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CHAPTER 18

Applications of Matching Models under Preferences

Péter Biró

18.1 Introduction

Matching problems under preferences have been studied widely in mathematics, computer science and economics, starting with the seminal paper by Gale and Shapley (1962). A comprehensive survey on this topic was published also in Chapter 14 of the Handbook of Computational Social Choice (Klaus et al., 2016), and for the interested reader we recommend consulting the following four comprehensive books on the computational (Gusfield and Irving, 1989; Manlove, 2013) and game-theoretical, market design aspects (Roth and Sotomayor, 1990; Roth, 2015) of this topic. In this chapter our goal is to give a general overview of the related applications.

The theory and practice of mechanism design for matching problems have always developed in an interactive way. Practical applications have motivated scientific research and theoretical results have been used for designing new applications and redesigning old ones. The college admission problem was first described and resolved by Gale and Shapley (1962). Their definition of *stable matching* proved to be the most crucial solution concept for two-sided markets. Furthermore, their algorithmic solution, the so-called *deferred acceptance* (DA) mechanism, has been used in many important applications, such as the centrally coordinated schemes for college admissions in Europe (Hungary, Spain, Turkey, Ireland, etc.), school choice programmes (in many US cities, Amsterdam, or Hungary) and for medical resident allocation (US, Japan, UK, etc.). But in fact, in some applications this matching mechanism has been invented and developed independently from the theoretical research. This is certainly the case for the US resident allocation program, NRMP, where the DA algorithm was already implemented in 1952 (Roth, 1984). Moreover, the special features of these applications have been constantly creating new theoretical problems. For an illustrative example we briefly describe the issue of couples in resident allocation problems.

As the proportion of women has increased over time in the medical field, so has the number of married couples among the residents. Most of these couples decided not to participate in the US centralised clearinghouse in the 1970s and

1980s, because they were afraid to get allocated to places far away from each other, which could ruin their partnership during the two-year internship period. So they were looking for pairs of positions outside of NRMP, which started to undermine the efficiency of the mechanism. Therefore in the 1980s the coordinators allowed the couples to submit joint applications within the programme. However, they treated these applications in a special way, asking the couples to nominate a leading person from the couple and only after allocating him/her they tried to find a place for the spouse. This was not satisfactory for the couples, and they kept staying out of the clearinghouse.

The scientific community working in this field took notice of this issue and started to investigate it. First, game theorists showed that if we were to allow the couples to submit joint preferences, which seemed to be inevitable, the existence of a stable matching is not guaranteed anymore (Roth, 1984). Later, the computer science community provided the further bad news that checking the existence of a stable solution is NP-complete (Ronn, 1990). Yet, the coordinators of NRMP decided to allow the couples to submit arbitrary joint preferences and asked two experts to redesign the matching mechanism. Roth and Peranson took an engineering approach and have succeeded to construct a new heuristic mechanism, based on the DA, that could deal with the issue of couples in the NRMP (Roth and Peranson, 1999). This algorithm has been used in many similar applications since (Roth, 2008). In the last two decades both economists and computer scientists have investigated the possible reasons for the tractability of this special feature in real applications, a survey on which was written by Biró and Klijn (2013).

In this chapter we try to give a comprehensive description on the important applications of matching models under preferences without monetary transfers. We focus on applications where central coordination has already been established, but we also mention applications which are not (yet) centrally coordinated, since central coordination can emerge in the future and matching models can also be useful to analyse the decentralised matching processes. We highlight the most relevant special features of the applications and we give pointers to the corresponding theoretical literature, related models and solutions concepts.

Personally, I always found it crucial to understand how the applications are working and what are the main issues to be resolved. Together with colleagues at University of Glasgow we started to collect descriptions of existing applications with references, and we continued this project from 2010 in Budapest (Website on matching practices at Hungarian Academy of Sciences, 2012). In 2010 the Matching in Practice research network has been established in Europe and we also started to collect descriptions of college admission, school choice and early labour allocation practices (Website of the Matching in Practice network, 2014). Finally, in a new COST Action project we are now trying to understand and describe how kidney exchange programmes are operating in Europe (Website of COST Action: European Network for Collaboration on Kidney Exchange Programmes, 2016).

There are also examples from outside of Europe for using a scientific approach to systematically review matching applications and to give policy design advice on their (re-)design. Al Roth has been leading this initiative with his informative blog (Website of Al Roth's blog, 2010), and his new book (Roth, 2015) by giving a comprehensive high level overview on market design together with many real examples. One particularly successful organisation which has emerged also through the work of Roth and his colleagues is the Institute for Innovation in Public School Choice (Website of Institute for Innovation in Public School Choice, 2014), which helps to advise US cities on their school choice practices.

The design of practical applications and the scientific research in multiple disciplines are developing together, and that is what is needed to obtain successful solutions. On the one hand, these successes validate the theoretical findings and boost further research in the newly emerging interdisciplinary fields of Computational Social Choice and Market Design, among others. On the other hand, the scientific results will ultimately lead to better mechanisms that provide fairness and optimality in a transparent and efficient way.

In Section 18.2 we start with the description of classical two-sided matching markets, such as college admission, school choice and resident allocation. We continue with two-sided markets where only one side has preferences in Section 18.3. In Section 18.4 we describe applications based on exchange models. Finally, we consider some further models in Section 18.5 and we study the related applications.

To publish papers in economics or computer science journals, theoretical results are strongly expected, so one cannot just publish a paper merely by describing applications. Indeed, most of the scientific papers cited in this chapter that are motivated by some applications typically focus only on one or two special features of the applications. Therefore, it is hard to judge how relevant these features are and whether there are some further crucial characteristics of the applications which are not mentioned or studied in the scientific papers referenced. So, whenever my descriptions are entirely based on scientific papers the reader must be warned that there may be some missing information. However, I can be more confident in the completeness of my notes when they are based on a description written by an expert and published, e.g., at the Matching in Practice website, or when the knowledge is mostly coming from my personal experience and research. To highlight the latter cases I will use framed environments whenever I had some personal information on the application.

Finally, we note that there are two closely related chapters in this book. In Chapter 15, Mattei and Walsh describe Preflib.org, a data library that also includes some data of matching applications. Cechlárová gives an overview on the theory and practice of trainee teachers in Slovakia in Chapter 19.

18.2 Two-Sided Markets

Stable matchings in two-sided markets were first studied by Gale and Shapley (1962) with title "College admissions and the stability of marriage". Their solution was based on the concept of *stability*. A matching is stable if there is no pair of agents who would be mutually better off by forming a new pair, after abandoning their previous partners. For college admissions the meaning of stability is that an applicant can only be rejected from a college if the college filled its quota with better applicants. This property is easy to check if the rankings by the colleges are based on scores and the so-called cutoff scores are published, which can be the scores of the weakest admitted applicants at the saturated colleges. Such cutoff systems are in use in many European countries, including Hungary, Ireland, Spain and Turkey.

Gale and Shapley gave an efficient algorithm for computing an applicant-optimal stable matching with the DA mechanism. This mechanism can be implemented in such a way that its running time is linear in the number of applications. Moreover, the mechanism is strategy-proof for the students. However, this mechanism is not Pareto optimal for the students, if we consider the set of all possible assignments, and not just the stable ones. The Top Trading Cycles algorithm used for two-sided markets achieves Pareto efficiency, it is strategy-proof, but fails to ensure stability. Finally, in many applications the so-called immediate acceptance (IA) or Boston mechanism is used, where the significant difference compared to DA is that the acceptances are final, so a college does not consider any new applications after its quota becomes filled. This mechanism is highly manipulable, and does not provide stable solutions, but for some special instances it can improve *ex ante* efficiency compared to DA. Nevertheless this first choice first priority system was banned in the UK for use in school choice (Pathak and Sönmez, 2013). See references and further theoretical findings in Section 1 of the chapter by Klaus et al. (2016).

To the best of our knowledge the first usage of the Gale-Shapley algorithm in a centralised matching programme was not for college admissions, but for resident allocation in the US as early as in 1952. Therefore we start our description of applications with resident allocation programmes and early labour applications in general.

Early Labour Markets

Allocating graduates to their first positions in a centrally coordinated way is typical in professions where internships are necessary for learning the practices and getting the licences. Therefore, most of these centralised intern allocation programmes have been established in the medical field, but there are also examples in law studies and army related professions. However, our first, very early example will be related to the famous engineering school of France that has been providing an elite education for the most talented students and a constant resource of trained professionals for the French ministries.

Sorting graduates at École polytechnique in 1806. This institution was founded in 1794, reorganised and named École polytechnique in 1805 under Napoleon, and it is still in operation. The students have always been selected by a highly competitive entrance exam, and after graduation they were sorted to ministries through a coordinated mechanism, called *classement sortie*. After the first three years of a decentralised system (1797-1799) a common final exam and sorting mechanism was established. Between 1799 and 1806 the students were asked to declare one service of their choice targeted at the beginning of their studies, however, the number of openings at the ministries varied, so many students had to find a different service in an after match period from what they had declared. To resolve this issue, the council approved a new system where the students could declare two choices, but at the end they modified this to a much simpler mechanism. In the implemented mechanism the students were commonly ranked in a master list, and they could choose their services one by one, without making any declaration beforehand. Thus, in mechanism design terms, the one-application system was replaced by a serial dictatorship mechanism based on a common ranking. See more details in Belhoste (2003).

National Resident Matching Program. The allocation of junior doctors to hospitals in the US used to work in a decentralised way until 1950 with various problems, as described in Roth (1984). In 1950 the market participants agreed to establish a central coordination, where the doctors submit their preferences, the hospitals submit their rankings and an algorithm computes the allocation. After the trial run in the academic year 1950/51 (which was not used in the real allocation) some students complained about the fairness of the mechanism, and so they agreed to adjust the mechanism, which was implemented in 1951 and was first used in 1952. This new algorithm was mathematically equivalent to the hospital-proposing Gale-Shapley algorithm (Roth, 1984; Gale and Sotomayor, 1985). In the early 1990s the trust in the central allocation system started to erode, party because of the spreading knowledge over the hospital-optimality (and thus doctor-pessimality and manipulability) of the mechanism and also because of the unsatisfactory way it treated the couples. Thus, the board of NRMP decided to redesign the mechanism on a scientific base, led by Roth and Peranson, as documented by Roth and Peranson (1999). The new algorithm, approved in 1997 and first used in 1998, switched to the applicant-proposing variant (although one of the main findings of Roth and Peranson was that the solutions produced by the two variants were not significantly different, suggesting that in large markets the set of stable solutions may be small in expectation). They also constructed a new heuristic algorithm, based on the Roth-Vande Vate process, to deal with the couples, and also to satisfy some further requirements. The algorithm has remained in operation since, and was subsequently used in many similar applications run by the National Matching Services Inc. (a private company based in Canada, founded by Peranson), see the details and the list of three dozen particular programmes by Roth (2008).

Resident allocation programmes in the UK. In the 1960s the British National Health Service recognised the issues with their decentralised resident allocation processes and thus recommended the usage of centralised schemes. However, the allocations were organised in regions separately, and NHS did not specify the allocation mechanism to be used, so almost every region adopted a different method (Roth, 1990, 1991). An interesting feature of the UK schemes was that the doctors had to apply for a pair of positions, a six-month medical and a six-month surgical post, and they could also indicate their preferences over the order they wanted to take these positions. Yet, if we assume responsive preferences for the doctors (i.e., the preference over two surgical positions is not influenced by the medical position assigned, and vica versa), then stable solutions can be obtained separately for the two kinds of positions leading to pairwise stable solutions for the many-to-two markets. However, if the allocation of either of the two positions fails to be stable then the overall solution is also bound to be blocked by a hospital-doctor pair. Roth has studied the allocation mechanisms of ten regions, and he found that four of them produced stable solutions and all four of them remained in use at the time of writing. However, from those six that could produce unstable solutions only two remained in operation, the other four were abandoned. Therefore this natural experiment provided strong evidence for the importance of stability in two-sided markets.

Scottish Foundation Allocation Scheme. The Scottish scheme was similar in characteristic to the above described UK schemes until 2005, when the programme was named SPA (Scottish PRHO Allocations). The solutions were computed from 2000 by Rob Irving with a sophisticated algorithm which not only provided stable solution for both medical and surgical positions separately, but also satisfied as many seasonal preferences as possible by a flow algorithm (Irving, 1998). In 2006 the programme was renamed SFAS (Scottish Foundation Allocation Scheme) and in the new scheme some characteristics also changed. Instead of the medical and surgical positions the doctors had to apply for a 2-year Foundation Programme that they conducted at one hospital. The hospitals' rankings were not provided independently, but they were deduced from a common scoring method and these also involved ties. The primary goal of the mechanism was to return a (weakly) stable matching of maximum size, which is an NP-hard problem even for master lists (Irving et al., 2008). The computation had been done by a sophisticated heuristic (Irving and Manlove, 2009). From 2009 the programme has also allowed the joint applications of couples, although in a slightly restricted form, so the matching algorithm had to be redesigned accordingly (Biró et al., 2011). In 2012 the Foundation Programme allocation in Scotland started to be handled by the UK Foundation Office, and unfortunately we have no information about the exact mechanism that they use.

Japanese Resident Matching Program. A new allocation system was introduced in 2009 to make the distribution of the residents more balanced in Japan (Kamada and Kojima, 2012). Regional caps have been imposed to reduce the number of doctors allocated in large cities and increase their numbers in rural areas. However, the implementation of the regional caps was strongly criticised from a market design perspective, see, e.g., Kamada and Kojima (2016, 2017), as the ministry introduced artificial quotas for all the hospitals (summing up to the regional quota in each region) which may lead to inefficient solutions with regard to the hard regional constraints.

Teacher allocation in France. Since 1999 the centralised (re-)allocation scheme is organised in two-rounds involving newly tenured teachers and also those teachers who want to change positions. In the first round they allocate the teachers to regions, and in the second round they do the (re-)allocation within each region separately. The priorities of the teachers for positions are based primarily whether they are moving to the region where their spouses are living and then by seniority. The matching mechanism is a combination of the school-proposing DA and the stable improvement cycles algorithm by Erdil and Ergin (2008). See more details by Terrier (2014) and a mechanism design analyses by Combe et al. (2015).

Allocation of lawyers in Germany. Every year more than 8000 graduating lawyers are assigned to legal internship positions, organised separately in each region (Bundesland). At the beginning of the process the set of admitted candidates is determined, where the majority are selected by their First State Exam grades, but some quotas are preserved for applicants with high waiting times and also with low social-economic status. The mechanism used in Berlin is conjectured to be the lawyer-oriented DA. A critical analyses of the mechanism is given by Dimakopoulos and Heller (2015).

Economics job market in the US. This is a partly centralised system, as the application platform is common and the majority of the interviews take place at the annual Allied Social Science Associations (ASSA) meeting, based on which the employers send out the flyouts. An interesting feature is that each candidate can send two signals to the employers, which help them to select the twenty applicants for interview from the hundreds of candidates (Coles et al., 2013). The final offer-rejection part of the process is decentralised.

Allocation in the US Navy. According to Short (2000), Robards (2001) and Yang et al. (2003) more than 300,000 personnel are (re-)assigned with the help of around 300 detailers in every period. Besides the strict eligibility criteria, the presence of couples is also an important feature recognised. Finally, unlike in the resident allocation, all the sailors have to be allocated and also some critical Navy billets must be filled. The latter requirement alone may prevent the existence of a stable matching.

Assigning positions in humanitarian organisations. Similar problems have been identified in the internal assignment process of humanitarian organisations (Soldner, 2014). Here also the goal is to find a complete matching, but instead of using orders, as in the army, they try to modify the preferences of both the staff and the job holders by costly negotiations. The problem of finding a complete stable matching for adjusted preferences with minimum negotiation costs is solved by heuristics and integer programming techniques.

Finally, note that we moved the description of the **resident allocation programme of Israel** to subsection 18.3, since the usage of a single item allocation model is more appropriate for that application.

College Admissions

College admission was the motivating application of Gale and Shapley (1962), and indeed an increasing number of countries have established centrally coordinated higher education admissions. In this section we describe some European, South American and Asian applications.

Europe. In many European countries the college admission is organised via a nationwide scheme involving all kinds of programmes. In **Spain**, cut-off scores have been computed by the student-proposing DA with limited preference lists and university rankings based on the students' grades and entrance exam scores according to Romero-Medina (1998). In **Turkey**, the cutoff score system is similar, but the college-proposing DA was used with tie-breaking by the age of the students, at the time of writing (Balinski and Sönmez, 1999). In **Ireland**, the cut-off scores are also published, but the ties are broken by lottery. The students can apply to at most 10 programmes and their scores are mainly coming from the Leaving Certificate Examinations, but the universities have freedom in choosing their scoring method. In the matching process essentially the college-proposing DA is applied in a manual form, where the students have to accept or reject the offers round by round every week. A further speciality is that the proposal-rejection mechanism is used separately for so-called level 6/7 degree programmes (2-3 year programmes) and for level 8 programmes (4 year programmes), so a student might be admitted to two places from which she can choose one at the end (Chen, 2012).

The **French** admission system is fragmented: there are several small schemes for admission to the elite *Grandes Écoles*, but there is a central clearinghouse, called Admission Post Bac (APB) (Frys and Staat, 2015). In theory, the universities participating in APB are not selective, but due to the overdemand they do select their students and prioritise the students coming from the local region and the first choice applications are also preferred to second choice applications. The latter speciality makes the mechanism similar to IA, thus causing strategic issues. The mechanism is applied in three rounds, and after each round the students can decide whether a) they accept their offer and withdraw their other applications, or b) just tentatively accept the offer and wait for a potentially better offer, or c) reject the offer and wait for better offer, or d) cancel all of their applications.

There are some European countries where the universities should provide open access to education for all students that satisfy the requirements, except in the use of some special fields of study with limited spaces, such as medicine, dentistry, veterinary medicine, and pharmacy. Among these countries, in **Belgium** the universities are divided by language. Flemish universities have common entrance exams for medicine and dentistry, whilst French-speaking universities have entrance exams for civil engineering studies (Cantillon and Declercq, 2012).

In **Italy** each field of study with caps has the entrance exams on the same day, so essentially every student can only apply to one university in each field of study (Merlino and Nicoló, 2013). **Germany** has also centralised admissions in medical studies. The first 20% of the seats are allocated to the students with the best grades, another 20% are allocated to students with long waiting times and the remaining 60% are according to the universities' rankings. In the first two groups the Boston mechanism is used, whilst for the last group the university-proposing DA is employed (Kübler, 2011). The flaws of this sequential mechanism have been studied in several scientific papers (Dwenger et al., 2010; Westkamp, 2013; Braun et al., 2014).

Hungary. In the Hungarian higher education admission scheme around 100,000-150,000 thousand students are allocated to programmes every year. From 1996 a heuristic based on the university-proposing DA was used, and in 2007 the student-oriented DA was implemented with some adjustments to deal with the special features. The students are ranked at most universities based on their exam scores, although different sets of subjects count for different programmes. The allocation is announced by publishing the cut-off scores. One special feature is that the ties are not broken, so students with equal scores are either all admitted or all rejected, which is a tractable feature by an extension of the DA (Biró and Kiselgof, 2015). The universities can also set lower quotas for the number of students admitted for each of their studies separately, which leads to an NP-hard problem, and thus requires a heuristic solution (Biró et al., 2010). Most Hungarian students are eligible for free studies in the majority of the programmes if they are willing to sign a contract, but it is also possible to attend most of the programmes under a different contract where tuition fee must be paid. These two kinds of contracts can be listed in the preference list of the students in any order. Another significant feature is that the government can set a national common quota in each field of study on the number of students admitted under state financed contracts. These common upper quotas together with the faculty quotas (applied to students under both kinds of contracts) make also the problem of finding a stable solution NP-hard (Biró et al., 2010). Paired applications are also allowed for teachers' programmes, which causes similar difficulties to the presence of couples in the resident allocation programmes. In a recent paper we investigated integer programming techniques to deal with the NP-hard cases, and the feature of lower quotas turned out to be tractable even for large real instances (Ágoston et al., 2016). New features of the scheme are the limited length preference lists since 2013, and the possibility of applying to dual university programmes since 2015, where a university programme is linked to an industrial internship. However, the admissions for these internship positions are organised in a decentralised way, causing discrepancies.

Brazil has a college admission system, where affirmative action policies are designated to secure place for racial minorities, low-income families and students coming from public highschools. However, the implementation of this policy is badly designed, separate quotas are devoted to students claiming each possible combination of privileges, sometimes leading to higher cut-off scores for students claiming some privilege than others who do not claim that. This results in unfairness, strategic manipulations on what privileges to claim, and unsatisfactory allocation with regard to the affirmative action goals (Aygün and Bó, 2015). In **Chile** the admission scheme is similar to the Hungarian one with respect to the feature that the ties are not broken, but interestingly their policy is more permissive, as they always admit the last group of students with whom the quota is violated (Ríos et al., 2014).

The largest centrally coordinated college admission system is operating in **China**, however the allocation mechanism runs separately with respect to the students applying from the same province. The universities are partitioned into tiers according to their prestige, and their rankings over the students are based on the national standardised test results. Regarding the allocation mechanism, the so-called sequential mechanism was replaced by the so-called parallel mechanism in 27 from the 31 provinces between 2002 and 2012 (Chen and Kesten, 2013). The sequential mechanism is essentially the IA mechanism used in each tier, whilst in the parallel mechanism the students can submit multiple ordered lists as choices, and for each of their choices the serial dictatorship is used separately.

In **South Korea** the admission system was redesigned in 1994. Until 1994 there were only two admission dates, and a student could only attend the exam of one university on each date, so essentially there were only two possible applications. Moreover, the elite universities all scheduled their exam on the first date. After 1994 the results of the comprehensive exam were published before the application period, and they also introduced an early admission period, where the students could apply to one university and they received binding offers according to their state exams. In the regular admission period each university could choose one from the four potential dates to conduct specialised exams. So the students could apply to four universities, but the elite universities decided to schedule their exams to one of the first two dates in an interesting pattern after an adjustment period. The game-theoretic reasons for the behaviour of the universities have been studied in Avery et al. (2014).

School Choice

Before going into the details of the applications, we note that the model behind school choice applications is considered to be slightly different from the Gale-Shapley college admission model. This is because the school seats can be seen as goods to be allocated, over which the children may have different priorities. Thus the main difference is that the schools are not necessarily strategic agents. The stability of a matching in a school choice setting can be therefore interpreted as the lack of *justified envy*, i.e. no student should have priority over another student assigned to a school if the former student is assigned to a less preferred place (or no place at all). For a comprehensive overview on school choice, we refer to Abdulkadiroğlu and Sönmez (2003).

However, we shall note that the usage of this modified model is not appropriate for every school choice application. In fact, as we will see in the descriptions, in Hungary the elementary and secondary schools are strategic players, as they can rank their applicants in any way they want (Biró, 2012), just like in Estonia (Lauri et al., 2014). Meanwhile, in almost all of the nationwide university admission systems that we have seen, the universities must use a commonly defined ranking method, together perhaps with some special priority structures and constraints imposed by the governments. So the universities cannot always be considered strategic players, as already observed in some early scientific papers on college admissions in Spain (Romero-Medina, 1998) and Turkey (Balinski and Sönmez, 1999). Yet, for university admissions the students are typically ranked by merit, so they can influence their rankings. This model was called *student-placement* problem by Balinski and Sönmez (1999).

To sum up the terminology, the Gale-Shapley college admission model refers to the case where both sides of the market are strategic. In the school choice and student placement models only the students are strategic, and the seats are considered as objects to be allocated. The slight difference between the latter two models is that in the school choice model we consider the priorities to be exogenously given, whilst in the student placement model the rankings are (partly) based on the students' performances, so the students can influence them.

We can actually give examples for the adequate use of each of the three models at any level of application, i.e., for school choice, college admissions and resident allocation. Note that the terminologies of the college admission and school choice models have probably been established with regard to the US context, but as we explained, one should use them with caution in other countries.

US city highschools. The decentralised **New York** school choice system switched to a centralised programme using the student-proposing DA with distance and sibling priorities and lotteries in 2004 (Abdulkadiroğlu et al., 2005a, 2013), immediately reducing the number of administratively assigned students from 30,000 to 3,000. The random tie-breaking was criticised, as the lotteries can create a seemingly artificial stability constraint. It was showed that around 2-3% of the students could have been assigned to a better school without harming anyone if only the original priorities would be considered as binding (Erdil and Ergin, 2008). However, removing the lotteries can cause strategic issues (Abdulkadiroğlu et al., 2009), so the lottery system remained in use. In **Boston** the IA mechanism was replaced with the DA in 2005 (Abdulkadiroğlu et al., 2005b). This was a scientifically proposed redesign that led to much discussion in the literature since. Note that in recent years the Boston school choice system has been transformed again, partly due to the request by the local authority to reduce busing costs (Shi, 2015). In the last decade at least a dozen US cities followed the advice and used the assistance of the Institute for Innovation in Public School Choice to establish a school choice system in the spirit of the successful New York programme. Most programmes adopted the student-proposing DA, except **New Orleans**, where the TTC algorithm was implemented in 2012 for the unified public school admissions (Abdulkadiroğlu et al., 2017).

European school choice. In **Hungary** a nationwide system has been in use for secondary school allocation since 2000. The students apply to study programmes with no limit on the number of applications, and the schools set their quotas for each study programme and rank their applicants strictly according to exam results and interviews (so the selection is merit based). The student-proposing DA is used to compute the solution, thus the classical Gale-Shapley college admission model is followed (Biró, 2012). Similarly, a nationwide system is used in **Finland**, where the schools rank their students separately based on their grades and entrance scores. Every student can apply up to five schools and the school-proposing DA is used for computing the solution (Salonen, 2014).

In **Spain** the admissions are organised separately in each municipality with no common rules (Calsamiglia, 2014). According to Calsamiglia (2011) the IA mechanism is applied everywhere with different priority systems. For elementary schools the location and sibling priorities are used (although in Madrid the distance-based priorities were removed), whilst for secondary schools the priorities are mostly coming from the grades of the students. In Barcelona there was a change in the distance priority criterion in 2007, when the catchment area system was replaced with a personal priority system where every child had priority for at least the six closest schools. This change enabled the study of the strategic aspects of the Boston mechanism (Calsamiglia and Güell, 2014). In **Germany**, school admissions are organised separately in each of the 16 states. In Berlin the IA mechanism is used with 10% of the seats reserved for low-income families first, then 60% of the seats are allocated on the basis of merit (Basteck et al., 2015), and the remaining 30% assigned by sibling priority and lottery.

The **Amsterdam** school choice programme was redesigned in 2015, replacing a sequential version of the IA mechanism by the DA with multiple tie-breaking, where no distance-based priorities are used (de Haan et al., 2015). **Paris** is divided into four sub-districts and the students are required to seek schools within their sub-district by getting 600 points in those schools. This number of points can be obtained for academic performance, whilst 300 points are given to low-income families, and 50 points are awarded for having siblings in the school. A centralized platform called Affelnet is used in all districts with the school-proposing DA, however, extra points are awarded in the first choice schools, so the overall mechanism is actually closer to IA (Hiller and Tercieux, 2013). A scientific paper on preference estimation based on data from one of the sub-districts has been published recently (Fack et al., 2015).

Kindergarten allocation in Estonia. The kindergarten allocation in Estonia is organised in cities usually based on a simple priority system with the application date being the main criterion. This creates unfair and inefficient solutions, since the preferences of the parents may change in the 3-4 years between the birth and the admission, and social objectives are also neglected. In 2015 Harku municipality agreed to redesign their mechanism on a scientific basis, documented by Veski et al. (2017). We tested seven different priority systems based on distance and sibling priorities combined with the student-proposing DA. In the simulation we used the data submitted by the parents in 2016, which may be assumed to be truthful. We also conducted a sensitivity analysis by adjusting the effect of the distance and sibling factors on the parents' preferences. The municipality decided to use a transitory priority system in 2016, where the importance of application date, age, distance and siblings vary across the seats in a rotating manner.

Further Applications

There are many further centrally coordinated applications of two-sided matching markets, some of which are of smaller size, and some others that are still in preparation. Due to the lack of space we only mention some interesting examples with future potential. In a coordinated **project allocation** university students are assigned to supervisors or to their offered topics (Abraham et al., 2007; Manlove and O'Malley, 2008). **Adoptions** could also be organised via more sophisticated allocation mechanisms, as studied by Slaugh et al. (2016). **Refugee allocation** under preferences is a timely issue discussed in some very recent mechanism design papers (Delacrétaz et al., 2016; Andersson and Ehlers, 2016). Finally, **dating markets** have always worked in a decentralised way, however due to some novel platforms the search and selection may be better coordinated, e.g. by signalling features (Lee and Niederle, 2015). We close this section with a description on an internship project allocation experience.

Internship allocation at Corvinus University of Budapest. As part of a Masters programme at Corvinus University the students are allocated to participating companies to conduct internship projects in groups. In 2016 the allocation mechanism was changed from IA to a stable matching mechanism by requiring the 25 students to rank all of the 5 companies and asking the companies to evaluate and score all of the candidates. The ties in the scores gave us more flexibility to find a (weakly) stable matching that also satisfies some distributional constraints, namely a lower quota on the number of students at each company and also a balanced distribution of foreign students. For the computation we used IP techniques (Ágoston et al., 2017).

18.3 Allocation of Indivisible Goods

Allocation of indivisible goods to strategic agents is a topic discussed in both the matching literature under the name of two-sided markets with one-sided preferences, and it also belongs to the field of fair division and resource allocation, extensively studied by the AI community. See Klaus et al. (2016) for the theoretical background.

Single-Item Allocation

When every agent can get at most one single item, then the classical mechanism is the *serial dictatorship* (SD), where the agents choose their objects one by one according to some specified order, based e.g. on seniority. This mechanism is used, e.g., for **dormitory allocation in Israel** (Perach et al., 2008). When the agents are ordered randomly we call the mechanism *random serial dictatorship* (RSD). RSD is strategy-proof and ex-post efficient, meaning that no set of agents can improve their situation with an after-match exchange. However, this mechanism is not ex-ante efficient: the agents may exchange their marginal probabilities and can sometimes improve their expected welfare. It is always possible to obtain an ex-ante efficient probability distribution by the so-called *Probabilistic Serial* (or eating) mechanism of Bogomolnaia and Moulin (2001), however this mechanism is manipulable.

In the **resident allocation program of Israel** RSD was used until 2014, but in the redesigned mechanism they conduct improving exchanges to increase the expected welfare of the residents, and in the meantime they randomise over assignments in such a way that every couple is guaranteed to get positions in the same hospital, without harming the chances of the single doctors (Bronfman et al., 2015b,a; Roth and Shorrer, 2015).

Online Matchings

In many situations the objects arrive one by one and have to be allocated immediately. This is the case in deceased organ allocation, organised with a common protocol in eight European countries by Eurotransplant and also in Nordic countries by Scandiatransplant. Social housing is another important application, where the recipients may accept or reject offers in a strategic way (Leshno, 2015; Bloch and Cantala, 2017). Further interesting applications are the allocation of parking slots (Ayala et al., 2012) and airport landing slots (Schummer and Abizada, 2017).

Course Allocation

How should we allocate the seats of the courses if some popular courses cannot accommodate all the interested students? The simplest way is perhaps the **serial dictatorship** (SD) mechanism, where students are ordered randomly and they can choose their bundles one by one. This is happening in most of the Hungarian universities, where the central administration website opens the enrollment process and the students can take their turn whenever they manage to log into the website. Perhaps it is more fair and less stressful to create a random order by a transparent lottery and let the students select their courses at scheduled intervals. The advantage of the SD mechanism is that it is Pareto-efficient and strategy-proof, since each student selects the best bundle from the set of available courses. However, the allocation is very unequal. One way to balance this over the years of studies is to have adjusted lotteries favouring those who were less fortunate so far. This is also the main intuition behind the **draft mechanisms**, used e.g. at the Harvard MBA courses (Budish and Cantillon, 2012), where the students are randomly ordered and they can choose their courses one by one, by switching the order of the picking sequence in the rounds. This results in more egalitarian solutions, however the result is not Pareto-optimal, and the students can benefit from strategic manipulations.

In **bidding mechanisms**, which are widely used in US colleges (Sönmez and Ünver, 2010; Krishna and Ünver, 2008), the students have a virtual budget that they can allocate over their targeted courses. The bids are ordered in a unique ranking decreasing in price and they grant the bids one by one whenever possible. The biggest issue with this mechanism is that the students' preferences may not be aligned with their strategically optimal bids, since the most-preferred course of a student may be less popular, so she could potentially get a seat there also by a minor bid, but if she is lucky with her higher bids for most popular, but personally least-preferred courses then she wont get her most-preferred course due to credit limits. A proposed improvement is to collect the preferences of the students together with her bids, which was also shown to be useful in experiments (Krishna and Ünver, 2008).

Finally, another kind of improvement for the bidding mechanism is the usage of a **competitive equilibrium** solution of a combinatorial auction, with the theoretical background prepared by Budish (2011), and implemented recently at Wharton college (Budish et al., 2016). Here the students report their cardinal utilities over the courses and the central coordinator computes the market clearing prices. In this approximate equilibrium solution every student gets her best possible bundle that is possible to buy from her budget under the market clearing prices. Further advantage of this system is that students can also express their valuations on sets of courses, making it possible to improve the selection of the best bundle by stating which courses are substitutes and which ones are complements of each other. However, it is challenging to elicit all the relevant preferences in advance, so the direct revelation mechanisms may have shortcomings when compared with sequential or decentralised mechanisms.

Priority mechanism at Eötvös Lóránd University (ELTE). Setting priorities at each course and then finding a student-optimal stable solution based on the students' submitted linear preferences over the courses has been proposed in the literature (Diebold et al., 2014). However, the course allocation method used at ELTE is decentralised: the students can optimise their bundles during a long enrollment period by seeing their possibilities at the administration website. The priorities of the students are based on reasonable factors, and multiple lotteries are used for breaking ties at every course separately. The students have to submit their ideal bundles in a preregistration phase; the university can use these to adjust their quotas before the enrollment starts, to better satisfy the demand. The only strategic issue might be that the priority at a course also depends on whether the student included that course in her submitted bundle at the preregistration. (This description was based on the presentation by Ferenc Zaka and Tamás Solymos (ELTE) at the 12th Workshop on Matching in Practice in December 2016, Budapest.)

Further Applications

The assignment of papers to reviewers at conferences is coordinated in many conference-administration websites, e.g. EasyChair, based on the reviewers' preferences, but the exact computational goals of these mechanisms are not known. A couple of reasonable approaches were studied theoretically by Garg et al. (2010). Foodbanks are responsible for allocation of unused food to people in need in many countries worldwide. In the US the largest organisation, Feeding America redesigned its allocation mechanism in 2005 following the advice of a group of economists. The new system is based on a combinatorial auction with virtual credits (Prendergast, 2016), and significantly improved the efficiency and overall welfare of the solution. Finally, a new approach was developed by Kurokawa et al. (2015) for allocating sets of indivisible goods via a randomised mechanism, which was successfully applied for room allocation to Charter schools in California.

18.4 Exchange Models

In exchange markets the agents are endowed with objects that they can exchange to improve everyone's situation without using monetary transfers. When every agent owns exactly one object then the so-called housing market always has a core solution, which can be obtained efficiently by Gale's Top Trading Cycles algorithm (Shapley and Scarf, 1974). However, the core property (also called as stability) may not be the only objective of the central organiser and constraints may also apply, e.g. on the lengths of the exchange cycles, so different models and solution concepts may be needed. Furthermore, when the agents own multiple indivisible goods then the exchange problem becomes significantly more complicated, even to decide the Pareto-efficiency of the solution can be challenging under strong restrictions on the preference domain (Aziz et al., 2016).

Exchange of Indivisible Goods

The exchange of houses or rental flats, the original motivating context of Shapley and Scarf (1974), has indeed been organised in ad-hoc exchange cycles without monetary payments in many countries. A Czechoslovakian film, *Ball lightning* from 1979 illustrates this, which also motivated a scientific paper on exchanges with marrying and divorcing couples (Cechlárová et al., 2016). Timeshare exchanges (Wang and Krishna, 2006) are often organised in a coordinated way. There are new platforms for exchanging homes for holiday, but typically allowing only pairwise exchanges which are negotiated in a decentralised way. Finally we shall note that a national exchange programme was established in 1943 for exchanging shoes among those who needed only a half pair or who had feet of significantly different sizes (Website of National Odd Shoe Exchange, 2016).

Kidney Exchange

Kidney exchange is certainly the most important application of the exchange model with indivisible goods, where patients with kidney failure wish to exchange their incompatible donors for compatible ones. The first single-centre exchange programme was established in South-Korea in 1991 and the Netherlands began the first nationwide programme in 2004 (de Klerk et al., 2008). For an overview we refer to two recent survey papers on this topic (Glorie et al., 2014; Ferrari et al., 2015). The most important features of these centrally coordinated schemes are that the lengths of the exchange cycles are bounded, e.g. in the Netherlands (respectively UK) up to four (respectively three) patient-donor pairs can be involved in an exchange cycle. The other distinguishing feature is that the goal of the programmes are typically to save as many patients as possible, so the optimal solution has maximum size under some constraints and priority criteria.

In the **USA** the New England Kidney Exchange Program was the first centralised scheme established in 2005 (Roth et al., 2005a). Currently the three major nationwide programmes are organised by the United Network for Organ Sharing (UNOS), the National Kidney Registry (NKR) and the Alliance for Paired Donation (APD) (Fumo et al., 2015). An important challenge in the US is that the large transplant centres are very influential, and they conduct the majority of the US kidney exchange transplants internally (e.g. the San Antonio centre). They have a tendency not to report their easy-to-match patient-donor pairs to the national schemes, but wait for suitable exchange partners within the hospital, which results in the accumulation of hard-to-match pairs and highly sensitised patients in the pool (Ashlagi and Roth, 2012). The usage of altruistic donors to trigger so-called never ending chains became a common practice in the US and it has improved the number of transplantations in the kidney exchange pool significantly (Stepkowski et al., 2015).

The Dutch and the UK programmes were established in **Europe** in 2004 and 2007, respectively. In recent years new programmes have been developed in Austria, Belgium, the Czech Republic, France, Italy, Poland, Portugal, Spain and Sweden. Moreover, there was already a transnational exchange between Austria and Czech Republic, and Sweden has also started to cooperate with Denmark and Norway via Scandiatransplant. In 2016 September a COST Action was started (Website of COST Action: European Network for Collaboration on Kidney Exchange Programmes, 2016) involving both practitioners and theoretical researchers in multiple disciplines, to study current practices, identify best practices and key challenges, and investigate the possibilities of further transnational collaborations. (A Handbook on the current practices is scheduled to be published in July 2017.) **Canada** and **Australia** also have well established programmes — see the comparison by Ferrari et al. (2015).

UK paired donation scheme. The scheme has been run by NHS Blood and Transplant since 2007 (Johnson et al., 2008) with matching runs conducted every three months. At the beginning only two-way exchanges were allowed, but since 2008 three-way exchanges have also been sought. The optimal solutions were first computed with an exact graph algorithm (Biro et al., 2009), which was later replaced by a sequential IP solution (Manlove and O'Malley, 2014). Altruistic donors have been used to start chains involving one or two patient-donor pairs from the exchange pool, with the last donor's kidney given to the waiting list. Overall, as if May 2017, from the 1676 patients registered, 1133 were identified for possible exchanges by the matching algorithm based on the virtual cross-match tests, out of which 687 received organs in the end.

Exchange of Multiple Indivisible Goods

Swapping used books without monetary payments is an potential application present in many countries (see e.g. ReadItSwapIt in the UK). The typical decentralised exchange process of such a scheme is to offer the goods first to the community for getting credits, and then spend the credits on other goods later on. This organisation is also typical for timebanks (see e.g. TimebankingUK), where the participants offer their time in the form of providing different services (e.g. teaching math) and receive the same amount of time back in the form of other services (e.g. French class or haircutting). There are many sites also for exchanging babysitting services (e.g. BabysitterExchange).

A more serious application is the exchange of deceased organs among countries. Eurotransplant is an organisation involving eight European countries who share their deceased donors among themselves in a fair way, and Scandiatransplant is a similar organisation for Scandinavian countries. In their online allocation policy it is crucial to find the best suitable recipient(s) for the newly available organs, but in the long run they also ensure the balancedness of the allocation, that is, every participating country should receive about the same number of organs as they offered.

The balancedness of the solution is also crucial in the tuition exchange programmes in the US, such as The Tuition Exchange, Inc, where member colleges award the dependents of their staff with free tuitions that they can use at other colleges. Thus essentially the colleges exchange their students. Although the coordination of the exchanges is semi-centralised, the number of incoming and outgoing students should be roughly equal for every college (Dur and Ünver, 2016). A similar framework is applied in the Erasmus programme, where European higher education institutions exchange their students. These exchanges are based on bilateral contracts, but in the long run every university cares about the balance of the incoming and outgoing students, since every student pays the tuition at her home university.

18.5 Further Matching Models

In this last section we mention some further models that have applications.

Roommates Problem

The roommates problem was defined by Gale and Shapley (1962) as the extension of the marriage problem for non-bipartite graphs – this is also referred to as a one-sided market in the economic literature. Here a stable matching may not exist, but we can decide the existence in linear time. Regarding the corresponding applications, finding roommates for twin rooms is a reasonable one, although we are not aware of any recorded application of this sort. Note that the pairwise exchange problem, i.e. where the length of exchange cycles is bounded by two, is equivalent to the roommates problem, if stability of the solution is the main concern. Therefore, for instance, the initial kidney exchange papers were also concerned with matching problems on non-bipartite graphs, where the Edmonds algorithm can find maximum size or maximum weight matching efficiently (Roth et al., 2005b). A very recent application of the roommates model is speed networking at conferences (Vaggi et al., 2014), where the participants are scheduled for one-to-one discussions on a scientific basis. Finally, we describe the classical application of chess pairings below.

FIDE chess pairings. Chess pairings are used in both individual and team tournaments for allocating the competitors into pairs based on their performance. The most well-known pairing system is the Swiss system, invented by Dr. Julius Müller of Brugg, which was first used in a chess tournament at Zurich in 1895. The goal of a chess tournament is to determine the winner and rank the others based on their results achieved in a fixed number of rounds, typically nine. The common practice in all of the variants used by the World Chess Federation (FIDE) is to try to match the players or teams with those who are similarly ranked based on their performance so far. However, the completeness of the pairing is also required (at most one player can be unmatched, if the number of players is odd), and no two players can play twice against each other. The colours are also important in individual tournament, no player can play with the same colour three times in a row. The main pairing rule used in the Swiss-tournaments has now four official variants, the Dutch, Lim, Dobov, and Burnstein systems (Website of the World Chess Federation, 2016). The description of the pairing methods states that every arbiter should be able to conduct the matching by hand, although official pairing software applications are widely used. For the above reasons, the descriptions of current processes contain some highly inefficient exhaustive search procedures, as the arbiters cannot be expected to find a complete matching, e.g., by using Edmonds' algorithm. Yet, it is an interesting question as to whether the solutions returned by the described processes can be obtained by efficient algorithms, which can be implemented and used in a software tool (Biró et al., 2017).

Coalitional Matchings

Matching problems can be extended to so-called coalition formation problems, where the groups of agents can agree on cooperations. An example that we have already seen is the matching problem with couples, where a couple can form a coalition with two hospitals. Stable 3D-matchings are also widely studied, where the set of agents can be separated into three parties (rather than two, as in the stable marriage problem), see e.g. (Biró and McDermid, 2010). In the group activity selection problem the agents are assigned to activities and they may have preferences also over the number of partners with which they participate in a given activity, see Chapter 5 or this book. Finally, in some national grant agencies (such as the Hungarian and the French ones) researchers can be involved in multiple project proposals, but their contributions must be declared and no one can be involved in projects with a total contribution exceeding 100%. These applications can be described in a general game-theoretic framework, see e.g. Biró and Fleiner (2016).

Matching with Contracts

In many applications two agents can choose from different contracts when co-operating. For instance, as we have seen, in the Hungarian higher education admission scheme the students may be admitted to the same programme under two different contracts, mainly differing in the tuition fees. Cadet branch allocation in the US is another important application, where the number of service years may differ in optional contracts (Sönmez and Switzer, 2013). If multiple contracts are possible in a many-to-one market, then the unit-capacity agents should have the opportunity to order the contracts in their applications in any way, as it is the practice in Hungary (but what is not allowed in the cadet branch matching). With regard to the capacitated side, the preferences of the agents on that side may be more complex over the sets of contracts, but as long as they satisfy the so-called substitutability condition, stable solutions bound to exist (Fleiner, 2003; Hatfield and Milgrom, 2005).

Matching with Payments (not covered)

In this survey we do not cover models where payments are allowed between the agents, such as coordinated auctions, since this would probably double the length of this chapter. However, we note that these applications are based on very similar matching models, with common generalisations including the matching with no payments models, see e.g. Kelso Jr and Crawford (1982). Indeed, some important applications, such as the Google adwords auctions, can also be studied with such hybrid models (Aggarwal et al., 2009). Furthermore, there are also examples for solving the allocation of indivisible goods with virtual auctions to increase the overall efficiency and the fairness of the solution, as we have seen for course allocation (Budish et al., 2016) and in the allocation process of foodbanks (Prendergast, 2016).

18.6 Conclusion

Discovering and describing the current applications based on matching models under preferences is an important task that our research community has also recognised, e.g., in the European research network on Matching in Practice. These findings can help us understand what the main features of the applications are and what the main questions, issues and challenges are. Scientists in several disciplines can then join forces to analyse the performance of the current mechanisms, and come up with ideas for possible improvements. Note though that in many cases the optimisation criteria are not entirely clear, there may be multiple solution methods to consider, and the best practices may highly depend on the context. But our ultimate goal as scientists should be to help the decision makers in the (re-)design of their applications to make them efficient, fair and transparent, by serving the best interests of society.

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