On associative, idempotent, symmetric, and nondecreasing operations

Jimmy Devillet and Bruno Teheux

University of Luxembourg, Esch-sur-Alzette, Luxembourg
{jimmy.devillet, bruno.teheux}@uni.lu

Abstract

The study of aggregation functions defined on finite ordinal scales (i.e., finite chains) encounters an increasing interest since the last decades (see, e.g., [1–3, 5, 7, 9–16]). Among these functions, discrete t-norms, t-conorms, uninorms, and nullnorms are binary operations that play an important role in fuzzy logic. In particular, these operations share the properties of being associative, symmetric, and nondecreasing (in each variable).

It is known that the class of associative, idempotent, and symmetric binary operations is in one-to-one correspondence with the class of partial orders of semilattices (see, e.g., [6]). We provide a full description of the class of associative, idempotent, symmetric, and nondecreasing binary operations defined on a finite chain in terms of properties of the Hasse diagram of the corresponding semilattice. In particular, given an operation belonging to the latter class, we provide a recursive construction of the corresponding semilattice. We also provide an associativity test for idempotent, symmetric, and nondecreasing operations. Moreover, the enumeration of the class of associative, idempotent, symmetric, and nondecreasing operations leads to a new occurrence of the Catalan numbers and provides another construction of this sequence.

References

1. M. Couceiro, J. Devillet, and J.-L. Marichal. Characterizations of idempotent discrete uninorms. Fuzzy Sets and Systems, 334:60–72, 2018.
2. B. De Baets, J. Fodor, D. Ruiz-Aguilera, and J. Torrens. Idempotent uninorms on finite ordinal scales. Int. J. of Uncertainty, Fuzziness and Knowledge-Based Systems, 17(1):1–14, 2009.
3. B. De Baets and R. Mesiar. Discrete triangular norms. In Topological and Algebraic Structures in Fuzzy Sets, A Handbook of Recent Developments in the Mathematics of Fuzzy Sets, Trends in Logic, eds. S. Rodabaugh and E. P. Klement (Kluwer Academic Publishers), 20, pp. 389–400, 2003.
4. J. Devillet and B. Teheux. Associative, idempotent, symmetric, and nondecreasing operations on chains. Working paper.
5. J. Fodor. Smooth associative operations on finite ordinal scales. IEEE Trans. Fuzzy Systems, 8:791–795, 2000.
6. G. Grätzer. General Lattice Theory, Second edition. Birkhäuser, 2003.

7. G. Li, H.-W. Liu, and J. Fodor. On weakly smooth uninorms on finite chain. Int. J. Intelligent Systems, 30:421–440, 2015.

8. J. Martín, G. Mayor, and J. Torrens. On locally internal monotonic operations. Fuzzy Sets and Systems, 137:27-42, 2003.

9. M. Mas, G. Mayor and J. Torrens. t-operators and uninorms on a finite totally ordered set. Int. J. Intelligent Systems, 14:909–922, 1999.

10. M. Mas, M. Monserrat and J. Torrens. On bisymmetric operators on a finite chain. IEEE Trans. Fuzzy Systems, 11:647–651, 2003.

11. M. Mas, M. Monserrat and J. Torrens. On left and right uninorms on a finite chain. Fuzzy Sets and Systems, 146:3–17, 2004.

12. M. Mas, M. Monserrat and J. Torrens. Smooth t-subnorms on finite scales. Fuzzy Sets and Systems, 167:82-91, 2011.

13. G. Mayor, J. Suñer and J. Torrens. Copula-like operations on finite settings. IEEE Trans. Fuzzy Systems, 13:468-477, 2005.

14. G. Mayor and J. Torrens. Triangular norms in discrete settings. In Logical, Algebraic, Analytic, and Probabilistic Aspects of Triangular Norms, eds. E. P. Klement and R. Mesiar (Elsevier, Amsterdam), pp. 189–230, 2005.

15. D. Ruiz-Aguilera and J. Torrens. A characterization of discrete uninorms having smooth underlying operators. Fuzzy Sets and Systems, 268:44-58, 2015.

16. Y. Su and H.-W. Liu. Discrete aggregation operators with annihilator. Fuzzy Sets and Systems, 308:72-84, 2017.

17. Y. Su, H.-W. Liu, and W. Pedrycz. On the discrete bisymmetry. IEEE Trans. Fuzzy Systems. To appear. DOI:10.1109/TFUZZ.2016.2637376