

# Second Law Analysis of Ion Slip Effect on MHD Couple Stress Fluid

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**Abstract**— This paper is concerned with the numerical investigation of entropy generation in viscous incompressible MHD couple stress fluid in a rotating frame of reference. An approximate solution of the dimensionless velocity and temperature profiles are obtained and used to calculate the entropy generation rate and Bejan number. The influences of the governing parameters on velocity, temperature, entropy generation and Bejan number are presented with the aid of graphs.

**Keywords**—Adomian decomposition method, Couple stress fluid, Entropy generation, Ion slip, MHD, Bejan number

## I. INTRODUCTION

COUPLE stress fluid theory was proposed by Stokes [1] to demonstrate its microrotation effects which could not be described by the classical Navier-Stokes theory. This type of fluid has attracted the attention of many researchers due to its numerous applications in various engineering and industrial processes such as extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, colloidal solutions, synovial joints (shoulder, hip, knee and ankle), geophysics, chemical engineering and astrophysics. Since then extension has been done, to include studies of hydrodynamic couple stress fluid [2-5], blood flows [6-7], journal bearings [8-9], squeeze films [10-11], couple irreversibility [12-18] where various factors associated with couple stress irreversibility are investigated. Others are [19-21]. The aim of this paper is to analyse the influence of Ion slip on the entropy generation of MHD couple stress fluid using the rapidly convergent Adomian decomposition method to solve the dimensionless velocity and temperature profiles obtained from the governing equations. This technique has been applied to various flow problems [22-25] and other linear and non-linear models [26-30].

The organization of the paper is as follows: section presents formation of the governing equations, Adomian decomposition method of solution is treated in section 3 and section 4 is based on

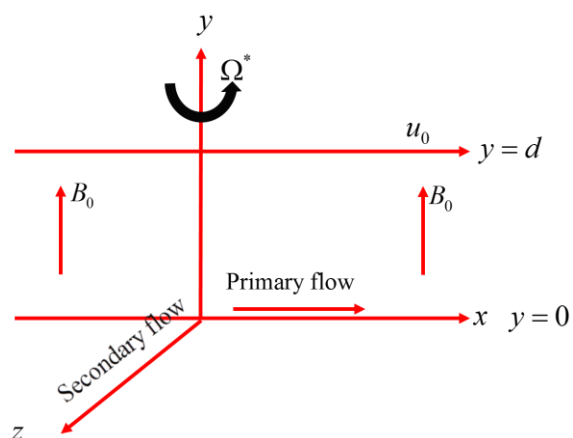
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the discussion of results and final remarks are made in section 5.

## II. PROBLEM FORMULATION

Consider the viscous incompressible and electrically conducting couple stress fluid bounded by two infinite horizontal parallel plates with convective heating separated by a distance  $d$ . Let a cartesian co-ordinate system be chosen such that the  $x$ -axis is along the lower plate in the direction of the flow, the  $y$ -axis is perpendicular to the plates while the  $z$ -axis is normal to  $xy$ -plane. Assumption of relatively high electron-atom collision frequency is taken so that the impact of ion slip cannot be neglected. The entire system is rotating with an angular velocity  $\Omega^*$  about the normal to the plate. The flow is fully hydrodynamically and thermally developed, therefore the fluid velocity and the temperature in the channel are functions of  $y$  only. Following the assumptions above, the governing equations for the flow can be written [31] as



**Figure 1:** Geometry of the problem

$$2\Omega^* w^* = -\frac{dp}{dx} + \mu \frac{d^2 u^*}{dy^{*2}} - \lambda \frac{d^4 u^*}{dy^{*4}} - \sigma B_0^2 u^*; \quad (1)$$

$$u^*(0) = \frac{d^2 u^*}{dy^{*2}}(0) = 0 = u^*(d) = \frac{d^2 u^*}{dy^{*2}}(d),$$

$$-2\Omega^* u^* = \mu \frac{d^2 w^*}{dy^{*2}} - \lambda \frac{d^4 w^*}{dy^{*4}} - \sigma B_0^2 w^*; \tag{2}$$

$$w^*(0) = \frac{d^2 w^*}{dy^{*2}}(0) = 0 = w^*(d) = \frac{d^2 w^*}{dy^{*2}}(d),$$

$$0 = k \frac{d^2 T^*}{dy^{*2}} + \mu \left[ \left( \frac{du^*}{dy^*} \right)^2 + \left( \frac{dw^*}{dy^*} \right)^2 \right] + \lambda \left[ \left( \frac{d^2 u^*}{dy^{*2}} \right)^2 + \left( \frac{d^2 w^*}{dy^{*2}} \right)^2 \right] + \tag{3}$$

$$\sigma B_0^2 (u^{*2} + w^{*2}); T^*(0) = T^*(d) = 0.$$

where  $u$  and  $w$  are the velocity components in the  $x$  and  $z$  directions respectively,  $\mu$  is the dynamic viscosity,  $\lambda$  is the coefficient of couple stresses,  $\sigma$  is the electrical conductivity,  $B_0$  is the magnetic parameter,  $k$  is the thermal conductivity,  $T$  is fluid temperature and  $d$  the channel width.

The dimensionless variables introduced are as follows

$$y = \frac{y^*}{d}, u = \frac{u^*}{U}, w = \frac{w^*}{U}, \theta = \frac{T^* - T_0}{T_d - T_0}, a = \mu \frac{d^2}{\lambda} \tag{4}$$

then equations (1-3) become

$$2K^2 w = G + \frac{d^2 u}{dy^2} - \frac{1}{a^2} \frac{d^4 u}{dy^4} - M^2 u; \tag{5}$$

$$u(0) = u''(0) = 0 = u(1) = u''(1),$$

$$-2K^2 u = \frac{d^2 w}{dy^2} - \frac{1}{a^2} \frac{d^4 w}{dy^4} - M^2 w; \tag{6}$$

$$w(0) = w''(0) = 0 = w(1) = w''(1),$$

$$\frac{d^2 \theta}{d\eta^2} = -Br \left[ \left( \frac{du}{dy} \right)^2 + \left( \frac{dw}{dy} \right)^2 \right] + \frac{Br}{a^2} \left[ \left( \frac{d^2 u}{dy^2} \right)^2 + \left( \frac{d^2 w}{dy^2} \right)^2 \right] + \tag{7}$$

$$BrM^2 (u^2 + v^2); \theta(0) = 0, \theta(1) = 1.$$

where  $G = -\frac{d^2}{\mu U} \frac{dP}{dx}$  is the pressure gradient parameter,

$\Omega = \frac{T_d - T_0}{T_0}$  is the temperature difference parameter,

$Br = \frac{\mu U^2}{k(T_\infty - T_0)}$  is the Brinkman number,  $K^2 = \frac{\Omega^* d^2}{\mu}$  is

the rotation parameter,  $M^2 = \frac{\sigma B_0^2 d^2}{\mu}$  is the Hartman

number,  $a^2 = \frac{\mu d^2}{\lambda}$  is the couple stress parameter.

### III. ADOMIAN DECOMPOSITION METHOD OF SOLUTION

Application of Adomian decomposition method requires that the differential equations (5) – (7) are written in the integral forms as follows

$$u(y) = \alpha_1 y + \frac{\alpha_2}{3!} y^3 + \tag{7}$$

$$a^2 \int_0^y \int_0^y \int_0^y \int_0^y \left\{ 2K^2 w + G - \frac{d^2 u}{dY^2} + M^2 u \right\} dY dY dY dY,$$

$$w(y) = \beta_1 y + \frac{\beta_2}{3!} y^3 + \tag{8}$$

$$a^2 \int_0^y \int_0^y \int_0^y \int_0^y \left\{ -2K^2 u - \frac{d^2 w}{dY^2} + M^2 w \right\} dY dY dY dY,$$

$$\theta(y) = f_1 +$$

$$\int_0^y \int_0^y \left\{ -Br \left( \frac{du}{dY} \right)^2 + \left( \frac{dw}{dY} \right)^2 + \frac{Br}{a^2} \left[ \left( \frac{d^2 u}{dY^2} \right)^2 + \left( \frac{d^2 w}{dY^2} \right)^2 \right] + BrM^2 (u^2 + w^2) \right\} dY dY. \tag{9}$$

Where  $\alpha_1, \alpha_2, \beta_1, \beta_2$  and  $f_1$  are constants to be computed at  $y = 1$ . Assuming following solution for equations (7)-(9)

$$u(y) = \sum_{n=0}^{\infty} u_n(y), \quad w(y) = \sum_{n=0}^{\infty} w_n(y), \tag{10}$$

$$\theta(y) = \sum_{n=0}^{\infty} \theta_n(y),$$

Substituting (10) in (7)-(9) yields the following

$$\sum_{n=0}^{\infty} u_n(y) = \alpha_1 y + \frac{\alpha_2}{3!} y^3 + \int_0^y \int_0^y \int_0^y \int_0^y \{ G \} dY dY dY dY; \tag{11}$$

$$\sum_{n=0}^{\infty} u_{n+1}(y) =$$

$$a^2 \int_0^y \int_0^y \int_0^y \int_0^y \left\{ 2K^2 w_n - \frac{d^2 u_n}{dY^2} + M^2 u_n \right\} dY dY dY dY, n \geq 0$$

$$\sum_{n=0}^{\infty} w_0(y) = \beta_1 y + \frac{\beta_2}{3!} y^3;$$

$$\sum_{n=0}^{\infty} w_{n+1}(y) =$$

$$a^2 \int_0^y \int_0^y \int_0^y \int_0^y \left\{ \frac{-2K^2 u}{dY^2} + M^2 w_n \right\} dY dY dY dY, n \geq 0$$

$$\sum_{n=0}^{\infty} \theta_0(y) = f_1; \sum_{n=0}^{\infty} \theta_{n+1}(y) =$$

$$\left\{ -Br \left( \frac{du}{dY} \right)^2 + \left( \frac{dv}{dY} \right)^2 + \right.$$

$$\left. - \int_0^y \int_0^y \left\{ \frac{Br}{a^2} \left( \left( \frac{d^2 u}{dY^2} \right)^2 + \left( \frac{d^2 v}{dY^2} \right)^2 \right) + \right. \right\} dY dY, n \geq 0$$

$$\left. BrM^2 (A_n + B_n) \right\}$$

Where  $A_n$  and  $B_n$  are the Adomian polynomials that can be computed as follows

$$A_0 = u^2, A_1 = 2u_0 u_1, A_2 = u_1^2 + 2u_0 u_2;$$

$$B_0 = w^2, B_1 = 2w_0 w_1, B_2 = w_1^2 + 2w_0 w_2$$

The final stage of implementation of Adomian technique is the coding of equations (11)-(13) in a computer algebra package known as Mathematica. The graphical results are presented in Figures 2-5.

IV. RESULTS AND DISCUSSION

To study the influence of rotation and convective heating parameters on the dimensionless velocity, temperature, entropy generation expression and Bejan number, some plots are presented in Figures 2-5.

Figures 2a and 2b display the rotation effect on fluid velocity; the plots indicate a decrease in primary velocity while secondary velocity is enhanced. This is physically correct since rotation is known to enhance secondary fluid velocity in the flow-field by retarding the primary fluid velocity. The reason is that Coriolis force is dominant in the region near to the axis of rotation.

In Figures 2c and 2d it is shown that both primary and secondary velocities decay with a rise in magnetic parameter. The Lorentz force effect is attributed to this phenomenon; the applied magnetic field clutches fluid particles together and thus impedes fluid flow.

In Figures 3a, 3b and 3c the temperature response for several values of rotation parameter ( $K^2$ ), lower Biot number ( $Bi_1$ ) and upper Biot number ( $Bi_2$ ) are displayed. It is seen that temperature rises as rotation parameter and lower Biot number increase; while it decreases with a rise in

convective cooling. The rise in temperature is expected due to exchange of heat between the lower wall and fluid particles where there is convective heating, however a reverse phenomenon is experienced at the upper plate with Newtonian cooling as depicted in Fig 3c.

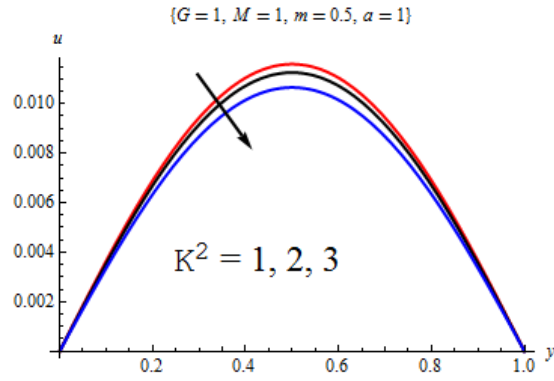


Figure2a: Primary velocity for different  $K^2$

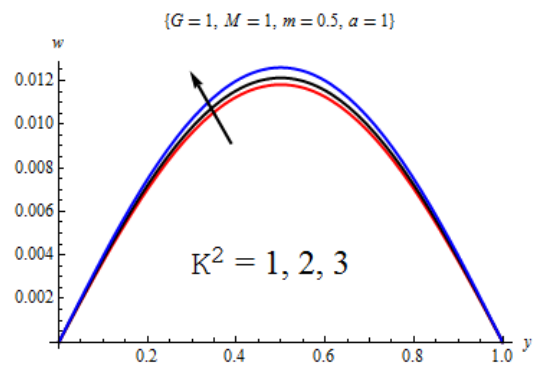


Figure2b: Secondary velocity for different  $K^2$

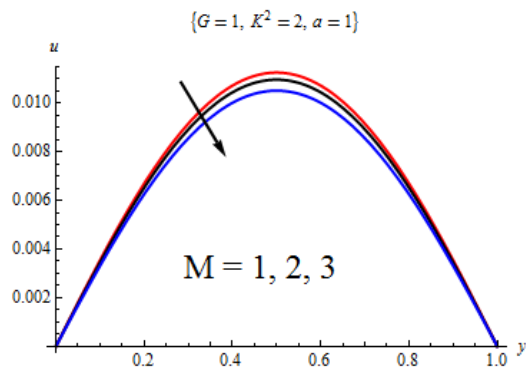


Figure2c: Primary velocity for different  $M$

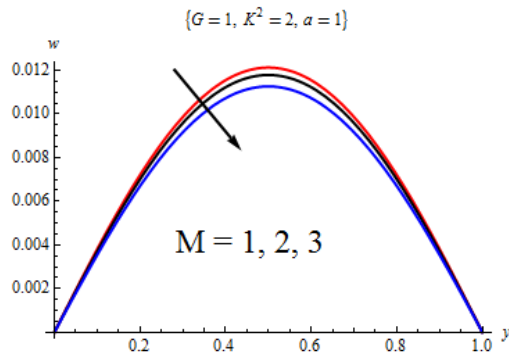


Figure 2d: Secondary velocity for different  $M$

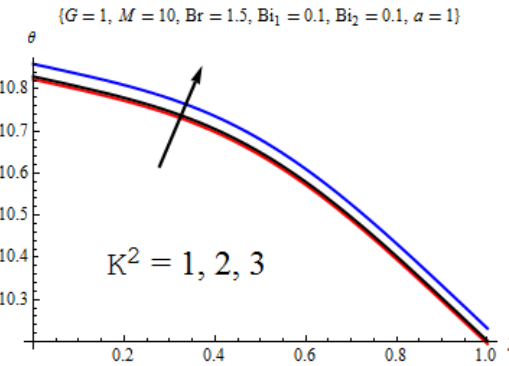


Figure 3a: Temperature profile for different  $K^2$

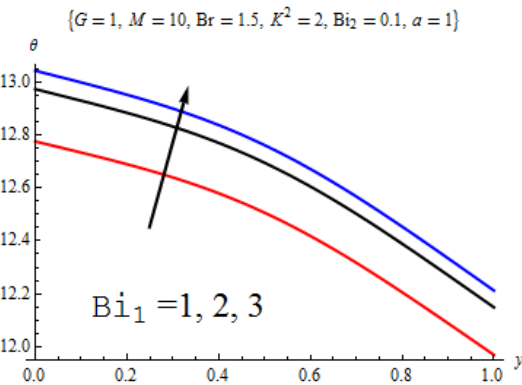


Figure 3b: Temperature profile for different  $Bi_1$

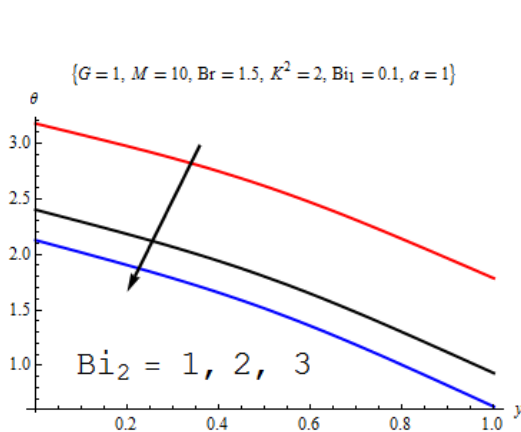


Figure 3c: Temperature profile for different  $Bi_2$

ENTROPY GENERATION ANALYSIS

The entropy generation expression (see Bejan [32]) for the flow is written as

$$E_G = \frac{k}{T_0^2} \left( \frac{dT^*}{dy^*} \right)^2 + \frac{\mu}{T_0} \left[ \left( \frac{du^*}{dy^*} \right)^2 + \left( \frac{dw^*}{dy^*} \right)^2 \right] + \frac{\lambda}{T_0} \left[ \left( \frac{d^2u^*}{dy^{*2}} \right)^2 + \left( \frac{d^2w^*}{dy^{*2}} \right)^2 \right] + \frac{\sigma B_0^2}{T_0} (w^{*2} + u^{*2}), \tag{15}$$

Entropy generation expression in equation (15) consists of heat transfer irreversibility, fluid friction irreversibility, couple stress irreversibility and magnetic field irreversibility respectively.

Substituting (4) in equation (15) yields the dimensionless entropy generation expression as

$$N_s = \left( \frac{d\theta}{d\eta} \right)^2 + \frac{Br}{\Omega} \left[ \frac{1}{a^2} \left( \left( \frac{d^2u}{dy^2} \right)^2 + \left( \frac{d^2w}{dy^2} \right)^2 \right) + M^2 (w^2 + u^2) \right], \tag{16}$$

Where  $N_s = \frac{T_0^2 h^2 E_G}{k (T_d - T_c)^2}$

The ratio of heat transfer irreversibility to fluid friction irreversibility can be represented as

$$\Phi = \frac{N_f}{N_h} \tag{17}$$

where

$$N_h = \left( \frac{d\theta}{d\eta} \right)^2;$$

$$N_f = \frac{Br}{\Omega} \left[ \frac{1}{a^2} \left( \left( \frac{d^2u}{dy^2} \right)^2 + \left( \frac{d^2w}{dy^2} \right)^2 \right) + M^2 (w^2 + u^2) \right] \tag{18}$$

An alternative irreversibility distribution ratio parameter is the Bejan number, it represents the ratio of heat transfer irreversibility to the total irreversibility due to heat transfer and fluid friction.

$$Be = \frac{N_h}{N_s} \tag{19}$$

In equation (19)  $Be$  is  $0 \leq Be \leq 1$ .  $Be = 0$  gives the limit at which fluid friction irreversibility dominates entropy generation while  $Be = 1$  denotes the dominance of heat transfer irreversibility over fluid friction irreversibility and  $Be = 0.5$  is the case when heat transfer and fluid friction entropy generation rates are equal.

Figures 4 and 5 present the influence of rotation parameter ( $K^2$ ), lower Biot number ( $Bi_1$ ) and upper Biot number ( $Bi_2$ ) on entropy generation and Bejan number. It is observed from Figure 4a that entropy generation decreases at plate  $y = 0$  while it rises at plate  $y = 1$  with increase in the rotation parameter. Figures 4b and 4b depict that entropy production is enhanced with increase in Biot numbers.

Furthermore, Figure 5a depicts that Bejan number reduces at plate  $y = 0$  while it rises at  $y = 1$ . Figures 5b and 5c show that Bejan number increases significantly at the middle of the channel but the increase is not noticeable at the channel walls. Generally, entropy generation is dominated by heat transfer irreversibility between  $0.3 \leq y \leq 1$ .

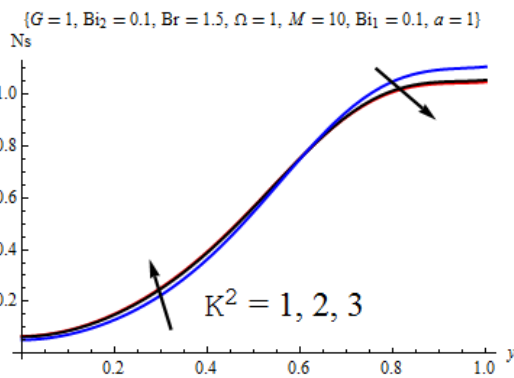


Figure 4a: Entropy generation for different  $K^2$

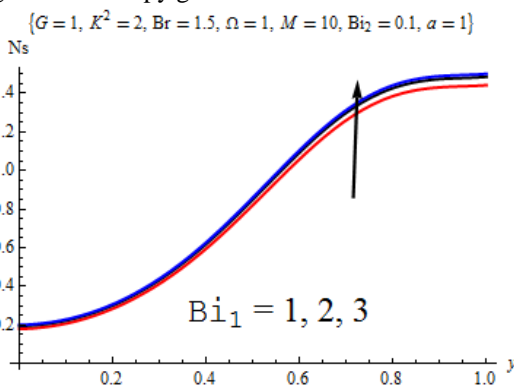


Figure 4b: Entropy generation for different  $Bi_1$

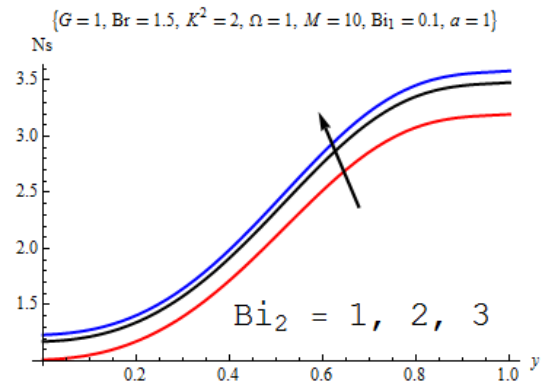


Figure 4c: Entropy generation for different  $Bi_2$

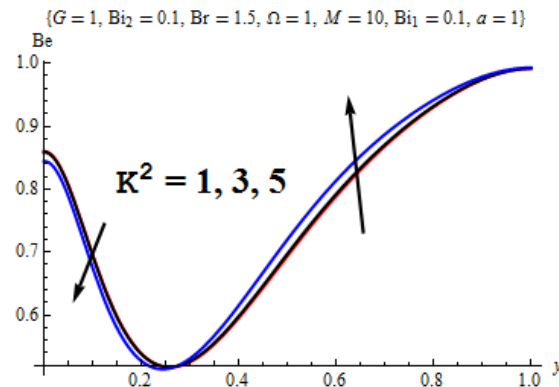


Figure 5a: Bejan number for different  $K^2$

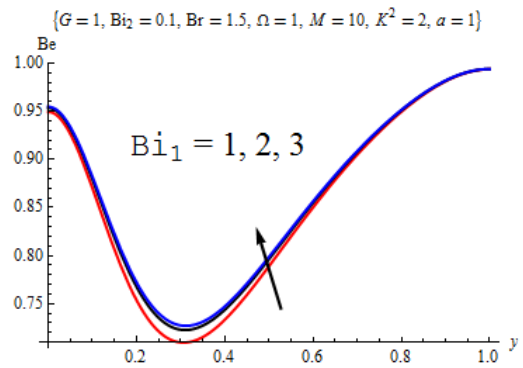


Figure 5b: Bejan number for different  $Bi_1$

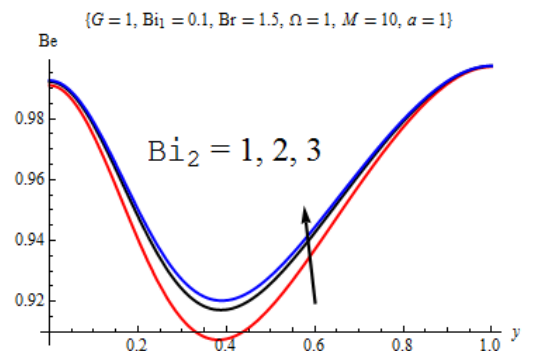


Figure 5c: Bejan number for different  $Bi_2$

## V. CONCLUSIONS

A numerical investigation into the rotation effect on the entropy generation of steady, viscous and incompressible MHD couple stress fluid has been conducted. The findings are;

- Rotation parameter retards primary fluid flow while secondary velocity is accelerated
- Magnetic parameter retards fluid motion,
- Rotation parameter and lower Biot number increase fluid speed while fluid temperature is reduced by upper Biot number
- Rotation parameter and Biot numbers increase temperature.
- Rotation parameter reduces Bejan number at the lower wall but increases it at the upper wall,
- The Biot numbers increase Bejan number.

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