Soil Dynamics and Earthquake Engineering 101 (2017) 153–156

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

# Correlations between the stress paths of a monotonic test and a cyclic test under the same initial conditions

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ABSTRACT

In most experimental studies on liquefaction, cyclic loadings are applied on specimens with various initial conditions. However, few studies compared cyclic test results with monotonic results under the same initial conditions. The relation between monotonic tests and cyclic tests is crucial for understanding liquefaction mechanics and liquefaction resistance. This work compares the stress paths of a monotonic test with those of a cyclic test under the same initial conditions, and concluded that the stress path of monotonic tests envelops the stress path of cyclic tests under the same initial conditions. In addition, a new parameter, Level of Liquefaction Index (LI) is proposed to evaluate the liquefaction resistance of specimens under various initial conditions, and a linear relationship between LI and number of cycles at failure is found.

## 1. Introduction

In current practice, liquefaction resistance is determined by using the procedure proposed by Seed and Harder [1]. In this method, the cyclic resistance of a sample with an initial confining stress of 100 kPa is first determined in level ground condition. Then two empirical correction factors $K_\sigma$ and $K_\alpha$ are used to modify the cyclic resistance of sand at an arbitrary vertical stress ($\sigma_{vc}$) and initial static shear stress ($\tau_{st}$), $CRR_{\sigma,\alpha}$, is given by:

$$CRR_{\sigma,\alpha} = CRR_{100,0} \times K_\sigma \times K_\alpha$$

Where CRR is the cyclic liquefaction resistance ratio, defined as $\frac{\tau_{cyc}}{\sigma'_{vc}}$. $\tau_{cyc}$ is the amplitude of cyclic shear stress, and it is taken when liquefaction is triggered at the 10th cycle, which represents a magnitude M = 6.75 earthquake [2]. $CRR_{100,0}$ is the liquefaction resistance determined in laboratory test, under $\sigma_{vc}$ of 100 kPa and $\tau_{st}$ of 0. $\alpha$ is the level of initial static shear stress, defined as $\frac{\tau_{st}}{\sigma'_{vc}}$. Empirical correction factor $K_\sigma$ is used to consider the effect of an arbitrary $\sigma_{vc}$, and $K_\alpha$ is used to consider the effect of an arbitrary $\tau_{st}$.

However, this method is not recommended in practice [3], due to the lack of coverage and consistency in the current understanding of the correction factor $K_\alpha$. In addition, the directional of the initial static shear stress, loading mode, and type of sand also affect the correction factor, and the correction factor does not consider those effects [4]. Many studies have been conducted in recent 20 years to improve the usability and reliability of $K_\alpha$ [5–7]. However, it is difficult to consider all those effects in one single correction factor. In addition, most experimental liquefaction studies only use cyclic tests with various initial conditions, few studies considered the shear behaviour of a monotonic test under the same initial conditions [8]. This study aims to find a possible way of determining liquefaction resistance by using the relationship between the monotonic and cyclic stress paths of dry Leighton Buzzard sand by a series of bi-directional cyclic simple shear tests.

## 2. Methods

Five series of tests are carried out using the first commercially available Variable Direction Dynamic Cyclic Simple Shear system (VDDCSS). More details about this apparatus are given by Li et al. [9]. Leighton Buzzard sand (Fraction B) is used in this study. Its maximum and minimum void ratios are 0.79 and 0.46, respectively [10]. A cylindrical specimen with 70 mm in diameter and 17 mm in height is tested. A stack of low–friction Teflon coated rings with 1.16 mm high each is placed outside the membrane of the specimen. Specimens are prepared by using dry deposition technique [11]. A vertical stress of 200 kPa is used to consolidate samples, and the relative density after consolidation is carefully controlled around 48%. The change of

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http://dx.doi.org/10.1016/j.soildyn.2017.07.023
Received 14 February 2017; Received in revised form 11 May 2017; Accepted 30 July 2017
Available online 07 August 2017

**Table 1**
Testing conditions for tests on the envelope effect.

| | The direction of initial static shear stress | SSR | Amplitudes of cyclic shear stress (kPa) |
|---|---|---|---|
| Group 1 | None | 0 | 6.49, 7.79, 8.31, 9.09 |
| Group 2 | 0° | 0.03 | 5.19, 5.45, 5.71, 6.49 |
| Group 3 | 0° | 0.05 | 2.60, 3.90, 4.42, 5.19, 7.79 |
| Group 4 | 90° | 0.03 | 5.19, 5.71, 6.49, 7.79 |
| Group 5 | 90° | 0.05 | 5.19, 5.45, 5.71, 6.49, 7.79 |

vertical stress under zero vertical strain in a dry specimen is assumed equivalent to the excess pore water pressure generated when a saturated specimen is tested under true undrained conditions [12–14]. In the five series of test, different initial static shear stress ratio (SSR, $\frac{\tau_{st}}{\sigma_{vc}}$), and the amplitude of cyclic shear stress are considered, the details of performed tests are summarized in Table 1. Initial static shear stress is applied along the 0° (primary shearing direction) and 90° (secondary shearing directions that prependicular to the primary shearing direction) directions of the VDDCSS. This is achieved by using two shear actuators that act prependicular to each other in the horizontal plane of the VDDCSS. To better control the stress path, stress control is adopted in this study.

## 3. Results

### 3.1. The envelope effect

In test results, it is showed that the stress path of a cyclic tests is below the stress path of a monotonic test under the same initial condition, and this effect is termed as "envelope effect" in this study. Three pairs of tests are used to demonstrate the envelope effect, which have different SSRs from 0 to 0.05 and the same amplitude of cyclic shear stress of 7.79 kPa.

Fig. 1 shows the stress paths of two tests with no initial static shear stress and cyclic shear stress of 7.79 kPa. Two horizontal lines show the peaks and troughs of applied normalized cyclic shear stress, $\frac{\tau_{st} \pm \tau_{cyclic}}{\tau_m}$ of 0.039 and –0.039, which show a good agreement with the measured cyclic shear stress. It should be noted that the measured cyclic shear stress may differ from applied cyclic shear stress under unstable states. It can be seen that the effective vertical stress decreases consistently from the beginning to the end. The rate of decreasing is lower in the middle of the cyclic shearing. Compared with the stress path of the corresponding monotonic test, it can be seen that when the stress path of the cyclic test is close to or beyond the stress path of the corresponding monotonic test, the effective vertical stress decreases dramatically. Similar relation is also reported in triaxial test. It was found that when the effective stress path of a cyclic test is close to the peak strength envelope obtained from monotonic compression test, failure is triggered [15,16]. Mao and Fahey [8] also report the difference in stress path between a cyclic test and its corresponding monotonic test at the beginning of shearing. In addition, Sivathayalan and Ha [4] observed the dramatic decrease of effective vertical stress at the end of shearing, and it will be discussed later using an example.

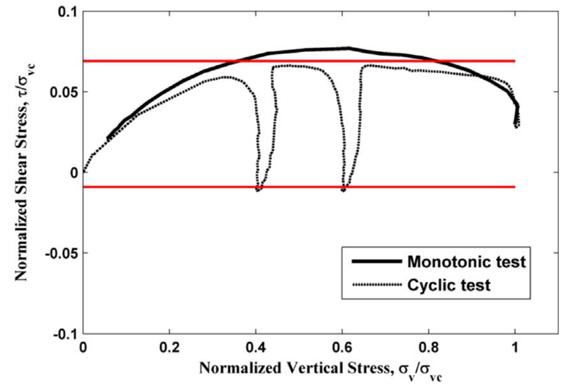

**Fig. 2.** Stress paths of tests with the consolidation shear stress ratio of 0.03.

Fig. 2 shows the stress paths of a pair of tests with the SSR of 0.03. Two horizontal lines show the peaks and troughs boundaries of applied normalized cyclic shear stress, at the $\frac{\tau_{st} \pm \tau_{cyclic}}{\tau_m}$ of 0.069 and –0.009, show an adequate agreement with the measured cyclic shear stress. The stress path of the cyclic test is closer to the stress path of the corresponding monotonic test, compared with that in Fig. 1. In the first quarter of the first cycle, the effective vertical stress drops dramatically, and the measured maximum shear stress is smaller than the applied maximum shear stress. Similar to the stress path showed in Fig. 1, the dramatic change of effective vertical stress in the cyclic test occurs when its stress path is above or near the stress path of the monotonic test.

Fig. 3 shows the stress paths of a pair of tests with the consolidation shear stress ratio of 0.05. Two horizontal lines show the peaks and troughs boundaries of applied normalized cyclic shear stress, at the $\frac{\tau_{st} \pm \tau_{cyclic}}{\tau_m}$ of 0.089 and 0.019, show a great difference compared with the measured cyclic shear stress. The stress path of the cyclic test is close to the stress path of the corresponding monotonic test, and the applied cyclic shear stress is not reached. In those three tests, the stress paths of monotonic tests are always above their corresponding cyclic tests when effective vertical stress is gradually decreased without a sudden drop, and the envelope effect is well demonstrated. Dyvik et al. [17] reported similar relations in a group of tests including monotonic and cyclic tests on clay, and stated that the cyclic load capacity is smaller than the static load capacity. This is due to the changing of soil fabric caused by cyclic loadings. Soil structure is broken down during the cyclic loadings, which causes a tendency of volumetric reduction in the soil sample. In undrained simple shear tests, the volumetric reduction is reflected by the decreasing of effective vertical stress [16]. This is well illustrated by

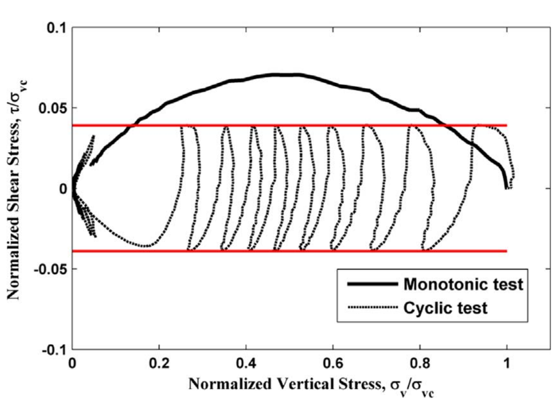

**Fig. 1.** Stress paths of tests with no consolidation shear stress.

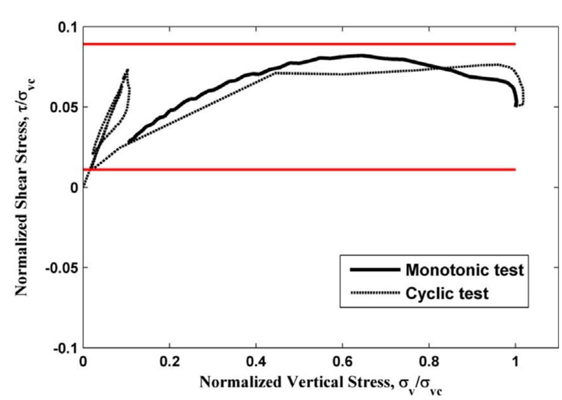

**Fig. 3.** Stress paths of tests with the consolidation shear stress ratio of 0.05.

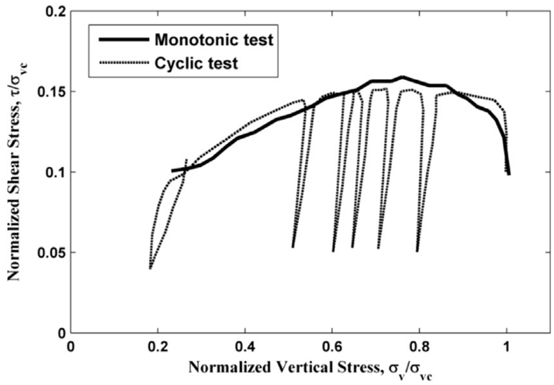


**Fig. 4.** Stress paths of tests with the consolidation shear stress ratio of 0.1 [4].

the effective stress paths of monotonic and cyclic tests in Fig. 3.

Similar relation is also found by Sivathayalan and Ha [4]. Fig. 4 shows the stress paths of tests with the SSR of 0.1, using data retrieved from the study conducted by Sivathayalan and Ha [4]. The two tests were conducted on specimens under a consolidation shear stress of 10 kPa and an effective vertical stress of 98 kPa with a similar relative density. The cyclic shear stress is 5 kPa. It can be seen that the normalized vertical stress drops dramatically in the first quarter of the first cycle, and the corresponding stress path is above the stress path of the monotonic test. In addition, the normalized vertical stress drops a lot in the 6th cycle, and the corresponding stress path is also slightly above the stress path of the monotonic test. It should be noted that the specimen failed at the 6th cycle, and a large shear deformation of 7% developed at failure. It clearly shows that when the stress path of the cyclic tests is above the stress path of the corresponding monotonic test, the effective vertical stress decreases dramatically.

From the above results, several conclusions can be drawn. First, at the beginning of loading, when the cyclic shear stress is greater than the shear stress of its corresponding monotonic test that has the same initial conditions, the effective vertical stress may drop significantly until the stress path of the cyclic test is inside that of the monotonic test. Second, in the process of loading, when the stress path of a cyclic test reaches or passes the stress path of its corresponding monotonic test, the specimen may fail in few cycles. To better investigate the relations, a new parameter, LI, is proposed:

$$LI = \frac{\tau_{st} + \tau_{cyclic}}{\tau_m} \quad (1)$$

In which LI is Liquefaction Index, it shows the possibility of liquefaction. It is independent of stress level and stress path. $\tau_{cyclic}$ is the amplitude of applied cyclic shear stress, $\tau_{st}$ is initial static shear stress, and $\tau_m$ is the peak shear stress in its corresponding monotonic test.

### 3.2. Linear relationship between LI and number of cycles

In tests Group 1 to Group 5, the effect of different magnitudes and directions of consolidation shear stresses were considered. The SSRs of 0, 0.03, and 0.05 were tested in the 0° (primary shearing direction) and 90° (secondary shearing directions that prependicular to the primary shearing direction) directions of the VDDCSS, and various cyclic shear stress amplitudes were tested. It should be noted that different amplitudes were used to get a number of cycles around 3–20 at liquefaction. In this study, 3–20 is considered as the effective number of cycles.

Fig. 5 summarizes the relationships between LI–number of cycles in 0° tests with different SSRs, together with a group of tests without SSR. It shows a linear relation between LI and number of cycles in each group of tests. At SSR = 0, the LI value at 10th cycle is 0.52, and the LI value is increased to 0.79 and 0.85 at the SSRs of 0.03 and 0.05, respectively. It can be concluded that with increasing SSR, LI is increased,

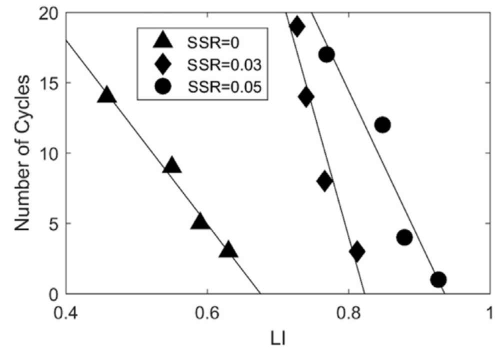


**Fig. 5.** LI versus the Number of cycles in 0° tests.

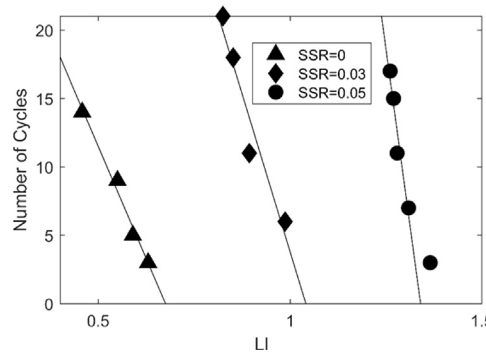


**Fig. 6.** LI (used total shear stress) versus the number of cycles in 90° tests.

and the liquefaction resistance is decreased.

Fig. 6 summarizes the relations between LI–number of cycles in all 90° tests with different SSRs, together with a group of tests without SSR. LI is calculated using the combined shear stress, which combines the shear stress vectors in the X and Y directions. It also shows a linear relation between LI and number of cycles. At SSR = 0, the LI value at 10th cycle is 0.52, and the LI value is increased to 0.95 and 1.3 at the SSRs of 0.03 and 0.05, respectively. It can be concluded that with increasing SSR, LI is increased, and the liquefaction resistance is decreased. Generally, in both 0° tests and 90° tests, increasing SSR, increases the LI and decreases liquefaction resistance.

The linear relationship between the number of cycles at liquefaction and LI can be helpful in determining the CRR (cyclic liquefaction resistance ratio, defined as $\frac{\tau_{cyc}}{\sigma'_{vc}}$). The $\tau_{cyc}$ is the amplitude of cyclic shear stress taken when liquefaction is triggered at the 10th cycle, and it can be difficult to obtain in experiment. Once the linear relation is obtained (by experiment) or proposed (by experience), the $\tau_{cyc}$ can be calculated using Eq. (1) and CRR can be calculated using $\frac{\tau_{cyc}}{\sigma'_{vc}}$. In addition, the $\tau_{cyc}$ at any number of cycles can be predicted using this relationship.

## 4. Conclusion

The correlation between the stress path of a cyclic test and its corresponding monotonic tests has been investigated in this study. Results showed that the stress path of a monotonic test envelopes that of its corresponding cyclic test. At the beginning of loading, when the cyclic shear stress is greater than the shear stress of its corresponding monotonic test under the same initial conditions, the effective vertical stress may drop until a stable state is reached. In the process of loading, when the applied cyclic shear stress is near or greater than the peak shear stress in its corresponding monotonic test, the specimen may fail quickly. To better investigate the relations, a new parameter, LI, is proposed. A linear relationship between LI and the number of cycles is found in tests with consolidation shear stresses in different magnitudes and directions. The linear relationship can be used to estimate the number of cycles at liquefaction in any given testing condition, and liquefaction resistance at 10th cycle.

## Acknowledgments

The support of the National Natural Science Foundation of China (NSFC Contract No. 11172312/A020311) and the International Doctoral Innovation Centre (IDIC) scholarship scheme is gratefully acknowledged. We also acknowledge the support from the Ningbo Education Bureau, Ningbo Science and Technology Bureau, China's MoST, and the University of Nottingham. This work was supported by the Engineering and Physical Sciences Research Council [Grant number EP/L015463/1], providing partial support.

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