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Blasius: A life in research and education

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Abstract The Blasius boundary layer solution is a basic feature of fluid mechanics, and the first application of Prandtl's boundary layer concept as a significant issue in engineering. This work highlights the contributions of Blasius to hydrodynamics over a period of only six years, which marked the initiation of boundary layer theory. His main papers relating to aerodynamics and smooth turbulent pipe flow are also reviewed, and a biography puts his career into the general environment of Germany over the period of two World Wars. A complete bibliography of Blasius is provided.

1 Introduction

Nineteenth-century advances in hydrodynamics were directed mainly towards the understanding of ideal fluid flow. Accordingly, successful research was directed towards wave hydrodynamics. William Froude (1810–1879) introduced a criterion named after him, according to which scale models involving free surface waves follow his similarity law. Shortly afterwards, Osborne Reynolds (1842–1912) formulated a law for viscous fluid flow. After Leonhard Euler had presented equations for ideal fluid flow in differential form, Navier, Saint-Venant and Stokes generalized his equations by accounting for viscosity. Mathematically, these relations are highly involved, however, and only special solutions such as for laminar pipe flow were amenable. In order to advance knowledge, therefore, appropriate simplifications had to be introduced.

Ludwig Prandtl (1875–1953) presented his benchmark paper on boundary layers in 1904 (Prandtl 1904), postulating that flow around a smooth body could be subdivided into two regions: (1) Close to the boundaries, fluid viscosity governs flow, for which the Navier–Stokes equations simplify to what we currently refer to as the boundary layer equations; and (2) away from the boundaries, viscosity has a small effect and the flow is influenced mainly by Euler's potential flow theory. The solutions pertaining to these two domains are suitably matched at the interface, and this approach was the key to advancing the questions of turbulence.

While Prandtl at that time presented no direct application of his approach, his first Ph.D. student Heinrich Blasius was able to outline the significance of the novel formulation. The boundary layer solution for the flat plate at once demonstrated the power of Prandtl's concept. The solution was subsequently tested in a variety of flow configurations, such as in naval engineering and in aerodynamics, and substantial agreement was noted. Blasius then considered problems of potential flow theory, in a way a step back from what he did earlier. However, his solutions were another addition to hydrodynamics, involving free surface flow and the improvement of the Pitot tube. Another noteworthy contribution was published in 1912 (Blasius 1912a), relating to the Blasius friction coefficient for turbulent smooth pipe flow. As the present author was able to find the daughter of Blasius, a biography is also presented. This note, therefore, would like to highlight a mathematical hydraulician, known by name to all those working in fluid mechanics, but not the history and biography of one of the founders of modern fluid mechanics (Fig. 1).

2 Blasius' 1907 paper

Hardly had Prandtl begun research at Göttingen university when his first Ph.D. student Blasius began working on a problem that was the foundation of both Prandtl's and Blasius' reputation. The Blasius' aim was to overcome the enigma of turbulence by considering the phenomenon of boundary layer flow. As demonstrated by Prandtl in 1904, the 2D Navier–Stokes equations may be simplified for the flat plate boundary layer as

$$u(\partial u/\partial x) + v(\partial u/\partial y) = \nu\left(\partial^2 u/\partial y^2\right) \tag{1}$$

$$\partial u/\partial x + \partial v/\partial y = 0 \tag{2}$$

subject to the boundary conditions $u(y{=}0){=}v(y{=}0){=}0$ and $u(y \to \infty){=}U$. Here, u and v are the streamwise and the transverse velocity components in the directions x and y, respectively, ν is the kinematic viscosity, and U is the free stream velocity. Blasius (1907) reduced the system of partial differential equations (1) and (2) to an ordinary third-order differential equation with the scalings

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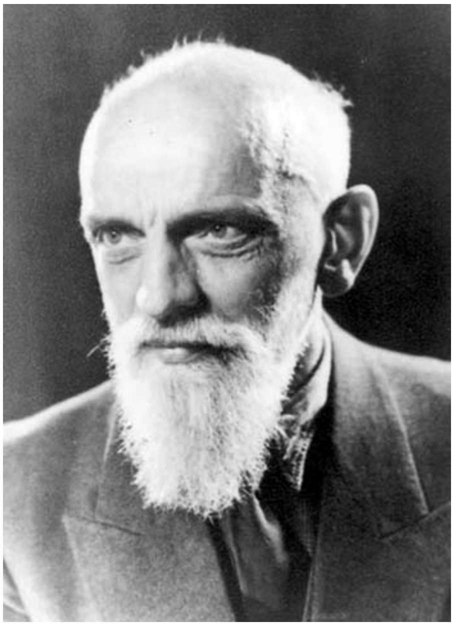

Fig. 1. Blasius in 1962, after retiring

$$\xi = (y/2)(U/\nu x)^{1/2} \quad \psi = (Ux/\nu)^{1/2}\zeta \tag{3}$$

where u=(1/2)$U\zeta'$ with ()′=d()/dζ. Then, the governing equation is

$$\zeta\zeta'' + \zeta''' = 0 \tag{4}$$

This equation was solved with a power series instead of a direct solution, resulting in a constant of integration α between 1.326 and 1.327. It should be noted that Prandtl (1904) had already investigated this problem and estimated α to be 1.1. The force F on the plate of width b and length L is

$$F = \alpha\rho b\left(\nu LU^3\right)^{1/2} \tag{5}$$

where ρ is fluid density. In discussing that paper, Toepfer outlined a simpler procedure for determining α, which he fixed to 1.32824. Note that the force varies as $F \sim U^{3/2}$ instead of the linear variation as for small Reynolds numbers. The concept of the "Blasius boundary layer profile" has had far-reaching consequences, because it represents a basic solution for viscous flow along a smooth boundary, as noted by Schlichting (1965).

Additional problems considered by Blasius in his 1908 paper (Blasius 1908) were flow separation behind a circular cylinder, development of boundary layer flow due to sudden initiation of flow, and separation from a cylinder for unsteady flow. This latter benchmark paper demonstrated the potential of Prandtl's new boundary layer concept and its engineering application towards understanding turbulent flow. It also reveals Blasius' impact on future research developments in turbulent flow.

3
Blasius' 1910 papers

A second key paper (Blasius 1910b) involved the classical potential theory applied to two cases, first to the force exerted on a body immersed in a fluid flow, and second to potential flow over weirs. For the latter, conformal mapping was applied to free surface flow, a task so far not considered. However, as described in the introduction, Blasius' results were not directed to application, but his method opened the way to general problems in fluid dynamics. Blasius (1911a) re-considered mathematical methods applied to potential flow, and derived an expression for the force of an obstacle positioned in a stream. This equation is referred to as the *Blasius theorem* in aerodynamics.

Blasius (1910c) considered the laminar flow of a discharge Q in a channel of variable width $B(x)$ and constant height (Fig. 2). For a diverging channel, separation occurs because of the adverse pressure gradient for a product of Reynolds number R=(Q/νH) times the change of width dB/dx larger than

$$\mathrm{R}(\mathrm{d}B/\mathrm{d}x) = 35/2 \tag{6}$$

Accordingly, separation is influenced by both the Reynolds number and the increase in width, such that a small R allows for a relatively large angle, and vice versa. For a maximum Reynolds number of R=2,000, say, one would have a maximum angle of 0.5°. For turbulent flow, larger angles are known to result, as was later investigated by Prandtl's scholar Johann Nikuradse (1894–1979).

4
Blasius' 1911 papers

Blasius (1911b) investigated the curved airfoil, using the Kutta method, according to which the lift force is F=2$\pi\rho$Uc, with circulation 2πc and the undisturbed approach velocity U. For a small ratio of c/U, Blasius re-derived the Kutta expression according to which $F=\zeta\rho \mathrm{F}U^2$, where $\zeta=2\pi(f/L)$, with f the height of the airfoil and L its width, ρ fluid density, and F wing surface. Wings of finite height were considered, and a method was outlined to find optimum wing characteristics.

Blasius (1911c) investigated flow in turbines using potential flow theory again. By assuming an appropriate stream function, an expression for the circulation $C=2\pi\theta(\nu_{-\infty}-\nu_{+\infty})$, where θ is a length scale and ν_∞ the free

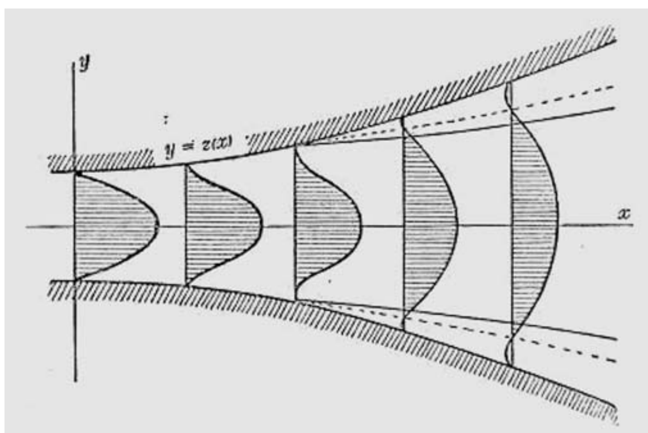

Fig. 2. Separation of laminar flow in a diverging channel (Blasius 1910c)

stream velocity, was derived, in agreement with the circulation theory for the force component perpendicular to the flow direction

$$K_y = \rho u_{\pm\infty} C \qquad (7)$$

Blasius demonstrated ability with conformal mapping applied to a variety of basic wing profiles. His results were highlighted particularly by Grammel (1917), together with those of Nikolai E. Zhukovsky (1847–1921) and Wilhelm Kutta (1867–1944), given his basic contribution to the theory of plane wing flow.

5
Blasius' 1912 papers

Whereas the 1907/1908 papers were related to internal flow features close to a smooth boundary, Blasius (1912a, 1913a) added to the understanding of pipe flow features. After Henry Darcy (1803–1858) and Henry Bazin (1829–1917) had presented excellent data sets relating to both pipes and open channels, and Osborne Reynolds (1842–1912) had introduced a number bearing his name to distinguish between laminar and turbulent flows, Blasius was the first to derive a law relating to so-called turbulent smooth pipe flows. Let the hydraulic gradient J in pipe flow of diameter D be

$$J = \left(V^2/2g\right)(f/D) \qquad (8)$$

with cross-sectional velocity V, gravitational acceleration g, and friction factor f. Equation (8) involves essentially Froude similarity, with the pipe Froude number $\mathrm{F}=V/(gD)^{1/2}$. However, as stated by Blasius (1912a), viscosity may be significant in addition. Therefore, the Reynolds number $\mathrm{R}=VD/\nu$ must also be included in the set of variables as $f=f(\mathrm{R}, \mathrm{F})$, whereas the Froude number accounts for relative roughness. For smooth pipe flow, a unique relation exists between f and R.

By plotting f as a function of R, Blasius correlated the data of various sources and proposed for Reynolds numbers $3\times10^3<\mathrm{R}<2\times10^5$ (Fig. 3)

$$f = 0.3164\mathrm{R}^{-0.25} \qquad (9)$$

Nobody would have assumed anythings so simple, especially after the numerous and complex proposals made in the 19th century. Blasius was again the first to realize the significance of hydraulic similitude, combined with basic quantities derived from the governing set of hydrodynamic equations. He would also dare to say that (9) not only governs water flow in pipes, but any Newtonian fluid flow such as air, as was demonstrated two years later by the Englishmen Stanton and Pannell. For smooth pipes, temperature thus has an effect on friction, a fact noted much earlier but never deduced physically. With this finding, the race was on to establish the effect of roughness on fluid flow, again with Prandtl greatly involved in this question. Until the mid-1920s, he indeed thought that (9) applied universally. Based on theoretical developments (Prandtl 1927; von Karman 1930) and the experiments of Nikuradse (1932), he realized that the solution was more complex than had been proposed by Blasius. The ideas of Blasius, Prandtl and von Karman are highlighted by Durand (Prandtl 1935) and led to the lasting reputation of the Göttingen School. The final answer to the pipe flow problem was presented by the two Englishmen Colebrook and White in 1937.

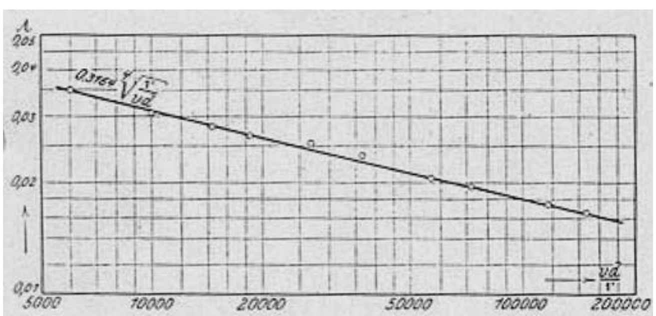

Fig. 3. Equation (9) tested with the Nusselt data set (Blasius 1912a)

Another paper (Blasius 1912c) related to the difference between the Froude and Reynolds similarity laws. According to Froude similarity, velocities V are scaled to $\lambda^{-1/2}$, with λ as the geometrical scale factor between model and prototype, whereas Reynolds similarity involves $V\sim\lambda^{+1}$. These laws apply for water and air, and advantages may result from using different fluids in model and prototype. In the discussion, a question related also to surface tension effects, and Prandtl responded that no relation was yet available. Moritz Weber (1871–1951) was present at that meeting, and, some years later, the Weber number was introduced to honor his lasting contribution to similarity theory.

According to Rotta (1990), Blasius presented a paper during the First Conference on the Science of Flight in November 1911 at Göttingen. The title of his paper was "Air resistance and Reynolds' number", where he mentioned that his findings would be published in 1912. It is noteworthy that Blasius was invited to that meeting and that his approach was discussed by the specialists previously mentioned.

6
Blasius' biography

Paul Richard Heinrich Blasius was born on 9 August 1883 in Berlin. After studies at the universities of Marburg and Göttingen from 1902 to 1906, he was a scientific collaborator with Ludwig Prandtl (1875–1953), and from 1908 a research assistant at the hydraulics laboratory of Berlin technical university. From 1912, he was a teacher at the technical college of Hamburg. Accordingly, Blasius spent only six years in science, and moved then to teaching, which he loved perhaps more than he loved research. Poggendorff (1936) lists a total of 11 quotations (Fig. 4).

During the first Berlin year, Blasius (1909) started working on the Pitot tube, a basic hydraulic instrument. After presenting seven different designs from the Berlin laboratory, he described mathematically the flow pattern across the tube, again using potential flow theory. Yet, a final design was not proposed, until later by Prandtl, whose name it bears, allowing combined measurement of static and dynamic pressure heads. One year later,

Fig. 4. Blasius (*left*) playing chess, around 1915

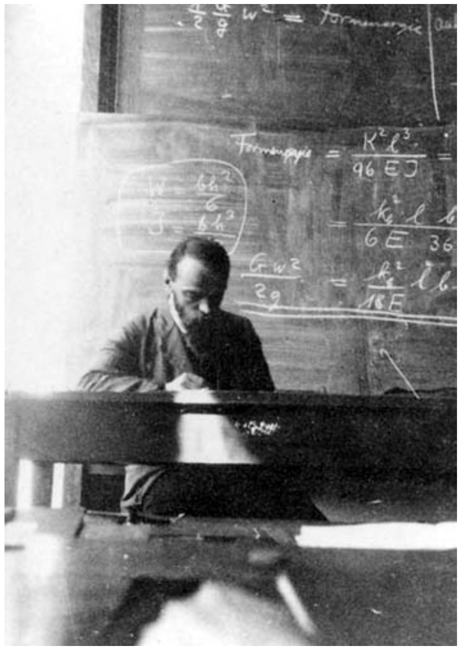

Fig. 5. Blasius at Hamburg engineering college, in 1920

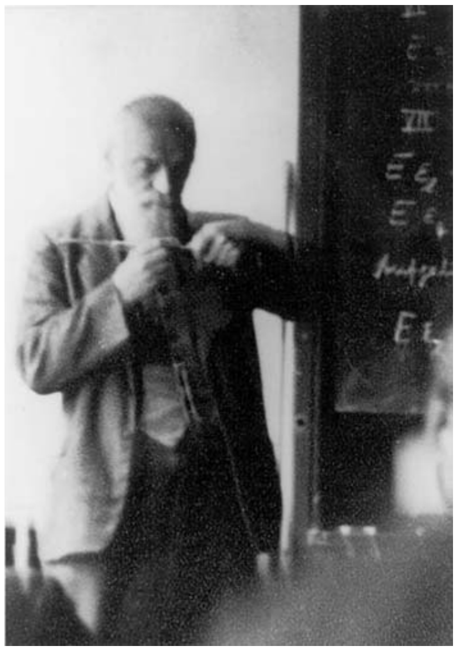

Fig. 6. Blasius during lecturing, 1925

Blasius (1910d) even presented a paper on sediment riffles and banks in loose boundary hydraulics. Whereas banks were known to occur in river bends, they had so far not been observed in straight rectangular channels, except in a description by Hubert Engels (1854–1945). Blasius attributed the effect to a flow instability. Whereas parallel riffles occurred for small Froude numbers, he noted oblique bed features for supercritical flow. Although these observations may be regarded as an early description of the complex flow configurations with sediment transport, the time had not arrived for a lasting contribution.

Blasius (1910a) also considered the hydraulic ram, forces on sluice chambers (Blasius 1912b), two problems in elasticity (Blasius 1913b, 1919), and in 1925 a flutter problem of wings (Blasius 1925). Early work on the latter problem was conducted towards the end of World War I, but Blasius published his results only in 1925 in a commemoration volume to Prandtl's 50th birthday. The research related to the lower wing of the double-decker Albatros D III type. That machine became known for dangerous incidents resulting in catastrophes for both the passengers and the aircraft. Blasius thus was able to point to an instability such that the problem could be technically removed (Fig. 5).

His undergraduate books on heat transfer followed in 1931 (Blasius 1931) and on mechanics in 1934 (Blasius 1934), and two notes published later were directed at college students. These include a description of an experiment by Galilei–Mach (Blasius 1936a) and a demonstration of internal stresses (Blasius 1936b). A basic math book is also quoted by Poggendorff (1953). According to his daughter, Blasius must have been at one with his college, regarding which a background is presented by Baensch (1955). Blasius was specially acknowledged for having rebuilt both lecture rooms and laboratories after devastation during World War II. Officially, he stayed at the mechanical engineering department from 1912 to 1950, and headed it from 1945 to 1950. Blasius continued lecturing, because he gained complete satisfaction from teaching (Anonymous 1962). A rich life came to an end on 24 April 1970, when Blasius passed away in Hamburg (Anonymous 1970) (Fig. 6).

7
Blasius' legacy

Mrs Elfriede Blasius handed me a document written by Blasius in 1962 after he retired from the engineering college. The following is a translation into English:

> At Easter 1902 I had sat my final college examination, and was now studying mathematics. This was not simple for me: Although one can see what follows from certain algebraic manipulations, why would this be done? "By mathematics, you are convicted, instead of convinced", according to the phi-

losopher Schopenhauer. After some time, I realized that, for me, mathematics was a collection of tricks. While my friends were busy with ions, with electrons, the Zeeman-effect and more, I stayed with the basics, with questions relating to philosophy and psychology, and remained unsatisfied. Accordingly, I changed to astronomy, but this was too subtle, and I continued with physics finally. I was asked to work on inhomogeneous waves generated due to total reflection in a second medium, as a Ph.D. topic. This was of interest to Prof. Voight but not to me! I continued with Prof. Prandtl and submitted a Ph.D. thesis on boundary layer flow in July 1907. I also added two or three cases for his theory. He told me after the exam that I hadn't known all that he wanted. But he said that what I knew I had understood, and I was satisfied.

I stayed for a short period with Prandtl as an auxiliary assistant, and then moved to (Preussische) Versuchsanstalt für Wasserbau und Schiffbau at Berlin, where I worked on fluid friction in pipe flow, among other topics. In addition, I also submitted scientific works, such as on flow around wings and turbine blades using complex variables. All this was unprofitable, without success. I tried to continue working in turbulence, again without success, and finally told myself: "You are not a scientific." Also, I admit that I did not have enough interest in the subject, but more for the approach. Prandtl was slightly disappointed because I was not interested in his wind tunnel, nor did I drive to the Rhön valley to glide model airplanes. I was asked why I did not look for a position as a professor. Evidently, you dream of this position as a student, and maybe I would have made it. At Aachen, I once achieved second ranking. My friend Barkhausen wanted to propose me for Dresden university, but I answered: "Let it be, I would disappoint you anyway!" Also, I was no more interested in science than before, and I do not regret it to this day. I feel my place is here at the engineering college. In any case, when you decide not to strive for the top position where you could have problems, choose the second position, where you can really add to progress. Therefore: "Blasius, who once worked in science, has been dead for a long time." It was good having once been a physicist, but now I am a teacher, not corresponding to a scale model of practice but to its foundation.

At Easter 1912, I started at Hamburg engineering college. After the war, which I survived without harm, I started with my books. I wanted to present a heuristic approach by accounting for the three impressions of the human mind: perception of ideas, elaboration of methods, and description of phenomena. Learning, working and order are thus the three essences of the mind. From 1920 to 1930, I mainly collected examples to present the basics of a problem. Finally, the heat transfer book appeared in 1931 (Blasius 1931), and then three booklets on mechanics by 1935. At the age of almost 78 years, Blasius (1961) published a work on lubrication.

I distinguish between exact knowledge and learning. As a good teacher, you should allow students to use their notes during exams, and ask questions on their integral knowledge. My concern is specialization, and – forgive me – the present trend in increasing lectures in other branches enhances this. Finally, consider our students: Precious people often interested in knowledge. Certainly, they would also like to earn money, but by working. They are willing to learn; care for them, for they trust you and are devoted to advance! Why at the end I am still here? Certainly not because of interest in the subject or because of money. The only reason I stayed for so long is because of my affection for youth, whom I should like to help in solving problems, and to assist into a profession. Here I am, and here I stay, until I am thrown out!

8
Impact of Blasius

Having reviewed the main papers of Blasius, I note that within a short period of only six years, a number of problems in fluid dynamics were successfully considered. These include boundary layer flow as a direct application of what Prandtl had set out some years earlier, and the definition of boundary layer separation in diverging flow, with the first definite result which clarified at least the basic issues of turbulence. Further, Blasius worked on potential flow, by solving problems of engineering concern relating to wing theory, the result of which is known as the Blasius theorem in aerodynamics. Finally, the Blasius pipe friction coefficient was defined using a consequent application of the Reynolds approach and resulting in the *1/4 power law* of turbulent smooth pipe flow. Today, the results relating to boundary layer flow have certainly maintained the greatest impact, given their significance in modern fluid mechanics. His potential flow approaches have lost their attraction because of numerical modeling. The Blasius friction law for turbulent pipe flow had a major impact in the 1910s and 1920s, because this relation was one of the few definite issues besides the Hagen–Poiseuille laminar flow formula. It took another 25 years before the effects of roughness and viscosity were fully understood.

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