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Second-Order Sensitivity in Applied General Equilibrium

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Abstract In most policy applications of general equilibrium modeling, cost functions are calibrated to benchmark data. Modelers often choose the functional form for cost functions based on suitability for numerical solution of the model. The data (including elasticities of substitution) determine first and second order derivatives (local behavior) of the cost functions at the benchmark. The functional form implicitly defines third and higher order derivatives (global behavior). In the absence of substantial analytic and computational effort, it is hard to assess the extent to which results of a particular model depend on third and higher order derivatives. Assuming that a modeler has no (or weak) empirical foundation for her choice of functional form in a model, it is therefore a priori unclear to what extent her results are driven by this choice. I present a method for performing second-order sensitivity analysis of modeling results with respect to functional form. As an illustration of this method I examine three general equilibrium models from the literature and demonstrate the extent to which results depend on functional form. The outcomes suggest that modeling results typically do not depend on the functional form for comparative static policy experiments in models with constant returns to scale. This is in contrast to an example with increasing returns to scale and an endogenous steady-state capital stock. Here results move far from benchmark equilibrium and significantly depend on the choice of functional form.

Keywords Sensitivity analysis · Out-of-sample behavior · CGE · Flexible functional forms

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1 Introduction

As general equilibrium models have grown in popularity as a guide to policy design, concerns about the robustness of modeling results have arisen. It was argued that the modeling results could not be trusted without further examination, because certain parameters of model calibration—especially elasticities of substitution—can not be reliably determined. Only after considering the variance of modeling results induced by this uncertainty can one properly assess the conclusions of a modeling experiment. A thorough treatment of these ideas was provided by Pagan and Shannon (1985) who systematically analyzed the effect of uncertain calibration parameters on modeling results. A numerical application of such sensitivity analysis is given by Harrison and Vinod (1992).

For econometric analysis of substitution elasticities, researchers employ flexible functional forms (FFFs). The free parameters of these functional forms allow the estimated function $C(\mathbf{p})$ to reproduce any set of factor demands $\bar{x}_i$ and elasticities of substitution σ_{ij} at any one given set of factor prices $\bar{\mathbf{p}}$. By assumption of cost minimization, the restrictions on the parameters of $C(\mathbf{p})$ are more specifically given by Shephard's lemma for factor demands $\bar{x}_i$ ($\bar{x}_i = C_i(\bar{\mathbf{p}}) := \partial C(\bar{\mathbf{p}})/\partial p_i$) and the definition of Allen-Uzawa elasticities of substitution ($\sigma^A_{ij} = C(\bar{\mathbf{p}})C_{ij}(\bar{\mathbf{p}})/[C_i(\bar{\mathbf{p}})C_j(\bar{\mathbf{p}})]$). These restrictions determine the first and second derivatives of $C(\bar{\mathbf{p}})$. Two FFFs calibrated to the same benchmark data therefore are asymptotically identical for $\mathbf{p} \to \bar{\mathbf{p}}$. But as $||\mathbf{p} - \bar{\mathbf{p}}|| \to 1$, higher order terms of the Taylor series expansion

$$C(\mathbf{p}) = C(\bar{\mathbf{p}}) + \nabla C(\bar{\mathbf{p}})(\mathbf{p} - \bar{\mathbf{p}}) + \frac{1}{2}(\mathbf{p} - \bar{\mathbf{p}})^T \nabla(\nabla C(\bar{\mathbf{p}}))(\mathbf{p} - \bar{\mathbf{p}}) + \mathcal{O}(||\mathbf{p} - \bar{\mathbf{p}}||^3)$$

with coefficients proportional to 3rd and higher order derivatives of $C(\mathbf{p})$ become important. But these are determined by the specific FFF and will normally differ across functional forms, even when the first two terms of the Taylor series expansion are identical.

In applied general equilibrium modeling, it is rarely the case that econometrically estimated FFFs are employed as cost functions. This can on the one hand be explained by the fact that econometric estimations of cost functions may not be available for all sectors in question. On the other hand cost functions estimated from several FFF are not necessarily globally regular (non-decreasing and concave in prices). Unfortunately, only general equilibrium models featuring cost functions that are globally regular can be guaranteed to have counterfactual equilibria for any desired tax experiment (Shoven and Whalley 1995).

If a modeling method is to be restricted to using only regular functional forms to represent cost in the models rather than the FFFs used to estimate elasticities of substitution in the first place, two assumptions need to be made. First, the regular functional form should be flexible in the sense that it can be calibrated to the benchmark demands and elasticities of substitution.[1] Secondly, such a method implicitly

[1] Perroni and Rutherford (1995) present such a "regular-flexible functional form" which is convenient to use in modeling, but unwieldy for econometric estimation.

assumes that changing FFFs from the originally estimated one to the regular one does not change the modeling results significantly.

The argument for the second assumption is that local coincidence of functions with benchmark data is enough to obtain reliable results, because solutions to counterfactual experiments normally remain close enough to the benchmark. In line with these assumptions, the examples given by Pagan and Shannon (1985) concentrate on the problem of possible misspecification of elasticities of substitution. Yet Despotakis (1986) rightfully raises the issue of misspecification of FFF and thus of higher order derivatives at the benchmark: he presents a case where calibration of two different FFFs to the same benchmark produces different experimental outcomes. And even if Despotakis' results have been weakened by the corrective note by Kittelsen (1989), Kittelsen still insists on the conclusion that "differences in economic performance of FFF, and accordingly in results of economic models that employ FFF in partial or general equilibrium, can well be substantial."

This indicates that the sensitivity analysis of Pagan and Shannon should be extended to a second-order sensitivity analysis, which takes into account sensitivity to third and higher order derivatives of cost functions. Such a second-order sensitivity analysis could *ex post* determine if results are influenced by the choice of FFF. I propose a method of second-order sensitivity analysis, where the results using a regular-flexible functional form are compared to the modeling results obtained from using three FFFs commonly used for econometric estimation. The thus observed sensitivity is what interests a modeler that uses the regular-flexible functional form and assumes that the elasticities of substitution he inserted have been estimated using one of the three alternative FFFs.

Normally, formulating a model with four different FFFs for cost functions basically requires four times the work involved in formulating one single model. This is because the cost functions can enter all equations of a general equilibrium model and the calibration process has to be repeated for each cost function of each FFF. The work required to do this by hand can be prohibitively large. Only automation of calibration makes second-order sensitivity analysis practicable for a wider application to general equilibrium models. The script for automatizing calibration that has been used for producing the results of this paper along with some documentation can be obtained from the author's web site.

The remainder of the paper covers the following: The next section discusses why only local properties of cost functions drive modeling results for small distortions of the benchmark situation while big distortions require considering the global properties as well. Sect. 3 gives a detailed description of the calibration process, which is the core of the presented method of second-order sensitivity analysis. In Sect. 4, the sensitivity analysis is applied to three examples published in the literature in order to illustrate its relevance. The experiments in this paper are intended to give an overview of the impact that changing FFFs can have on different models. I will conclude that choice of functional form should not influence conclusions on a wide range of realistic tax experiments in constant-returns-to-scale models. Only under conditions of very big shocks do global properties have a considerable effect on the overall results. By contrast, one finds relevant sensitivity in models in which increasing returns to scale and endogenous investment decisions allow for far-reaching deviations from the benchmark. In such a case, second-order sensitivity analysis is a valuable tool to appraise the reliability of the model results.

2 Global versus Local Behavior of Functions

The implicit assumption in general equilibrium policy experiments is that the model results are mainly driven by first and second order derivatives of cost functions rather than higher order derivatives. Thus, calibrating cost functions to benchmark demands and elasticities of substitution should determine the result with sufficient precision, and the higher order derivatives implicit to the choice of FFF should not significantly influence the results.

The intuition from Taylor series expansion of analytical cost functions tells us that this is true if experimental results stay close enough to the benchmark data but becomes increasingly difficult to defend when counterfactual experiments substantially move away from the initial situation. The following gives a mathematical sketch of why this intuition is usually right and illustrates such a situation using the proposed method of sensitivity analysis.

I want to look at a standard general equilibrium tax experiment to illustrate the sensitivity of results for small tax changes in different models employing functional forms. In this model, a representative household is endowed with factor quantities Ω, which are processed by sectors to meet final good demand $\mathbf{d}$, which the household buys with its income M. Sectors j incur cost $c_j(\mathbf{p})$ for one unit of output and earn p_j from selling it if prices are $\mathbf{p}$. By Shephard's lemma, the cost-minimizing demand for producing y_j units of output is $y_j \nabla c_j(\mathbf{p})$. In policy experiments, government raises taxes $t_{j,i}$ on the good i input of sector j and hands the tax revenue to the representative household. Taxes on sector j are combined in a matrix

$$\mathbf{T_j} = \begin{bmatrix} t_{j,1} & & 0 \\ & \ddots & \\ 0 & & t_{j,I} \end{bmatrix},$$

where I is the number of goods in the economy. Let $\mathbf{T}$ be the vector of matrices $\mathbf{T_j}$.

The general equilibrium is given by the solution to the system of equations

$$\begin{aligned} -\Pi_j(\mathbf{p}) := c_j(\mathbf{p} + \mathbf{T_j}\mathbf{p}) - p_j \geq 0 \quad \forall j \qquad & \text{(zero profits)} \\ \sum_j y_j \nabla \Pi_j(\mathbf{p}) + \Omega \geq \mathbf{d}(\mathbf{p}, M) \qquad & \text{(market clearance)} \\ \mathbf{p} \cdot \Omega + \sum_j y_j (\nabla c_j)^T \mathbf{T_j}\mathbf{p} = M \qquad & \text{(income balance).} \end{aligned} \tag{1}$$

If the weak inequalities hold with equality at the equilibrium, this can be written in the form of $F(\mathbf{p}, \mathbf{y}, M; \mathbf{T}) = 0$, a system of as many equations as variables. [2] It is assumed here that the solutions $\mathbf{y}^*$,$\mathbf{p}^*$,M^* to (1) are continuously differentiable functions of $\mathbf{T}$ around benchmark tax rates $\mathbf{T} = \mathbf{T}^0$ (if $\det(\nabla_{p,y,M}F|_{T^0}) \neq 0$ the implicit function theorem guarantees this). Thus, changes in solution prices d$\mathbf{p}$ are proportional to d$\mathbf{T}$, if those are infinitesimal.

Assume now a reformulation of the model with different cost functions $\tilde{c}_j(\mathbf{p})$, which we shall denote $\tilde{F}(\mathbf{p}, \mathbf{y}, M; \mathbf{T}) = 0$. I assume c and $\tilde{c}$ to agree in 0th to 2nd derivative at $\mathbf{p}^*(\mathbf{T}^0)$, i.e. for small $\mathrm{d}\mathbf{p} = \mathbf{p} - \mathbf{p}^*(\mathbf{T}^0)$, $\mathrm{d}c_j(\mathbf{p}) = [c_j(\mathbf{p}) - \tilde{c}_j(\mathbf{p})]$ is of the form $\sum_{i,j,k\in I} a_{ijk}\mathrm{d}p_i\mathrm{d}p_j\mathrm{d}p_k + \mathcal{O}(\mathrm{d}\mathbf{p}^4)$. So for small tax variations d$\mathbf{T}$, $||\mathrm{d}c_j(\mathbf{p})||_\infty$ goes to zero like $||\mathrm{d}\mathbf{p}||^3_\infty \sim ||\mathrm{d}\mathbf{T}||^3_\infty$ ($||\mathrm{d}\mathbf{T}||_\infty = \max_{j,i}\{t_{j,i}\}$). By construction of F and $\tilde{F}$ the $\mathrm{d}c_j(\mathbf{p}) \sim ||\mathrm{d}\mathbf{p}||^3_\infty$ result in

$$
\begin{aligned}
&||\mathrm{d}F(\mathbf{y}^*(\mathbf{T}), \mathbf{p}^*(\mathbf{T}), M^*(\mathbf{T}); \mathbf{T})||_\infty \\
:= &||\tilde{F}(\mathbf{y}^*(\mathbf{T}), \mathbf{p}^*(\mathbf{T}), M^*(\mathbf{T}); \mathbf{T}) - F(\mathbf{y}^*(\mathbf{T}), \mathbf{p}^*(\mathbf{T}), M^*(\mathbf{T}); \mathbf{T})||_\infty \\
\sim &\max_j ||\nabla_{\mathbf{p}}\mathrm{d}c_j(\mathbf{p})||_\infty \sim ||\mathrm{d}\mathbf{p}||^2_\infty
\end{aligned}
$$

going to zero like $||\mathrm{d}\mathbf{T}||^2_\infty$. By arguing that the difference between the solution $(\mathbf{y}^*, \mathbf{p}^*, M^*)$ to $F(\mathbf{y}, \mathbf{p}, M; \mathbf{T}) = 0$ and the solution $(\tilde{\mathbf{y}}^*, \tilde{\mathbf{p}}^*, \tilde{M}^*)$ to $\tilde{F}(\mathbf{y}, \mathbf{p}, M; \mathbf{T}) = 0$ is proportional to $\mathrm{d}F(\mathbf{y}^*, \mathbf{p}^*, M^*; \mathbf{T})$, I conclude that the differences between the solutions of the two models go to zero like $||\mathrm{d}\mathbf{T}||^2_\infty$, while the solution $\mathbf{p}^*(\mathbf{T})$ itself only goes to zero like $||\mathrm{d}\mathbf{T}||_\infty$.

The following model that obeys Eq. 1 shall illustrate the results of the above mathematical considerations. The basic model is formulated using CES functions to describe production and utility. Second-order sensitivity analysis is applied by replacing the CES functions with three alternative functional forms. The alternative functional forms are calibrated to have the same first and second derivatives at the benchmark as the original CES functions, but will gradually deviate from them as prices move away from the benchmark situation.

In the model, 3 sectors produce sector specific goods j from factors labor and capital. The representative agent buys and consumes these goods according to his Cobb-Douglas preferences. The social accounting matrix in Table 1 gives the benchmark supply and demand that were used in this model. The elasticity of substitution between labor and capitals is 3 for all sectors. In different policy experiments, a tax of up to 500% on capital input of sector 2 is raised.

Figure 1 shows the reaction of the price of good 2 to different tax rates. The results reflect the behavior predicted by the above mathematical considerations: the results from different calibrated FFFs converge faster to one another than to their benchmark values as the tax approaches the benchmark tax. If the counterfactual tax rate is too different from the benchmark however, the results depend to an increasing degree on

[2] Walras' law says that in (1), one market clearance equation is implied by the others. We therefore have one fewer independent equation than listed. This is compensated for by the fact that one price can be fixed as a numeraire (Cost functions are required to be homogeneous of degree one in prices. Then, if $(\mathbf{y}, \mathbf{p})$ describes an equilibrium, so does $(\mathbf{y}, \lambda\mathbf{p})$, $\forall\lambda \in \mathbb{R}^+$).

Table 1 Social accounting matrix for the generic example model

Sectors buy labor and capital from the respective markets and sell their output to the representative agent

	Sector 1	Sector 2	Sector 3	Labor	Capital	Rep. agent
Sector 1						2
Sector 2						4
Sector 3						6
Labor	1	2	3			
Capital	1	2	3			
Rep. agent				6	6	

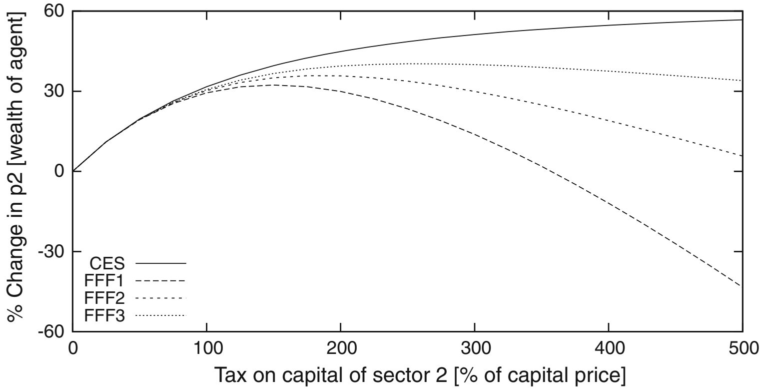

Fig. 1 Price of the good of the taxed sector in a simple generic model. For taxes below 100%, the sensitivity to FFFs is small compared to the impact of the tariff

the FFF that was used. In terms of order of magnitude, the second-order sensitivity analysis of this model reveals that if the tax does not exceed 100%, the predicted changes in market price of good 2 are relatively stable with respect to exchanging FFFs in the model formulation.

3 Functional Forms

As mentioned in the introduction, the implementation presented here of second-order sensitivity analysis considers one regular-flexible functional form appealing to general equilibrium modelers and three FFFs that are commonly used for econometric estimation of substitution elasticities. The regular-flexible functional form is the N-stage nested CES (NNCES) presented by Perroni and Rutherford (1995). The work of Perroni and Rutherford shows that a nested CES cost function of N prices is guaranteed to be flexible if its depth of nesting can be up to N and each nest is allowed to have N subnests. While the constructive proof that the authors provide is useful for calibration of NNCES to given benchmark data, the general NNCES functions contain parameters far in excess of what is needed for econometric estimation of benchmark demand and substitution elasticities alone.

The remaining three FFFs on the other hand are prominent examples of functional forms that have been custom-made for estimating the above benchmark data. These three FFFs are the Translog (TL) (Christensen et al. 1973), the Generalized Leontieff (GL) (Diewert 1971), and the Normalized Quadratic (NQ) (Diewert and Wales 1987). Table 2 displays the general form of TL, GL, and NQ cost functions.

The automated process I pursue takes the given formulation of a model with NNCES cost functions (of which simple CES cost functions are a special case), computes the value shares $\theta_i = \bar{p}_i C_i(\bar{p})/C(\bar{p})$ and the Allen-Uzawa elasticities of substitution (AUES) $\sigma^A_{ij} = C_{ij}(\bar{p})C(\bar{p})/(C_i(\bar{p})C_j(\bar{p}))$ at prices $\bar{p}$, and then calibrates the FFFs TL, GL, and NQ to reproduce these benchmark data. Appendix A explains how to systematically compute Allen-Uzawa elasticities of substitution for a given NNCES cost function. The calibration of TL, GL, and NQ unit cost functions to benchmark values of $\bar{p}$, θ_i, and σ^A_{ij} is given in Table 2.

Unfortunately, the reformulation of a model in terms of different FFFs can have undesirable effects on the computability or even on the existence of the model's solution. As Perroni and Rutherford (1998) have found, "the Translog, Generalized Leontieff and Normalized Quadratic forms are all prone to loss of regularity[3], particularly when they are calibrated to large cross-elasticity values, " while NNCES are globally regular (i.e. non-decreasing and concave in prices everywhere in price space). However, global regularity of cost functions is required to warrant a solution to the respective general equilibrium problem (Shoven and Whalley 1995). This implies that the replacement of NNCES cost functions with cost functions of different FFFs might convert a solvable model into a model that has no solution.

But the lack of global regularity does not only affect the existence of a general equilibrium for counterfactual policies. It is a basic economic assumption that cost functions should be regular. Cost functions that are not globally regular therefore seem somewhat dubious building blocks of an economic model.

One can still defend the use of cost functions that are not globally regular, if a model containing such cost functions does yield solutions for policy experiments. The cost functions might actually accurately represent cost for the price ranges in question and only deviate from regularity for prices away from benchmark and counterfactual solution. But even in this case, the danger exists that a numerical solver, on his way to find this solution, still evaluates cost functions at points where they are not regular and thus fails to find the existing general equilibrium.

Generally, if a reformulation with a certain FFF yields a model that does not solve, I do not consider this reformulation and its failure to solve for the sensitivity analysis. In such a case, the solver apparently evaluates cost functions at prices where they are not regular and therefore not credible representations of cost. Such a reformulation therefore risks to display non-regular cost functions at combinations of prices that are relevant for our economic analysis and then should be thought of as a 'wrong formulation'.

[3] In a 2006 GTAP conference paper, Gohin, A. and Laborde, D. discuss how by introducing the notion of virtual prices, NQ can always be viewed as being regular. Here, this idea is not further pursued. The paper can be found at https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=2108.

Table 2 The FFF used for sensitivity analysis and how they are calibrated to benchmark data

FFF	Parameter calibration to $\bar{C} = C(\bar{\mathbf{p}})$, $\bar{p}_i, \theta_i, \sigma_{ij}^A$	Cost function $C(\mathbf{p})$	Demand function $C_i(\mathbf{p}) = \frac{\partial C(\mathbf{p})}{\partial p_i}$
TL	$b_0 = \bar{C}/L(\bar{\mathbf{p}})$, $b_i = \theta_i - \sum_j a_{ij} \ln \bar{p}_j$, $a_{ij} = \theta_i \theta_j (\sigma_{ij}^A - 1),\ i \neq j; \quad a_{ii} = -\sum_{j \neq i} a_{ij}$	$b_0 \underbrace{\prod_i p_i^{b_i} \prod_{ij} p_i^{a_{ij} \ln p_j/2}}_{=:L(\mathbf{p})}$	$\left(b_i + \sum_j a_{ij} \ln p_j\right) C(\mathbf{p})/p_i$
GL	$a_{ij} = 4\theta_i \theta_j \bar{C}(\bar{p}_i \bar{p}_j)^{-1/2} \sigma_{ij}^A, \quad i \neq j$ $a_{ii} = 2\theta_i \bar{C}/\bar{p}_i - \sum_{j \neq i} a_{ij} (\bar{p}_j/\bar{p}_i)^{1/2}$	$\frac{1}{2} \sum_{ij} a_{ij} (p_i p_j)^{1/2}$	$\frac{1}{2} \sum_j a_{ij} (p_j/p_i)^{1/2}$
NQ	$a_{ij} = \bar{C} \frac{\theta_i \theta_j}{\bar{p}_i \bar{p}_j} \left(\sigma_{ij}^A \sum_k b_k \bar{p}_k + \bar{p}_i + \bar{p}_j\right), \quad i \neq j;$ $a_{ii} = \frac{1}{\bar{p}_i^2} \left[\theta_i \bar{C} \left(\sum_k b_k \bar{p}_k + \bar{p}_i\right) - \sum_{j \neq i} a_{ij} \bar{p}_i \bar{p}_j\right]$	$\frac{1}{2} \frac{\sum_{ij} a_{ij} p_i p_j}{\sum_i b_i p_i}$	$\frac{\sum_j a_{ij} p_j - b_i C(\mathbf{p})}{\sum_j b_j p_j}$

4 Examples from the Literature

4.1 A Tariff Experiment: Miller and Spencer (1977)

In 1977, an important issue in UK politics was the accession of the UK to the European Economic Community (EEC). In order to base the decision for or against accession on scientific grounds, economists looked for reliable predictions as to what the gains or losses for the members of the customs union would be. Given that the complexity of such a trade union forbids detailed algebraic analysis of the situation, many economists at that time argued that the only way of finding out the effects of a trade union would be to turn to empirical measurements. But unfortunately, trade unions seldom resemble each other enough so that historical conclusions about earlier trade unions could be applied to predict the effects of future ones.

Miller and Spencer decided to use CGE in order to get a more situation specific forecast for the changes at hand. By simplifying the representation of participants and turning to computational methods, it became possible to take into account all the effects implied by neoclassical trade theory.

The authors stylized the situation as follows: Tariffs between countries varied depending on region of import and export and depending on the product class. The regions UK, EEC, Common Wealth and the Rest of the World were chosen. Production was split into the two sectors 'food' and 'non-food,' both produced from given factors of capital and labor according to a Cobb-Douglas production function. The two factors were assumed to be perfectly mobile between sectors, but immobile between regions. Trade of goods between regions was modeled to be frictionless and conducted in terms of world prices. The consumers of each region filled their basket of commodities with food and non-food goods, badly substituting one for the other (CES assumption, elasticity of substitution: $\sigma = 0.1$). Both food and non-food goods are modeled as CES Armington aggregates between imported and domestically produced versions of the good with an elasticity of substitution $\sigma_{armington}$ of 3.0.

The accession of the UK to the EEC was modeled as a change in tariff regimes. Table 3 shows how tariffs were due to change in the course of accession. In the paper the post accession (POST) situation is further divided into a scenario where there is a transfer of UK tariff revenues to the EEC as intended by the customs union, and a scenario where these transfers would not be paid (POST(NT)). As a third counterfactual, the authors have considered global free trade (FT), where no tariffs are imposed by any region. The resulting changes in the models welfare index (level of composite consumption) for UK and Commonwealth are given in the rows labeled CES in Table 4.

For second-order analysis of the model results, the functional forms for description of production, Armington aggregation, and consumption bundling were replaced by TL, GL, and NQ. The effects on the results of changing functional forms can be observed in the respective rows of Table 4.

The results in the lower half of the table were obtained by setting $\sigma = \sigma_{armington} = 1$. Miller and Spencer (1977) used this alteration to make a rough sensitivity analysis of their results with respect to elasticities of substitution. I assume that the authors chose these values because they cover a range of elasticities that corresponds to their uncertainty about these values. I also assume that the cost functions yielding such

Table 3 (UK model) Tariff rates before EEC accession and how they were due to change in the course of accession

imposing\paying	UK	EC	CW	RW
		GOOD X		
UK		$0.15 \rightarrow 0$	$0 \rightarrow 0.15$	0.15
EC	$0.15 \rightarrow 0$		0.15	0.15
CW	$0.05 \rightarrow 0.2$	0.2		0.2
RW	0.125	0.125	0.125	
		GOOD Z		
UK		0	$0 \rightarrow 0.2$	$0 \rightarrow 0.2$
EC	0.2		0.2	0.2
CW	0	0		0
RW	0.2	0.2	0.2	

Additionally, 30% of UK tariff revenues on X-imports and 90% on Z-imports were to be transferred to the EEC after accession

Table 4 (UK model) Equivalent variation in percent of benchmark wealth

		POST(NT)	POST	FT
		$\sigma_{armington} = 3, \sigma = 0.1$		
UK	CES	−0.012	−1.824	0.696
	GL	−0.077	−1.917	0.597
	NQ	−0.117	−1.967	0.469
	TL	−0.034	−1.850	0.650
CW	CES	0.161	0.164	0.450
	GL	0.173	0.172	0.471
	NQ	0.175	0.172	0.488
	TL	0.159	0.160	0.454
		$\sigma_{armington} = \sigma = 1$		
UK	CES/TL	0.573	−1.898	0.652
	GL	0.594	−1.883	0.672
	NQ	0.644	−1.822	0.637
CW	CES/TL	−0.136	−0.122	0.106
	GL	−0.138	−0.135	0.120
	NQ	−0.137	−0.177	0.177

The figures illustrate sensitivity to FFF as well as to elasticities of substitution

elasticities could be adequately estimated by at least one of the FFFs considered in the second-order sensitivity analysis. Under these assumptions, one can now compare the effect on modeling results of plausible variations in elasticities with the effect of plausible variations of FFF. Comparing the effects of these simple 1st and 2nd order sensitivity analyses, one can summarize that results in this experiment are generally more sensitive to the change in elasticities than to the change in functional form.

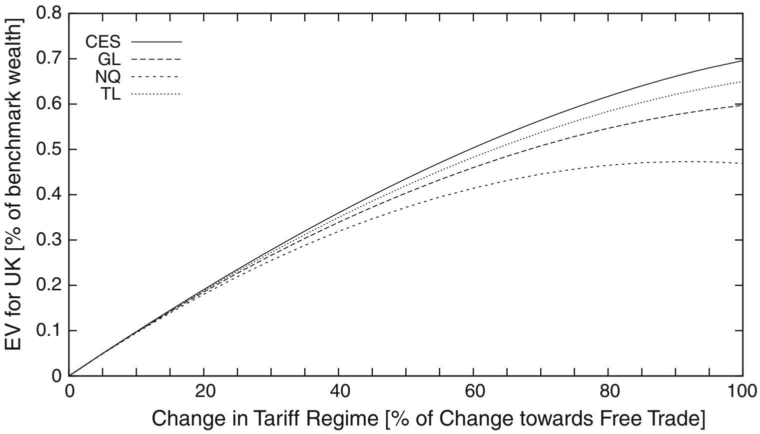

Fig. 2 (UK model) FFF dependent equivalent variation (EV) resulting from different mixtures between benchmark (left hand side) and global free trade (right hand side). Functional forms hardly matter close to the benchmark.

In order to show again how 2nd order sensitivity depends on policy impact, Fig. 2 illustrates how in the limit of the counterfactual approaching the benchmark, the results for different FFFs converge faster towards each other than each of them converges to the benchmark value. In this example, the tariff regime was continuously shifted from the benchmark case (tariffs have initial values listed in Table 3) to global free trade (no tariffs at all).

4.2 Discrete Resource Shock: Condon et al. (1987)

Condon et al. (1987) use CGE to make predictions for the economic future of Cameroon after the discovery of substantial oil reserves on its national territory. They were the first to use the General Algebraic Modeling System (GAMS) for implementing a CGE model. The model was formulated as a nonlinear constrained optimization program, and vacuously optimizes an objective function which represents welfare of households.

At the time, the most important sector of Cameroonian economy was agriculture, accounting for 32% of GDP. Cash crops (mainly coffee and cocoa) made up 72% of export earnings, which in turn constituted 20% of GDP (Condon et al. 1987). This fundamental source of productivity was seen as imperiled by the effects of the ‘Dutch disease’, which were expected to follow the inflow of foreign capital.

The term ‘Dutch disease’ describes the paradoxically adverse effects that a temporary increase of revenues from natural resources can have on a country’s economy. The inflow of foreign capital from the sales of the natural resources creates an appreciation of the real exchange rate (a rise in domestic price levels in the case of Cameroon: Cameroon’s nominal exchange rate was fixed to the French franc). This makes imports more attractive in the domestic market and exports less attractive in the world market. Thus, domestic sectors of traded goods lose profitability and shrink. If the country finally runs out of the natural resource, foreign capital stops flowing in. The shrunk

Table 5 (Cameroon model) Percentage change in output for the base scenario with oil revenues

	CES	GL	NQ	TL
Food crops	2.75	2.77	2.83	2.83
Cash Crops	−14.17	−14.26	−14.26	−14.63
Forestry	−6.66	−6.78	−6.74	−6.89
Food Processing	−7.39	−7.26	−6.72	−7.57
Consumer goods	0.91	1.10	1.38	0.88
Intermediate goods	−2.67	−2.72	−2.58	−2.78
Cement & Base Metals	−4.71	−4.86	−4.88	−5.94
Capital Goods	10.17	10.25	7.39	7.98
Construction	23.17	23.10	23.96	22.98
Private Services	−0.68	−0.69	−0.61	−0.64
Public Services	0.41	0.41	0.39	0.41

sectors of tradable goods cannot take over to employ labor or generate GDP immediately. With temporary revenues gone and formerly well working sectors crippled, the economy is now clearly worse off than before the appearance of the natural resource.

The Cameroonian model divides the economy in 12 sectors, each producing a single good. Sectoral production, consumer utility and Armington aggregation between imports and domestic products are all described by CES functions. The model assumes the new oil wealth to affect the Cameroonian economy only though revenue from oil sales, which is justified by the fact that oil extraction mainly employs highly skilled foreign labor. The value of annual oil sales after the resource shock was set to $500 million (approximately 5% of Cameroonian GDP). The investment sector then uses this wealth to invest into new production-specific capital, which creates increased demand, and subsequently the Dutch disease.

Besides analyzing the effect on the economy of the increased oil reserves and the Dutch disease per se, the work of Condon et al. focuses on the discussion of two different tariff policies and their effectiveness in alleviating the Dutch disease. One is a doubling of tariffs on food crops in order to sustain food self-sufficiency, the other a doubling of those on intermediate goods and construction materials in order to protect those sectors. The authors tested both policies and found them to have little effect compared to the shock created by the capital inflow from oil sales.

The results of Condon et al. (1987) for domestic production of the different sectors after the resource shock are reproduced by the data in the first column of Table 5. The model reformulations used for sensitivity analysis replaced the CES functions for production, utility, and Armington aggregation by functions of the FFFs GL, NQ, and TL. Results for the resource shock scenario are displayed in the respective columns of Table 5.

Apparently, second-order sensitivity of these results is negligible (maybe with the exception of the capital goods sector) for shocks of this magnitude. In order to see what role second-order sensitivity can play for bigger oil sale shocks, Fig. 3 shows results from annual oil sales of up to 50% of GDP[4]. It displays the FFF-dependent impacts of

[4] For comparison: According to http://en.wikipedia.org the petroleum sector accounted for roughly 50% of Saudi Arabian GDP in 2008.

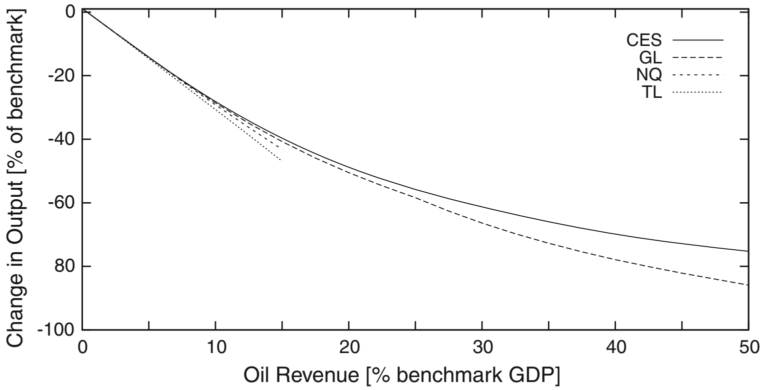

Fig. 3 (Cameroon model) Domestic production of cash crops as a function of oil revenues. The effects of the Dutch disease are more severe for high oil revenues. Oil revenues assumed in the original paper were roughly 5% of GDP.

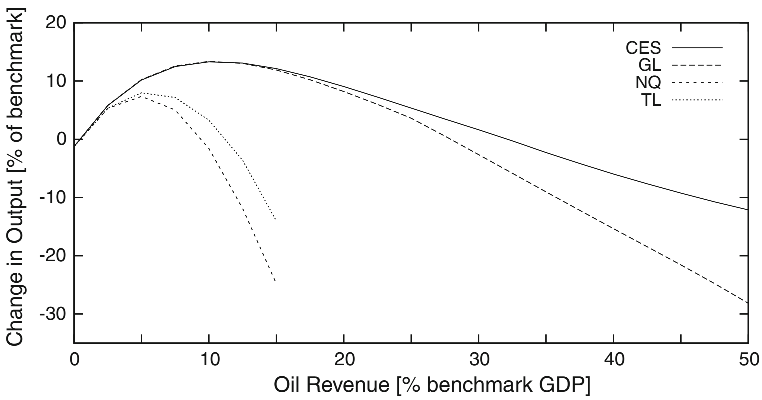

Fig. 4 (Cameroon model) Domestic production of capital goods as a function of oil revenues. The prospects for the capital goods sector highly depend on the FFF used to model the situation.

such oil revenues on one of the most vulnerable sectors, the cash crops sector. Unfortunately, the formulations using the FFFs NQ and TL cease to produce results for oil revenue shocks above 15% of GDP. If we look at the sectoral activity in the domestic capital goods sector, we see that the modeling results from different FFFs diverge as revenue shocks approach 15% of GDP (see Fig. 4). Given that prices and demand for aggregate capital goods are relatively stable, this indicates that the calibrated cost functions of different FFF on the demand side of domestically produced capital goods are of very different shapes, and it is very likely that the NQ and TLversions of these cost functions are the ones whose irregularities make the solution for big revenue shocks impossible. And indeed, the Armington aggregate for capital goods is highly dominated by imports at the benchmark, which leads to this extreme sensitivity to FFF. On the other hand, the fact that domestically produced capital goods make up only a small fraction of the Armington aggregate makes the aggregate barely sensitive to the price of the domestically produced part. So aggregate price and demand remain stable even though price and demand for domestic capital goods production are sensitive to FFF. That way the sensitivity remains isolated in the relatively small capital goods sector.

The general result—that the cash crops sector is indeed hit hard by the Dutch disease—is invariant under exchange of FFFs in model construction. But if we look at the actual remainder of the cash crops sector in the case of an oil revenue shock of 50% of GDP, it becomes difficult to pin down exact numbers. CES and GL indicate that results have to be expected to vary between 15% and 25% of initial size. And while the cash crops results for the different FFFs are consistent for a wide range of oil sale shocks, the capital goods sector becomes very unpredictable for shocks that exceed 10% of GDP.

4.3 Economies of Scale: Balistreri et al. (2009)

In the context of the Doha Development Agenda, Kenya has been requested to lower barriers against foreign investment in business services. In their work, Balistreri et al. (2009) argue that "in practice, the Kenyan regulatory regime imposes even higher inefficiency costs on a non-discriminatory basis". They interpret as regulatory barriers all impediments against offering a wide variety of services to businesses. Important examples are time-consuming administrative procedures at borders and ports, market restrictions in the telecommunications sector, and severe problems accessing credit for smaller enterprises caused in part by regulations in both the banking and insurance sector.

The expected benefits of deregulation are decreasing cost of providing services on the one hand and increasing productivity through better fitting business services on the other. The increase in business service quality with the number of service providers was modeled within the Dixit-Stiglitz framework (Dixit and Stiglitz 1977). This gives the utility of business services increasing returns to scale, which makes the model harder to solve but also interesting for second-order sensitivity analysis: it offers the possibility to examine the effect of exchanging functional forms in the context of increasing returns to scale.

In order to model the reduction of regulatory barriers, Balistreri et al. devised a "full reform" experiment. Full reform consists of cutting in half all regulatory barriers against investment (both foreign and domestic) in business services. Furthermore all tariffs are set to a uniform level which leaves tariff revenue constant. Besides this full reform package, the authors also consider the effects of partial reform: reducing only those barriers that are non-discriminatory, reducing only discriminatory barriers, reducing all barriers, and only changing the tariff system to uniform tariffs.

From the aspect of modeling technique, the full reform scenario was also run without variety-induced productivity gains (largely but not completely reducing welfare gains) on the one hand and with steady state capital stock adjustment (drastically increasing the welfare effects of the reform) on the other hand. The drastic deviation from the benchmark (see Tables 6 and 7) is again an interesting aspect of Balistreri et al.'s model to the analysis presented in this article.

Table 6 (Kenya model) Summary of results for different scenarios

Scenario definition	Full Reform	All services barriers[a]	Only non-discriminatory services barriers[a]	Only barriers against FDI in services[b]	Only uniform tariffs[a]	CRTS[a]	Steady State[b]
Liberalization of regulatory barriers for all services firms	Yes	Yes	Yes	No	No	Yes	Yes
Liberalization of discriminatory barriers on foreign services firms	No	Yes	No	Yes	No	No	No
Uniform import tariffs?	Yes	No	No	No	Yes	Yes	Yes
Steady-state capital stock	No	No	No	No	No	No	Yes
Dixit-Stiglitz variety-induced productivity gains	Yes	Yes	Yes	Yes	Yes	No	Yes
Aggregate welfare							
Welfare (EV as % of consumption)	**8.5** to 12.0	2.8 to 8.3	−0.7 to 6.5	**2.1** to 2.2	**0.6** to 0.8	**2.9** to 3.0	**31.2** to 52.3
Welfare (EV as % of GDP)	**7.1** to 10.1	2.4 to 7.0	−0.6 to 5.4	**1.7** to 1.8	**0.5** to 0.6	**2.5** to 2.6	**26.2** to 43.9
Government budget							
Tariff revenue (% of GDP)	2.7 to **2.8**	**2.9**	**2.9**	**2.9**	**2.9**	**2.9**	2.0 to **2.4**
Tariff revenue	**0.0**	3.7 to 4.4	0.9 to 3.4	1.2 to **1.3**	**0.0**	**0.0**	**0.0**
Aggregate trade							
Real exchange rate	0.4 to **1.4**	1.0 to 2.5	0.8 to 2.4	**0.5**	0.2 to **0.3**	0.2 to **0.4**	1.4 to **3.5**
Aggregate exports	−11.2 to **0**	−1.0 to 5.6	−1.6 to 7.7	1.5 to **1.6**	−4.2 to **−3.0**	−3.0 to **−1.4**	14.2 to 47.6

Source: Authors' estimates

Unless otherwise stated, numbers are percentage change from initial equilibrium. The minimal and maximal values obtained using different FFF are given. Bold print indicates NNCES results

[a] Model with the 'Translog' functional did not yield a solution for this scenario

[b] Model with the 'normalized quadratic' functional did not yield a solution for this scenario

Table 7 (Kenya model) Continuation of Table 6

Scenario definition	Full Reform[a]	All services barriers[a]	Only non-discriminatory services barriers[a]	Only barriers against FDI in services[b]	Only uniform tariffs[a]	CRTS[a]	Steady State[b]
Liberalization of regulatory barriers for all services firms	Yes	Yes	Yes	No	No	Yes	Yes
Liberalization of discriminatory barriers on foreign services firms	No	Yes	No	Yes	No	No	No
Uniform import tariffs?	Yes	No	No	No	Yes	Yes	Yes
Steady-state capital stock	No	No	No	No	No	No	Yes
Dixit-Stiglitz variety-induced productivity gains	Yes	Yes	Yes	Yes	Yes	No	Yes
Factor Earnings							
Skilled labor	**9.1** to 15.9	**8.4** to 17.8	6.4 to 15.2	**2.6** to 2.7	**0.2** to 0.6	2.3 to 2.6	**28.7** to 44.6
Semi-skilled labor	**5.3** to 11.1	**4.9** to 10.7	**3.1** to 7.4	**0.4**	**0.8** to 1.1	**1.1** to 1.3	**25.5** to 42.0
Unskilled labor	**11.7** to 15.8	8.5 to 10.4	5.6 to 8.0	**3.4** to 3.5	**1.3** to 1.6	**5.0** to 5.3	34.1 to 56.9
Capital	**9.5** to 13.7	**8.2** to 9.2	**6.3** to 6.5	**2.6**	**1.0** to 1.2	**3.7** to 4.0	−4.5 to 0.5
Land	−**7.6** to 4.5	4.3 to **7.4**	3.8 to **4.9**	2.3 to **2.5**	−2.4 to −**1.7**	−2.3 to −**1.2**	1.8 to **3.9**
Factor adjustments							
Skilled labor	8.1 to 10.7	9.1 to 13.2	7.7 to 12.2	4.2 to 4.6	1.6 to **1.9**	**4.4** to 4.7	**9.3** to 10.6
Semi-skilled labor	**9.9** to 11.7	**8.9** to 12.0	**7.9** to 10.6	4.6 to 4.8	**2.1** to 2.2	**4.7** to 5.0	**10.3** to 12.6
Unskilled labor	**2.1** to 3.2	**2.0** to 3.7	**1.7** to 4.4	**0.8**	**0.6**	**0.9** to 1.0	**3.9** to 5.6
Capital	**3.5** to 5.1	**3.3** to 5.3	**2.8** to 4.4	1.3 to **1.4**	**0.9** to 1.1	**1.7** to 1.9	**1.2** to 1.3
Land	19.1 to **24.4**	22.0 to **25.1**	20.9 to **22**	13.6 to 14.8	2.7 to **3.7**	**12.7** to 13.1	26.8 to 30.2
Capital stock and investment							32.4 to 63.1

Source: Authors' estimates
[a] Model with the 'Translog' functional did not yield a solution for this scenario
[b] Model with the 'normalized quadratic' functional did not yield a solution for this scenario

Table 6 shows the range of outcomes for some characteristic variables in the cases of the different scenarios. Policy recommendations based on the welfare indices are mostly stable with respect to functional form (however, the model using the Normalized Quadratic functional attributes negative welfare effects to solely removing discriminatory barriers). The basic message of the paper—namely that the biggest gain for Kenya would come from a general reduction of regulatory barriers—remains unaffected by exchanging FFF in the model structure. If we want to use the model to estimate how a full reform package would affect single sectors, however, second-order sensitivity analysis indicates that results from any single FFF cannot be relied upon too much. Especially predictions for aggregate exports and the earnings from land use look very unreliable in light of this sensitivity analysis.

5 Conclusion

This paper introduced the notion of second-order sensitivity analysis for applied general equilibrium models. The method varies the global properties of cost functions by changing their underlying functional forms. The effect of this on modeling results gives the modeler an estimate of the uncertainty in results that originates from choosing an FFF for representation of cost without knowing if another FFF would estimate the 'real' cost structure more precisely.

In the first order sensitivity analysis by Pagan and Shannon (1985), the analyst can decide on the probabilistic distribution of the uncertain parameters and get a distribution of modeling results in return. My method of second-order sensitivity analysis only allows for implicit 'control' of the distribution of uncertain parameters by choosing the set of FFFs that are cycled through. This however suffices to verify whether the use of one of the analyzed FFF is defensible, given that the cost function estimated by an econometrician would be of an FFF that is also included in the set of analyzed FFFs.

The paper presents the results of applying the sensitivity analysis to three applied general equilibrium models from the literature. The found variance in results never provides a reason to refute the principal messages of the original works. But if benchmark and counterfactual results turn out to display drastic differences, there may be sectors for which even the direction of change may depend on which FFF was used. In such cases, an automated framework for second-order sensitivity analysis proves valuable to appraise the reliability of modeling results.

While NNCES cost functions are globally regular, possible irregularities in cost functions of the three alternative FFFs can make the corresponding model unsolvable. In such cases the second-order sensitivity analysis is restricted to the FFFs for which a solution can be found. In the various tests that were run on three models in this paper, such insolvabilities were encountered in two of the models. Only for one model did the problems with irregularities appear in the counterfactual experiments proposed by the original work.

Finally, it should be noted that a rigorous analysis of model sensitivity in the spirit of Pagan and Shannon should not only include changing the functional form of all cost functions at once, but also changing it for single sectors only. The economic intuition for an efficient analysis is that sectors that experience high change in relative factor prices should be the first to be analyzed with different FFFs.

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Appendix A: Extracting AUES from NNCES

The following explains the functioning of the automated process of second-order sensitivity analysis used to obtain the results presented in this paper. An executable version along with some documentation is available on the authors web site (http://flandis.shorturl.com). The implemented approach uses the NNCES cost functions that MPS/GE assumes in its model, and calibrates cost functions of the FFFs GL, NQ, and TL to the same benchmark data. The benchmark data needed for calibration are $\bar{C}$, $\bar{p}_i$, θ_i, and σ^A_{ij}. While they are not explicitly given by the MPS/GE formulation of the model, they are implicit to the cost functions used by MPS/GE. The following describes the nature of NNCES cost functions and how to extract benchmark data from them.

If p_i, $i \in \mathscr{I} = \{1, ..., N\}$ are all prices appearing in a given NNCES cost function, it can be constructed from level ℓ nests

$$c^\ell_k(\mathbf{p}) = \left[\sum_{k' \in \mathscr{S}^\ell \cup \mathscr{I}} \alpha^\ell_{kk'} c^{\ell+1}_{k'}(\mathbf{p})^{1-\gamma^k} \right]^{1/(1-\gamma^k)}, \quad \alpha^\ell_{kk'} \geq 0, \quad \sum_{k'} \alpha^\ell_{kk'} = 1 \tag{2}$$

where $\mathscr{S}^\ell$ is the set of all level $\ell + 1$ subnests. $c^{\ell+1}_{k'}$ is again of the form (2) if $k' \in \mathscr{S}^\ell$ or a terminal nest ($k' \in \mathscr{I}$):

$$c^{\ell+1}_{k'}(\mathbf{p}) = p_{k'}/\bar{p}_{k'}. \tag{3}$$

All sets $\mathscr{S}^\ell$ shall be finite and there shall exist an $\ell < N$ for which all $\mathscr{S}^\ell$ are empty. In no nest shall the number of subnests $|\mathscr{S}^\ell \cup \mathscr{I}|$ be greater than N. If $C^{\ell+1}_k(\mathbf{p})$ is the cost incurred through the goods entering the level-($\ell + 1$) subnest k',

$$\alpha^\ell_{kk'} := \frac{C^{\ell+1}_{k'}(\bar{\mathbf{p}})}{\sum_{h \in \mathscr{S}^\ell \cup \mathscr{I}} C^{\ell+1}_h(\bar{\mathbf{p}})}$$

and

$$c^{\ell+1}_k(\mathbf{p}) = \frac{C^{\ell+1}_k(\mathbf{p})}{C^{\ell+1}_k(\bar{\mathbf{p}})}.$$

An NNCES cost function is then written as $\bar{C}$ times a level 0 NNCES-nest

$$C(\mathbf{p}) = \bar{C}c^0(\mathbf{p}) = \bar{C}\left[\sum_k \alpha_k^0 c_k^1(\mathbf{p})^{1-\gamma}\right]^{1/(1-\gamma)}.$$

Example 1 For the MPS/GE production sector[5]

```
$PROD:X s:5 va:6
O:PX    Q:15
I:P1    Q:1
I:P1    Q:2      va:
I:P2    Q:3 P:4 va:
```

the NNCES cost function is

$$C(\mathbf{p}) = \bar{C}\left[\alpha_1^0 c_1^1(\mathbf{p})^{1-5} + \alpha_{va}^0 c_{va}^1(\mathbf{p})^{1-5}\right]^{1/(1-5)}$$

with

$$\begin{aligned}\bar{C} &= 1 + 2 + 3 \cdot 4 = 15\\ \alpha_1^0 &= 1/15\\ \alpha_{va}^0 &= (2 + 3 \cdot 4)/15\end{aligned}$$

and

$$\begin{aligned}c_1^1(\mathbf{p}) &= p_1\\ c_{va}^1(\mathbf{p}) &= \left[\alpha_{va,1}^1 c_1^2(\mathbf{p})^{1-6} + \alpha_{va,2}^1 c_2^2(\mathbf{p})^{1-6}\right]^{1/(1-6)}\end{aligned}$$

with

$$\begin{aligned}\alpha_{va,1}^1 &= 2/14\\ \alpha_{va,2}^1 &= 3 \cdot 4/14\\ c_1^2 &= p_1\\ c_2^2 &= p_2/4.\end{aligned}$$

Let us now turn to the task of extracting benchmark data from a given NNCES cost function. Value shares θ_i can be either computed directly from given MPS/GE figures or recursively from the NNCES formulation: If we define θ_{ki}^ℓ as the value share of good i in the level-ℓ subnest k

[5] For information on MPS/GE syntax and its interpretation, please refer to section B.4 in http://www.mpsge.org/mpsge/syntax.pdf.

$$\theta_{ki}^{\ell} := \frac{\bar{p}_i}{c_k^{\ell}(\bar{\mathbf{p}})} \frac{\partial c_k^{\ell}(\bar{\mathbf{p}})}{\partial p_i} = \sum_{k' \in \mathscr{S}^{\ell} \cup \mathscr{I}} \alpha_{kk'}^{\ell} \theta_{k'i}^{\ell+1}, \quad k \in \mathscr{S}^{\ell}$$

$$\theta_{ki}^{\ell} := \delta_{ki} = \begin{cases} 1 \text{ if } i = k \\ 0 \text{ else} \end{cases}, \qquad k \in \mathscr{I},$$

θ_i is just the level 0 value share of good i, θ_i^0. It remains to compute the AUES σ_{ij}^A. This is again done by recursively calculating the AUES of the level-ℓ nest k:

$$\sigma_{kij}^{\ell}(\bar{\mathbf{p}}) = \gamma^k + \frac{1}{\theta_{ki}^{\ell} \theta_{kj}^{\ell}} \sum_{k' \in \mathcal{S}^{\ell+1} \cup \mathscr{I}} (\sigma_{k'ij}^{\ell+1} - \gamma^k) \alpha_{kk'}^{\ell} \theta_{k'i}^{\ell+1} \theta_{k'j}^{\ell+1}, \quad i \neq j$$

and thus obtaining the benchmark AUES $\sigma_{ij}^A = \sigma_{ij}^0(\bar{\mathbf{p}})$.

Given θ_i and σ_{ij}^A, calibration of the remaining FFFs is then just a matter of applying the information in Table 2 due to Perroni and Rutherford (1998).

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