Article:

Onishchenko, O.G., Fedun, V. orcid.org/0000-0002-0893-7346, Smolyakov, A. et al. (3 more authors) (2018) Tornado model for a magnetised plasma. Physics of Plasmas, 25 (5). ISSN 1070-664X

https://doi.org/10.1063/1.5023167

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Citation: Physics of Plasmas **25**, 054503 (2018); doi: 10.1063/1.5023167
View online: https://doi.org/10.1063/1.5023167
View Table of Contents: http://aip.scitation.org/toc/php/25/5
Published by the American Institute of Physics

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(Received 22 January 2018; accepted 18 April 2018; published online 8 May 2018)

A new analytical model of axially-symmetric magnetic vortices with both a twisted fluid flow and a magnetic field is proposed. The exact solution for the three-dimensional structure of the fluid velocity and the magnetic field is obtained within the framework of the ideal magnetohydrodynamic equations for an incompressible fluid in a gravitational field. A quasi-stationary localised vortex arises when the radial flow that tends to concentrate vorticity in a narrow column around the axis of symmetry is balanced by the vertical vortex advection in the axial direction. The explicit expressions for the velocity and magnetic field components are obtained. The proposed analytic model may be used to parameterise the observed solar tornadoes and can provide a new indirect way for estimating magnetic twist from the observed azimuthal velocity profiles. https://doi.org/10.1063/1.5023167

The formation of steady state magnetic vortices is one of the fundamental problems of plasma physics.[1–6] The magnetic flux tubes with vortical motion are a fundamental model for understanding solar tornadoes,[7–10] astrophysical jets[11,12] and disks around rotating magnetised stars and black holes[13] as well as some configurations of confined laboratory plasmas.[14–16]

Recent solar observations from the Atmospheric Imaging Assembly (AIA) on board NASA's space-based Solar Dynamics Observatory (SDO) and the Crisp Imaging Spectropolarimeter (CRISP) on the ground-based, Swedish 1-m Solar Telescope have generated a lot of interest in solar magnetic tornadoes as possible channels for energy transfer into the solar corona.[9,10] **The acoustic gravity tornadoes can also appear in the form of twisted density structures**.[17] In the present work, we propose a new model of axially-symmetric steady-state magnetic vortex with helical motion. In the framework of an ideal MHD, it is assumed that the vortices are generated in convectively unstable plasmas in a gravitational field where the upward moving plasma penetrates the surface of the Sun from the interior as a result of thermal convection and is accelerated by the vertical pressure gradient. The accelerating vertical plasma flows generate converging radial fluxes of plasma due to incompressibility. The dynamics of the developing vortices can be conditionally divided into three stages: the rapidly developing generation stage, the vortex in the quasi-stationary stage, and the vortex damping due to dissipative processes. The generation stage is similar to the one discussed previously by Ref. 18, see also the references therein. In this paper, we focus on the quasi-stationary stage by developing an axially symmetric model of magnetic vortices derived from the ideal MHD equations.

This model may be used as a basis for parameterising observed solar tornadoes[19,20] and estimating, otherwise difficult to measure, magnetic twist profiles from azimuthal velocities.

In the course of investigating axially-symmetric structures, we introduce the cylindrical coordinate system (r, ϕ, z) with the z-axis in the vertical direction and considering $\partial/\partial\phi = 0$. As the initial equations, we use stationary (i.e., $\partial/\partial t = 0$) magnetohydrodynamic equations for an ideal incompressible fluid in the gravitational field

$$-\mathbf{v}\times(\nabla\times\mathbf{v}) + \frac{1}{\rho\mu_0}\mathbf{B}\times(\nabla\times\mathbf{B}) = -\frac{1}{\rho}\nabla\left(p+\frac{\mathbf{v}^2}{2}\right)+\mathbf{g} \tag{1}$$

and

$$\nabla\cdot\mathbf{v}=0,\quad \nabla\cdot\mathbf{B}=0,\quad \mathbf{v}\times\mathbf{B}=0. \tag{2}$$

Here, $\mathbf{v} = (v_r, v_\phi, v_z)$ is the velocity field, $\mathbf{B} = (B_r, B_\phi, B_z)$ is the magnetic field, ρ is the constant density, p is the pressure, $\mathbf{g} = -g\hat{\mathbf{e}}_z$ **is the gravity acceleration,** $\hat{\mathbf{e}}_z$ **is the unit vector directed along the** z**-axis**, and μ_0 is the permeability of free space. Chandrasekhar's[1] equipartition solution of Eqs. (1) and (2) corresponds to

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$$\mathbf{v} = \pm \frac{\mathbf{B}}{(\rho\mu_0)^{1/2}} \tag{3}$$

and from the r and z components of (1), we obtain the generalised Chandrasekhar[1] solution under hydrostatic pressure

$$p = p_0\left(1 - \frac{z}{L}\right) + \frac{B^2}{2\mu_0}, \tag{4}$$

where $L = p_0/\rho g$ is the vertical atmosphere scale.

The most general divergence-free flow velocity $\mathbf{v}$ can be decomposed into its poloidal $\mathbf{v}_p = (v_r, 0, v_z)$ and toroidal $v_\phi \hat{\mathbf{e}}_\phi$ parts, i.e., $\mathbf{v} = \mathbf{v}_p + v_\phi \hat{\mathbf{e}}_\phi$. Here, $\mathbf{v}_p = \nabla \times (\psi \cdot \nabla\phi) = \nabla\psi \times \nabla\phi$. Furthermore, $\psi(r, \phi, z)$ is the stream function

$$v_r = -\frac{1}{r}\frac{\partial\psi}{\partial z}, \quad v_z = \frac{1}{r}\frac{\partial\psi}{\partial r}. \tag{5}$$

Similar to Ref. 18, as the model vortex stream function, we use the following representation:

$$\psi = v_0 r^2 \frac{z}{L} \exp\left(-\frac{r^2}{r_0^2}\right), \tag{6}$$

where $v_0 = const$ is the characteristic vortex velocity and $r_0 = const$ is the characteristic vortex radius. Making use of Eqs. (5) and (6), we have

$$\frac{v_r}{v_0} = -\frac{r}{L}\exp\left(-\frac{r^2}{r_0^2}\right) \tag{7}$$

and

$$\frac{v_z}{v_0} = 2\frac{z}{L}\left(1 - \frac{r^2}{r_0^2}\right)\exp\left(-\frac{r^2}{r_0^2}\right). \tag{8}$$

The magnetic field is also decomposed into its poloidal and toroidal components, i.e., $\mathbf{B} = \mathbf{B}_p + B_\phi \hat{\mathbf{e}}_\phi$. The poloidal magnetic field is represented by $\mathbf{B}_p = \nabla \times \mathbf{A}$ with the vector potential $\mathbf{A}(r, z)$, where the vector potential possesses only a ϕ component

$$\mathbf{A} = (0, A_\phi(r, z), 0). \tag{9}$$

In this model, we consider that

$$\frac{A_\phi}{B_0} = r\frac{z}{L}\exp\left(-\frac{r^2}{r_0^2}\right), \tag{10}$$

where $B_0 = const$ stands for the characteristic magnetic field strength. Then, we find the following expressions for the magnetic field components:

$$\frac{B_r}{B_0} = -\frac{1}{B_0}\frac{\partial}{\partial}A_\phi = \frac{v_r}{v_0} = -\frac{r}{L}\exp\left(-\frac{r^2}{r_0^2}\right) \tag{11}$$

and

$$\frac{B_z}{B_0} = \frac{1}{rB_0}\frac{\partial}{\partial r}(rA_\phi) = \frac{v_z}{v_0} = 2\frac{z}{L}\left(1 - \frac{r^2}{r_0^2}\right)\exp\left(-\frac{r^2}{r_0^2}\right). \tag{12}$$

The ϕ component of the vector Eq. (1) can be represented as

$$\frac{v_r}{r}\frac{\partial}{\partial r}(rv_\phi) + v_z\frac{\partial v_\phi}{\partial z} = \frac{1}{\mu_0\rho}\left(\frac{B_r}{r}\frac{\partial}{\partial r}(rB_\phi) + B_z\frac{\partial B_\phi}{\partial z}\right). \tag{13}$$

Equation (13) is an equation describing the dynamics of the vertical vorticity. The term proportional to v_r describes the effect of the radial flow that tends to localise the vorticity $\omega_z = (1/r)\partial(rv_\phi)/\partial r$ in a narrow column around the axis of symmetry and the term $v_z\partial v_\phi/\partial z$ describes the vorticity advection along z. The stationary magnetised plasma vortex arises when these two effects balance each other and Eq. (13) determines the toroidal components of the velocity v_ϕ and the magnetic field B_ϕ. From Eqs. (7), (8), (11) and (12), we obtain the following relations between the toroidal magnetic field and the velocity components:

$$\frac{B_\phi}{B_{\phi 0}} = \frac{v_\phi}{v_{\phi 0}} = \frac{r}{r_0}\frac{z}{L}\exp\left(-\frac{r^2}{r_0^2}\right), \tag{14}$$

where $B_{\phi 0}$ and $v_{\phi 0}$ are the characteristic toroidal magnetic field strength and the velocity, respectively. According to Eq. (3), the characteristic velocity and the magnetic field are related to each other as $v_0 = B_0/\sqrt{\rho\mu_0}$ and $v_{\phi 0} = B_{\phi 0}/\sqrt{\rho\mu_0}$. The structure described by the obtained B_r, B_ϕ and B_z components of the magnetic field is shown in Fig. 1.

Equations (4), (11), (12) and (14) are the exact vortex solution of the MHD equilibrium equations for an ideal incompressible plasma in a gravitational field. In the solar context, relative to direct magnetic field measurements, it is often easier to measure the radial profile of the toroidal velocity of solar tornadoes (off-limb with Doppler velocities and on-disc with Local Correlation Tracking (LCT) of intensity). Hence, Eq. (14) could be exploited to indirectly estimate the radial toroidal magnetic field profile, which can be very challenging to measure directly, e.g., from the Zeeman

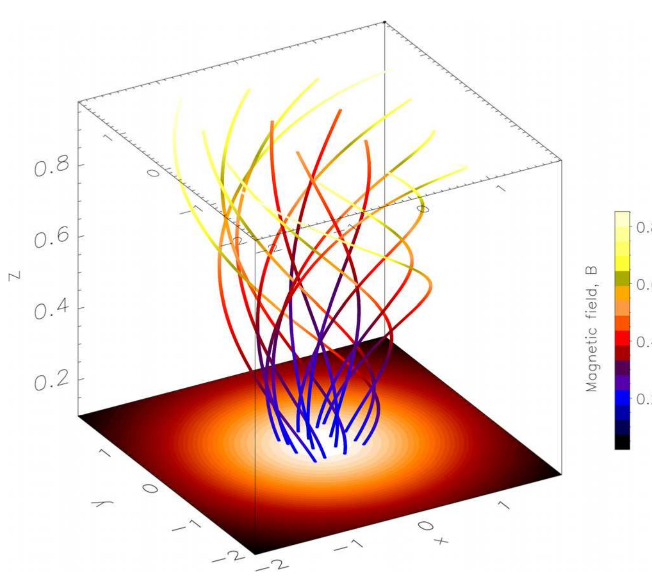

FIG. 1. Three-dimensional structure of the magnetic field plotted using Eqs. (11), (14), and (12) for B_r, B_ϕ, and B_z magnetic field components, respectively. Here, we used $B = B/B_0$; $X, Y = (x, y)/r_0$; and $Z = 10z/L$ to satisfy $z/L \ll 1$.

or Hanle effects, especially if the desired magnetic field components are mostly perpendicular to the line-of-sight.

In summary, modelling the dynamics of solar tornadoes is key to understanding their energy transfer between the lower and upper atmospheres.[10] The elementary model described in this work of a steady state incompressible ideal plasma vortex with twisted magnetic fields and flows is the exact solution of the MHD equilibrium equations. As in a Burgers vortex,[21] the inward, radial flow tends to localise the vorticity in a narrow column around the symmetry axis. At the same time, the vertical flow tends to spread the vorticity in the z-direction. The stationary vortex arises when the two effects are balanced. Although an actual solar tornado is likely to be more complicated than this axially symmetric model, the proposed four parameters (v_0 and $v_{\phi 0}$ are the characteristic fluid motion velocities, r_0 is the vortex radius, L is the characteristic atmospheric scale height), serve as the theoretical foundation to help us start parameterising the observed solar tornadoes.

O.O.G. and O.A.P. are thankful to the Program of Prezidium of the Russian Academy of Sciences No. 28 **and the state task of IPE RAS** for the partial financial support. W.H. offers thanks for partial support of the U.S. Department of Energy under Grant No. DE-FG02-04ER54742 and the Space and Geophysics Laboratory at The University of Texas in Austin. V.F. and G.V. are grateful for support from The Royal Society (International Exchanges Scheme), STFC Consolidated Grant and ISSI (International Space Science Institute) through the team "Towards Dynamic Solar Atmospheric Magneto-Seismology with New Generation Instrumentation."

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