A novel comparison of Møller and Compton electron-beam polarimeters

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ABSTRACT

We have performed a novel comparison between electron-beam polarimeters based on Møller and Compton scattering. A sequence of electron-beam polarization measurements were performed at low beam currents (< 5 μA) during the Q_{weak} experiment in Hall-C at Jefferson Lab. These low current measurements were bracketed by the regular high current (180 μA) operation of the Compton polarimeter. All measurements were found to be consistent within experimental uncertainties of 1% or less, demonstrating that electron polarization does not depend significantly on the beam current. This result lends confidence to the common practice of applying Møller measurements made at low beam currents to physics experiments performed at higher beam currents. The agreement between two polarimetry techniques based on independent physical processes sets an important benchmark for future precision asymmetry measurements that require sub-1% precision in polarimetry.

1. Introduction

Polarized electrons have become an essential tool in nuclear and particle physics experiments seeking to understand the fundamental forces of nature. They are used to address a wide variety of topics ranging from the internal structure of nucleons to precision tests of the Standard Model. Typically these studies need longitudinal beam polarization, where the electron spin is orientated parallel or anti-parallel to the beam momentum. Over the last several decades technological progress has resulted in dramatic improvement in the precision of experiments that utilize polarized beams. Knowledge of the beam polarization is an important source

of experimental uncertainty in these experiments. Future measurements [1–3] will require 0.5% precision in the beam polarization. Advances in electron-beam polarimetry are a key driver for the future improvements in precision spin physics.

All techniques used to determine the electron-beam polarization at accelerator energies exploit the spin dependence of a scattering process by measuring the difference in the electron scattering rate for two possible helicity configurations. The ratio of this difference to the sum of the scattering rates is called the asymmetry, and is directly proportional to the beam polarization and the analyzing power of the scattering process. Hence, for scattering processes whose analyzing power can be precisely calculated, we can determine the polarization of the beam by measuring the asymmetry.

It is desirable that the spin-dependent scattering process has a large rate and a slowly varying analyzing power. When polarized targets are used, they should be stable and highly polarized with easily measurable polarization. It is also desirable that the polarimetry technique be non-invasive to the physics experiment and use the same beam, i.e. the same energy, current and location, as the physics experiment. The two processes most commonly employed in electron-beam polarimetry are Møller scattering, from spin-polarized electrons in magnetic materials, and Compton scattering, from circularly polarized laser photons.

None of the readily available polarimetry techniques have all of the desirable properties. For example, polarimeters based on Møller scattering can be operated only at low currents making this technique invasive to the experiment, while the analyzing power in Compton scattering varies rapidly (even changing sign) as a function of the energy of the scattered particles. Thus, in order to achieve the desired high accuracy, multiple independent and high precision polarimeters have to be used in concert. The most recent experiment to employ both Møller and Compton polarimeters was the Q_{weak} experiment, a parity-violating electron scattering experiment in Hall-C at Jefferson Lab (JLab) [4–6]. The Q_{weak} experiment aims to test the Standard Model of particle physics by providing a first precision measurement of the weak vector charge of the proton, from which the weak mixing angle will be extracted with the highest precision to date away from the Z^0 pole. Knowledge of the electron-beam polarization is one of the largest experimental corrections for the Q_{weak} experiment. To achieve the desired precision the experiment used an existing high-precision Møller polarimeter and a new Compton polarimeter to continuously monitor the electron-beam polarization. The Møller polarimeter was used intermittently throughout the experiment, operating at a beam current of a few microamps, while the Compton polarimeter monitored the beam polarization at the exact running conditions of the Q_{weak} experiment, which included beam currents of up to ~ 180 μA, at a beam energy of 1.16 GeV.

The two polarimeters have very different analyzing powers and systematic uncertainties. A precise comparison of the polarization measured by the Møller and Compton, in quick succession and at the same beam currents provides an essential cross-check between the two techniques. Further, comparison of the measured beam polarization at different beam currents can be used to verify the often used assumption that the electron-beam polarization is independent of beam current. In this letter we report the results from a series of measurements where Møller, followed by Compton, followed by Møller measurements were performed at identical low beam currents in rapid succession. In addition the sequence of measurements was bracketed by Compton measurements at high currents.

A previous comparison of multiple electron-beam polarimeters at Jefferson Lab has been reported in Ref. [7]. In that study, measurements were made using a Mott polarimeter in the accelerator
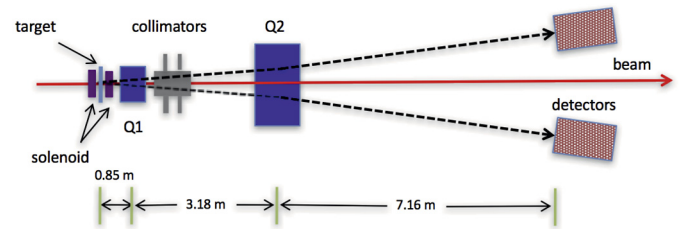

Fig. 1. Schematic of the JLab Hall-C Møller polarimeter. Note that the quadrupole positions are slightly different than described in Ref. [8].

injector, Møller polarimeters in Halls A, B, and C, and a Compton polarimeter in Hall A (the Hall-C Compton polarimeter did not exist at that time). While the polarizations extracted by the various devices were deemed compatible within their systematic uncertainties, only the Hall-C Møller polarimeter was capable of making measurements with a systematic uncertainty better than 1%. In addition, the measurements using the Møller polarimeters were performed at different beam conditions than the Compton polarimeter.

The measurement described in this letter is the first comparison of two polarimeters capable of sub-1% systematic errors, performed in the same experimental hall under identical beam conditions. The entire sequence of measurements allows us to compare the Compton and Møller measurements at low currents, as well as to compare them to the Compton measurements at high current.

2. The Hall-C Møller polarimeter

The Hall-C Møller polarimeter is designed to provide an absolute polarization measurement with a statistical precision of better than 0.5% within a few minutes. Fig. 1 depicts a schematic of the polarimeter. A 3.5 T superconducting solenoid magnet is used to polarize a 1–4 μm thick, pure iron foil out-of-plane with an applied magnetic field well above the 2.2 T required for magnetic saturation of pure iron. The maximum of the analyzing power for Møller scattering occurs at a scattering angle of 90° in the center-of-mass frame and the polarimeter is designed to optimize the acceptance for these kinematics. A pair of quadrupole magnets focus the scattered and recoiling atomic electrons onto the detectors. Detection of both electrons in coincidence reduces backgrounds due to Mott scattering. A set of movable collimators placed between the quadrupole magnets is employed to further reduce the Mott background without affecting the acceptance of Møller electrons. The effective analyzing power for the Hall-C Møller polarimeter is ≈ 0.064 at 1.16 GeV, with some variation (typically less than 0.5%) due to beam position variation. The details of the JLab Hall-C Møller polarimeter are described in Ref. [8,9]

3. The Hall-C Compton polarimeter

A schematic of the Compton polarimeter in Hall-C at JLab is shown in Fig. 2. The JLab Hall-C Compton polarimeter was designed to continuously monitor the beam polarization at high beam currents with better than 1% statistical uncertainty per hour. It consists of four identical dipole magnets forming a magnetic chicane that displaces a 1.16 GeV electron beam vertically downward by 57 cm. A high intensity (~ 1–2 kW) beam of ~ 100% circularly polarized photons is provided by an external low-gain Fabry–Pérot laser cavity which consists of an 85 cm long optical cavity with a gain between 100 and 200, coupled to a green (532 nm), continuous wave, 10 W laser (Coherent VERDI). The laser light is focused at the interaction region ($\sigma_{waist} \sim 90$ μm), where it is larger than the electron beam envelope ($\sigma_{x/y} \sim 40$ μm). The laser is operated
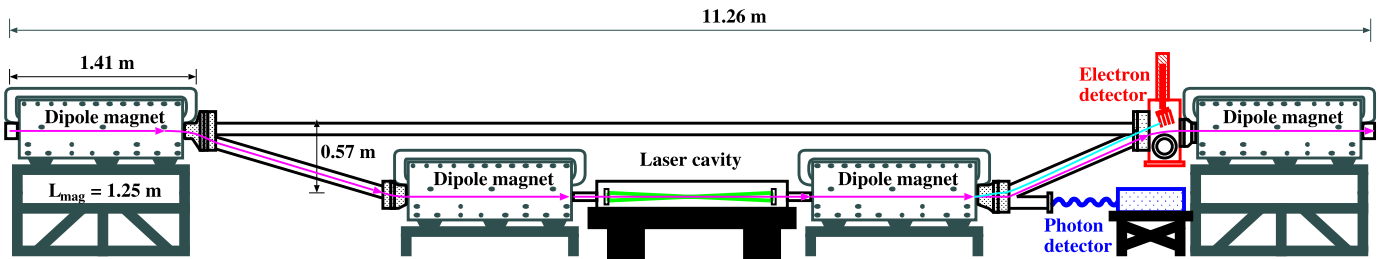
Fig. 2. Schematic of the JLab Hall-C Compton polarimeter. Figure taken from [6].

in 90 second cycles, where it is active for 60 seconds (laser on period) and blocked off (laser off period) for the rest of the cycle. The laser off data are used to measure the background.

At the electron beam energy of 1.16 GeV used for these studies, the maximum scattered photon energy is approximately 46 MeV, while the maximum separation between the primary electron beam and the Compton scattered electrons, just upstream of the fourth dipole, is ∼ 17 mm. The displacement of the scattered electron with respect to the primary electron beam is detected by a set of four diamond micro-strip detectors. The proximity of the detectors to the primary beam allows them to capture most of the energy spectrum of the scattered electrons. The data analysis technique exploits track finding, the high granularity of the electron detector and its large acceptance to fit the shape of the measured asymmetry spectrum to the precisely calculable Compton asymmetry.

A calorimeter consisting of a 2×2 matrix of 3 cm $\times$ 3 cm $\times$ 20 cm $PbWO_4$ scintillating crystals attached to a single photo-multiplier tube was used to measure the scattered photon energy. The signal from the photon detector is digitally integrated with no thresholds over a full helicity state (~ 1 ms) using a 200 MHz flash analog to digital converter.

Details on the Compton polarimeter can be found in Ref. [5,6, 10–12]. Since the electron detector provided the highest precision and most reliable measurements of the beam polarization from the Compton polarimeter, the results discussed here are from the electron detector only.

4. Data analysis and results

The electron beam helicity was reversed at a rate of 960 Hz in a pseudo-random sequence, using an electro-optic Pockels cell in the laser optics of the polarized electron photoemission gun [13,14]. In addition, a half-wave plate in the polarized source laser optics was inserted or removed about every 8 hours to reverse the beam helicity as a systematic check. These reversals change the sign of the beam polarization, but the magnitude of the polarization was found to be independent of the half-wave plate. The analysis procedure to extract beam polarization from the Møller measurements is described in Ref. [8,9] while the procedure for the Compton measurements is described in Ref. [6].

Typically, Møller measurements are conducted at low beam currents (< 2 μA) to minimize the foil depolarization due to beam heating, whereas Compton polarimeters achieve their best statistical precision when operated at the maximum beam current. Comparing the two under identical conditions required finding a suitable "compromise" current. At this beam current, target foil depolarization effects due to beam heating would be minimal for the Møller while still enabling the Compton to achieve adequate statistics in a reasonable time. This study consisted of two Møller current scans, with an 8-hour series of Compton measurements in-between.

The temperature increase in the Møller target foil due to the power deposited by the electron beam was determined by solving the 1D-radial heat equation using the known (temperature dependent) value of the thermal conductivity of iron and knowledge of the approximate size of the electron beam. The calculation assumed that radiative cooling effects were small and that the primary cooling mechanism was via conduction through the target holder.

After calculating the target's temperature rise, the target depolarization was calculated using empirical fits which were shown to agree well with previously published measurements of the temperature dependence of the magnetization of iron [15].

At beam currents of 1 μA or smaller, the foil depolarization due to beam heating was estimated to be less than 0.14%. At the highest currents used in this study, the depolarization grows to almost 1%.

Dead time is the only other effect that depends significantly on beam current. Corrections for target heating and dead-time, and their effect on the measured polarization, are shown in Fig. 3 (note that only statistical uncertainties are shown in the top panel of Fig. 3). The uncertainty on the target heating correction is estimated to be about 30% of the size of the correction. At 4.5 μA, the maximum current used for the Møller and Compton comparison, the impact of the target heating uncertainty is $\Delta P/P = 0.24\%$ – this value is used when comparing the Møller and Compton results.

The largest systematic uncertainty for the Møller measurement comes from the effect of atomic Fermi motion of the electrons (the Levchuk effect) [16]. Other significant uncertainties come from beam position and angle on target, which were determined from special systematic studies of the analyzing power dependence on beam position. These are discussed in [17].

The full list of systematic uncertainties for the Møller polarimeter during this study is given in Table 1. A discussion of the typical conditions is given in [5]. The major difference is the absence of a rather large uncertainty of 0.50% for extrapolating to higher currents. Since both polarimeters ran at approximately the same current (< 5 μA) for this cross-calibration, a high-current extrapolation was not necessary.

For the Compton measurements, as described in Ref. [10], the yield asymmetry measured by the electron detector for each ~ 1 hour long interval (run) was compared to the theoretical Compton asymmetry for each detector strip, with the beam polarization (P_e) and the non-integer strip number corresponding to the maximum displaced electrons (n_{CE}), or the Compton edge, as the two independent parameters.

The typical asymmetry obtained from the electron detector fit to the calculated asymmetry is shown in Fig. 4. The upper panel shows the asymmetry and fit for high current, while the bottom panel shows the asymmetry and fit at low current.

DAQ inefficiency, from such things as dead-time and the trigger-forming algorithm, was determined using a simulation of the DAQ
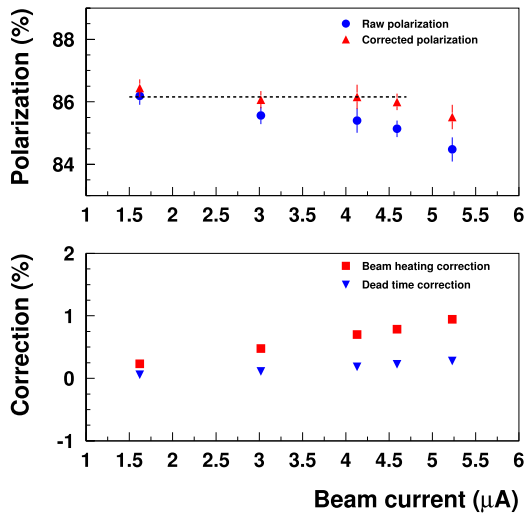
Fig. 3. Top: Electron-beam polarization as determined by the Møller polarimeter as a function of current. The blue circles are the raw measurements, while the red triangles include corrections for dead time and target heating effects. Error bars denote statistical uncertainties only. The dashed line is the best fit of the corrected data for all points at currents less than 5 μA. The average is $86.16 \pm 0.15\%$ (stat), with a χ^2/dof of 0.5 (associated probability 0.69). Bottom: Size of correction vs. current, in percent. Red squares are the beam heating correction, while blue inverted triangles are the dead time correction. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 1
Systematic uncertainties of the Møller polarimeter.

Source	Uncertainty	$\Delta P/P\%$
Beam position X	0.2 mm	0.14
Beam position Y	0.2 mm	0.28
Beam angle X	0.5 mrad	0.10
Beam angle Y	0.5 mrad	0.10
Q1 current	2%	0.07
Q3 current	2%	0.05
Q3 position	1 mm	0.10
Multiple scattering	10%	0.01
Levchuk effect	10%	0.33
Fixed collimator positions	0.5 mm	0.03
Beam heating of target	30%	0.24
B-field direction	2 degrees	0.14
B-field strength	5%	0.03
Spin polarization in Fe	–	0.25
Electronic D.T.	100%	0.045
Solenoid focusing	100%	0.21
Solenoid position (x, y)	0.5 mm	0.23
Monte Carlo statistics	–	0.14
Total		**0.71**

system [6]. The DAQ simulation was used to determine a correction factor for the detector yield based on the aggregate detector rate. The corrections were negligible for the low current data and $< 1\%$ for the high current data. An independent analysis of a separate data stream, using only raw hits in the electron detector rather than triggers that met the tracking definitions, was used to validate these results over a wide range of rates. The two analyses, in principle, have very different rate sensitivities, and their comparison was used to demonstrate the validity of these rate-dependent corrections. The DAQ-related systematic uncertainties are listed in Table 2.

A full Monte Carlo simulation of the Compton polarimeter using the GEANT3 [18] detector simulation package was used to validate the analysis procedure and to study a variety of sources of systematic uncertainties [10]. The list of contributions is shown in Table 2. From these simulation studies we have determined that
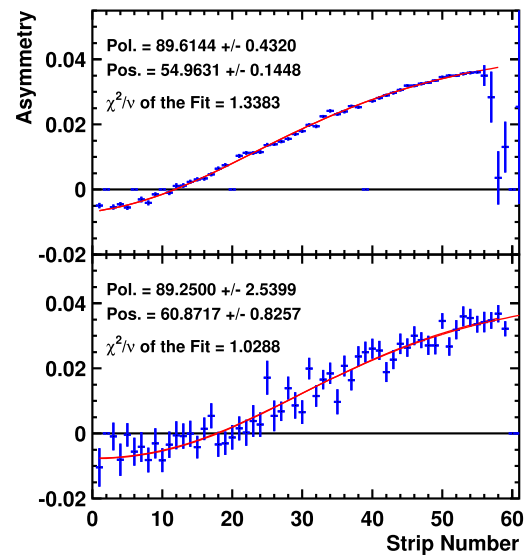

Fig. 4. The typical Compton asymmetry (blue) and fit (red) for high current (upper) and low current (lower). The extracted polarization, location of the Compton edge and the χ^2 per degree of freedom for the fit are also indicated in the figure. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Systematic uncertainties of the Compton polarimeter.

Source	Uncertainty	$\Delta P/P\%$
Laser Polarization	0.18%	0.18
Magnetic field	0.0011 T	0.13
Beam energy	1 MeV	0.08
Detector z position	1 mm	0.03
Trigger multiplicity	1–3 plane	0.19
Trigger clustering	1–8 strips	0.01
Detector tilt (w.r.t x, y, z)	1 degree	0.06
Detector efficiency	0.0–1.0	0.1
Detector noise	up to 20% of rate	0.1
Fringe field	100%	0.05
Radiative corrections	20%	0.05
DAQ inefficiency correction	40%	0.3
DAQ inefficiency pt.-to-pt.	–	0.3
Beam vert. pos. variation	0.5 mrad	0.2
Helicity correl. beam pos.	5 nm	< 0.05
Helicity correl. beam angle	3 nrad	< 0.05
Spin precession in chicane	20 mrad	< 0.03
Total		**0.59**

the net systematic uncertainty of the extracted beam polarization is 0.59% [6].

At low beam currents, the measurement in the Compton polarimeter is hampered by poor statistics (as seen in Fig. 4). On the other hand, increasing the beam current results in larger systematic uncertainties in the Møller measurement, due to target heating. As mentioned earlier from the current scan of the Møller polarimeter, it was determined that the highest beam current where the heating effects do not dominate is ~ 4.5 μA. Therefore, the Møller–Compton–Møller (MCM) sequence of measurements described here, were performed at a beam current of 4.5 μA. In order to keep the electron-beam optics identical for the Møller and Compton measurements, the beam was transported through the Compton chicane for both polarimeters during the MCM comparison. The beam transport parameters were identical during the comparison measurements. A slow position feedback loop, which locked the electron-beam position at the laser interaction point, was used for the Compton measurements but disabled for the
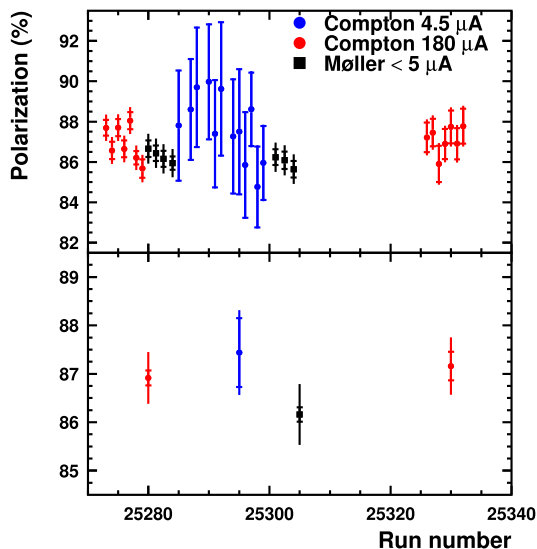
Fig. 5. (Top) Polarization measured at 4.5 µA along with neighboring high current runs and the Møller measurements taken at beam currents less than 5 µA. (Bottom) The average of high current and low current Compton and the neighboring Møller measurements.

Table 3
Mean polarization measurements during cross calibration. The first Møller entry is averaged over all beam currents less than 5 µA, while the second is at a fixed beam current of 4.5 µA.

Type	Current (µA)	Mean pol. (%)	Stat. uncert.	Total uncert.
Compton	180	86.92	0.15	0.53
Møller (avg.)	3.3	86.16	0.15	0.63
Møller	4.5	86.00	0.27	0.67
Compton	4.5	87.44	0.71	0.88
Compton	180	87.16	0.29	0.59

Møller measurements. Fig. 5 shows the polarization extracted from the consecutive Møller, Compton and Møller measurements using the same low current (~ 4.5 µA for the Compton measurements and ~ 1.5–4.5 µA for the Møller measurements) beam. The results demonstrate that the polarization measured by the Møller and Compton polarimeters are consistent within experimental uncertainties (about 1% relative for the Compton and 0.73% for the Møller) at low beam current. Also plotted in Fig. 5 are the two adjacent high current (~ 180 µA) Compton measurements, during which we do not expect the electron-beam polarization to have changed. The consistency between the low current and high current measurements indicate that within experimental uncertainties the beam polarization does not vary with beam current. The polarization obtained from the Compton measurements for both high current runs and low current runs were averaged separately and are shown in the lower panel of Fig. 5 and listed in Table 3. In these plots the inner error bar shows the statistical uncertainty while the outer error bar shows the total uncertainty given by the quadrature sum of statistical and systematic uncertainties. A detailed analysis of the Compton and Møller data, properly accounting for correlated systematic uncertainties for the Compton measurements, indicates a 1-σ upper limit of 0.98% (relative) for the change in beam polarization between 4.5 and 180 µA. This result does not rule out some small dependence of electron-beam polarization on current, but does demonstrate that polarization measurements made at low currents can safely be applied at higher currents without a significant increase in systematic uncertainty.

5. Conclusions

We have compared the polarization obtained from consecutive measurements using a Møller and a Compton polarimeter at low beam currents. These low current measurements were bracketed by the regular high current operation of the Compton polarimeter. All measurements were found to be consistent within experimental uncertainties, demonstrating that the electron-beam polarization does not depend on beam current to better than 1% for a beam current range of 175 µA. These results give confidence in the use of Møller measurements made at low beam currents for physics experiments carried out at much higher current. In addition, the demonstration of the consistency of two high-precision electron-beam polarimeters under identical beam conditions is a significant step forward in verifying the accuracy of electron-beam polarimetry at the level required by future high-precision measurements of spin-dependent asymmetries, such as the SOLID and MOLLER experiments at Jefferson Lab or the P2 experiment at Mainz.

Acknowledgements

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