DISCUSSION

Response by the Authors to S. Sakurai's Discussion of the Paper "On a Paradox of Elasto-Plastic Tunnel Analysis"

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1 Introduction

In a recent publication, hereinafter referred to as the "Paper", we investigated the reasons for a counter-intuitive feature of the behaviour exhibited under certain conditions by widely used computational models (Cantieni and Anagnostou 2011). The discussion by Sakurai (2011), hereinafter referred to as the "Discussion", did not question the existence of the paradox or the possibility that the identified reasons (time-dependency of ground behaviour, convergences of excavated profile) may play a role. The Discussion seems, however, to believe that the main reason for the paradox is rather that the underlying computational model neglects the gravitational body force. The Discussion bases this belief upon qualitative considerations rather than on computational evidence. The central argument of the Discussion is that the computations of the Paper may overestimate the amount of stress relief in the ground ahead of the face because they do not pay due account to the gravitational body forces. The later must always be preserved, because they are caused by the weight of the materials (cf. Discussion, Section 4, Paragraph 2 and Section 5).

The present response explains the nature of the simplifying assumption of zero gravity (Sect. 2), shows computationally by means of a simplified model that the paradox persists even in the presence of body forces (Sect. 3) and comments on some other points of the Discussion (Sect. 4).

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2 Simplifying Assumption

In order to reduce the number of dimensions of the tunnel advance problem from 3 to 2 (the axisymmetric problem), the Paper made the simplifying assumption that the initial stress field is homogeneous. The initial stress was taken equal to 10 MPa, which is the geostatic pressure prevailing at the elevation of a 400 m deep tunnel (10 MPa = 400 m × γ, where the unit weight $\gamma = 25$ kN/m^3). Line bb' in Fig. 1 shows the assumed initial stress. The assumption of a constant initial stress disregards the stress variation caused by the gravitational body force b of 25 kN/m^3 (Fig. 1, line aa'). The error introduced by this assumption is zero at the tunnel axis and increases with distance from the tunnel. In the tunnelling influence zone (i.e. at points located up to a distance of few tunnel diameters away from the tunnel), the error amounts only to few percent.

Figure 4d of the Paper shows the longitudinal distribution of the load that develops upon the lining for two values of the uniaxial compressive strength f_c of the rock. The final load amounts to about 3.5 MPa for $f_c = 1$ MPa, but increases to 4.1 MPa in the case of the higher strength of $f_c = 3$ MPa. The computational model underlying this paradox assumes a constant initial stress of 4 MPa, which, as explained above, deviates by only a few percent from the geostatic initial stress. We did not expect that such a small difference in the initial stress would change the results of the Paper's Figure 4 significantly and this is why we did not investigate the influence of gravitational body force in the Paper.

3 Computations with Body Force

As the Discussion raised the issue of gravity effect, however, we carried out a comparative computation, which
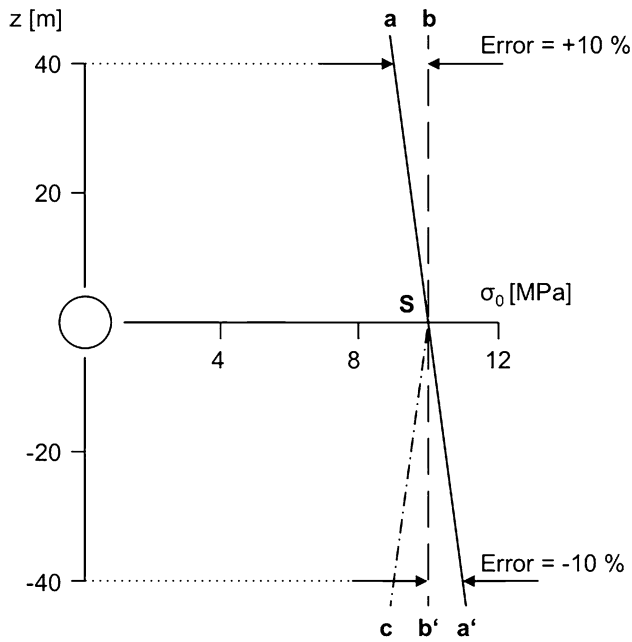
Fig. 1 Vertical distribution of the actual initial stress (*solid line* aa′), the initial stress of the Paper (*the dashed line* bb′) and the initial stress of the comparative calculation of the present response (*the dashed line* aSc)

confirmed that this effect really is negligible. The comparative calculation is based upon the axisymmetric model of the Paper, but also takes into account a radial body force of 25 kN/m³. The assumed initial stress distribution (line aSc in Fig. 1) obviously deviates from the actual distribution of the tunnelling problem (line aa′ in Fig. 1), but is appropriate for quantifying the possible effect of the body forces. Figure 2 compares the rock load distribution obtained when considering body forces (marked points) with the rock load distribution obtained when assuming a
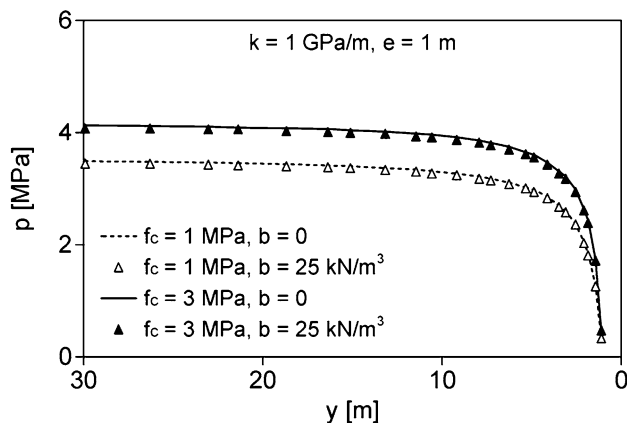
Fig. 2 Longitudinal distribution of the rock loading after Figure 4d of the Paper, supplemented by the results of a comparative calculation of the present response

constant initial stress (curves). The difference in the results really is negligible.

It should be noted that the computational assumption of constant initial stress (i.e. following line bb′ in Fig. 1 instead of line aa′) is the only simplification made in the Paper with respect to gravity. The Discussion also contains several remarks on how to carry-out elasto-plastic analyses with gravity (and also about possible errors in such analyses). These points will be addressed below.

4 Other Points

4.1 Gravitational Body Force Replaced by Surface Force

According to the Discussion (Section 2, Paragraph 1), in a finite element analysis the "gravitational body force is replaced by the surface force (Cauchy stress), so that in the course of the numerical analysis of a tunnel, only the surface traction vectors are considered, although the tunnel is situated in the gravitational field".

In the iterative solution of the non-linear, elasto-plastic tunnel excavation problem, the stress state at each Gauss sampling point is calculated by integrating the incremental, elasto-plastic stress–strain equations. The initial stress state prevailing at each sampling point serves as initial condition of the integration:

$$\sigma_{ij} = \sigma_{ij,0} + \Delta\sigma_{ij}, \tag{1}$$

where

$$\Delta\sigma_{ij} = \int D_{ijkl} d\varepsilon_{kl} \tag{2}$$

and $\sigma_{ij,0}$, $\Delta\sigma_{ij}$, D_{ijkl} and ε_{kl} denote the initial stress, the excavation-induced stress change, the elasto-plastic stress–strain tensor and the excavation-induced strain, respectively.

Since the initial stress field depends directly on the gravitational body force (the vertical gradient of the initial vertical stress is equal to the body force) and the body force also appears in the equilibrium equation, we do not agree that the numerical analysis considers only the surface traction vectors: in a proper non-linear analysis, the full initial stress field (including its gradient due to gravity) is permanently present.

4.2 Validity of Equation (1)

Equation (1) is identical to Eq. (1) of the Discussion. According to the final paragraph of Section 2 of the Discussion, "Equation (1) is valid only for a tunnel being excavated in an elastic ground. In a ground consisting of elasto-plastic materials, however, Equation (1) may be

questionable, because the principle of superposition is debatable for use with non-linear problems like an elasto-plastic tunnel analysis. Furthermore, there is no guarantee that the uniqueness of the solution is valid in an elasto-plastic analysis,…".

On account of the stress-path dependency of the elasto-plastic stress–strain equations, the statement that there is no guarantee of solution uniqueness is correct (cf. Cantieni and Anagnostou 2008). It is also true that superposition is inadmissible in the case of non-linear material behaviour. Equation (1) has, nevertheless, nothing to do with super-position. Equation (1) merely expresses the trivial fact that the stress is equal to the initial stress plus the excavation-induced stress change. This is valid independently of the constitutive behaviour of the ground and applies, of course, to elasto-plastic behaviour as well.

4.3 No tunnel failure in numerical analysis

According to Section 4 of the Discussion, "the stress in the plastic zone will always remain within the yielding criterion. This implies, from a numerical analysis point of view, that no tunnel failure ever occurs. In reality, however, a tunnel often fails […]. This is also a paradox of elasto-plastic tunnel analyses, although this is not addressed in the Paper".

The alleged paradox does not exist. The fact that the stress state does not violate the yield criterion does not mean that failure is impossible in a numerical model. Failure in a numerical analysis is the inability to find a stress field that simultaneously satisfies the boundary conditions, the yield criterion and the equilibrium condition. This may happen for the following two basic reasons.

The first reasons is associated with body forces, it may occur even in deep tunnels and it can easily be illustrated by considering the equilibrium at the crown of a circular unsupported tunnel (Fig. 3a). The equilibrium condition in polar co-ordinates reads as follows:
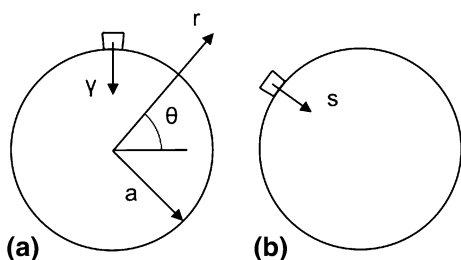
$$\frac{d\sigma_r}{dr} + \frac{1}{r}\frac{d\tau_{r\theta}}{d\theta} + \frac{\sigma_r - \sigma_\theta}{r} + \gamma = 0, \tag{3}$$

where σ_r, σ_θ and γ denote the radial stress, the tangential stress and the unit weight of the ground, respectively. In the case of an unsupported tunnel, the boundary tractions are equal to zero, i.e.

$$\sigma_r|_{r=a} = \tau_{r\theta}|_{r=a} = 0, \tag{4}$$

where a is the tunnel radius, and, consequently, the equilibrium condition at the tunnel crown reads as follows:

$$\frac{d\sigma_r}{dr}\bigg|_{r=a} = \frac{\sigma_\theta|_{r=a}}{a} - \gamma. \tag{5}$$

As the tangential stress at the excavation boundary of an unsupported tunnel cannot be higher than the uniaxial compressive strength f_c of the ground,

$$\frac{d\sigma_r}{dr}\bigg|_{r=a} \leq \frac{f_c}{a} - \gamma. \tag{6}$$

If

$$f_c \leq a\gamma, \tag{7}$$

the right-hand side of the inequality (6) becomes negative, which means that equilibrium is impossible unless the ground exhibits tensile strength. For a typical traffic tunnel radius of $a = 5$–6 m and a unit weight γ of 20–25 kN/m³, the critical uniaxial strength amounts to 100–150 kPa.

Fig. 3 Unsupported opening **a** with gravitational body force γ or **b** with a radial seepage force s due to seepage flow
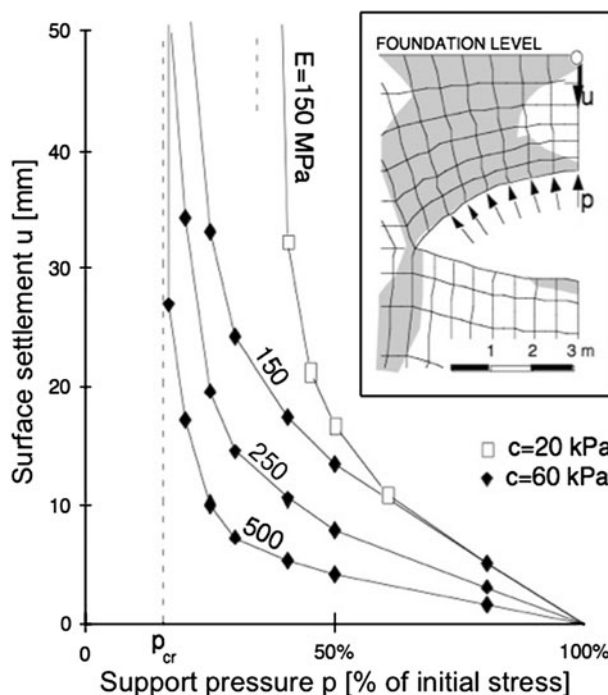

Fig. 4 Surface settlement over tunnel support pressure (from Anagnostou et al. 1997)

Similar results can be obtained when considering the body force due to seepage flow (Fig. 3b, cf. Anagnostou 2006).

The other failure reason is associated with the impossibility of stress re-distribution when the plastic zone reaches the boundary of the computational domain. This mechanism is relevant for shallow tunnels because of the vicinity of the free soil surface. Failure manifests itself as an asymptotic increase in the deformations as the system approaches limit state. This is a phenomenon well known from numerical analyses of soft ground tunnelling (cf., e.g., Vermeer et al. 2002). Figure 4 presents the results of such an analysis. The diagram shows the relationship between the support pressure p and the settlement u at the soil surface. When the support pressure approaches a critical value p_{cr}, the settlement becomes infinite and the plastic zone reaches the soil surface (the hatched area in the inset of Fig. 4). A further reduction in the support pressure is impossible without a collapse.

In conclusion, numerical analyses are able to reproduce failure.

5 Conclusions

Fulfilling the equilibrium condition is a central requirement for any analytical or numerical solution. The Discussion's argument (that the gravitational body forces are caused by the weight of the materials and must always be preserved) is therefore surely true. The argument has nevertheless no bearing for the question under consideration. The paradox exists even in relation to a heterogeneous initial stress field.

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