a radius can be easily understood when fitting a circle through three points. The inscription of a circle into three points being not affected by uncertainty is mathematically unique. This circle is represented in Fig. 6 as the black circle. If each point is now affected by the uncertainty U_n perpendicular to the circle line, it is possible to draw a maximum and a minimum circle into the fitting points which deviate in their radius and in their centre point position from the true circle. The largest possible circle is represented by the red dashed line, while the smallest possible circle is depicted by a blue dotted line. In this example, the uncertainties have the same value but are independent of each other. For three equally distributed points within an angular range of 90°, a true circle radius of $r=20$ μm and a point uncertainty of

$U_n=1$ μm, the possible circle radii vary between $17.5 < r < 23.5$ μm. This simple example shows how the point uncertainty affects the radius uncertainty by a multiple of its own value.

The effect of this propagated error usually becomes smaller when increasing the angular range of the points used to fit in a circle. Increasing the number of data points used for the fitting decreases the radius uncertainty as well.

Based on the calculations of [27], the influence of point dispersion, angular range and amount of points on the radius uncertainty as well as centre point uncertainty can be determined. For a least square circle fitting, the uncertainty factors that have to be multiplied with the dispersion of the measured points are tabulated in Table 1. The numbers are valid for an uncertainty range of 95% provided that the point distances are constant and only random errors and no systematic errors influence the measurement.

If 100 points are taken for a circle fitting over an angular range of 90°, the resulting error in radius contributes 2.02 times the dispersion of the used points.

The number of points and the point dispersion are directly related to the measurement method. For accurate measurements, as many points as possible should be determined along a curvature with a point dispersion as small as possible. Typically, the feature of interest has radii between 2 and 100 μm. Especially in the lower region of that range, where the resolution of some measurement systems might have the same order of magnitude as the feature to be characterised, an accurate determination of the rounded cutting edge radius is difficult.

For the other fitting algorithms, a determination of the functional relation between the dispersion of all measurement points and propagated error is difficult, as the fitting result is only dependent on a few points of all measurement points: for the determination of the minimum circumscribed or the maximum inscribed circle, only three points are relevant. The minimum zone circle is described by 4 points [28].

Taking the above mentioned aspects into consideration, only the least square circle fitting is to be used for the characterisation of rounded cutting edges by a radius. However, the influence of uncertainty on the fitting quality must be kept in mind and should thus always be specified.

2.2 Method proposed by Denkena

To be able to describe the shape of an asymmetrically rounded cutting edge, Denkena et al. [4, 13] developed a characterisation method which uses four respectively five parameters to express the shape of a cutting edge rounding. Its individual steps and the resulting parameters are the following according to [13]. The characterisation procedure is illustrated in Fig. 7 with the numbers indicated.
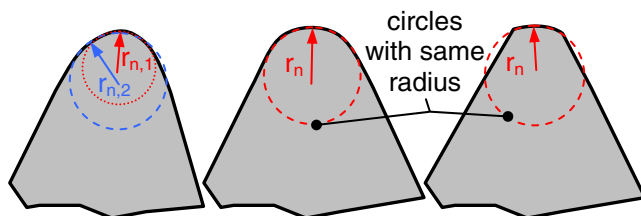
Fig. 5 Characterisation of the same cutting edge by different radii r_n (*left*) and different shapes by the same radius r_n (*right*)

Fig. 6 Influence of point uncertainty U_n on the minimum radius $r_{n,min}$ and maximum radius $r_{n,max}$ of a circle fitted through three points
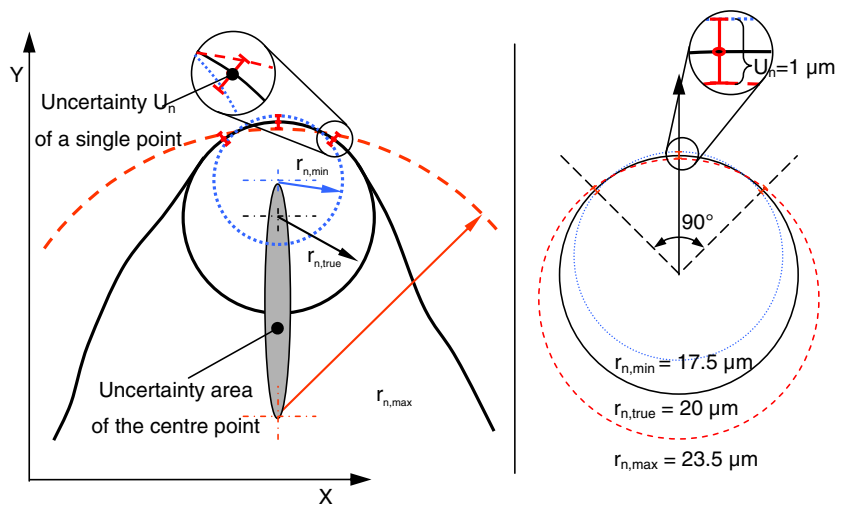
(1) Build flank and face tangents and find their intersection.
(2) Determine the wedge angle bisector.
(3) Find the points at which the cutting edge begins to separate from the flank and face.
(4) Measure the distances between the tangent intersection and the separating points. These distances are labelled S_γ for the face and S_α for the flank.
(5) Determine the distance Δr, which is the shortest way from the intersection of the lines representing the flank and face to the cutting edge profile.
(6) Determine the angle φ between wedge angle bisector and a straight line through the highest point of the profile.
(7) Calculate the ratio K between S_γ and S_α
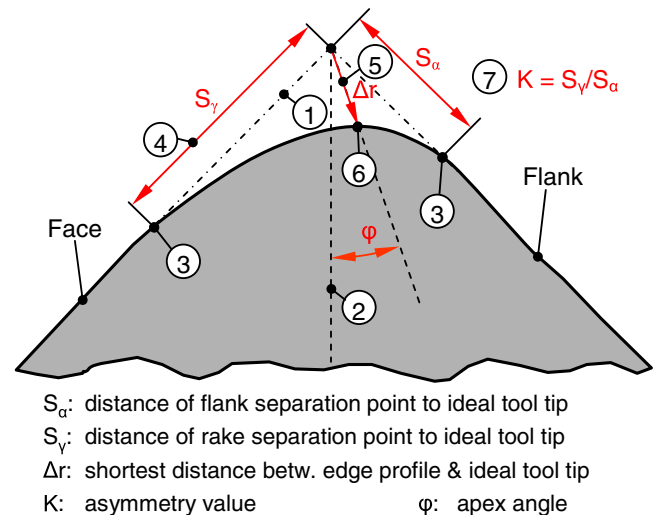
The values S_γ and S_α and thus their ratio K indicate the asymmetry of the curvature. A ratio of $K=1$ specifies a symmetrically rounded edge. Values below $K=1$ characterise waterfall-shaped profiles. Cutting edges with K values larger than 1 are often referred to as trumpet style profiles, q. v. Fig. 2. The distance Δr indicates the general magnitude of the rounding. The smaller the Δr value is, the sharper the edge is. The angle φ describes the shift from the profile apex to the wedge angle bisector and is thus another value for the asymmetry of the edge profile. In recently published papers, in which this characterisation method is used [14, 29], the angle φ is not employed anymore. A similar, earlier published approach to determine asymmetrical cutting edge can be found in [21], which characterises a cutting edge rounding by its radius r_n (no information is given about fitting area and algorithm) and the two distances S_γ and S_α.

Due to their simplicity, the parameters S_γ and S_α are easy to understand. Together with Δr these values give three grid points by which the approximate shape of the edge profile becomes imaginable. Besides being able to principally indicate an asymmetric rounding, this method has the

Table 1 Uncertainty factors u_r for the circle radius, dependent on angular range and number of points, after [27]

Number of points	Angular range [°]				
	30	90	180	270	360
4	411.48	41.53	8.95	6.35	6.35
5	132.83	13.61	2.98	1.94	1.93
6	94.12	9.71	2.15	1.31	1.30
8	69.96	7.29	1.61	0.93	0.91
10	60.53	6.32	1.40	0.79	0.75
20	41.90	4.41	0.98	0.52	0.48
32	32.90	3.47	0.77	0.40	0.36
100	19.06	2.02	0.45	0.23	0.20
320	11.11	1.17	0.26	0.13	0.12
1000	6.31	0.67	0.15	0.07	0.07

S_α: distance of flank separation point to ideal tool tip
S_γ: distance of rake separation point to ideal tool tip
Δr: shortest distance betw. edge profile & ideal tool tip
K: asymmetry value φ: apex angle

Fig. 7 Characterisation steps and results of the method proposed by Denkena et al. [4]

advantage that the general description of the magnitude of the rounding is less sensitive regarding point uncertainties than a radius that has been fitted into the profile. Assuming that the position of the intersection point of the straight lines is determined with high accuracy (see further below), the uncertainty of the value Δr corresponds to the same uncertainty as the point of the profile apex is affected with as shown in Fig. 8.

When using the profile flattening Δr instead of a rounded cutting edge radius r_n to describe the rounding magnitude, it has to be kept in mind that the relationship between these two values is nonlinear and dependent on the wedge angle. For two different wedge angles β, the same Δr describes different cutting edge radii r_n. The relation between cutting edge radius and Δr is visualised in Fig. 9. The mathematical relation between the flattening Δr, the radius r_n of a circle touching both fitting lines and the wedge angle is given by:

$$\Delta r(r,\beta) = r_n \cdot \left(\sqrt{1 + \frac{1}{(\tan\beta/2)^2}} - 1 \right) \tag{12}$$

2.2.1 Influence of macro geometry and choice of fitting area on the characterisation result

In order to determine the intersection point p_c, line fits are to be done with the profile areas representing the face and flank. A high repeatability in fitting can be achieved with the least square algorithm. In the case of perfect flat faces, the determination of fitting lines is simple and easily repeatable as the points used for the fitting are all ideally lying on one straight line. However, many cutting tools have curved surfaces rather than plane surfaces. The orientation of the straight fitting lines and the intersection point p_c is then strongly dependent on the points chosen for the line approximation according to Fig. 10.

Unfortunately, no details are mentioned about the proper choice of the fitting areas. Thus, depending on the user's
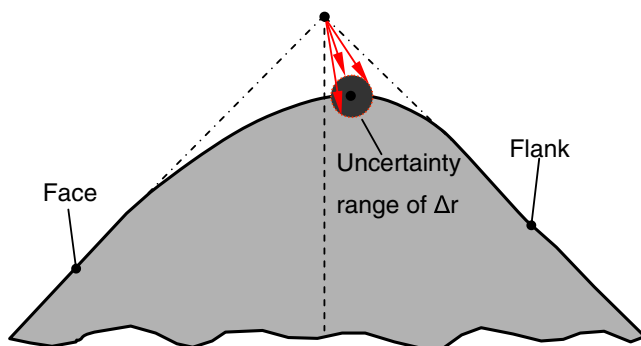

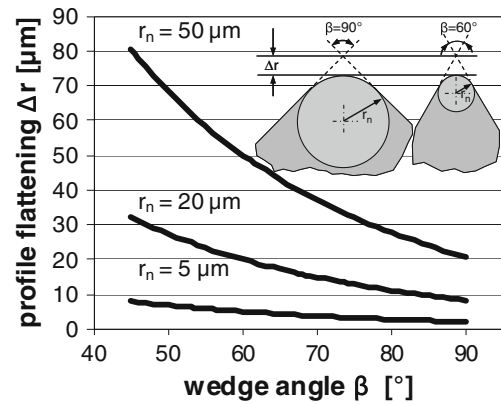Fig. 9 Interrelation of wedge angle β, profile flattening Δr and rounded cutting edge radius r_n

decision the characterisation results will be individually different. Figure 10 also points out a second difficulty in the application of this characterisation method. Depending on the points used to fit in a least square straight line, no tangential transition point exists. The fitting lines might cross the edge profile (see $p_{c,1}$ and $p_{c,2}$ in Fig. 10). The determination of S_γ and S_α, as implied originally, becomes impossible.

2.2.2 Influence of form error and measurement uncertainty

Assuming that the macroscopic form of the flank and face is a plane, the real surfaces can still have form errors due to manufacturing errors or due to tool wear, which might have a significant effect on the determination of the distances S_γ and S_α. While at first glance the separation points might be visually easy to determine for a perfect flat face, in practise they are difficult to approximate for a surface having form errors and uncertainty affected points. The method is very sensitive regarding point uncertainties. Minor deviations normal to the surface can cause major differences in the values of S_γ and S_α.

In a user individual, visual determination of the parameters S_α and S_γ, the result will strongly be influenced by form errors, chosen magnification and illustrated line thickness, see Fig. 11.

Fig. 8 Influence of point uncertainty on the shortest distance Δr between cutting edge profile and the *tip* of an ideal sharp cutting edge

Fig. 10 Influence of curved face and flank and chosen fitting area on straight fitting lines and their intersection point p_c
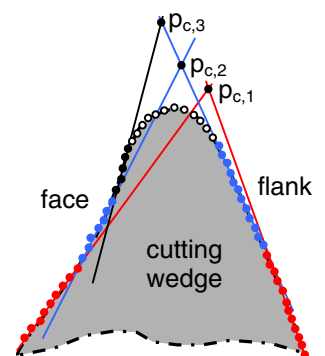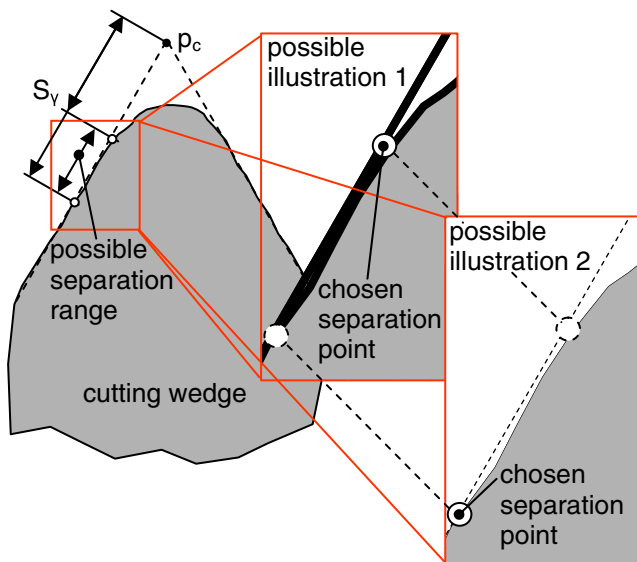
Fig. 11 Influence of form error and illustrated line thickness on the user individual determination of the separation points of cutting edge profile and straight lines

p_{int}: intersection of wedge angle bisector & edge profile

p_α, p_γ: intersection of horizontal line & best-fit straight lines

p_1, p_3: intersection of circle & edge profile

r_0: reference radius $r_{1...4}$: transition radii between tangents

Fig. 12 Rounded cutting edge characterisation according to Tikal and Holsten [20]
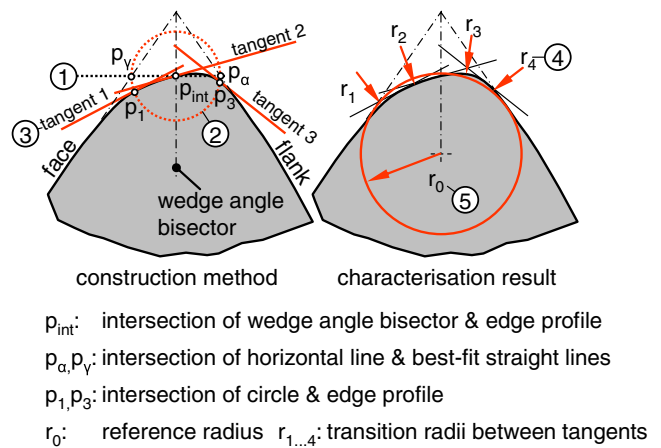
For an accurate identification of the separation points, a mathematical method is needed. Ideally, the profile is approximated by a function. For this function, the curvature in every point of the profile can be derived. If the curvature of the contour line starting from the rake or flank falls below a certain limit (which might be chosen as a function of the macroscopic curvature of flank and rake), the point is marked as separation point. However, real cutting edge profiles show form errors that sometimes are in the same magnitude as the radius of the rounded cutting edge to be characterised. For such cutting edges, a proper characterisation by the parameters mentioned is difficult.

2.3 Methods proposed by Tikal

Only little information is available about the method originally proposed by Tikal. Tikal's method describes the shape of a rounded cutting edge by determining the radii at different positions along the curvature. As shown in Fig. 12, the method depicted in [20] determines the radii r_1 to r_4 of the curvature in four areas as well as a reference radius r_0.

The selected areas are transition areas between three different tangents, the flank and the rake. No exact procedure is given for the characterisation, but it seems to be based on the following steps:

(1) At the intersection point p_{int} of wedge angle bisector and cutting edge profile, draw a line perpendicular to the wedge angle bisector. The line crosses the best-fit lines of flank and face. The intersection points are called p_α and p_γ.

(2) Draw a circle with a centre point in p_{int} and a radius corresponding to the distance p_{int} to p_α.

(3) Build the tangents to the cutting edge profile in the point p_{int} and in the intersection points p_1 and p_3 of the circle and the cutting edge profile.

(4) Determine the radii r_1 to r_4 in the transition curves between the different tangents or tangents and face or flank, respectively.

(5) Determine the reference radius r_0 of the circle which touches the best-fit straight lines of flank and face and has its centre on the bisector of the wedge angle.

The steps are illustrated by the circled numbers given in Fig. 12. The result of this complex characterisation method is a detailed description of the cutting edge rounding by five different radii. A straight imagination of how the actual shape looks like seems to be difficult. A further difficulty of the characterisation method arises due to the small regions that are chosen for the curve fitting. As mentioned above, the influence of measurement uncertainty on the fitting result of circles becomes larger with a decreasing angular range. In case of fitting several radii along a curvature, which consists of a limited number of data points, the uncertainty might become larger than the nominal values of the four circle radii r_1 to r_4 inscribed. Moreover, the number of parameters that this characterisation method generates impairs its significance. The method further indicates a precision of manufacturing methods for the generation of cutting edges which is currently not given.

In [22], a modification of this method is given, which also determines the angle φ of the tangent touching the edge profile at the intersection with the wedge angle bisector. The method is illustrated in Fig. 13. The parameter B_f describes the width of the cutting edge tip as the distance between the two intersection points of a line crossing the flank and rake best-fit straight lines perpendicular to the wedge angle bisector. This modification gives a further parameter—the apex angle φ—for the asymmetry of a cutting edge profile
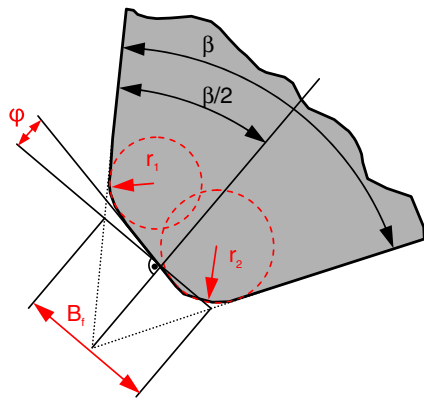
Fig. 13 Characterisation of a chamfered and rounded cutting edge by transition radii r_1 and r_2, apex angle φ and apex width B_f, as proposed by Tikal and Holsten [22]

and seems especially suitable for chamfered cutting edges with rounded transitions to flank and face. Form errors might impede a representative line fitting for the determination of the apex angle φ.
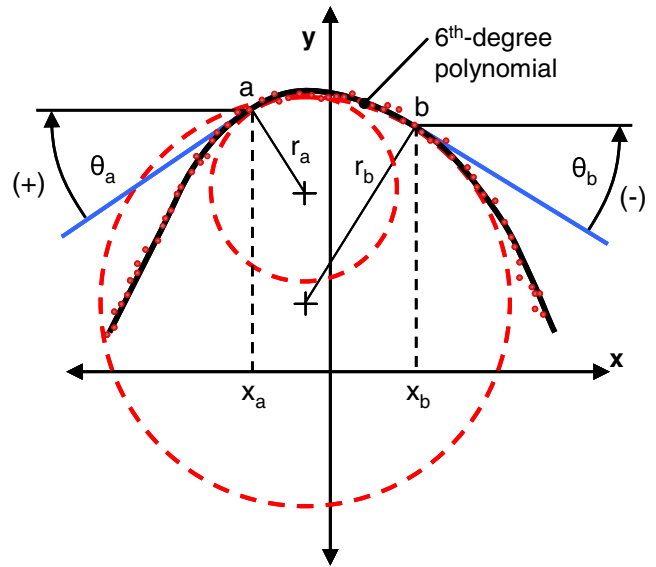
2.4 Method proposed by Cortés Rodríguez

The method of [15] is an enhancement of the method proposed earlier in [20]. In a first step, the profile is approximated by a sixth-degree polynomial. Based on this polynomial, a whole series of geometry describing parameters can be calculated for every point on the curvature. The parameters to be considered are according to [15]: curvature function, radius function, turning angle function, effective contour, effective rake, position of nose tip, nose radius and asymmetry as well as a best-fit radius.

The polynomial approximation of the edge profile and radii r_a and r_b as well as turning angles θ_a and θ_b for two individually chosen points a and b along the curvature are depicted in Fig. 14.

To determine the asymmetry of a profile, Cortés Rodríguez searches the apex n along the curvature (see Fig. 15) [15]. Through this point, he draws a straight line perpendicular to the wedge angle bisector. The ratio S_c of the distances p and q from that point to the lines representing flank and face is a measure for the asymmetry:
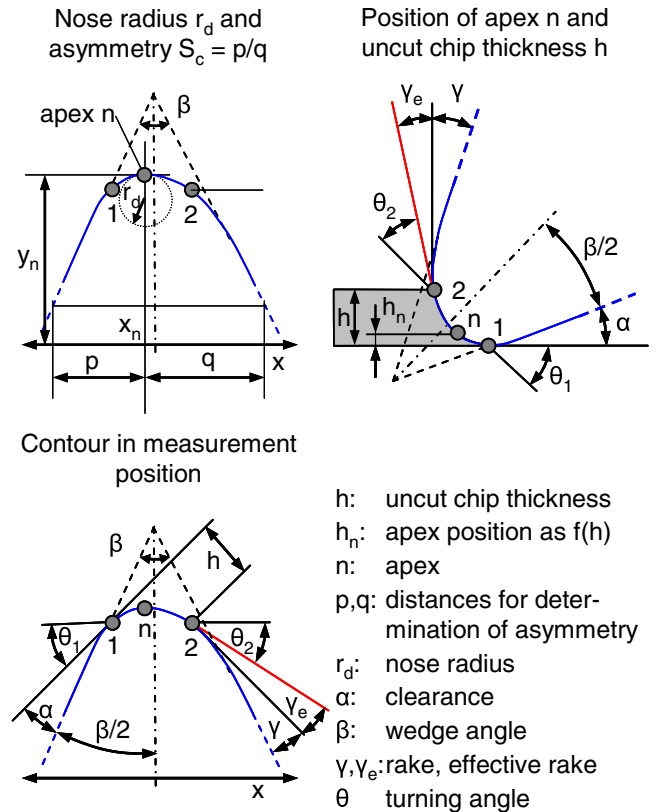
$$S_c = \frac{p}{q} \tag{13}$$

In the next step—this is the biggest difference to the other mentioned characterisation methods—Cortés Rodríguez takes the effective working conditions into consideration. An effective rake angle γ_e at the point representing the uncut chip thickness h and the turning angle θ_1 at the cutting edge point that touches the machined workpiece surface are defined [15], as shown in Fig. 15. According to [15], the position of n is influencing the material flow.

Fig. 14 Characterisation of a cutting edge profile by a sixth-degree polynomial and determination of turning angle θ and radius r for two randomly chosen points a and b, as proposed by Cortés Rodríguez [15]

Therefore, its distance h_n to the machined surface is important to know:

$$h_n = \sqrt{(y_n - y_1)^2 + (x_n - x_1)^2} \cdot \left[\sin\left(\theta_1 - \arctan\left(\frac{(y_n - y_1)}{(x_n - x_1)} \right) \right) \right] \tag{14}$$

h: uncut chip thickness
h_n: apex position as f(h)
n: apex
p,q: distances for determination of asymmetry
r_d: nose radius
α: clearance
β: wedge angle
γ,γ_e: rake, effective rake
θ   turning angle

Fig. 15 Cutting edge characterisation proposed by [15]

For symmetrical roundings, i.e. $S_c \approx 1$, [15] considers a least square circle fitting which is subject to being tangential to the lines representing flank and face.

Applying this method [15] transfers the macroscopic cutting tool features (rake angle, clearance) into the microscopic range. The difficulty in separating the edge features from the tool flank and face is avoided by defining the areas as a function of the working conditions.

An analysis of the error influence caused by fitting a sixth-grade polynomial is difficult to generalise, as the result is always dependent on the distribution of the measurement points representing the actual shape of the cutting edge. The approximation by a sixth-grade polynomial impairs the risk of oscillating. The higher the grade n of a polynomial is, the higher is the tendency to oscillate between its fitting points (known as Runge's phenomenon), which increases towards the boundary of the set point multiply. As a consequence, large deviations between fitted function and real profile may arise, which should be outside of the range where the polynomial represents the cutting edge. The deviations are certainly dependent on the real shape of the edge and the area chosen for the polynomial fitting. For a representative polynomial fitting of a high grade, the fitting area should be limited to the micro geometry area. For that in turn, this area has to be determined in advance.

In some cases, the sixth-grade polynomial fitting result might be worse than a circle fitting; in other cases, it might be of higher accuracy, meaning that the maximum deviation between real cutting edge profile and fitted function is smaller. The problem of oscillating can be avoided by using spline functions. Thus, an even better approximation and a better basis for subsequent calculations may be achieved.

Even though extensive, this method is certainly powerful in relating machining behaviour to the actual shape of the cutting edge. However, for the general description of cutting edges, independent of the cutting conditions they will be applied to, the set of parameters that the method proposes might be unmanageable. The characterisation method lacks comprehensibility and comparability. The approximation by a polynomial indicates a precision which cannot be manufactured with currently used manufacturing methods for the generation of rounded cutting edges.

3 Requirements for alternative characterisation methods

The existing characterisation methods are partly difficult to apply. The methods are mostly based on some separation of macro and micro geometry. In practise, especially the determination of the transition between both implies difficulties, e.g. the area which is to be used for a circle fitting is in most cases not clearly indicated. Form errors and point uncertainties make the application of the mentioned methods more difficult, as the methods are mostly based on ideal geometric shapes. Therefore, a characterisation method is necessary, that is either independent of the transition between micro and macro geometry or that is defining parameters which explicitly separates the fitting areas.

Furthermore, a characterisation method should be simple. In best case the whole geometry is characterised by only few parameters which are easy to identify, representative for the actual shape of the cutting edge and which are manufacturable and relevant for the process behaviour.

The characterisation method should indicate the magnitude of the rounding and, if applicable, give information about the asymmetrical shape. In best case the parameters can be used for both rounded as well as chamfered cutting edges.

Every characterisation is based on data which are subject to uncertainty. None of the available methods consider this influence on the characterisation result. The influence of point uncertainties on the characterisation result should be kept to a minimum. To indicate the quality of a characterisation result, if possible the uncertainty of the parameters used for a characterisation should be evaluated as well.

4 Discussion of alternative characterisation methods

Considering the above-mentioned requirements, different possible characterisation methods are revised. To generally keep the user influence low, two strategies can be considered. Either, an algorithm is used that recognises the area of the cutting edge micro geometry autonomously and does subsequent fittings only within the autonomously determined boundaries, or the area crucial for the micro geometry characterisation is iteratively defined by a function of macroscopic features.

4.1 Characterisation algorithms without predefined fitting areas

So far, the fitting areas for the determination of fitting elements have always been chosen individually. This leads to a user-dependent characterisation without a unique solution. A possibility to uncouple the characterisation result from individual influences is to use an algorithm that determines its boundary conditions for the least square fitting of elements autonomously.

One way to implement such a method is to consider the whole cutting edge profile including flank and face as possible fitting area for a circle. For every combination of adjoined points of the edge profile the algorithm fits in a circle that fulfils certain boundary conditions, e.g. minimum number of

points n, maximum allowable radius $r_{n,\max}$, minimum coefficient of determination $R_{\min}{}^2$ etc. The circle that fulfils the different criteria and has the lowest radius uncertainty is then chosen as the cutting edge radius. Figure 16 shows the graphical evaluation of such an algorithm.

4.2 Characterisation algorithms with predefined or iteratively defined fitting areas

4.2.1 Fitting an ellipse into the cutting edge profile

By least square fitting of an elliptical arc into the cutting edge profile, the rounding magnitude as well as the asymmetry can be indicated. Preferably, the fitting area is either known, or defined as proposed further below. An ellipse is described by the position of the centre point, the angular position ρ of major axis, and the lengths a and b of major and minor axis. For the simplest case, i.e. an ellipse with the centre in (0/0) and a major axis matching the x-axis of the coordinate system, the equation is given by:

$$\frac{x^2}{a} + \frac{y^2}{b} = 1 \tag{15}$$

Thus, the geometry and orientation are described by three parameters. To describe the position, two parameters (e.g. Δx and Δy) are needed plus a reference point on the wedge angle bisector, see Fig. 17. The amount of parameters already signifies that a simple, easy imaginable characterisation is not feasible by an ellipse. Therefore, this approach is not further pursued.

4.2.2 Suggestion of a new characterisation method for rounded cutting edges with iteratively determined fitting area

For obvious reasons, it is most plausible to use a radius to dimension the magnitude of a rounded cutting edge. Due to

different causes, the characterisation by a circle radius is not yet robust enough. The radius uncertainty resulting from fitting a circle to a set of uncertainty affected points has been mentioned above. In case of characterising a rounded cutting edge by a circle, following uncertainty factors influence the result:
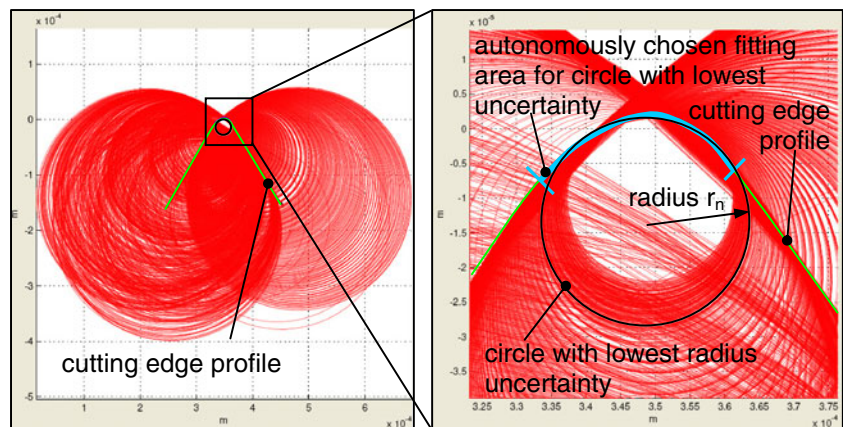
- Point uncertainty
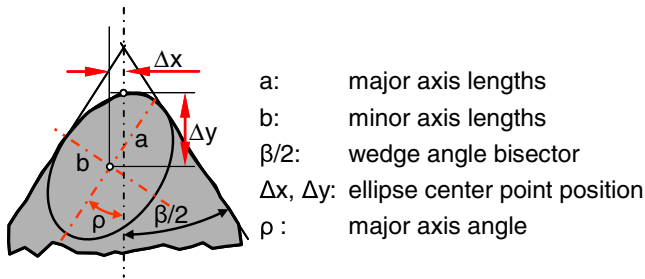- Fitting algorithm
- Fitting area

For a robust characterisation of a rounded cutting edge, the uncertainty of a measured point should be as small as possible. This is to be ensured by the user by choosing a proper measurement method and correct measurement settings. The tabulated values in Table 1 may serve as a guideline to identify the minimum resolution and maximum instrumental uncertainty of a measurement system which is necessary to keep the uncertainty of the fitted element low.

Even though probably used in the other methods mentioned (though never explicitly specified) as fitting algorithm, a least square fitting is suggested. As discussed, this method has a high repeatability and is unsusceptible regarding outliers.

Choosing the wrong fitting area can lead to a circle fitting which is not representative for the actual profile of a cutting edge. As a result the same edge is either characterised by different circle radii, or differently shaped edges might be characterised by the same radius. This difference in results has already been criticised in [13]. To combat its cause, the choice of fitting area has to be separated from the individual user. By making the fitting area user independent, the repeatability increases. As one uncertainty driver—definition of cutting area—is eliminated, the resulting characterisation uncertainty is reduced. The mathematical determination of the fitting areas is accomplished iteratively proposing the following algorithm, which is also shown in Fig. 18:

Fig. 16 Example of a not-area-limited, least square circle fitting on a cutting edge. For every combination of adjoined measurement points of the cutting edge profile, the algorithm calculates a least square fitted circle (*red circles*), determines the radius, uncertainty and coefficient of determination and then chooses the radius with the lowest radius uncertainty (*black circle*) as cutting edge radius
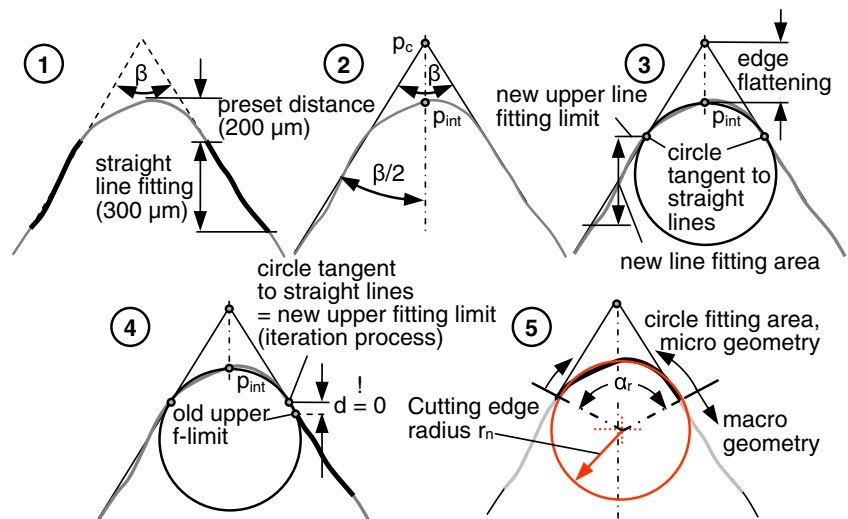
Fig. 17 Description of a rounded cutting edge by fitting an elliptical arc into the profile

a:	major axis lengths
b:	minor axis lengths
β/2:	wedge angle bisector
Δx, Δy:	ellipse center point position
ρ :	major axis angle

(1) A least squares straight line fitting is accomplished on flank and face over an area preset to a certain distance to the nose tip. An eligible value for the distance corresponds to the maximum of the cutting edge rounding expected. In this example, a value of 200 μm is chosen. If a non-realistic value is chosen, e.g. 0 μm or several mm, the error will be compensated within the next iterative steps. The fitting length should be chosen in such a way that a macro geometrical curvature of flank and face is not causing an inappropriate fit and still represents the effective working geometry. Thus, the considered length should correspond with the maximum uncut chip thickness the tool is proposed to be used for. In this case, a fitting length of 300 μm is chosen.

(2) The least squares fitted straight lines cross in point p_c. The angle inscribed by these straight lines is the wedge angle β. The wedge angle bisector gives as intersection with the cutting edge profile point p_{int}.

(3) Draw a circle that intersects point p_{int} and is tangent to both straight fitting lines. The points where the circle touches the fitting lines represent the new upper limit for the least squares straight line fitting of flank and face.

(4) Steps (2) and (3) are repeated until the distance between the points where the straight lines are tangent to the circle and the upper fitting limit of the foregoing step is approximating zero. These points are the limit finally representing the transition from macro to micro geometry.

(5) Generate a least squares reference circle using all points within the micro geometry limit. The circle does not necessarily need to touch the fitting lines nor is its centre necessarily on the wedge angle bisector. The radius r_n of the fitted circle represents the radius of the rounded cutting edge.

Following this algorithm a unique solution for the characterisation of a rounded cutting edge by its radius r_n is achieved.

Based on the equations of [27] and provided that the point uncertainty is known, the radius uncertainty of the fitted circle can be derived. For an ideal radius of r_n=50 μm, 100 evenly distributed measurement points over an angular range of α_r=90° and a measurement uncertainty for one point of U=0.5 μm, the resulting radius uncertainty is 2% of the diameter, based on an uncertainty range of P=95% (k=2). A calculated low radius uncertainty, however, is not necessarily representing a low deviation between fitted circle and data points. Therefore, additionally the coefficient of determination R^2 can be calculated for the fitted circle. The coefficient of determination R^2 indicates the goodness of fit, i.e. how well the points used for the circle fitting are actually represented by the fitted circle and its radius:

$$R^2 = \frac{SS_{reg}}{SS_{tot}} = \frac{\sum_{i=1}^{n}(\hat{r}_i - \overline{r})^2}{\sum_{i=1}^{n}(r_i - \overline{r})^2} \tag{16}$$

The estimated sum of squares SS_{reg} is the deviation between predicted radius $\hat{r}_i$ and observed mean radius $\overline{r}$.

Fig. 18 Proposed method for the characterisation of rounded cutting edges

The total sum of squares SS_{tot} corresponds to the deviation between observed radius r_i and observed mean radius $\bar{r}$. If R^2 equals one, the observed values are perfectly predicted by the fitted curve. That means for the present case, every point used to approximate the circle lies on the circle line. Together, the radius uncertainty and coefficient of determination can be used as an indicator for the quality of the fitting procedure and suitability of the describing geometry for the real cutting edge.

The choice of the fitting area is deduced from a perfectly rounded cutting edge. Thus, the fitting quality is dependent on the actual cutting edge profile. However, it is now possible to uniformly describe the magnitude of the rounding by the proposed procedure.

To determine the dimension of the cutting edge preparation independently of its actual shape (chamfered or rounded), it is proposed to carry out the steps (1) to (4) of the procedure mentioned above. In the last step, the distance between the straight line point of intersection p_c to the intersection between the wedge angle bisector and the real cutting edge p_{int} is determined and serves as a general indicator for the edge flattening Δr of the cutting edge profile (see Fig. 18). The smaller Δr is, the closer the edge is to a pointy profile. The relation between profile flattening, ideal radius and wedge angle are illustrated above in Fig. 9. If the fitting areas for the least square straight lines of flank and rake are preset, the position of the intersection point is only affected by a low uncertainty. The characterisation of the flattening by Δr gives the smallest possible uncertainty, as only the point uncertainty in one dimension (direction of wedge angle bisector) affects this value.

In case of a poor quality of fit, i.e. if the coefficient of determination falls below a preset limit of, e.g. $R^2 < 0.9$, the cutting edge should not be characterised by a circle radius. In such a case, a line fitting over the micro edge geometry should be considered. As illustrated in Fig. 19, the data
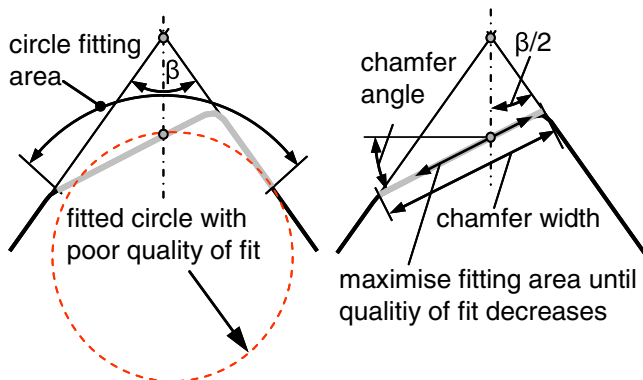
points for starting a least square straight line fitting are those close to the intersection of wedge angle bisector and cutting edge. The fitting area should be maximised until the coefficient of determination decreases. Subsequently, the angle and width of the best fitting chamfer are to be calculated.

4.2.3 Evaluation of the cutting edge asymmetry

To determine the asymmetry of a cutting edge, the following method is proposed: The first five steps are identical with the method mentioned above. The method is illustrated in Fig. 20.

(6) Draw a line through the intersection point p_{int} perpendicular to the wedge angle bisector. At its intersections with the least square straight lines, determine the distance to the cutting edge profile in the direction parallel to the wedge angle bisector. The values are called D_γ or D_α, respectively. The ratio of these two values indicates the asymmetry S.

In comparison to using tangential transition points as proposed by Denkena et al. [4], the concept of using distances between intersection points gives the advantage of a reduced sensitivity regarding point uncertainty and deviations of single points.

Local form errors might distort the asymmetry values though. To assess this influence, the asymmetry value S is compared with an area based evaluation of the asymmetry. The area-based algorithm considers the distances from the cutting edge profile to the horizontal line of all points that are lying between the points used for the determination of the asymmetry value S above. The ratio of the areas left and right to the wedge angle bisector give the area-based asymmetry values S_a. By taking more points into consideration, the influence of local form deviations and measurement uncertainties on the asymmetry value relativises based on the assumption that they are statistically evenly distributed.

Figure 21 shows the asymmetry evaluation of cutting edges that were abrasive jet machined at different blasting

Fig. 19 The characterisation of a chamfered cutting edge by a circle radius results in a poor quality of fit (*left*). In such a case, the microgeometry is to be fitted by a straight line (*right*)

Fig. 20 Asymmetry determination of a rounded cutting edge by building the ratio S of the distances D_γ on face side and D_α on flank side
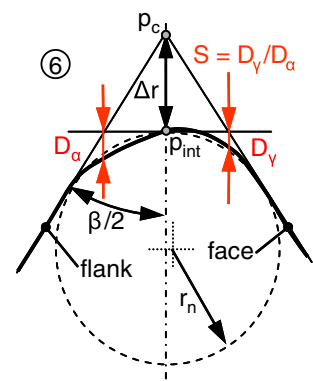
Fig. 21 Comparison of line-based and area-based asymmetry characterisation of cutting edges micro-abrasive jet machined at different blasting angles. Every bar gives the average value of 10 cutting edges machined with the same parameters
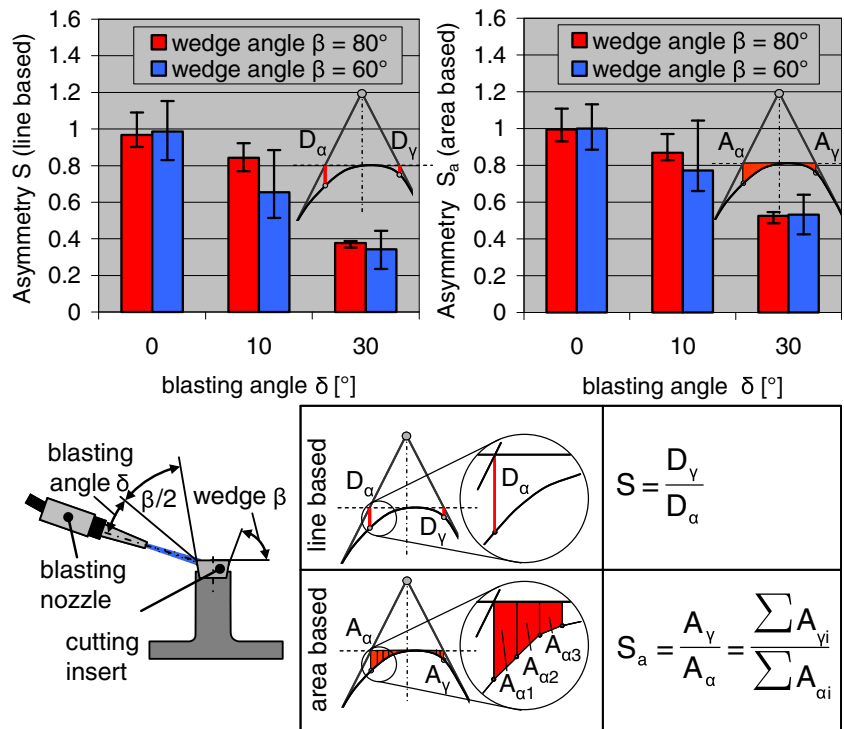

angles δ relative to the wedge angle bisector. Both asymmetry parameters S and S_a show the same trend. The larger the blasting angle, the lower is their value. Furthermore, the dispersions of the two methods are within the same range. Thus, local form errors do not significantly distort the line based asymmetry evaluation.

For that reason, the area-based asymmetry characterisation is not recommended over the line-based variant, as the values resulting from this method are not directly convertible into a defined measure that gives the actual profile of the edge. The method only specifies if an asymmetry is existent and indicates its magnitude.

Using the line-based asymmetry definition together with the value Δr, three points along the curvature are known. These three grid points are all based on intersection of elements, which are only affected by little uncertainties. The positions of the resulting grid points are easily traceable and the general shape of the cutting edge can be reconstructed. A further advantage of the three parameters Δr, D_α and D_γ is their validity for all cutting edge profiles: rounded as well as chamfered cutting edges or combinations thereof can be characterised by the values.

Applying statistical values which are used to describe symmetries of distributions like skewness or kurtosis to the characterisation of asymmetries is not recommended. The skewness gives an indication whether the cutting edge rounding is more trumpet or waterfall shaped, but it is not as descriptive and traceable as the proposed values of D_α and D_γ. Furthermore, skewness and kurtosis give no dimensional information.
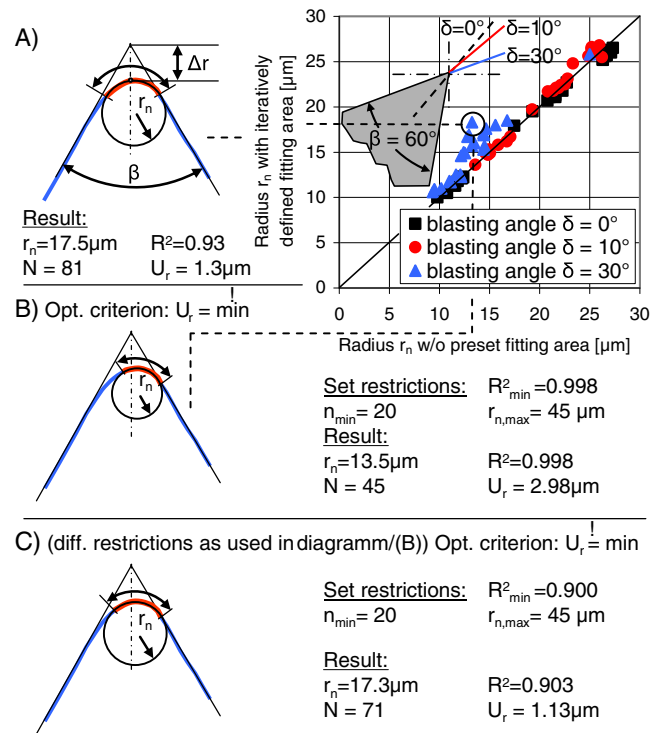
Fig. 22 Comparison of characterisation results when applying a fitting algorithm with iteratively defined fitting area (**A**) and without preset fitting area and a minimum radius uncertainty U_r as optimisation criterion (**B**). The set restrictions (number of points n, max radius $r_{n,max}$ and min coefficient of determination R_{min}^2) for the illustrated comparison are given in (**B**). (**C**) shows the influence of different restriction values as in (**B**) on the radius r_n, number of points N and radius uncertainty U_r (based on point uncertainty U_n=1 μm)

4.3 Comparison between a least square circle fitting with iteratively determined fitting area and without predefined fitting area

Using differently prepared cutting edges, the new characterisation method for rounded cutting edges that is suggested above (Fig. 18) is compared with the circle fitting algorithm without any predefined fitting areas (Fig. 16). The characterised cutting edges were prepared by microabrasive jet blasting. The blasting angle δ between wedge angle bisector and nozzle axis varied between 0°, 10° and 30°. Blasting times differed as well. The characterisation results are illustrated in Fig. 22. The radii r of the algorithm without any predefined fitting zone and a minimum radius uncertainty as optimisation criterion is shown on the horizontal axis. The set restrictions are given in (B). The radii r_n of the proposed method with an iteratively defined area as function of profile flattening and wedge angle are depicted on the vertical axis. The depicted values are mostly in accordance with each other. Deviations can especially be observed for asymmetrically rounded cutting edges (blasting angle $\delta=30°$). The influence of the characterisation method on the determined radius is exemplified using the cutting edge that shows the worst radius agreement when using the different methods: Obviously, the optimisation criterion and boundary conditions ($R_{\min}^2=0.998$) cause a fitting of a circle into the top area of the cutting edge rounding (B). This radius corresponds to the smallest circle that still fits best in terms of uncertainty and coefficient of determination. If the restrictions are changed (Fig. 22(C)) to a similar minimum coefficient of determination $R_{\min}^2$ as the resulting R^2 in the suggested characterisation method (A), the radii of the two circles equalise. Thus, if the coefficient of determination is set high, the method without predefined fitting area tends to give the lower limit of radii that a cutting edge could still be dimensioned with. For this reason, the achievable radius uncertainty is typically larger, as fewer points are considered for the circle fitting. For symmetrically rounded cutting edges, the difference in radius between the two characterisation methods is, using the restrictions given in Fig. 22(B), smaller than 5% of the nominal values.

However, while the algorithm without predefined fitting area might find multiple circles with the same minimum uncertainty, the suggested method with an iteratively determined fitting area as a function of edge flattening Δr and wedge angle β always results in a unique solution.

5 Conclusions

The currently available methods to characterise the micro geometry of a cutting edge are difficult to apply. Either the methods are affected by an increased uncertainty, difficult to reconstruct or the number of generated parameters decreases their significance. Therefore, a new algorithm is proposed which ensures a unique determination of the cutting edge rounding by a least square circle fit over an area that is determined iteratively. The algorithm eliminates one uncertainty driver, which effectively reduces the resulting characterisation uncertainty and increases characterisation reproducibility and repeatability. To indicate the magnitude of the cutting edge preparation, a value is introduced that is ensuring a minimum uncertainty. To specify the asymmetry of the rounding, a characterisation is suggested which is insensitive to point uncertainties and form deviations and can be used irrespective of whether the edge is chamfered or rounded. Further methods to avoid an individual user influence are thinkable, but most probably more complicated in their application. The points mentioned in this paper shall give ideas for the elaboration of an urgently needed international standard to characterise the cutting edge micro geometry.

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