Decision support for design of reconfigurable rotary machining systems for family part production

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To remain competitive in currently unpredictable markets, the enterprises must adapt their manufacturing systems to frequent market changes and high product variety. Reconfigurable manufacturing systems (RMSs) promise to offer a rapid and cost-effective response to production fluctuations under the condition that their configuration is attentively studied and optimised. This paper presents a decision support tool for designing reconfigurable machining systems to be used for family part production. The objective is to elaborate a cost-effective solution for production of several part families. This design issue is modelled as a combinatorial optimisation problem. An illustrative example and computational experiments are discussed to reveal the application of the proposed methodology. Insight gained would be useful to the decision-makers managing the configuration of manufacturing systems for diversified products.

Keywords: reconfigurable machining systems; production system design; family part production; rotary machine; combinatorial design; combinatorial optimisation

1. Introduction

Today, manufacturing companies have to cope with increasing global competition and unpredictable market changes driven by the rapid introduction of new products and constantly varying product demand (ElMaraghy et al. 2013). One of the possible responses to the challenge of meeting customers' needs is offered by the introduction of reconfigurable equipment in the manufacturing process. The reconfigurable manufacturing equipment (RMSs) was invented to provide a rapid and cost-effective response to production requirements. This is accomplished through reconfiguring the system elements over the time for a diverse set of products often required in small quantities and with short delivery lead time (Koren and Shpitalni 2010).

In practice, different physical structures can support the physical reconfiguration of the system. The physical structure defines such core characteristics of RMS as modularity, scalability, convertibility and diagnosability (Singh, Khilwani, and Tiwari 2007; Gumasta et al. 2011). This paper considers reconfigurable machining systems with rotary transfer and turrets (Figure 1). The goal was to develop optimisation methods adapted to this physical structure that will help designers to select machining units and to match the system configuration with the production requirements of each particular part family.

The sectors of the rotary table, where parts are placed, correspond to the working positions of the machine. The table can serve at most m_0 working positions. Working positions can be reconfigured depending on the part family to be machined. Not all positions are used for machining each part.

At each working position, modular machining units (modules) are used for processing parts. In the considered design problem, the following machining units are distinguished:

(1) According to the number of machining units linked together:
  - (1.a) a spindle head which constitutes a single machining module that contains one or several spindles applied in parallel to the part being machined,
  - (1.b) a turret which holds several machining units activated in a given sequence as shown in Figure 1.

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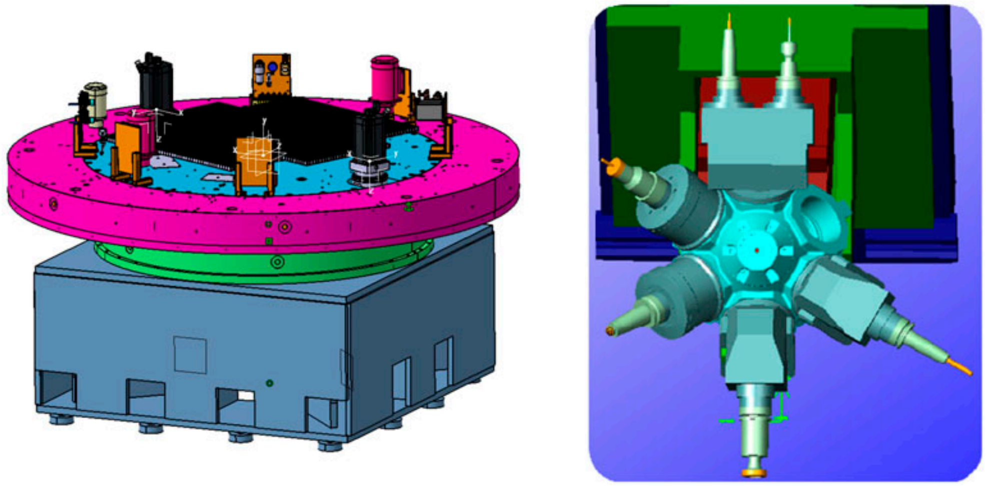

Figure 1. A rotary table and a turret with five machining units: one of them holds two spindles.

(2) According to the direction of machining process:
  (2.a) the vertical modules that are applied to machine vertically (*Z*-axis). Note that in the considered case, if a vertical turret is installed at one position, it is only used at this position. However, it is also possible to install a vertical spindle head common to several working positions.
  (2.b) the horizontal modules that are fixed and applied to the parts to machine horizontally (other axes).

The design of transfer machines with rotary or mobile tables was mostly studied for mass production (Szadkowski 1971; Dolgui, Guschinsky, and Levin 2009; Battaïa et al. 2012). For such machining systems, the reconfiguration process is not effortless, is costly and requires solving a specific optimisation problem as it was studied in (Makssoud, Battaïa, and Dolgui 2014). Usually, in the automotive industry, the reconfiguration of mass production transfer lines is made only every 7 years.

Since the introduction of the concept of reconfigurable manufacturing systems (RMSs) by Koren et al. (1999), the configuration and reconfiguration of such systems have been often discussed in the literature (Bi et al. 2008; Liu and Liang 2008; Dou, Dai, and Meng 2010; Gwangwava et al. 2014; Mpofu and Tlale 2014). However, these previous studies have considered different physical structures of RMS in comparison with that considered in the present paper. As a consequence, the existing optimisation methods cannot be applied directly to the design of reconfigurable rotary machining systems with turrets.

Different criteria were already used for design of reconfigurable and flexible manufacturing systems; in particular, the cost of the installation and their operation were assessed in (Saxena and Jain 2012; Tolio and Urgo 2013; Kristianto, Gunasekaran, and Jiao 2014). The rapid responsiveness and value creation of RMS have been discussed in detail in (Koren 2013; Koren, Wang, and Gu, forthcoming).

Other optimisation problems related to the use of RMS have been also revealed in the literature, namely measurement of operational capability (Goyal, Jain, and Jain 2012), recognition of appropriate sets of part families (Goyal, Jain, and Jain 2013), integrated process planning and scheduling for RMS (Bensmaine, Dahane, and Benyoucef 2014), production planning and performance optimisation (Abbasi and Houshmand 2011). Variety-oriented design of machining systems used for batch production was considered by Battaïa et al. (2015). An overview of artificial intelligence applications to the optimal design of dedicated and RMS was presented by Renzi et al. (2014).

This paper develops a novel decision support tool assisting designers in the design of reconfigurable rotary machining systems for production of part families. This design problem is formulated as a combinatorial optimisation problem. Section 2 introduces the general statement of the problem and provides a mathematical model for variables, constraints and the objective function. Section 3 presents the mathematical model and the solution approach. An industrial example is considered in Section 4. Concluding remarks are given in Section 5.

2. Problem statement

2.1 *Definitions*

The machine to be designed is employed for machining several families of similar parts. No set-up is required for different parts from the same family (Agrawal et al. 2013). However, a reconfiguration of the system may be required between different families.

At the design step, it is assumed that there are $\aleph$ families of parts to be produced with required output O_υ, $\upsilon = 1,2, \ldots, \aleph$. At the end of each family, the machine is reconfigured for machining the next family, i.e. the fixtures of parts are changed and some spindles are mounted or dismounted if necessary. In total, there are d_0 different types of parts. The parts of υ-th family are loaded in sequence $\pi_\upsilon = (\pi_{\upsilon 1}, \pi_{\upsilon 2}, \ldots, \pi_{\upsilon \mu_\upsilon})$ where $\pi_{\upsilon j} \in \{0, 1, 2, \ldots, d_0\}$, $j = 1, 2, \ldots, \mu_\upsilon$, μ_υ is a multiple to $m_0 + 1$ and $\pi_{\upsilon j} = 0$ means that no part is loaded.

Using sequences π_υ, we can define in one-to-one manner function $\pi_\upsilon(i,k)$, $i = 1,\ldots,O_\upsilon\mu_\upsilon + m_0 - 1$, of part number on the k-th working position after i turns of the rotary table in the following way:

$$\pi_\upsilon(i,k) = \begin{cases} \pi_{\upsilon\psi(i-k+1)}, \text{ if } i-k+1>0 \text{ and } i \leq O_\upsilon\mu_\upsilon, \\ 0, \text{ if } i-k+1 \leq 0 \text{ or } i > O_\upsilon\mu_\upsilon \text{ and } k < i - O_\upsilon\mu_\upsilon + 1, \\ \pi_{\upsilon(\mu_\upsilon - k + i - O_\upsilon\mu_\upsilon + 1)}, \text{ otherwise,} \end{cases}$$

where $\psi(a) = \mu_\upsilon$ if a is multiple to μ_υ and $\psi(a) = \mathrm{mod}(a, \mu_\upsilon)$ otherwise.

The machine to be designed should perform the set of machining operations $\mathbf{N} = \bigcup_{d=1}^{d_0} \mathbf{N}^d$ where $\mathbf{N}^d$ is the set of machining operations that should be performed for processing the d-th part, $d = 1, 2, \ldots, d_0$. They are required for machining elements (holes, faces, etc.) located on n_d sides of the d-th part. The side is defined by direction axis of the machined elements.

Only one side of each part can be accessible for the vertical spindle head or turret. All other operations have to be performed by horizontal spindle heads or turrets.

Each part d has several possible orientations represented by a matrix $\mathbf{H}(d) = (h_{rs}(d))_{r=1,s=1}^{r_d\ n_d}$ where $h_{rs}(d)$ is equal j, $j = 1,2$ if the elements of the s-th side of the part d can be machined by spindle head or turret type j. The execution of each operation depends on the part's orientation, i.e. set $H(p)$ of feasible orientations of the part (indexes $r \in \{1, 2, \ldots, r_d\}$ of rows of matrix $\mathbf{H}(d)$) for execution of operation $p \in N_s^d$ by spindle head or turret of type j (vertical if $h_{rs}(d) = 1$ and horizontal if $h_{rs}(d) = 2$).

Each operation $p \in$ is also characterised by its working stroke length $\lambda(p)$ (i.e. the distance to be run by the tool in order to complete operation p) and the range of feasible values of feed rate $[\gamma_1(p), \gamma_2(p)]$ which sets the limits of the machining speed.

To sum up, the following assumptions are considered at the design step:

- The families of parts to be machined are defined by required machining operations and required output.
- The number of working positions is defined.
- The loading sequence of families of parts is given.
- The orientation of the parts cannot be changed at any working position.

2.2 *Decisions to be taken*

The goal of the design problem is to define the configuration of the machining system. More precisely, the designer has to define the following:

(1) the orientation of each part d
(2) the machining modules (horizontal or vertical, spindle head or turret) to be installed at each working position and their use for each part d
(3) the set of operations N_{dkjl} to be performed by each machining module l ($l = 1, \ldots, b_{kj}$) of vertical ($j = 1$) or horizontal ($j = 2$) type on each part d at working position k
(4) feed per minute Γ_{dkjl} associated with N_{dkjl}

These decisions can be modelled in the following way:

(1) Let subset N_k, $k = 1, \ldots, m$ contain the operations from set $\mathbf{N}$ assigned to the k-th working position.
(2) Let sets N_{k1} and N_{k2} be the sets of operations assigned to working position k that are realised by vertical and horizontal machining, respectively.

(3) Finally, let b_{kj} be the number of machining modules (not more than b_0) of type j (vertical if $j = 1$ or horizontal if $j = 2$) installed at the k-th working position and, respectively, subsets N_{kjl}, $l = 1, \ldots, b_{kj}$ contain the operations from set N_{kj} assigned to the same machining module.

Taking into account these definitions, let $P = \langle P_1, \ldots, P_k, \ldots, P_{m_0} \rangle$ be a design decision with $P_k = (P_{1k11}, P_{2k11}, \ldots, P_{d_0k11}, \ldots, P_{1k1b_{k1}}, P_{2k1b_{k1}}, \ldots, P_{d_0k1b_{k1}}, P_{1k21}, P_{2k21}, \ldots, P_{d_0k21}, \ldots, P_{1k2b_{k1}}, P_{2k2b_{k1}}, \ldots, P_{d_0k2b_{k1}})$, $P_{dkjl} = (N_{dkjl}, \Gamma_{dkjl})$, $P_{dkj} = (P_{dkjl} | l = 1, \ldots, b_{kj})$, $P_{dk} = (P_{dkj} | j = 1,2)$, and $\mathbf{N}_j = \bigcup_{d=1}^{d_0} \bigcup_{k=1}^{m_0} \bigcup_{l=1}^{b_{kj}} N_{dkjl}$, $j = 1, 2$.

This decision has to respect a number of technological constraints that are known in the literature as *precedence*, *inclusion* and *exclusion* constraints.

Precedence constraints are represented by a directed graph $G^{OR} = (\mathbf{N}, D^{OR})$: if an arc $(p,q) \in D^{OR}$, then operation p has to be executed before operation q. It should be noted that if such operations p and q belong to different sides of the part, then they cannot be executed at the same position without violating the precedence constraint.

Inclusion constraints are represented by undirected graphs $G^{SP} = (\mathbf{N}, E^{SP})$, $G^{ST} = (\mathbf{N}, E^{ST})$, $G^{SM} = (\mathbf{N}, E^{SM})$ and $G^{SS} = (\mathbf{N}, E^{SS})$. If there is an edge $(p,q) \in E^{SS}$ ($(p,q) \in E^{SM}$, $(p,q) \in E^{ST}$, $(p,q) \in E^{SP}$), then operations p and q must be executed by the same spindle (the same machining module, turret, at the same position).

Exclusion constraints are represented by undirected graphs $G^{DM} = (\mathbf{N}, E^{DM})$, $G^{DT} = (\mathbf{N}, E^{DT})$ and $G^{DP} = (\mathbf{N}, E^{DP})$. If there is an edge $(p,q) \in E^{DM}$ ($(p,q) \in E^{DT}$), $(p,q) \in E^{DP}$)), then operations p and q cannot be executed by the same machining module (turret, at the same position).

Based on matrices $\mathbf{H}(d)$, $d = 1, 2, \ldots, d_0$, we can build matrix $\mathbf{H}$ of dimension $\prod_{d=1}^{d_0} r_d \times \sum_{d=1}^{d_0} n_d$. It can be modified with relation to the inclusion constraints on turrets, machining modules and spindles, i.e. row r of $\mathbf{H}$ is deleted if $h_{rs'} \neq h_{rs''}$ for $p \in N_{s'}^{d'}$, $q \in N_{s''}^{d''}$ and $(p,q) \in E^{SS} \cup E^{SM} \cup E^{ST}$. Each row of $\mathbf{H}$ defines in one-to-one manner partition of $\mathbf{N}$ to $\mathbf{N}_1$ and $\mathbf{N}_2$. Then, the optimal solution of the initial problem can be found as the best partition of corresponding $\mathbf{N}_1$ and $\mathbf{N}_2$.

2.3 *Machining time calculation*

The execution time $t^b(P_{dkjl})$ of operations from N_{dkjl} with the feed per minute $\Gamma_{dkjl} \in [\max\{\gamma_1(p) | p \in N_{dkjl}\}, \min\{\gamma_2(p) | p \in N_{dkjl}\}]$ is equal to

$$t^b(P_{dkjl}) = L(N_{dkjl}) / \Gamma_{dkjl} + \tau^a$$

where $L(N_{dkjl}) = \max\{\lambda(\mathrm{p}) | p \in N_{dkjl}\}$, and τ^a is a constant presenting an additional time for advance and disengagement of tools.

We assume that if a turret of type j is installed at the k-th position, then the execution time of operations from N_{dkjl} is equal to

$t^h(P_{dkj}) = \tau^g b_{kj} + \sum_{l=1}^{b_{kj}} t^b(P_{dkjl})$, $j = 1, 2$,

where τ^g is an additional fixed time for one rotation of turret (Battaïa et al. 2014). If the spindle head is installed, then $t^h(P_{dkj}) = t^b(P_{dkj1})$, $j = 1,2$. If all N_{dkjl} are empty, then $t^h(P_{dkj}) = 0$.

The execution time $t^p(P_{dk})$ is defined as

$t^p(P_{dk}) = \tau^r + \max\{t^h(P_{dkj}) | j = 1,2\}$,

where τ^r is an additional constant time for table rotation.

Then, the time $T(P)$ for machining all the families of parts is equal to

$T(P) = \sum_{\upsilon=1}^{\aleph} \sum_{i=1}^{O_\upsilon \mu_\upsilon + m_0 - 1} \max\{t^p(P_{\pi_\upsilon(i,k)k}) | k = 1, \ldots, m_0\}$.

The required productivity is provided, if the total time $T(P)$ does not exceed the available time T_0.

2.4 *Objective function*

Let C_1, C_2, C_3 and C_4 be the relative costs for one position, one turret, one machining module of a turret and one spindle head, respectively. Since the vertical spindle head (if it is present) is common to several positions, its size (and therefore the cost) depends on the number of positions to be covered. Let $k_{\min}^h$ and $k_{\max}^h$ be the minimal and the maximal position of the common vertical spindle head. Then, its cost can be estimated as $C_4 + (k_{\max}^h - k_{\min}^h) C_5$ where C_5 is the relative cost for covering one additional position by the vertical spindle head. If a vertical spindle turret is installed, its cost can be estimated by $C_2 + C_3 b_{k1}$.

In a similar way, the cost $C(b_{k2})$ for performing set of operations N_{k2} by associated b_{k2} machining modules can be assessed as follows:

$$C(b_{k2}) = \begin{cases} 0 \text{ if } b_{k2} = 0, \\ C_4 \text{ if } b_{k2} = 1, \\ C_2 + C_3 b_{k2} \text{ if } b_{k2} > 1. \end{cases}$$

In the next section, we present mixed integer programming (MIP) formulation for the design problem with the objective being to minimise total equipment cost.

3. Solution approach

3.1 *MIP formulation*

Let us introduce the following notation:

X_{pkl} 1 if operation p is assigned to l-th machining module of spindle head or turret of type j ($j = 1$ if $p \in \mathbf{N}_1$ and $j = 2$ if $p \in \mathbf{N}_2$) at position k

Y^d_{kjl} 1 if at least one operation is executed for part d by l-th machining module of the spindle head or turret of type j at position k

Y^d_{kj} 1 if at least one operation is executed for part d by a spindle head or turret of type j at position k

Y_{kjl} 1 if the l-th machining module of spindle head or turret type j is installed at the k-th position

$Y_{1\min}$ k if k is the first position covered by vertical spindle head or turret ($Y_{1\min} = 0$ if $\mathbf{N}_1 = \emptyset$)

$Y_{1\max}$ k if k is the last position covered by vertical spindle head or turret ($Y_{1\max} = 0$ if $\mathbf{N}_1 = \emptyset$)

Z_k 1 if at least one operation is assigned to k-th position

The following auxiliary variables are used for determining the execution time:

F^d_{kjl} for part d by l-th machining module of spindle head or turret type j at k-th position

F^d_k for part d at k-th position

$F^{\upsilon i}$ total time of all positions when processing of part $\pi_{\upsilon i}$ of υ-th family is finished

$F^{\upsilon i}_s$ total time of first i positions after the i-th turn of the rotary table for processing υ-th family

$F^{\upsilon i}_f$ total time of last i positions after the $O_\upsilon \mu_\upsilon + m_0$-$i$-th turn of the rotary table for processing υ-th family

t_{pq} minimal time necessary for the execution of operations p and q in the same machining module, $t_{pq} = \max(\lambda(p), \lambda(q))/\min(\gamma_2(p),\gamma_2(q)) + \tau^a$

It is assumed that $(p,q) \in E^{DM}$ if $\min(\gamma_2(p),\gamma_2(q)) < \max(\gamma_1(p),\gamma_1(q))$.

Since the vertical spindle head has a common feed rate, it can be determined in advance if it is possible to install a common vertical spindle head for all machined parts. It cannot be installed if $\max\{\gamma_1(p)|p \in \mathbf{N}_1\} > \min\{\gamma_2(p)|p \in \mathbf{N}_1\}$. The vertical turret cannot be installed if there exist operations $p \in \mathbf{N}_1$ and $q \in \mathbf{N}_2$ such that $(p,q) \in E^{SP}$ or operations $p \in \mathbf{N}_1$ and $q \in \mathbf{N}_1$ such that $(p,q) \in E^{DT} \cup E^{DP}$. If both cases (for spindle head and turret) are identified, then the problem has no solution.

The objective function representing the total cost of all equipment can be expressed as follows:

$$\text{Min } C_1 \sum_{k=1}^{m_0} Z_k + C_4 \sum_{k=1}^{m_0} Y_{k21} + (C_2 + 2C_3 - C_4) \sum_{k=1}^{m_0} \sum_{j=1}^{2} Y_{kj2} + C_3 \sum_{k=1}^{m_0} \sum_{j=1}^{2} \sum_{l=3}^{b_0} Y_{kjl} + C_4 Y_1 + C_5 (Y_{1\max} - Y_{1\min}) \quad (1)$$

If the horizontal turret is installed at position k, then $Y_{k21} = Y_{k22} = 1$ and $C_4 Y_{k21} + (C_2 + 2C_3 - C_4) Y_{k22} = C_2 + 2C_3$. If a horizontal spindle head is installed at position k, then $Y_{k2l} = 0$, $l = 2,\dots,b_0$, and $C_4 Y_{k21} + (C_2 + 2C_3 - C_4) Y_{k22} = C_4$. If the vertical turret is installed at position k, then $Y_{k11} = Y_{k12} = 1$, $Y_1 = 1$, $Y_{1\min} = Y_{1\max}$ and $(C_2 + 2C_3 - C_4) Y_{k12} + C_4 Y_1 + C_5 (Y_{1\max} - Y_{1\min}) = C_2 + 2C_3$. If the vertical spindle head is common for positions $k_1 = Y_{1\min}, \dots, k_v = Y_{1\max}$, then $Y_1 = 1$, $Y_{k1l} = 0$, $l = 2,\dots,b_0$, $k = 1,\dots,m_0$ and $C_4 Y_1 + (C_2 + 2C_3 - C_4) \sum_{k=1}^{m_0} Y_{k12} = C_4$.

Variables Z_k, $k = 1, \dots, m_0$ have to satisfy the following constraints:

$$Z_k \leq Y_{k11} + Y_{k21};\ k = 1, \dots, m_0 \quad (2)$$

$$Y_{k11} + Y_{k21} \leq 2Z_k;\ k = 1, \dots, m_0 \quad (3)$$

If $\mathbf{N}_1 \neq \emptyset$, variables $Y_{1\min}$ and $Y_{1\max}$ can be defined by the following constraints:

$$(m_0 - k + 1) Y_{k11} + Y_{1\min} \leq m_0 + 1;\ k = 1, \dots, m_0 \quad (4)$$

$$Y_{1\max} \geq kY_{k11}; k = 1, \ldots, m_0 \tag{5}$$

The number of variables and constraints can be reduced using set **N'** instead of **N**. The set **N'** is built based on graph G^{SSM}=(**N**, $E^{SSM} = E^{SS} \cup E^{SM}$). Let $G_i^{SSM} = (N_i^{SSM}, E_i^{SSM})$, i = 1,…,n^{SSM}, be connectivity components of G^{SSM} including isolated vertices. Only one vertex (operation) $\wp_i$ is chosen from each N_i^{SSM} and included into **N'**. Later $\chi(p)$=$\wp_i$ for all $p \in N_i^{SSM}$.

Each operation is assigned to one machining module

$$\sum_{k=1}^{m_0} \sum_{l=1}^{b_0} X_{pkl} = 1; p \in \mathbf{N'} \tag{6}$$

Precedence constraints:

$$\sum_{p \in Pred(q)} \sum_{k'=1}^{k-1} \sum_{l'=1}^{b_0} X_{\chi(p)k'l'} + \sum_{p \in Pred(q) \cap \mathbf{N}_j} \sum_{l'=1}^{l-1} X_{\chi(p)kl'} \geq |Pred(q)| X_{\chi(q)kl}; q \in \mathbf{N}_j;\ j = 1, 2 \tag{7}$$

where $Pred(q)$={$p \in$ **N**|(p,q) $\in D^{OR}$}.

For operations p and q that have to be performed in the same working position and turret

$$\sum_{l=1}^{b_0} X_{\chi(p)kl} = \sum_{l=1}^{b_0} X_{\chi(q)kl}; (p, q) \in E^{SP} \cup E^{ST}\ ; k = 1, \ldots, m_0 \tag{8}$$

For operations p and q that have to be executed in different working positions

$$\sum_{l=1}^{b_0} X_{\chi(p)kl} + \sum_{l=1}^{b_0} X_{\chi(q)kl} \leq 1, (p, q) \in E^{DP}; k = 1, \ldots, m_0 \tag{9}$$

For operations p and q that have to be executed in different turrets, but can be executed by the same spindle head

$$\sum_{l=1}^{b_0} X_{\chi(p)kl} + \sum_{l=1}^{b_0} X_{\chi(q)kl} + Y_{kj2} \leq 2, (p, q) \in E^{DT}; p, q \in \mathbf{N}_j; k = 1, \ldots, m_0; j = 1, 2 \tag{10}$$

For operations p and q that have to be executed with different machining modules

$$X_{\chi(p)kl} + X_{\chi(q)kl} \leq 1; (p, q) \in E^{DB}; k = 1, \ldots, m_0; l = 1, \ldots, b_0 \tag{11}$$

The following constraints define Y_{kjl}^d, Y_{kj}^d, and Y_{kjl}. These decision variables take 1 if and only if the corresponding sums are not equal to 0.

$$Y_{kjl}^d \leq \sum_{p \in \mathbf{N}_j \bigcap \mathbf{N}^d} X_{\chi(p)kl}; d = 1, \ldots, d_0; k = 1, \ldots, m_0; j = 1, 2; l = 1, \ldots, b_0 \tag{12}$$

$$\sum_{p \in \mathbf{N}_j \bigcap \mathbf{N}^d} X_{\chi(p)kl} \leq |\mathbf{N}_j \bigcap \mathbf{N}^d| Y_{kjl}^d; d = 1, \ldots, d_0;\ k = 1, \ldots, m_0; j = 1, 2; l = 1, \ldots, b_0 \tag{13}$$

$$Y_{kjl} \leq \sum_{d=1}^{d_0} Y_{kjl}^d; k = 1, \ldots, m_0; j = 1, 2; l = 1, \ldots, b_0 \tag{14}$$

$$\sum_{d=1}^{d_0} Y_{kjl}^d \le d_0 Y_{kjl}; k = 1, \ldots, m_0; j = 1, 2; l = 1, \ldots, b_0 \tag{15}$$

$$Y_{kj}^d \le \sum_{l=1}^{b_0} Y_{kjl}^d; d = 1, \ldots, d_0; k = 1, \ldots, m_0; j = 1, 2 \tag{16}$$

$$\sum_{l=1}^{b_0} Y_{kjl}^d \le b_0 Y_{kj}^d; d = 1, \ldots, d_0; k = 1, \ldots, m_0; j = 1, 2 \tag{17}$$

Empty machining modules are not allowed:

$$Y_{kjl-1} \ge Y_{kjl}; k = 1, \ldots, m_0; j = 1, 2; l = 2, \ldots, b_0 \tag{18}$$

A vertical turret cannot be combined with a horizontal one:

$$Y_{k12} + Y_{k21} \le 1; k = 1, \ldots, m_0 \tag{19}$$

If the vertical turret cannot be installed then the following equations should be satisfied:

$$Y_{k11} = 0; k = 1, \ldots, m_0; l = 2, \ldots, b_0 \tag{20}$$

The following constraints (23–26) define the execution time variables as introduced here below:

$$F_{kjl}^d \ge t_{qq} X_{\chi(q)kl}; q \in \mathbf{N}^d \cap \mathbf{N}_j; j = 1, 2; d = 1, \ldots, d_0; k = 1, \ldots, m_0; l = 1, \ldots, b_0 \tag{21}$$

$$F_{kjl}^d \ge t_{pq}(X_{\chi(p)kl} + X_{\chi(q)kl} - 1); p, q \in \mathbf{N}^d \cap \mathbf{N}_j; j = 1, 2; d = 1, \ldots, d_0; k = 1, \ldots, m_0; l = 1, \ldots, b_0 \tag{22}$$

If a vertical spindle head can be installed ($\max\{\gamma_1(p)|p \in \mathbf{N}_1\} \le \min\{\gamma_2(p)|p \in \mathbf{N}_1\}$), then

$$F_{k11}^d \ge (\lambda(p)/\gamma_2(q) + \tau^a)(X_{\chi(p)k1} + X_{\chi(q)k'1} - 1); p, q \in \mathbf{N}^d \cap \mathbf{N}_1; d = 1, \ldots, d_0; k, k' = 1, \ldots, m_0; k \ne k' \tag{23}$$

Otherwise

$$Y_{k11} = Y_{k12}; k = 1, \ldots, m_0 \tag{24}$$

$$F_k^d \ge \sum_{l=1}^{b_0} F_{kjl}^d + 2\tau^g Y_{kj2} + \tau^g \sum_{l=3}^{b_0} Y_{kjl} + b_0 \tau^g (Y_{kj}^d - 1); d = 1, \ldots, d_0; k = 1, \ldots, m_0; j = 1, 2 \tag{25}$$

If a turret of type j with b_{kj} machining modules is installed at the k-th position, then $F_k^d \ge \sum_{l=1}^{b_{kj}} F_{kjl}^d + b_0\tau^g$ if at least one operation from $\mathbf{N}^d$ is executed by the turret and $F_k^d = 0$ otherwise. If a spindle head of type j is installed at the k-th position, then $F_k^d \ge F_{kj1}^d$.

The required productivity is provided if

$$F^{\upsilon i} \ge F_k^{\pi_\upsilon(i,k)} + \tau^r; \upsilon = 1, \ldots, \aleph; i = 1, \ldots, \mu_\upsilon; k = 1, \ldots, m_0 \tag{26}$$

$$F_s^{\upsilon i} \ge F_k^{\pi_\upsilon(i,k)} + \tau^r; \upsilon = 1, \ldots, \aleph; i = 1, \ldots, m_0 - 1; k = 1, \ldots, i \tag{27}$$

$$F_f^{\upsilon i} \ge F_k^{\pi_\upsilon(O_\upsilon \mu_\upsilon + m_0 - i, k)} + \tau^r; \upsilon = 1, \ldots, \aleph; i = 2, \ldots, m_0; k = i, \ldots, m_0 \tag{28}$$

$$\sum_{\upsilon=1}^{\aleph}\left(O_\upsilon\sum_{i=1}^{\mu_\upsilon}F^{\upsilon i}+\sum_{i=1}^{m_0-1}F_s^{\upsilon i}+\sum_{i=2}^{m_0}F_f^{\upsilon i}-\sum_{i=\mu_\upsilon-m_0+2}^{\mu_\upsilon}F^{\upsilon i}\right)\leq T_0 \tag{29}$$

$$X_{pkl}\in\{0,1\};p\in\mathbf{N}^{'};k=1,\ldots,m_0;l=1,\ldots,b_0 \tag{30}$$

$$Y_{kj}^d\in\{0,1\};k=1,\ldots,m_0;d=1,\ldots,d_0;j=1,2 \tag{31}$$

$$Y_{kjl}^d\in\{0,1\};k=1,\ldots,m_0;j=1,2;l=1,\ldots,b_0;d=1,\ldots,d_0 \tag{32}$$

$$Y_{kjl}\in\{0,1\};k=1,\ldots,m_0;j=1,2;l=1,\ldots,b_0 \tag{33}$$

$$Y_{1\min},Y_{1\max}\in\{0,1,\ldots,m_0\} \tag{34}$$

$$Z_k\in\{0,1\};k=1,\ldots,m_0 \tag{35}$$

$$F_{kjl}^d\in[0,\overline{t}_k^d-\tau^r];k=1,\ldots,m_0;j=1,2;l=1,\ldots,b_0;d=1,\ldots,d_0 \tag{36}$$

$$F_k^d\in[0,\overline{t}_k^d-\tau^r];k=1,\ldots,m_0;d=1,\ldots,d_0 \tag{37}$$

$$F^{\upsilon i}\in[\underline{t}^{\upsilon i},\overline{t}^{\upsilon i}];\upsilon=1,\ldots,\aleph;i=1,\ldots,\mu_\upsilon \tag{38}$$

$$F_s^{\upsilon i}\in[\max\{\underline{t}^{\pi_\upsilon(i,k)}|k=1,\ldots,i\},\max\{\overline{t}_k^{\pi_\upsilon(i,k)}|k=1,\ldots,i\}];\upsilon=1,\ldots,\aleph;i=1,\ldots,m_0-1 \tag{39}$$

$$F_f^{\upsilon i}\in[\max\{\underline{t}^{\pi_\upsilon(O_\upsilon\mu_\upsilon-i,k)}|k=i,\ldots,m_0\}\max\{\overline{t}_k^{\pi_\upsilon(O_\upsilon\mu_\upsilon-i,k)}|k=i,\ldots,m_0\}];\upsilon=1,\ldots,N;i=2,\ldots,m_0 \tag{40}$$

where $\underline{t}^d=\min\{\lambda(p)/\gamma_2(p)+\tau^a+\tau^r|p\in\mathbf{N}^d\}$, $\underline{t}^{\upsilon i}=\max\{\underline{t}^{\pi_\upsilon(m_0-2+i,k)}|k=1,\ldots,m_0\}$, $\overline{t}^{\upsilon i}=(T_0-O_{\upsilon'}\sum_{\upsilon'=1,\upsilon'\neq\upsilon}^{\aleph}\sum_{i'=1}^{\mu_{\upsilon'}}\underline{t}^{\upsilon' i'})/O_\upsilon-\sum_{i'=1,i'\neq i}^{\mu_\upsilon}\underline{t}^{\upsilon i'}$ and $\overline{t}_k^d=\max\{\overline{t}^{\upsilon i}|\upsilon=1,\ldots,\aleph,i=1,\ldots,\mu_\upsilon,\pi_\upsilon(m_0-2+i,k)=d\}$.

Since the considered problem is a generalisation of the design problem for a single product, the considered optimisation problem is also NP-hard. As a consequence, a heuristic approach is needed for large-scale instances.

3.2 *Heuristic approach*

The overall heuristic approach is based on comparing two design solutions which use a spindle head or a turret for vertical machining. The second one is obtained by finding the best partitions of $\mathbf{N}_1$ and $\mathbf{N}_2$ to vertical and horizontal machining modules separately and then combining these partitions appropriately.

Ten versions of the algorithm named sequential assignment of operations (SAO) are developed in order to assign the machining operation to the machining modules. At each iteration, the algorithm creates machining modules of the current position step by step. At the beginning, taking into account precedence and exclusion constraints on positions, list *In* of operations which are potentially assignable to a current machining module is created. Then, list *In* is modified in accordance with the inclusion constraints.

Then, one operation or several operations (if required by inclusion constraints) are chosen to be assigned to a current machining module. If it is not possible, a new machining module is created. After the operation assignment, list *In* is modified and the assignment process is repeated. If list *In* is empty or b_0 machining modules have been already created,

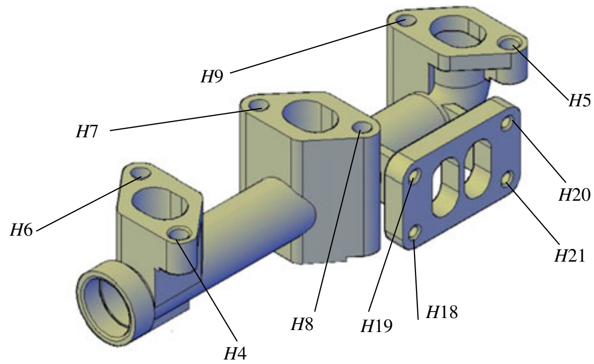

Figure 2. The first part to be machined.

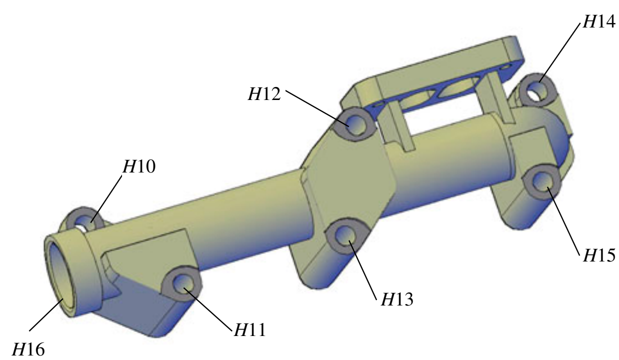

Figure 3. The second part to be machined.

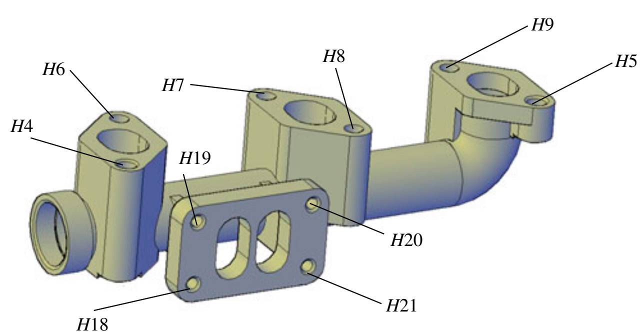

Figure 4. The third part to be machined.

the current position is closed and the productivity constraint is checked. If this constraint is not satisfied, the algorithm starts from the beginning (creation of the first position). The iteration is considered unsuccessful if after the creation of m_0 positions not all the operations from $\mathbf{N}$ have been assigned.

Let TR_{tot} be the current number of iterations, TR_{nimp} be the number of iterations without solution improvement, C be the cost of the current solution and $C_{\min}$ be the cost of the best solution. The following algorithm tries to assign operations from $\mathbf{N}_1$ to a vertical spindle head common for several positions and operations from $\mathbf{N}_2$ to horizontal spindle heads and turrets.

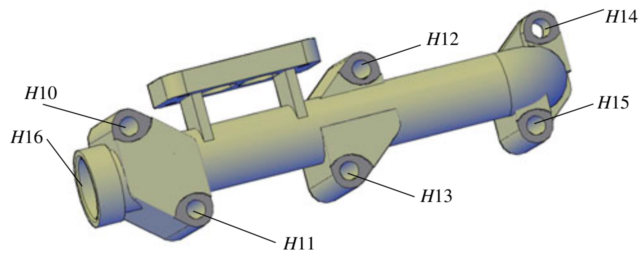

Figure 5. The fourth part to be machined.

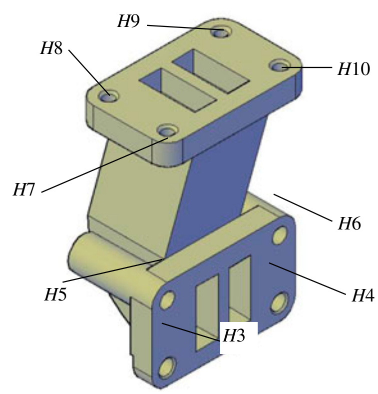

Figure 6. The fifth part to be machined.

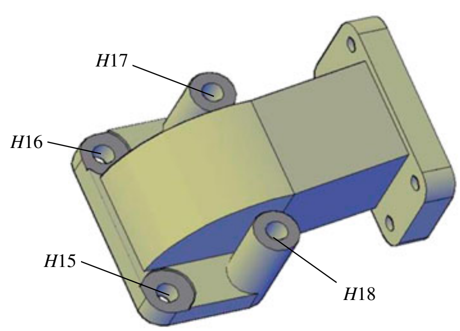

Figure 7. The sixth part to be machined.

Algorithm.

Step 1. Let $C_{\min} = \propto$, $TR_{tot} = 0$, $TR_{nimp} = 0$.

Step 2. Let $C = 0$, $N^a = \varnothing$, $m = 0$.

Step 3. Let $m = m + 1$. If $m > m_0$, then let $C = \propto$ and go to *Step* 12. Otherwise let $N_{m11} = N_{m21} = \varnothing$, $b_{m1} = b_{m2} = 0$, $N^{na} = \varnothing$.

Step 4. Put in list *In* all operations from $\mathbf{N} \backslash N^a$ without non-assigned predecessors. If list *In* is empty, then set $C = \propto$ and go to Step 12.

Step 5. Choose *op* from list *In*. Set $N = \{op\}$. Include into N all the operations which are linked with operation *op* by any inclusion constraints on position, turret, machining module or tool and all their predecessors. Save current state of b_{m2}, N_{m11} and $N_{m2\ l}$, $l = 1,..,b_{m2}$.

Table 1. Operations and their parameters.

p	Hole	Part	Side	$\lambda(p)$, mm	$\gamma_1(p)$, mm/min	$\gamma_2(p)$, mm/min	p	Hole	Part	Side	$\lambda(p)$, mm	$\gamma_1(p)$, mm/min	$\gamma_2(p)$, mm/min
1	H4	1	1	47	39.2	62.9	41	H9	3	1	77	22.8	81.3
2	H4	1	1	34	27.2	248	42	H9	3	1	75	44	86.5
3	H5	1	1	47	39.2	62.9	43	H18	3	2	29	24.6	83.6
4	H5	1	1	34	27.2	248	44	H18	3	2	9	28.3	106.3
5	H6	1	1	107	22.8	81.3	45	H19	3	2	29	24.6	83.6
6	H6	1	1	105	44	86.5	46	H19	3	2	9	28.3	106.3
7	H7	1	1	107	22.8	81.3	47	H20	3	2	29	24.6	83.6
8	H7	1	1	105	44	86.5	48	H20	3	2	9	28.3	106.3
9	H8	1	1	107	22.8	81.3	49	H21	3	2	29	24.6	83.6
10	H8	1	1	105	44	86.5	50	H21	3	2	9	28.3	106.3
11	H9	1	1	91	22.8	81.3	51	H16	4	1	35	50.2	170.1
12	H9	1	1	89	44	86.5	52	H16	4	1	19	31.9	197.1
13	H18	1	2	29	24.6	83.6	53	H16	4	1	19	26.9	161.6
14	H18	1	2	9	28.3	106.3	54	H16	4	1	18	26.7	160.2
15	H19	1	2	29	24.6	83.6	55	H10	4	2	7	35.2	105.6
16	H19	1	2	9	28.3	106.3	56	H11	4	2	7	35.2	105.6
17	H20	1	2	29	24.6	83.6	57	H12	4	2	7	35.2	105.6
18	H20	1	2	9	28.3	106.3	58	H13	4	2	7	35.2	105.6
19	H21	1	2	29	24.6	83.6	59	H14	4	2	7	35.2	105.6
20	H21	1	2	9	28.3	106.3	60	H15	4	2	6	35.2	105.6
21	H16	2	1	35	50.2	170.1	61	H3	5	1	34	37.7	63.4
22	H16	2	1	19	31.9	197.1	62	H3	5	1	22	27.8	249.5
23	H16	2	1	19	26.9	161.6	63	H4	5	1	34	37.7	63.4
24	H16	2	1	18	26.7	160.2	64	H4	5	1	22	27.8	249.5
25	H10	2	2	6	35.2	105.6	65	H5	5	1	72	22.8	81.3
26	H11	2	2	7	35.2	105.6	66	H5	5	1	70	48.7	91
27	H12	2	2	7	35.2	105.6	67	H6	5	1	72	22.8	81.3
28	H13	2	2	7	35.2	105.6	68	H6	5	1	70	48.7	91
29	H14	2	2	6	35.2	105.6	69	H7	5	2	24	24.6	83.6
30	H15	2	2	6	35.2	105.6	70	H7	5	2	9	28.3	106.3
31	H4	3	1	103	39.2	62.9	71	H8	5	2	24	24.6	83.6
32	H4	3	1	18	27.2	248	72	H8	5	2	9	28.3	106.3
33	H5	3	1	47	39.2	62.9	73	H9	5	2	24	24.6	83.6
34	H5	3	1	34	27.2	248	74	H9	5	2	9	28.3	106.3
35	H6	3	1	92	22.8	81.3	75	H10	5	2	24	24.6	83.6
36	H6	3	1	90	44	86.5	76	H10	5	2	9	28.3	106.3
37	H7	3	1	92	22.8	81.3	77	H15	6	1	5	42.7	128.2
38	H7	3	1	90	44	86.5	78	H16	6	1	5	42.7	128.2
39	H8	3	1	77	22.8	81.3	79	H17	6	1	5	42.7	128.2
40	H8	3	1	75	44	86.5	80	H18	6	1	5	42.7	128.2

Step 6. If set $N \cap \mathbf{N}_1 \cup N_{m11}$ cannot be assigned to the same machining module, then set $N^{na} = N^{na} \cup N$ and go to *Step* 9. Otherwise set $N_{m11} = N_{m11} \cup (N \cap \mathbf{N}_1)$.

Step 7. Divide set $N \cap \mathbf{N}_2$ into subsets N^{2i}, $i = 1, 2, \ldots, n_2$, which should be executed in one machining module or by the same tool. If set N^{2i} can be executed in one machining module with N_{m2l}, for some $l \in \{1,..,b_{m2}\}$, then let $N_{m2l} = N_{m2l} \cup N^{2i}$ and go to *Step* 8. If $b_{m2} = b_0$, then let $N^{na} = N^{na} \cup N$ and go to *Step* 9. Otherwise let $b_{m2} = b_{m2} + 1$ and $N_{m21} = N^{2i}$ for $l = b_{m2}$.

Step 8. Compute $T(P)$ for $N_{dkjl} = N_{kjl} \cap \mathbf{N}^d$ and $\Gamma_{dkjl} = [\min\{\gamma_2(p) | p \in N_{dkjl}\}$. If $T(P) > T_0$, then restore the saved state of b_{m2}, N_{m11} and N_{m2l}, $l = 1,..,b_{m2}$ as well as let $N^{na} = N^{na} \cup N$. Otherwise let $N^a = N^a \cup N$.

Step 9. Include in list *In* each operation *op* from $\mathbf{N} \backslash N^{na} \backslash N^a$ that satisfy precedence constraints for set N^a and exclusion constraints for set $\bigcup_{j=1}^{2} \bigcup_{l=1}^{b_{kj}} N_{mjl}$. If list *In* is not empty, then go to *Step* 5. Otherwise let $b_{m1} = 1$ if $N_{m11} \neq \varnothing$.

Step 10. If N^a does not include all the operations from $\mathbf{N}$, then go to *Step* 2.

Step 11. Compute $C = Q(P)$.

Step 12. If $C_{\min} > C$, then set $C_{\min} = C$, $TR_{nimp} = 0$ and keep the current solution as the best, set $TR_{nimp} = TR_{nimp} + 1$, otherwise.

Table 2. Precedence constraints.

Operation	Predecessors	Operation	Predecessors
2	1 3 31 33	40	5 7 9 11 35 37 39 41
4	1 3 31 33	42	9 11 39 41
6	5 7 9 35 37 39	44	43 45
8	5 7 9 35 37 39	46	43 45
10	5 7 9 11 35 37 39 41	48	47 49
12	9 11 39 41	50	47 49
14	13 15 17 19	52	21 51
16	13 15 17 19	53	22 52
18	13 15 17 19	54	23 53
20	13 15 17 19	62	61 63
22	21 51	64	61 63
23	22 52	66	65 67
24	23 53	68	65 67
32	1 3 31 33	70	69 71 73 75
34	1 3 31 33	72	69 71 73 75
36	5 7 9 35 37 39	74	69 71 73 75
38	5 7 9 35 37 39	76	69 71 73 75

Step 13. Set $TR_{tot} = TR_{tot} + 1$.
Step 14. Stop if one of the following conditions holds:

- a given solution time is exceeded;
- a given solution time is exceeded;
- TR_{tot} is greater than the maximum number of iterations authorised;
- TR_{nimp} is greater than a given value;
- $C_{\min}$ is lower than a given cost value.

Go to *Step* 2, otherwise.

This algorithm can also be applied for assigning operations from $\mathbf{N}_1$ to a vertical turret by employing in the algorithm $m_0 = 1$, $\mathbf{N}_2 = \mathbf{N}_1$ and $\mathbf{N}_1 = \varnothing$. Then, the obtained assignment should be combined with the assignment of $\mathbf{N}_2$ by checking precedence and productivity constraints.

If there are several operations in list *In* at *Step* 5, operation *op* can be chosen in different ways. In this paper, ten of them are tested and compared.

SAO1 selects any *op*;
SAO2 selects *op* with inclusion constraints;
SAO3 selects *op* with the maximal number of successors;
SAO4 selects *op* with the minimal number of successors;
SAO5 selects *op* with the maximal number of operations not to be executed in one machining module;
SAO6 selects *op* with the minimal number of operations not to be executed in one machining module;
SAO7 selects *op* with the maximal execution time;
SAO8 selects *op* with the minimal execution time;
SAO9 selects randomly one of rules 1–8;
SAO10 selects randomly one of rules 1, 2, 3, 5, 7.

If there is still a tie, then one of the equally ranked candidates is chosen at random. In the next section, an industrial example is considered.

4. An industrial example

The following 6 parts are to be machined (Figure 2–6). Elements of the first five parts are located on two sides, and elements of the sixth part are located on one side. Parameters of operations are presented in Table 1. The available time

Table 3. Incompatibility of operations in machining modules.

Operation	Incompatible operations	Operation	Incompatible operations	Operation	Incompatible operations	Operation	Incompatible operations
2	1	23	21 22	43	1–12 31–42	63	25–30 55–60
4	3	24	21–23	44	1–12 31–42 43	64	63
6	5	25–30	1 3 21–24	45	1–12 31–42	66	65
8	7	31	2 13–20 25–30	46	1–12 31–42 45	68	67
10	9	32	1 13–20 31	47	1–12 31–42	69	61–68
12	11	33	4 13–20 25–30	48	1–12 31–42 47	70	61–69
13	1–12	34	3 13–20 33	49	1–12 31–42	71	61–68
14	1–13	35	6 13–20	50	1–12 31–42 49	72	61–68 71
15	1–12	36	5 13–20 35	51	22–30	73	61–68
16	1–12 15	37	8 13–20	52	21 23–30 51	74	61–68 73
17	1–12	38	7 13–20 37	53	21 22 24–30 51 52	75	61–68
18	1–12 17	39	10 13–20	54	21–23 25–30 51–53	76	61–68 75
19	1–12	40	9 13–20 39	55–60	1 3 21–24 31 33 51–54	77 78	1 3 13 15 17 19 31 33 43 45 47 49 61 63 = **
20	1–12 19	41	12–20	61	25–30 55–60	79	**, 77
22	21	42	11 13–20 41	62	61	80	**, 78

Table 4. Incompatibility of operations in turrets.

Operations	Incompatible operations
13–20	1–12
25–30	21–24
31–42	13–20
43–50	1–12 31–42
51–54	25–30
55–60	21–24 51–54
69–76	61–68
77–80	21 51

Table 5. Incompatibility of operations in working positions.

Operations	Incompatible operations
21	1–20
25–30	1 3 21
31	21–30
32	21
33	21–30
34–50	21
51	1–20 25–50
55–60	1 3 21 31 33 51
61 63	25–30 55–60
69–76	21 51
77–80	1 3 13 15 17 19 31 33 43 45 47 49 61 63

Table 6. Operations to be assigned to the same machining module.

Operation	Operations to be in the same machining module	Operation	Operations to be in the same machining module
1	3	40	42
5	7 9	43	45
6	8 10	47	49
13	15 17 19	61	63
35	37	65	67
36	38	66	68
39	41	69	71 73 75

T_0 is 288 min. The first family consists of the first 4 parts with output $O_1 = 16$ and the loading sequence {1, 2, 3, 4}. The second family includes 5-th and 6-th parts with output $O_2 = 12$ and the loading sequence {5, 5, 6, 6}. Other parameters are $\tau^a = \tau^g = \tau^r = 0.1$ min, $C_1 = 10$, $C_2 = 5$, $C_3 = 2$, $C_4 = 4$, $C_5 = 2$. The possible orientations of the parts are the following: $\mathbf{H}(1) = \mathbf{H}(2) = \mathbf{H}(3) = \mathbf{H}(4) = \mathbf{H}(5) = \begin{pmatrix} 1,2 \\ 2,1 \end{pmatrix}$, $\mathbf{H}(6) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. The total number of feasible orientations of all parts is $64 = 2^6$.

Precedence constraints, exclusion constraints for machining modules, turrets and working positions are presented in Tables 2–5, respectively. Inclusion constraints for machining modules are given in Table 6. Operations to be executed by the same spindle are presented in Table 7.

The total number of feasible orientations of all the parts was reduced to 16 due to the inclusion constraints (Table 7). The academic version of solver CPLEX 12.2 was used to solve the corresponding problems (1)–(41) for each combination

Table 7. Operations to be executed by the same spindle.

Operation	Operations to be executed by the same spindle	Operation	Operations to be executed by the same spindle
1	31	12	42
2	32	21	51
3	33	22	52
4	34	23	53
5	35	24	54
6	36	25	55
7	37	26	56
8	38	27	57
9	39	28	58
10	40	29	59
11	41	30	60

Table 8. An optimal solution.

Set N_{dkjl}	Operations of N_{dkjl}	$L(N_{dkjl})$	γ_{dkjl}	$t^b(P_{dkjl})$
N_{1111}	13 15 17 19	29	83.6	0.45
N_{3111}	43 45 47 49	29	83.6	0.45
N_{5111}	69 71 73 75	24	83.6	0.39
N_{1121}	1 3 5 7 9 11	107	62.9	1.8
N_{3121}	31 33 35 37 39 41	103	62.9	1.74
N_{5121}	61 63	34	63.4	0.64
N_{1122}	2 4 6 8 10 12	105	86.5	1.31
N_{3122}	32 34 36 38 40 42	90	86.5	1.14
N_{6211}	79 80	5	83.6	0.16
N_{2221}	21	35	170.1	0.36
N_{4221}	51	35	170.1	0.36
N_{5221}	65 67	72	81.3	0.99
N_{2222}	22	19	197.1	0.2
N_{4222}	52	19	197.1	0.2
N_{2223}	23	19	161.6	0.22
N_{4223}	53	19	161.6	0.22
N_{5223}	62	22	249.5	0.19
N_{2224}	24	18	160.2	0.21
N_{4224}	54	18	160.2	0.21
N_{5224}	64 66 68	70	91	0.87
N_{1311}	14 16 18 20	9	83.6	0.21
N_{2311}	25 26 27 28 29 30	7	83.6	0.18
N_{3311}	44 46 48 50	9	83.6	0.21
N_{4311}	55 56 57 58 59 60	7	83.6	0.18
N_{5311}	70 72 74 76	9	83.6	0.21
N_{6311}	77 78	5	83.6	0.16

Table 9. Characteristics of the solution.

Position k	$t^p(P_{1k})$	$t^p(P_{2k})$	$t^p(P_{3k})$	$t^p(P_{4k})$	$t^p(P_{5k})$	$t^p(P_{6k})$
1	3.41	0.1	3.18	0.1	0.94	0.1
2	0.1	1.49	0.1	1.49	2.54	0.26
3	0.31	0.28	0.31	0.28	0.31	0.26

Table 10. Parameters of problems.

Parameters of problems	|**N**|	OSP	DM	DT	DP	SS	SM	$m0+1$	LS	NF
Parameters of problems for 4 parts										
Minimal value	44	0.034	0.064	0.026	0	0.027	0	4	8	2
Maximal value	95	0.525	0.659	0.659	0.242	0.051	0.016	8	16	2
Average value	69	0.106	0.373	0.348	0.024	0.036	0.004	6	12	2
Parameters of problems for 6 parts										
Minimal value	89	0.029	0.003	0.002	0	0.024	0	3	6	2
Maximal value	159	0.471	0.462	0.462	0.205	0.031	0.057	9	18	2
Average value	124	0.29	0.228	0.197	0.027	0.027	0.016	6	12	2
Parameters of problems for 8 parts										
Minimal value	118	0.023	0.003	0.002	0	0.024	0	3	8	2
Maximal value	216	0.456	0.438	0.417	0.214	0.033	0.057	10	20	2
Average value	165	0.288	0.197	0.168	0.025	0.028	0.017	6	12	2
Parameters of problems for 10 parts										
Minimal value	251	0.023	0.025	0.02	0	0.014	0	4	12	2
Maximal value	255	0.062	0.58	0.588	0.194	0.026	0.005	9	27	3
Average value	254	0.04	0.326	0.3	0.031	0.019	0.001	7	18	2.5

Table 11. Comparison of the results obtained with CPLEX and SAO1-SAO10.

	METH	NSOL	NOPT	AVT	AVED	MAXD
Four parts	SAO1	100	53	12.9	2.55	12.50
	SAO10	100	53	13.8	2.55	12.50
	CPLEX	100	100	50.4	0.00	0.00
Six parts	SAO1	100	27	26.3	5.38	19.44
	SAO9	100	27	29.8	5.37	19.44
	SAO10	100	27	27.3	5.29	19.44
	CPLEX	100	95	600.1	0.00	0.00
Eight parts	SAO1	100	20	10.9	5.90	18.52
	SAO9	100	20	71.0	6.03	18.52
	SAO10	99	20	43.5	5.93	18.52
	CPLEX	97	77	1285	0.01	1.12
10 parts	SAO1	100	21	98.9	2.48	11.34
	SAO9	100	21	96.6	2.63	11.76
	SAO10	100	21	125.4	2.48	11.34
	CPLEX	74	54	1564	1.00	17.71

of part orientations, but only one combination of part orientations resulted in a feasible system configuration. The obtained results are presented in Table 8. This solution was found in 0.56 s. The unfeasibility of 14 problems was discovered in 0.33 s on average. However, for one problem, 1.2 s was necessary to prove the unfeasibility of the problem. The total solution time was 6.5 s. The number of variables in MIP models was equal to 864.

The obtained optimal solution and its characteristics are presented in Tables 8 and 9. The vertical spindle head is common for positions 1, 2 and 3. Parts 1, 3, 5 are machined at position 1, part 6 is machined at position 2, and all the parts are machined at position 3. At position 1, there is a horizontal turret with 2 machining units (the first one is used for parts 1, 3, 5 and the second one is used for parts 1, 3). At position 2, there is a horizontal turret with 4 machining units which are used for machining parts 2, 4 and 5. The total time $T(P)$ is equal to $16(3.18 + 0.28 + 3.41 + 0.28) + (3.41 + 0.1) + (0.28 + 3.18) - (3.41 + 0.28) + 12(2.54 + 0.31 + 0.94 + 2.54) + (0.94 + 2.54) + (0.94 + 0.26) - (0.94 + 2.54) = 194.84$ min.

5. Experimental study

The purpose of this study was to evaluate the effectiveness of the proposed techniques. There were generated series of 100 test instances for batches of 4, 6, 8 and 10 parts. Their characteristics are presented in Table 10, where |**N**| is the number of operations, OSP is the order strength of precedence constraints, DM, DT, DP, SS and SM are the densities of graphs G^{DM}, G^{DT}, G^{DP}, G^{SS} and G^{SM}, respectively, LS is the sum of lengths of loading sequences, and NF is the number of families. Constraints were generated using tools developed previously (Battaïa et al. 2012). Experiments were carried out on ASUS notebook (1.86 Ghz, 4 Gb RAM) with academic version of CPLEX 12.2.

In Table 11, we compare results for CPLEX12.2 (maximal solution time 3600 s) with SAO1 – SAO8 for $TR_{nimp} = 500$, $C_{\min} = 0$. TR_{tot} was set to 1000 for SAO1 and SAO2 and 200 for SAO3-SAO8. Only the best heuristic results are provided in Table 11. In this table, NSOL is the number of problems with a founded feasible solution, NOPT is the number of problems with proven optimality, AVT is the average solution time (in sec), AVED and MAXD are average and maximal deviations (in percents) of the found value of the objective function from the best known, respectively. Minimal deviation was 0 for all instances.

These results show that the CPLEX solutions remain time-efficient for the problems with up to six different parts. For the problems with more parts to be produced, the heuristics can be used in the cases where CPLEX does not provide optimal solution or any solution at all. It can be noted that MAXD is superior for CPLEX solutions (a feasible solution found but not optimal) than for heuristic solutions. As a conclusion, both developed approaches are useful in practice to treat different industrial cases.

6. Conclusion

The use of reconfigurable machining equipment can be an efficient response to increasing global competition and unpredictable market changes. Due to the physical structure that can be easily changed, the machining configuration can be optimised for each particular part family. The use of the optimisation methods at this stage helps to reduce the total design time and to promptly discard unfeasible solutions. This paper proposed a decision support approach for the design of reconfigurable rotary machining systems with turrets used for producing several families of parts. The complex design constraints such as compatibility and productivity requirements as well as design objectives were modelled within a mixed integer programme. The model allows taking efficient decisions about part orientations, selection of machining modules and configuration/reconfiguration of working positions depending on the part families to be produced. The approach was validated on industrial case studies, and one of these industrial examples was illustrated in the paper. The conducted study showed that the solution time to find the best cost-efficient machine configuration respecting all given constraints remains acceptable for the machine designers. Further development will concern the design of reconfigurable machining lines consisting several reconfigurable machines. In order to evaluate the dynamic behaviour of the system, further studies should be conducted in order to develop appropriate simulation models and the integration scheme to combine optimisation and simulation techniques in an efficient design scheme.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by the Centre National de la Recherche Scientifique [PICS France-Belarus grant].

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