

# Changing Macroeconomic Dynamics at the Zero Lower Bound\*

Philip Liu  
*International Monetary Fund*

Haroon Mumtaz  
*Queen Mary University*

Konstantinos Theodoridis  
*Bank of England*

Francesco Zanetti  
*University of Oxford*

April 2017

## Abstract

This paper develops a change-point VAR model that isolates four major macroeconomic regimes in the US since the 1960s. The model identifies shocks to demand, supply, monetary policy, and spread yield using restrictions from a general equilibrium model. The analysis discloses important changes to the statistical properties of key macroeconomic variables and their responses to the identified shocks. During the crisis period, spread shocks became more important for movements in unemployment and inflation. A counterfactual exercise evaluates the importance of lower bond-yield spread during the crises and suggests that the Fed's large-scale asset purchases helped lower the unemployment rate by about 0.6 percentage points, while boosting inflation by about 1 percentage point.

*JEL codes:* E42, E52.

*Keywords:* change-point VAR model, global financial crisis, large-scale asset purchases.

---

\*The paper is best viewed in color. We would like to thank Sophocles Mavroeidis and seminar participants at the Bank of England, the International Monetary Fund, and the University of Oxford for extremely helpful comments and suggestions. Please address correspondence to: [pliu@imf.org](mailto:pliu@imf.org), [h.mumtaz@qmul.ac.uk](mailto:h.mumtaz@qmul.ac.uk), [konstantinos.theodoridis@bankofengland.co.uk](mailto:konstantinos.theodoridis@bankofengland.co.uk), and [francesco.zanetti@economics.ox.ac.uk](mailto:francesco.zanetti@economics.ox.ac.uk). The views expressed in this work are those of the authors and do not necessarily reflect those of the Bank of England or the International Monetary Fund.

# 1 Introduction

Beginning in the summer of 2007, money markets around the world experienced sustained periods of dysfunction with sharply higher short-term interest rates for commercial paper and interbank borrowing. This intense liquidity squeeze led the Federal Reserve (Fed) to substantially lower its federal funds rate (FFR) and act as the liquidity provider of last resort to supply funds to banks and the broader financial system via its Term Auction Facility (TAF).<sup>1</sup> The FFR, the Fed's traditional policy instrument, reached its effective zero lower bound (ZLB) in December 2008, and the Fed faced the challenge of how to further ease the stance of monetary policy as the economic outlook deteriorated. The Fed responded in part by expanding its monetary policy toolkit to purchase substantial quantities of public and private sector securities with medium and long maturities.<sup>2</sup>

While the FFR had reached its effective ZLB, the large-scale asset purchases (LSAPs), which reduced the supply of riskier long term assets and increased the supply of safer liquid assets (bank reserve), appear to have been effective in driving down private sector borrowing rates—the intermediate target of conventional monetary policy expansions. Gagnon et al. (2010) estimate that the LSAPs reduced the overall size of the 10-year term premium by somewhere between 30 and 100 basis points, with most estimates in the lower and middle third of this range. Furthermore, they find that the program had an even larger effect on reducing yields on riskier government-sponsored enterprise and mortgage-backed securities. Similarly, Neely (2015) also finds that the programs not only reduced long-term US bond yields but also significantly reduced long-term foreign bond yields and the spot value of the dollar. Baumeister and Benati (2013) find that a compression in the long-term yield spread exerted a powerful effect on output growth and inflation. Swanson and Williams (2014a) and Swanson and Williams (2014b) investigate the effect of the ZLB on the behavior of short- and long-run yields in the US, UK, and Germany and establish that the effectiveness of mon-

---

<sup>1</sup>Wu (2011) estimates the term auction facility helped lower the three-month Libor–OIS spread by 50 or 55 basis points during the crisis.

<sup>2</sup>On November 25, 2008, the Fed announced that it would purchase up to \$100 billion of government-sponsored-enterprise debt and up to \$500 billion in mortgage-backed security debt to reduce risk spreads on GSE debt and mitigate turmoil in the market for housing credit. On March 18, 2009, the Federal Open Market Committee (FOMC) press release announced that the Fed would purchase an additional \$750 billion of agency mortgage-backed securities, an additional \$100 billion in agency debt, and \$300 billion of longer-term Treasury securities. More recently, the FOMC announced at its November 2010 meeting the intention to purchase another \$600 billion in longer-term Treasury securities by the middle of 2011.

etary policy and the fiscal multiplier were close to normal during the crisis. However, yields became less responsive and the fiscal multiplier increased when the expected length of the ZLB increased. The underlying conjecture common across all these studies is that the lower long-term borrowing costs stimulated economic activity. However, none of the above-mentioned studies focuses on the effect of lower borrowing costs on real activities and unemployment.

The central focus of this paper is to assess the extent to which macroeconomic dynamics have changed under the Fed’s “non-standard” monetary policy, the LSAPs program, while its traditional policy instrument was at the ZLB.<sup>3</sup> The analysis proposes a novel method to investigate this issue. We estimate changes in macroeconomic dynamics by developing an innovative point-change vector autoregression (VAR) model that allows for different regimes throughout the sample period and identifies a variety of shocks (supply, demand, monetary policy, and the spread between long- and short-run maturities) from the theoretical reactions of an innovative general equilibrium model. This approach enables the VAR model to endogenously identify changes to the structure of the US economy as well as variations to the properties of exogenous shocks during the sample period. A wealth of studies has documented the presence of different regime shifts in the US economy.<sup>4</sup> However, as noted by Gagnon et al. (2010), these models are based primarily on the Great Moderation period, which could understate severely the incidence and the severity of ZLB events. Our change-point VAR model with non-recurrent states offers a novel way to estimate changes in the transmission mechanism of a variety of shocks over an extensive period.

The analysis isolates results that refer to the statistical properties of the series, the changes in the transmission mechanism of shocks, and the contribution of disturbances to explain movements in the data. The key findings are the following. First, important statistical properties of key macroeconomic variables have changed throughout the sample period. In particular, the analysis shows that the persistence of inflation and money growth has declined steadily. Interestingly, changes in the properties of these two series are remarkably similar across different time periods, providing strong statistical evidence of the link between money growth and inflation. On the

---

<sup>3</sup>We use the terms “non-standard” and “unconventional” monetary policy interchangeably.

<sup>4</sup>Among other studies, see those by Benati and Mumtaz (2007), McConnell and Perez-Quiros (2000), Cogley and Sargent (2005), Primiceri (2005), Baumeister et al. (2010), Rudebusch and Wu (2007), Mumtaz and Surico (2009), Mavroeidis (2010), and Bianchi (2013).

other hand, the persistence of the unemployment rate and the nominal interest rate has remained broadly similar across different regimes, although it increased slightly during the sample period. The unconditional variance of the unemployment rate, inflation, and stock price growth increased substantially during the crisis period.

Second, the model shows that the response of the economy to key macroeconomic variables to shocks changed throughout the sample period. In particular, the response of the nominal interest rate to demand and supply shocks decreased steadily, in line with studies related to the Great Moderation period, as in Stock and Watson (2003). The reaction of the bond-yield spread declines over the sample period while the response of inflation increases, going from 0.1 percentage points in the first regime to 0.2 percentage points in the fourth regime. We interpret the increase in the size of the response of inflation as a sign of improved effectiveness of the Fed's unconventional monetary policy since even small changes in the interest rate spread are effective in influencing the economy.

Third, the analysis shows that supply and monetary policy shocks explain the bulk of fluctuations in inflation whereas yield spread shocks are important for unemployment. The effect of the interest rate shock increases substantially during the late 1990s and mid-2000s, showing that the stance of monetary policy was important for the dynamics in the data and therefore suggesting that the policy was an important contributor to the Great Moderation period. In addition, the historical contribution of yield spread shocks to unemployment and inflation increases substantially from early 2008 onwards, suggesting that these shocks played a relevant role for the dynamics of these variables during the crisis period.

Finally, we use the estimated model to simulate a counterfactual scenario to examine the impact of the Fed's policies that led to compressed long-term borrowing costs proxied by the 10-year spread on the economic outlook. The counterfactual exercise simulates a higher bond-yield spread of 60 basis points, as suggested in Baumeister and Benati (2013). We find that a lower spread had significant impact in supporting economic activity and higher inflation. Without the spread compression, the unemployment rate is estimated to be 0.6 percentage points higher and inflation an average of 1 percentage point lower in 2010.

This study is linked to the empirical literature that investigates the effect of non-conventional

monetary policy on the macroeconomy. Chung et al. (2012) show that estimates from a variety of models indicate that past and projected expansion of the Fed's securities holdings since late 2008 lower the unemployment rate, relative to what it would have been absent the purchases, by 1(1/2) percentage points by 2012. Nakajima (2011) explores the transmission of monetary policy shocks using a time-varying VAR model with stochastic volatility in the context of Japan and finds that the ZLB has a sizeable effect on the response of the short-term nominal interest rate, but it has negligible effects on other key macroeconomic variables. Giannone et al. (2012) estimate a large VAR model on Euro Area data for different time horizons and establish that the reaction of key macroeconomic variables remains similar across time and countries. Kapetanios et al. (2012) use an array of econometric models, including a change-point structural VAR, to evaluate the effect of quantitative easing on output and inflation in the UK. They establish that the policy effectively stimulates output, despite considerable uncertainty surrounding the estimates.<sup>5</sup> Finally, a number of studies have documented strong links between the term structure of interest rates and the rest of the macroeconomy (for instance, Ang and Piazzesi (2003), Diebold and Li (2006) and Diebold et al. (2006)). Given that long-term interest rates were identified as the main transmission channel of the Fed's LSAPs to the rest of the economy, this paper focuses on the macro-financial linkage in the transmission of macroeconomic shocks.

This study also is related to the strand of the literature that develops general equilibrium models with financial frictions to investigate changes in transmission mechanism of macroeconomic shocks. Andres et al. (2004), Goodfriend and McCallum (2007), Curdia and Woodford (2010), Del Negro et al. (2016), and Harrison (2012) examine the impact of unconventional monetary policy on economic activity in models where a spread between long- and short-run maturities arises endogenously. This paper develops a general equilibrium model that uses portfolio frictions to generate a spread between short- and long-term interest rates, as in Andres et al. (2004) and Harrison (2012), and it extends the framework by embedding indivisible labor, as in Gali (2011),

---

<sup>5</sup>Our study differs from Kapetanios et al. (2012) in two fundamental ways. First, we develop a novel, microfounded general equilibrium model to derive internally-consistent sign restrictions that identify the distinct effect of structural shocks on macroeconomic variables. We find that the theoretical restrictions are consistent with a broad class of macroeconomic models in the literature. Second, we address methodological issues related to the development and implementation of Bayesian change-point VAR methodology for the study of the transmission mechanisms of shocks.

and wage rigidities. In this way, the theoretical model is able to track the dynamics of the interest rate spread and unemployment in addition to inflation, real money balances, stock prices, and the nominal interest rate of standard New Keynesian models, thereby providing theoretical restrictions in the point-change VAR model for a wider set of variables.

Section 2 describes the theoretical model and details the sign restrictions from the theoretical model. Section 3 sets up the change-point VAR model and details the estimation and identification procedures. In Section 4, we discuss the results from the estimated model, we present the results from a counterfactual scenario that isolates the impact of the spread shock. Section 5 offers a summary and conclusion.

## 2 The Theoretical Model and Sign Restrictions

This section outlines the theoretical model and discusses the sign restrictions. Appendix A provides a detailed description of the theoretical model and describes the solution and calibration.

We base the model on the simplest version of the New Keynesian framework as developed by Ireland (2011), which accounts for the dynamics of inflation, the short-term nominal interest rate, money balances, and stock prices. We enrich this framework in two ways, first, by embedding portfolio frictions that make short- and long-term bonds imperfect substitutes and generating a spread between short- and long-term interest rates, as in Andres et al. (2004) and Harrison (2012). Second, we introduce nominal wage rigidities using quadratic adjustment costs on wages and unemployment based on the indivisible labor framework developed by Zanetti (2007) and Gali (2011). In this way, the model also accounts for fluctuations in the interest rate spread and unemployment, whose dynamic responses are important in identifying shocks in the change-point VAR model.<sup>6</sup> The model comprises a continuum of household, a representative finished-goods-producing firm, a continuum of intermediate-goods-producing firms, the government, and a central bank that sets the short-term nominal interest rate using a Taylor rule.

---

<sup>6</sup>The derivation of sign restrictions by a unified DSGE model provides internally consistent restrictions that are in line with a broad class of models, as we discuss below. Note that the change-point VAR model is overparametrized compared to the DSGE model and therefore has more degrees of freedom to match the data.

We use the theoretical model to generate robust variable responses of shocks to monetary policy, bond yields, and supply and demand, which are needed to identify these shocks in the empirical model. To derive the sign restrictions to impose on the change-point VAR model, we use the theoretical framework to determine how each variable reacts to shocks. To produce robust responses to a one positive percentage point increase in each shock that is robust across a broad range of the parameters' calibration, we simulate the theoretical model by drawing 10,000 times from parameters' values that are uniformly and independently distributed over a wide range of plausible values. The range value for each parameter includes a wide range of plausible values and is reported in Table 3 of Appendix A. As in Pappa (2009), Canova and Paustian (2011), and Mumtaz and Zanetti (2012, 2015), we discard the regions of the two distributions below and above 2.5 and 97.5 percentiles, respectively, to eliminate extreme responses. In this section, we restrict focus on the variables used in the empirical investigation and therefore show responses of the short-term nominal interest rate ( $r_t$ ), stock prices ( $q_t$ ), unemployment rate ( $u_t$ ), money holdings ( $m_t$ ), price inflation ( $\pi_t$ ), and the interest rate spread ( $r_{L,t} - r_t$ ).

To implement the identification scheme, we impose the sign restrictions, as summarized in Table 1, on the first-period reaction of the VAR model.<sup>7</sup> Subsequently, the data can freely inform the dynamics of the response. Note that by using these restrictions, we are able to disentangle the effect of these four shocks in the data.

The theoretical model enables us to produce internally consistent restrictions that uniquely identify the structural disturbances. These restrictions are consistent with a broad class of macroeconomic models. For instance, the sign restrictions on monetary policy, demand, and supply shocks are in line with the responses in Smets and Wouters (2007), and the restrictions on the spread shock are in line with Baumeister and Benati (2013) and references therein.

---

<sup>7</sup>To incorporate the insensitivity of the nominal interest rate to shocks during the crisis period, we impose that the nominal interest rate does not react to shocks during the financial crisis.

Table 1: Sign restrictions in the benchmark model.

Shock	Variable					
	$r_t$ (*)	$r_{L,t} - r_t$	$u_t$	$\pi_t$	$m_t$	$q_t$
Monetary policy shock	$\geq$	$<$	$>$	$<$	$<$	$<$
Spread shock	$<$	$>$	$>$	$<$	$>$	$<$
Demand shock	$>$	$>$	$<$	$>$	$<$	$>$
Supply shock	$\geq$	$\leq$	$\leq$	$\leq$	?	$\geq$

Notes: Entries show the sign of the first period responses of the variables to shocks. The sign ‘ $\geq$ ’ refers to a positive response, ‘ $\leq$ ’ to a negative response, and ‘?’ to an undetermined response as the sign can be either positive or negative, depending on the calibration of the model.

### 3 Change-point VAR Model

In this section, we describe the empirical VAR model, the sampling procedure for the estimation, and the derivation of the marginal likelihood of the change-point VAR model. We then discuss the identification scheme based on sign restrictions.

To examine possible regime changes, we estimate the following VAR model,

$$Z_t = c_S + \sum_{j=1}^K B_S Z_{t-j} + \Omega_S^{1/2} \varepsilon_t, \quad (1)$$

where the data matrix  $Z_t$  contains monthly data on the federal funds rate, the 10-year government bond-yield spread (defined as the 10-year yield minus the FFR), the unemployment rate, annual CPI inflation, annual M2 growth, and annual change in stock prices.  $B_S$  and  $\Omega_S$  are regime dependent autoregressive coefficients and reduced form variance covariance matrices.

The VAR model allows for  $M$  breaks at unknown dates, as in Chib (1998), and we model the breaks via the latent state variable,  $S$ . This state variable is assumed to follow an  $M$  state Markov chain with restricted transition probabilities,  $p_{ij} = p(S_t = j | S_{t-1} = i)$ , given by

$$\begin{aligned} p_{ij} &> 0 \text{ if } i = j \\ p_{ij} &> 0 \text{ if } j = i + 1 \\ p_{MM} &= 1 \end{aligned} \quad (2)$$



$$p_{ij} = 0 \text{ otherwise.}$$

For example, if  $M = 4$ , the transition matrix is defined as

$$\tilde{P} = \begin{pmatrix} p_{11} & 0 & 0 & 0 \\ 1 - p_{11} & p_{22} & 0 & 0 \\ 0 & 1 - p_{22} & p_{33} & 0 \\ 0 & 0 & 1 - p_{33} & 1 \end{pmatrix}.$$

Equations (1) and (2) define a Markov switching VAR with non-recurrent states where transitions are allowed in a sequential manner. For example, to move from Regime 1 to Regime 3, the process has to visit Regime 2. Similarly, transitions to past regimes are not allowed. As discussed in Sims et al. (2008), this structure is similar to a Markov Switching model, but it models structural breaks as multiple change points where the state can either remain at the current regime or switch to the subsequent regime. Since the state is not allowed to switch back to the preceding regime, the analysis precludes the case of recurrent regimes. Our structure implies that any new regimes are given a new label rather than being linked explicitly to past states (as in a standard Markov switching model). We believe that this approach is advantageous over standard Markov switching models since it internalizes the long-lasting effect of structural changes by preventing frequent and quick regime reversals.<sup>8</sup> As we discuss below, this form of regime switching allows us to isolate periods of interest (for example, the period of the financial crisis) and adapt our shock identification scheme accordingly.

---

<sup>8</sup>To compare methods, we have estimated a MS-VAR model. Given the large number of regimes, we restrict the transition matrix as shown in Sims (2010). Similarly, given the large number of parameters, we follow the methodology in Sims et al. (2008) and obtain the posterior mode by breaking the maximisation of the log likelihood of the model into three blocks (autoregressive, covariance and probabilities). We find that despite the marginal likelihood is larger in the MS-VAR model, the dynamic properties of the models are similar across shocks and regimes. Therefore the specific structure imposed by the change-point VAR model on the transition matrix does not affect the economic predictions of the model. An appendix that details the findings is available on request to the authors.

### 3.1 Estimation and Selection of the Number of Change Points

We follow Chib (1998) and adopt a Bayesian Gibbs sampling approach to the estimation of the change-point VAR models. Appendix B provides a detailed description of the prior and appendix C describes the main steps of the algorithm. We estimate the change-point VAR model using 200,000 replications of the Gibbs sampler and discard the first 190,000 as burn-in.<sup>9</sup>

The choice of the number of breakpoints is a crucial specification issue. We select  $M$  by comparing the marginal likelihood across different models with  $M = 1, \dots, 3$ . The limit of  $M = 3$  as the maximum number of breaks is largely driven by computational concerns and the limited number of observations covering the current financial crisis. Allowing for a larger number of breakpoints could result in some regimes with few observations and thus rendering estimates of the VAR coefficients unreliable. Similarly, the number of lags also could play an important role for the model's results. Therefore, we select the number of lags, ranging from 3 to 6, by comparing the models' marginal likelihood. The maximum lag length is set to 6 to ensure all regimes will last more than three years given the restriction that each regime must have at least  $N \times K + 2$  observations.<sup>10</sup> A higher number of lags would automatically rule out any breaks associated with an economic event lasting less than three years, a period of paramount interest for this study.

#### 3.1.1 Marginal Likelihood and the Identification of Structural Shocks

As described in Chib (1998) and Bauwens and Rombouts (2012), we estimate the marginal likelihood for the change-point model  $m$  by considering the following identity:

$$\ln G(Z_t | m) = \ln f(Z_t | m, \Theta, \tilde{P}) + \ln p(\Theta, \tilde{P} | m) - \ln g(\Theta, \tilde{P} | Z_t, m). \quad (3)$$

Equation (3) relates the marginal likelihood,  $\ln G(Z_t | m)$ , to the likelihood function,  $\ln f(Z_t | m, \Theta, \tilde{P})$ ,

---

<sup>9</sup>An appendix showing that the autocorrelation of the retained draws is fairly low and providing evidence of convergence to the ergodic distribution is available on request to the authors.

<sup>10</sup>In models with a large number of regimes and lags, there are instances when the estimation algorithm leads to regimes with a limited number of observations, letting the the prior heavily influencing the estimation. In order to prevent the issue, we limit the number of observations per regime to be equal to the number of coefficients in the VAR plus one, i.e.  $N \times K + 2$ . This choice is arbitrary, but it conforms to the number of observations required to estimate a VAR model equation by equation using ordinary least squares.

the prior distribution of the VAR parameters,  $\Theta \ln p(\Theta, \tilde{P} | m)$ , and the posterior distribution,  $\ln g(\Theta, \tilde{P} | Z_t, m)$ . We obtain this equation by simply re-arranging the Bayes rule and taking logs for computational convenience. Note that as  $\ln G(Z_t | m)$  does not depend on the parameters of the model, equation (3), in theory, can be evaluated at any value of the parameters. Following standard practice, we evaluate the marginal likelihood at the posterior mean. The first two terms on the right-hand side of equation (3) are easily evaluated whereas the normalizing constant of the posterior density,  $\ln g(\Theta, \tilde{P} | Z_t, m)$  is unknown. Evaluating this final term requires more work. As described in detail in Bauwens and Rombouts (2012), this term can be evaluated by considering reduced Gibbs runs on an appropriate factorization of  $g(\Theta, \tilde{P} | Z_t, m)$ . We use 10,000 additional Gibbs replications to evaluate  $g(\Theta, \tilde{P} | Z_t, m)$  at the posterior mean.

The identification scheme, based on sign restrictions, is implemented using the technique recently developed by Arias et al. (2014), which shows how to efficiently draw from the uniform distribution with respect to the Haar measure on the set of orthogonal matrices conditional on zero restrictions. The authors illustrate that this step is an important one, allowing the user to draw from the posterior distribution of structural parameters conditional on the sign and zero restrictions. Specifically, the matrix  $\Omega_S^{1/2}$  is a product of the Choleski factor ( $C_S$ ) of the state dependent variance-covariance matrix of the VAR residuals ( $\Sigma_S$ ) and the orthonormal matrix ( $Q_S Q_S' = I$ ), where  $I$  is the identity matrix

$$\Omega_S^{1/2} = C_S Q_S. \tag{4}$$

The matrix  $Q$  is drawn using Algorithm 4 in Arias et al. (2014).<sup>11</sup>

## 4 Results

This section focuses on our findings. First, we consider the model specification, the determination of the number of regimes, and changes in the statistical properties of the data across regimes. Second, we discuss the changes in macroeconomic dynamics across regimes. Third, we investigate

---

<sup>11</sup>Following an insightful comment from an anonymous referee, Appendix D shows how to use the model to validate predictions from theory.

the extent to which each shock contributes to the movements in the variables at different horizons, and we provide historical shock decomposition to study how shocks contributed to the dynamics of key macroeconomic data throughout the sample period. Finally, we consider a counterfactual simulation to evaluate the importance of spread shocks and monetary policy interventions during the crisis period.

## 4.1 Model Specification and Estimated Regimes

To implement the estimation, before using the theoretical restrictions from the theoretical model, we need to specify the variables for the change-point VAR model. To maintain the closest mapping between the theoretical and the empirical models, we set up a VAR model that includes the main variables that enter into the theoretical model, thereby using the short-term interest rate, long-term interest rate, unemployment rate, price inflation, money holdings and asset prices. We collect data for the effective FFR, 10-Year treasury bond yield at constant maturity, civilian unemployment rate, consumer price index (CPI), M2 definition of the money supply, and the average monthly closing price of the Dow Jones Industrial index. We draw data from the St. Louis FRED database, which are part of the monthly series that covers the period 1965:M4 to 2011M3.<sup>12</sup> The unemployment rate, CPI, and M2 are seasonally adjusted. The interest rate spread is defined as the 10-year yield minus the FFR. We use the 12-month percentage change to compute inflation, the growth rate of M2, and the growth rate of stock prices.<sup>13</sup>

Table 2 presents the estimated log marginal likelihood for the change-point VAR model across a different number of regimes and lag lengths. To allow the model to explore whether a large number of breaks and lags could potentially be associated with a high marginal-likelihood function and therefore provide a better fit to the data, we allow for six lags and five regimes.<sup>14</sup> The log marginal

---

<sup>12</sup>The data series end in 2011M3 since the focus of the analysis is on the outset of the financial crisis to investigate the effect of initial policies aimed at providing liquidity to the broad financial system to lower borrowing costs. Subsequent policy measures were aimed at stimulating specific sectors of financial markets (i.e., the housing market). Results are robust to extending the sample period to 2016M12. An appendix that shows the findings for the extended data sample is available on request to the authors.

<sup>13</sup>We have estimated the model using monthly changes in the data series and established that the results are quantitatively robust. An appendix that presents robustness analyses is available on request to the authors.

<sup>14</sup>We limit the maximum number of lags to six since it is difficult to estimate a five regime model with a large number of lags (beyond six) as the number of observations in each regime becomes low.

Table 2: Log Marginal Likelihood

Number of Breaks	3 lags	4 lags	5 lags	6 lags
0	-592.48	-568.22	-553.78	-508.63
1	-671.57	-641.85	-642.68	-648.04
2	-503.53	-509.01	-520.46	-518.98
3	-655.02	-643.27	-548.14	-490.34
4	-673.41	-674.72	-676.73	-700.03

Notes: The table shows the log marginal likelihood estimates across different regimes and lag lengths.

likelihood estimates show that the VAR(6) model with three breaks (i.e., four regimes) delivers best fit of the data and therefore is strongly preferred to alternative specifications.

Figure 1 presents the probability of each regime,  $\Pr(S_t = j)$ , for  $j = 1, \dots, 4$ . Given the  $M$  draws of  $S_t$ , we easily can estimate this probability (for the  $j^{\text{th}}$  regime) as  $\frac{1}{M} \sum_{m=1}^M I[S_t = j]$ , where  $I[S_t = j]$  is an indicator variable equal to one if  $S_t = j$ . We estimate the first breakpoint to occur in the early 1990s, with the probability of the first regime being less than 0.5 in January 1992. Several studies detect a structural break in the series in the mid-1980s and early-1990s.<sup>15</sup> The estimate for the second breakpoint is February 2000 while the final break estimate of September 2007 coincides with beginning of the recent financial crisis.<sup>16</sup>

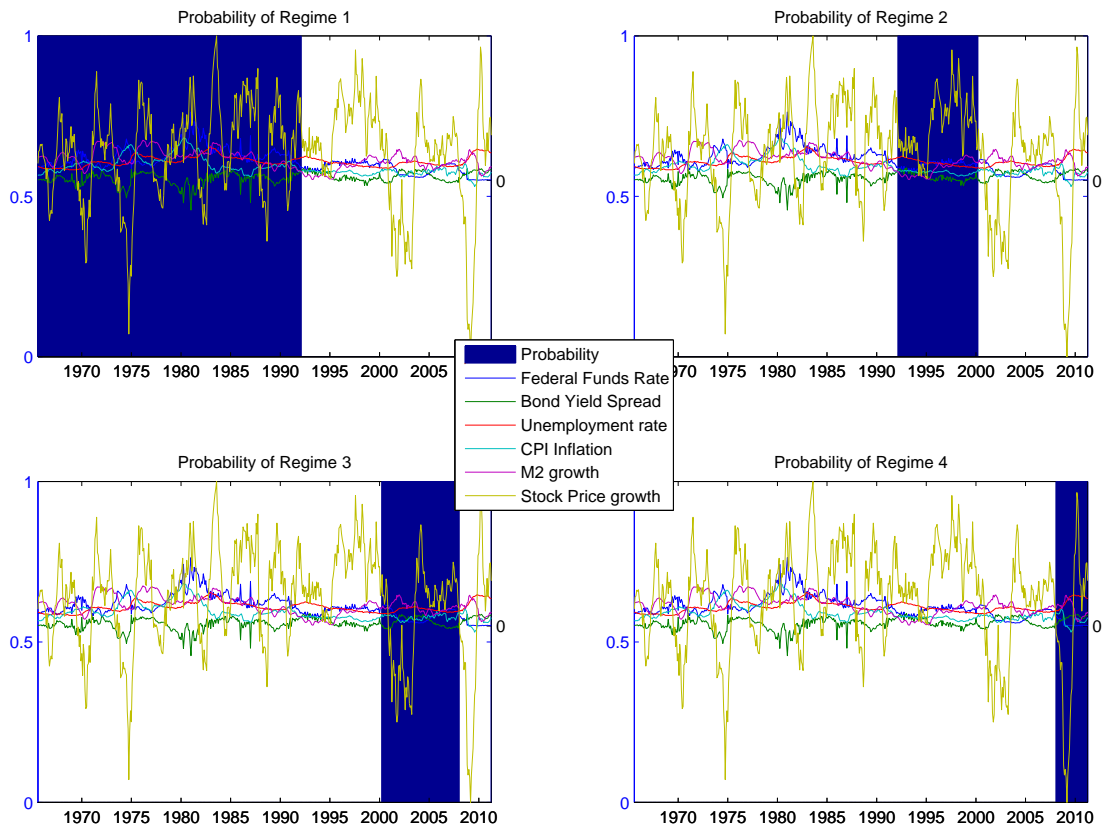
In the second half of 2007, a financial turmoil, triggered by a subprime mortgage meltdown, swept over the US and other major economies. The crisis quickly spread to major financial markets, and the cost of short-term funding on the interbank money market rose sharply. As strains in money markets persisted and worsened in early December 2007, the Fed lowered the FFR and established the term auction facility (TAF) to provide liquidity support to the broader financial system.<sup>17</sup> As the spillover from distress in the financial markets fed through to the real economy, the Fed lowered

<sup>15</sup>Similar to us, Benati and Goodhart (2010), Bianchi (2012) and Fernandez-Villaverde and Rubio-Ramirez (2008) establish that important structural changes in the systematic response of monetary and fiscal policies occurred in the early 1990s. Strachan and Dijk (2013) also detect important differences in the time series properties in the mid-1980s.

<sup>16</sup>These breakpoints are consistent with the findings in Benati and Goodhart (2010), who detect important changes in the response of monetary policy to the 9/11 terrorist attack and the Nasdaq/tech bubble and bust in the mid-2000s.

<sup>17</sup>Through this facility, the Fed auctioned preannounced amounts of credit, twice a month, to eligible depository institutions in sound financial condition for a term of one month instead of overnight. The Term Auction Facility (TAF) accepted the same kinds of collateral as the discount window. The TAF, initially set at \$20 billion for each auction, gradually increased to \$150 billion in January 2009 before being scaled back. The final auction was held March 8, 2010.

Figure 1: The estimated probability of each regime

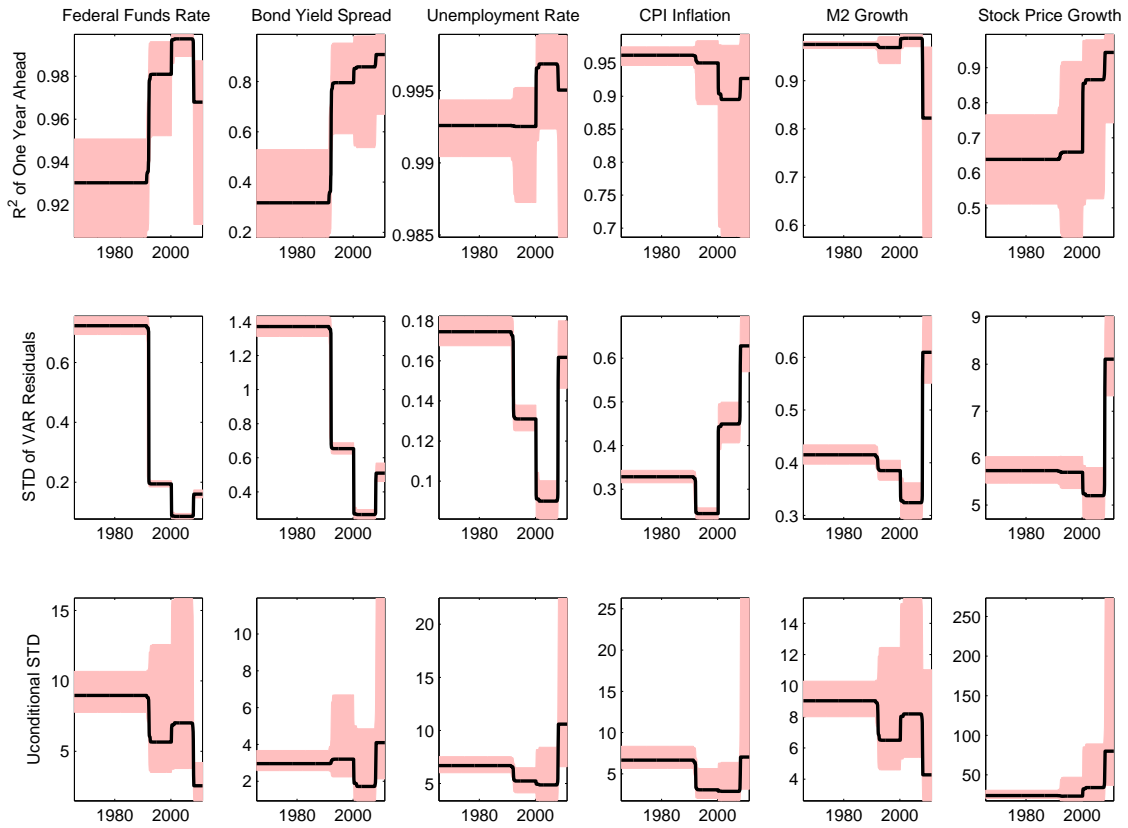


Notes: The four regimes correspond to the periods of January 1962-January 1992, February 1992-February 2000, March 2000-December 2007 and January 2008-March 2011.

its FFR to its effective ZLB in December 2008. To further stimulate economic activity, the Fed announced that it would purchase substantial quantities of assets with medium and long maturities in an effort to drive down private borrowing rates, particularly at longer maturities. The last regime of our baseline model coincides with the period corresponding to these extraordinary events and policy interventions.

To tie these breakpoints to changing macroeconomic dynamics, figure 2 plots some key reduced form summary statistics from the change-point VAR. Note that these are estimated separately in each regime, and averages are computed across regimes using  $S_t$  as the weight.

Figure 2: Regime dependent summary statistics



Notes: The four regimes correspond to the periods of January 1962-January 1992, February 1992-February 2000, March 2000-December 2007 and January 2008-March 2011. The shadow areas show the 68% confidence band.

The top panel of figure 2 plots the estimated multivariate,  $R_t^2$ . This measure is defined as  $R_t^2 = 1 - \left[ \sum_{h=0}^{11} \tilde{B}_S^h \text{var}(\varepsilon_{t+h}) \tilde{B}_S^{h'} / \sum_{h=0}^{\infty} \tilde{B}_S^h \text{var}(\varepsilon_{t+h}) \tilde{B}_S^{h'} \right]$ , where  $\tilde{B}_S$  denotes the VAR coefficients in companion form. As discussed in Cogley et al. (2010), this metric can be thought of as a measure of persistence of the endogenous variables (in deviations from trend). A few interesting patterns emerge. First, the  $R_t^2$  of inflation and money growth have declined throughout the sample period, and the series show a quantitative similar persistence across regimes. We interpret this similarity as further statistical evidence that inflation and money growth have remained linked throughout the whole sample period. Second, the  $R_t^2$  of the bond-yield spread has steadily increased throughout the different regimes, starting at approximately 0.3 in the first regime and reaching approximately 0.85 in the fourth regime. The  $R_t^2$  of the federal funds rate and unemployment rate have remained substantially the same across different regimes, with values around 0.96 and 0.99, respectively. Finally, the persistence of stock price growth has changed throughout the four regimes, reaching its highest value during the crisis period. However, the statistical uncertainty surrounding these estimates is high across the different regimes.

The second row of figure 2 plots the diagonal elements of the error covariance matrix,  $\Omega_S^{1/2}$ , estimated in each regime. The volatility of the reduced-form errors declined for all variables as the system moves to Regime 2, indicating the first breakpoint that marks the start of the Great Moderation period. Note that the timing of this breakpoint in January 1992 is somewhat later than that suggested in past studies and is due possibly to the high volatility of the stock market index during the mid-1980s, a variable often neglected in previous studies. The volatility of the reduced-form errors to all variables, except inflation, shows a sharp decrease during the third regime. The fourth regime is characterized by a sharp increase in the volatility of shocks to all variables, with the volatility of shocks to inflation, money growth, and the stock price index at historical highs.

The final row of the figure plots the estimated regime-dependent, unconditional volatility of each variable calculated as  $\text{vec}[VAR(Z_t)] = \text{vec}(\Omega_S) / \left( I - \tilde{B}_S^h \otimes \tilde{B}_S^h \right)$ . This result shows a similar pattern to the reduced form shock variance. Regime 2 is associated with the initial decline in the unconditional variance (that falls further in regime 3) while the final regime marks a return to a high variance state for most variables.



## 4.2 Macroeconomic Dynamics across Regimes

The empirical framework is particularly well-suited to investigate changes in macroeconomic dynamics across the sample horizon since the change-point VAR model allows the coefficients in the model to vary across regimes. Figures 3 to 6 plot the impulse response functions (IRFs) of the six endogenous variables to a one-standard-deviation shock for the four identified shocks across the four regimes.<sup>18</sup> We obtain the median and 68% confidence bands based on 5,000 retained Gibbs replications.

Figure 3 shows the responses of the variables to a contractionary monetary policy shock (i.e. an increase in the nominal interest rate).<sup>19</sup> The figure shows that the reaction of the bond-yield spread significantly declines across the three regimes. During the first regime, a contractionary monetary policy shock decreases the bond-yield spread by approximately 0.6 percentage points whereas during the third regime, the magnitude of the change was approximately 0.2 percentage points lower. The figure shows that the reaction of CPI inflation is significantly lower in the second regime, almost half the size compared to the first and third regimes. Similarly, the response of the unemployment rate declines from the first regime to the second and third regimes. Overall, the IRFs highlight that the transmission of monetary policy shocks significantly changes throughout the different regimes.

Figure 4 shows the responses of the variables to a negative interest rate spread shock. To implement the analysis, we impose that the short-term interest rate is exogenous to the spread shock in the fourth regime.<sup>20</sup> The figure shows that the reaction of the bond-yield spread significantly declines over the different regimes, with the interest rate spread shock decreasing the bond-yield spread by 0.5 percentage points in the first regime compared to approximately 0.2 percentage points

---

<sup>18</sup>The six variables are the short-term interest rate, the 10-year interest rate spread, the unemployment rate, the inflation rate, the growth rate of money, and the growth rate of stock prices; the four identified shocks are the monetary policy shock, the 10-year interest rate spread shock, the demand shock, and the supply shock; the four regimes correspond to the periods of January 1962-January 1992, February 1992-February 2000, March 2000-December 2007, and January 2008-March 2011.

<sup>19</sup>For this shock, the fourth regime is absent since monetary policy deliberately maintained the nominal interest rate at approximately zero during the fourth regime period.

<sup>20</sup>The aim is to mimic the impact of the Fed's non-conventional monetary policies undertaken during the crisis period aimed at compressing the long-term interest rate spreads while leaving the conventional monetary policy instrument, the FFR, unchanged.

during the fourth regime, with a stable decline between regimes. At the same time, the response of inflation increases, going from approximately 0.1 percentage points in the first regime to approximately 0.2 percentage points in the fourth regime. The change in the responses may be interpreted as evidence on the improved effectiveness of the Fed's unconventional policies since even small changes in the interest rate spread are effective in influencing the economy. The figure also shows that a negative interest rate spread shock decreases the unemployment rate. While the responses are largely similar across the first three regimes, its impact is larger and more persistent during the crisis period (the median peak impact is approximately 0.1 percentage points). However, there is uncertainty around this response, due to the sizeable confidence interval around the estimate. This finding is consistent with that of Baumeister and Benati (2013), who also find an increase in the response of output growth during the crisis.

Figure 5 shows the responses of the variables to an expansionary demand shock that decreases the unemployment rate. The demand shock has a highly persistent effect on the unemployment rate during the crisis regime, with the median estimate staying below zero for more than five years.

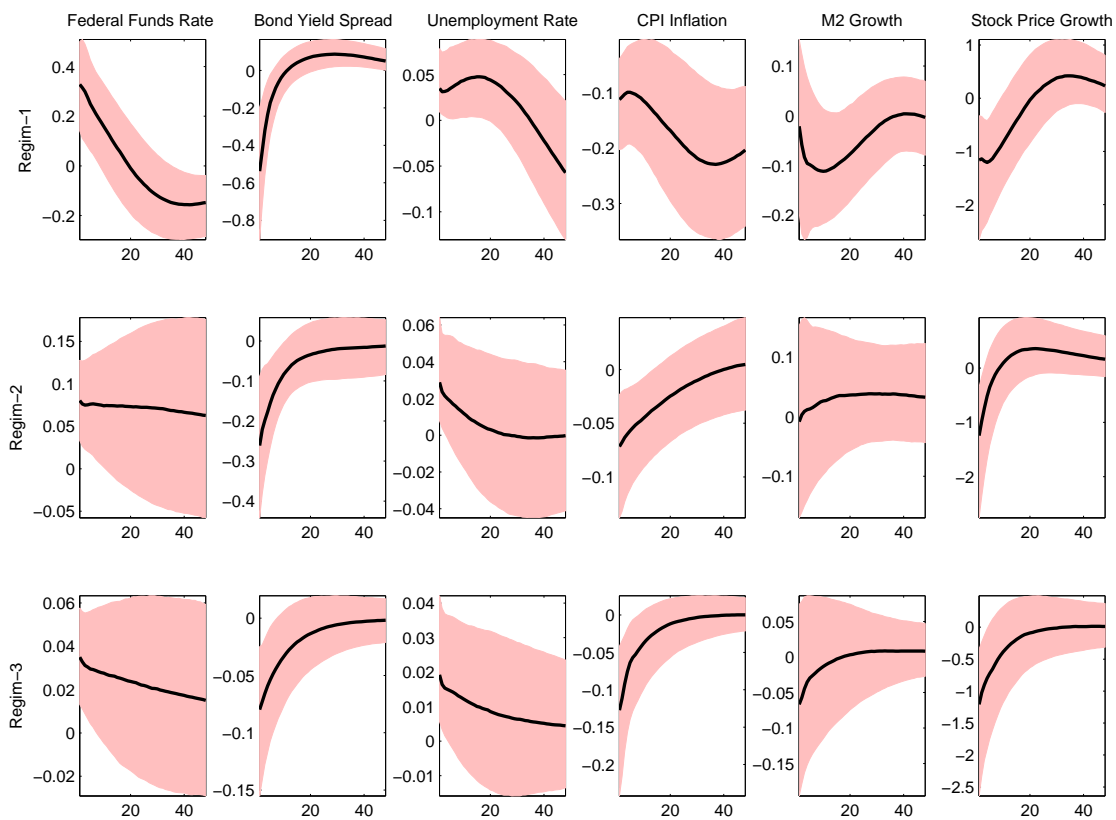
Figure 6 shows the responses of the variables to an expansionary supply shock that decreases inflation. Similar to the case of a demand shock, the reaction of the bond-yield spread is generally insensitive to the shock across regimes. The response of the nominal interest rate to demand and supply shocks decreases throughout the sample period, going from approximately 0.2 (-0.1) percentage points in the first regime to approximately 0.04 (-0.02) percentage points in the third regime for demand (supply) shocks.<sup>21</sup>

Looking across all these impulse responses suggests that the transmission mechanism of the different shocks has changed across the four regimes. One interesting pattern is the decreased response of the nominal interest rate to the shocks across the four regimes, which, as mentioned, echoes the findings related to the Great Moderation period.

---

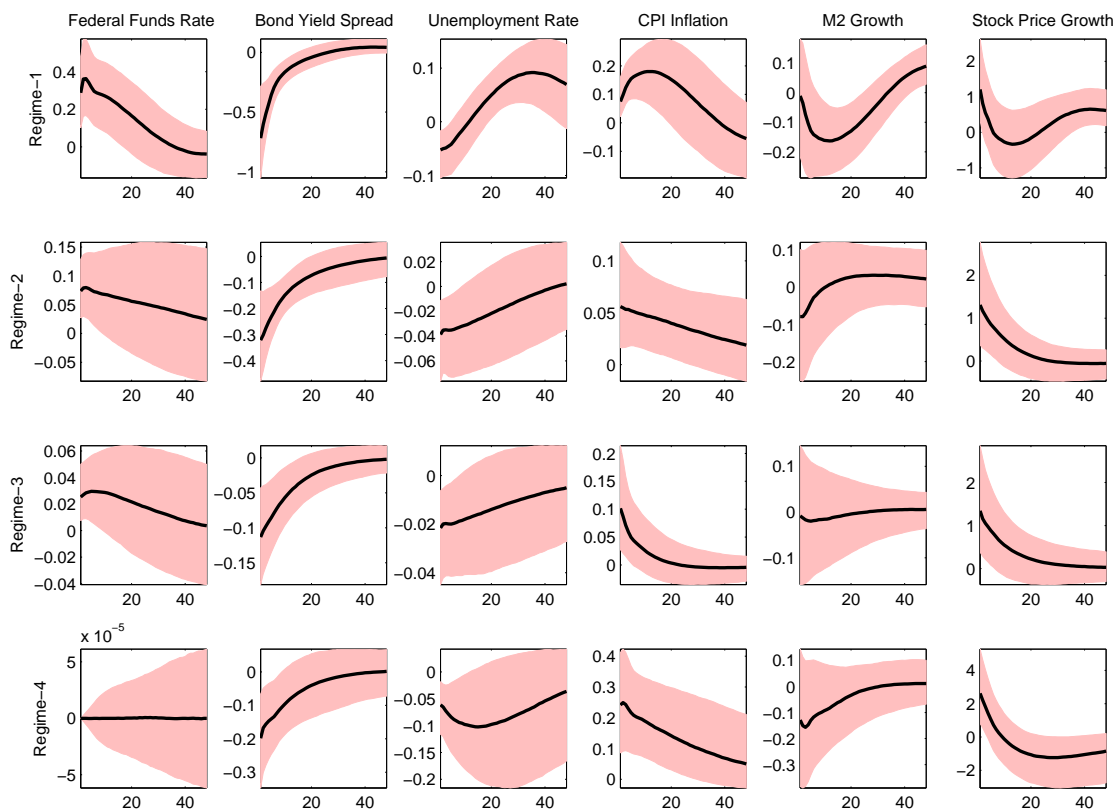
<sup>21</sup>Note that this result is isomorphic to changes in the coefficients of a Taylor rule.

Figure 3: Impulse response functions to a contractionary monetary policy shock



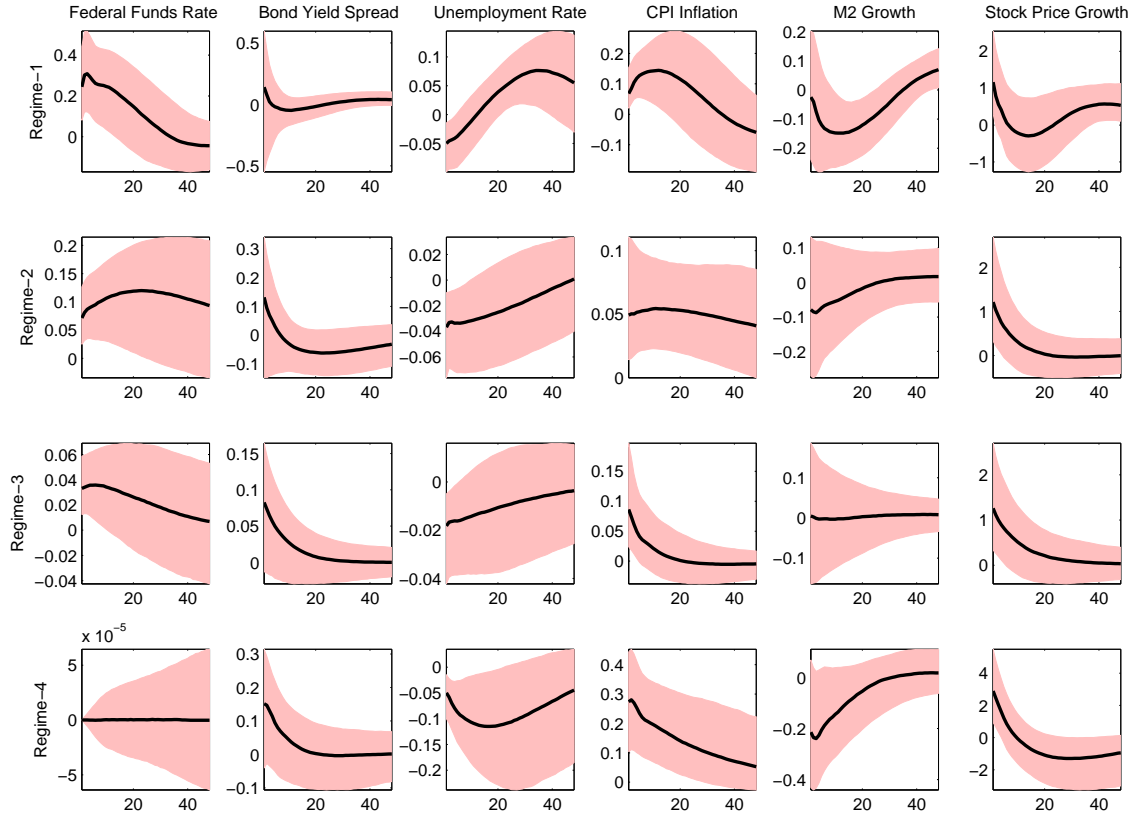
Notes: The four regimes correspond to the periods of January 1962-January 1992, February 1992-February 2000, March 2000-December 2007 and January 2008-March 2011.

Figure 4: Impulse response functions to a negative interest rate spread shock



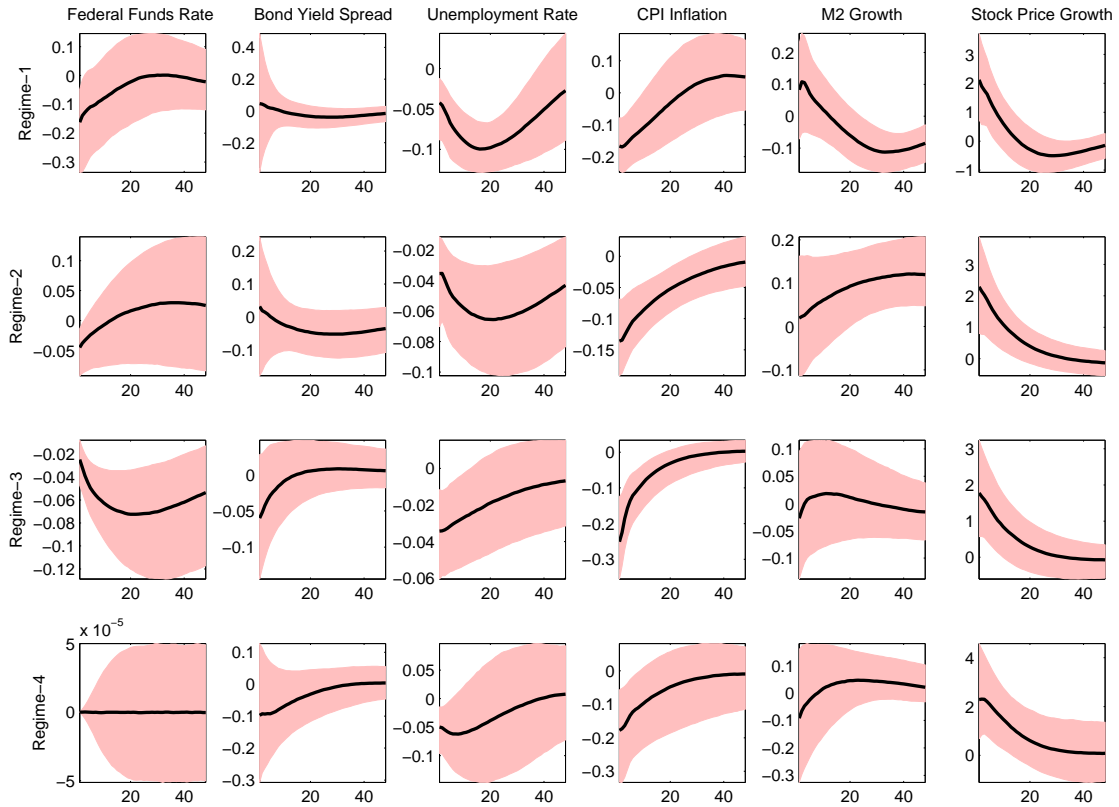
Notes: The four regimes correspond to the periods of January 1962-January 1992, February 1992-February 2000, March 2000-December 2007 and January 2008-March 2011.

Figure 5: Impulse response functions to an expansionary demand shock



Notes: The four regimes correspond to the periods of January 1962-January 1992, February 1992-February 2000, March 2000-December 2007 and January 2008-March 2011.

Figure 6: Impulse response functions to an expansionary supply shock



Notes: The four regimes correspond to the periods of January 1962-January 1992, February 1992-February 2000, March 2000-December 2007 and January 2008-March 2011.

### 4.3 Forecast Error Variance Decomposition and Historical Shock Decomposition

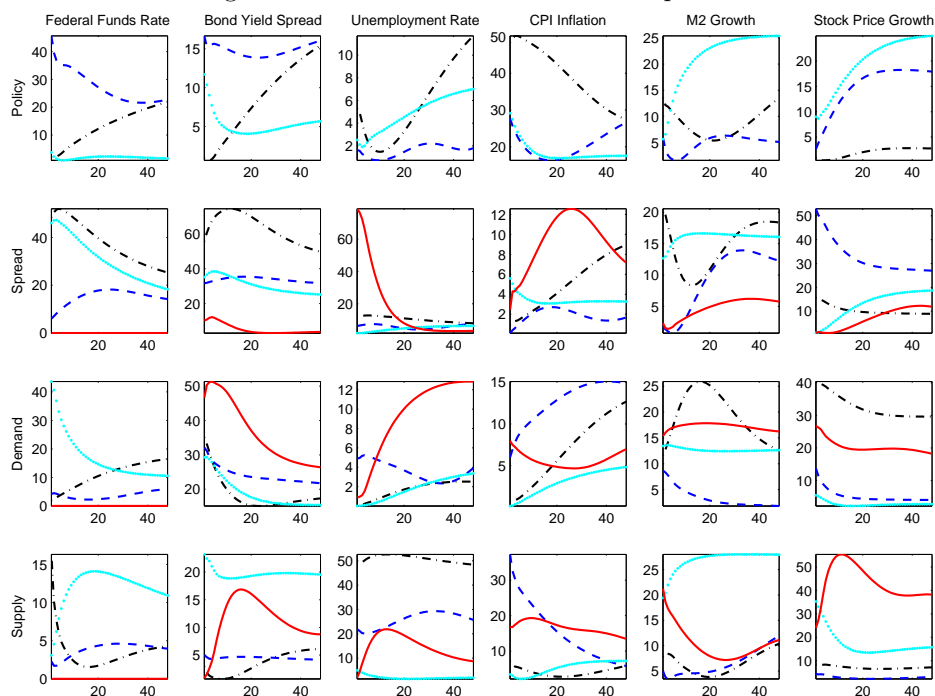
To understand the extent to which movements of each variable are explained by each shock and how the contribution of shocks has changed across regimes, figure 7 reports the forecast error variance decompositions of the six endogenous variables for each of the four shocks. The dashed blue line refers to Regime 1, the dashed-dotted black line refers to Regime 2, the dotted cyan line refers to Regime 3, and the solid red line refers to Regime 4.<sup>22</sup> The results show that spread shocks are important across the four regimes as they explain the bulk of fluctuations in bond-yield spread, the unemployment rate, and stock price growth, and they also play a competing role with other shocks in explaining fluctuations in money growth and the nominal interest rate. Similarly, supply shocks explain the bulk of fluctuations in stock price growth and the unemployment rate, and they compete with spread shocks to explain fluctuations in inflation. Monetary policy shocks explain most of the fluctuations in inflation whereas they play a supporting contribution to movements in the nominal interest rate and bond-yield spread. The figure also provides insights on how the contribution of shocks has changed in the fourth regime. During the fourth regime, the contribution of bond-yield spread shocks to all the variables, except the unemployment rate and inflation, remains broadly stable. Similarly, the contribution of demand shocks to all the variables, except the unemployment rate and inflation, remains broadly stable at different horizons whereas the contribution of supply shocks increases at short horizons. In particular, supply shocks explain approximately 20% of short-run fluctuations in unemployment in the fourth regime whereas the contribution is around 9% at long horizons.

The historical shock decomposition is an alternative useful metric to evaluate the importance of the various shocks in driving the variation of the key observed macro variables across the different regimes. Since we use a change-point VAR, we first briefly outline how we produce the historical shock decomposition and then discuss the findings. To derive the historical structural shocks, we

---

<sup>22</sup>Note that in the fourth regime, the contribution of the shocks on the short-term interest rate is constant because of the restriction that the policy rate does not respond to lagged changes in the other endogenous variables. In addition, the spread shock has no impact on the short-term interest rate because of the zero contemporaneous restriction imposed in the fourth regime.

Figure 7: Forecast error variance decomposition



Notes: The four regimes correspond to the periods of January 1962-January 1992 (dashed blue line), February 1992-February 2000 (dashed-dotted black line), March 2000-December 2007 (dotted cyan line) and January 2008-March 2011 (solid red line).



re-write the change-point VAR model as follows:

$$y_t = B_0 \xi_t + B_1 (\xi_t \otimes I_s) y_{t-1} + \dots + B_k (\xi_t \otimes I_s) y_{t-k} + A_0 (\xi_t \otimes I_s) \omega_t \quad (5)$$

$$= [B_0 \ B_1 \ \dots \ B_k] \begin{bmatrix} \xi_t \\ (\xi_t \otimes I_s) y_{t-1} \\ \vdots \\ (\xi_t \otimes I_s) y_{t-k} \end{bmatrix} + A_0 (\xi_t \otimes I_s) \omega_t \quad (6)$$

$$= \tilde{B} X_t + A_0 (\xi_t \otimes I_s) \omega_t, \quad (7)$$

where  $\xi_t$  is the  $s$  column of the  $I_4$  matrix,  $y_t$  is the vector of endogenous variables,  $\omega_t$  is the vector of structural shocks, and coefficients are defined as

$$B_i = [B_i(s=1) \ B_i(s=2) \ B_i(s=3) \ B_i(s=4)] \text{ for } i = 0, \dots, k$$

$$A_0 = [A_0(s=1) \ A_0(s=2) \ A_0(s=3) \ A_0(s=4)].$$

From equation (7), we derive  $\omega_t$  as

$$\omega_t = [A_0 (\xi_t \otimes I_s)]^{-1} (y_t - \tilde{B} X_t). \quad (8)$$

Intuitively, this approach amounts to computing the structural shocks based on the reduced-form errors using the identification matrix that corresponds to each individual regime. With the identified structural shocks, one can decompose the endogenous variables in terms of the structural shocks. Figures 8-10 plot the historical decomposition (deviations from the mean) for the 10-year spread, unemployment rate, and inflation in terms of the monetary policy, spread, demand, and supply shocks.<sup>23</sup>

Figure 8 shows the historical shock decomposition of the 10-year spread. The sharp compression in the 10-year spread in the early 1970s was driven largely by spread shocks. Meanwhile the falls in the spread in the late 1970s and early 1980s can be largely attributed to a mix of monetary policy

<sup>23</sup>We label the unidentified component as other shocks.

and spread shocks. The model attributes the persistent decline in the spread since the mid-1990s to other shocks that our model did not identify while demand shocks acted in the opposite direction. More recently during the financial crisis, both spread and other shocks helped keep the 10-year spread elevated.

Figure 8: Historical shock decomposition: 10-year spread deviation from the mean

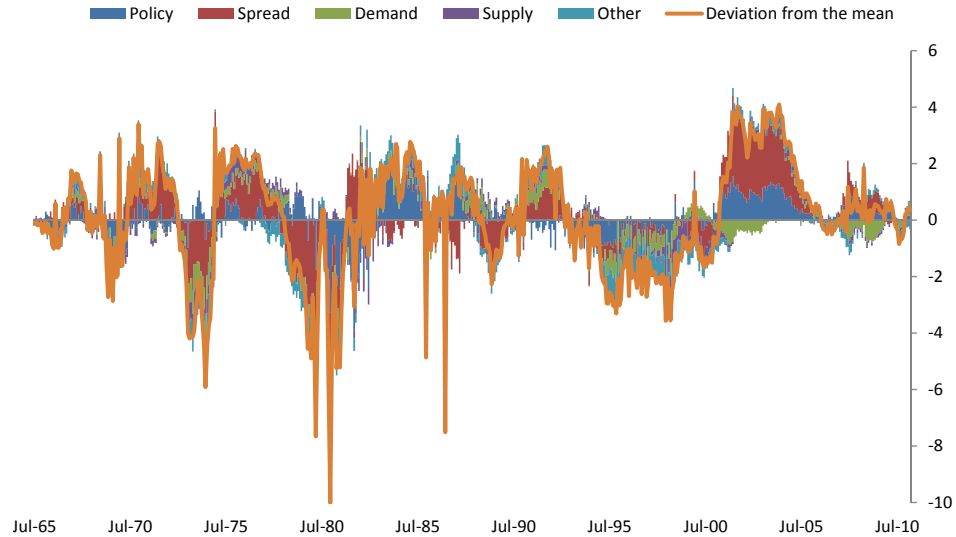
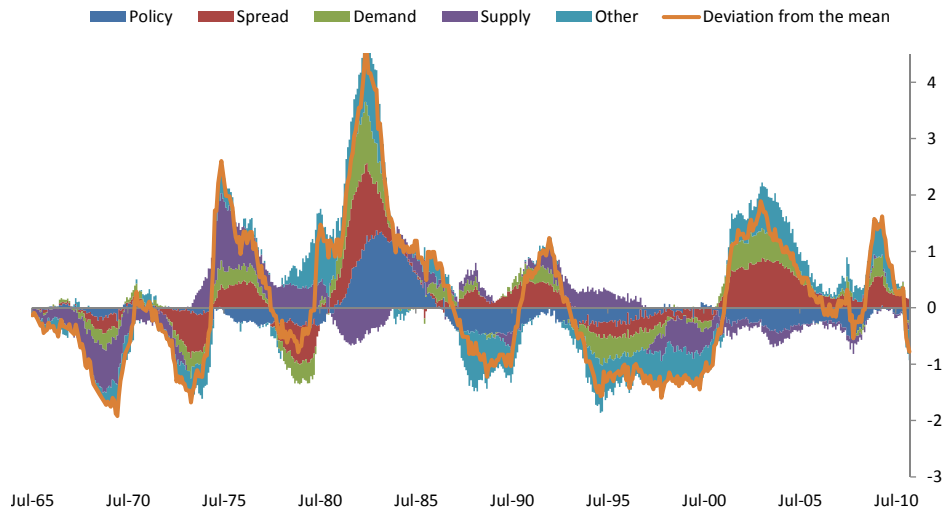


Figure 9 shows the historical shock decomposition of the unemployment rate. The unemployment peak in 1975 was attributed largely to supply and spread shocks where both the 10-year spread and the short-term interest rates increased sharply. The subsequent decline in the unemployment rate was driven by monetary policy and spread shocks. The second spike in unemployment in the early 1980s was attributed to negative monetary policy shocks as well as negative demand and spread shocks. In the 1990s, favorable monetary policy shocks contributed negatively to the unemployment rate while supply shocks had the opposite effect. Spread and demand shocks dominated the sharp increase in unemployment during the crisis. However, unlike previous episodes, monetary policy shocks did not contribute to the increase in unemployment.

Figure 10 shows the historical shock decomposition of inflation. The two spikes in inflation in the mid-1970s and early 1980s were due largely to demand, monetary policy, and spread shocks. The model identifies negative supply shocks as the key contributors to rising inflation in the early

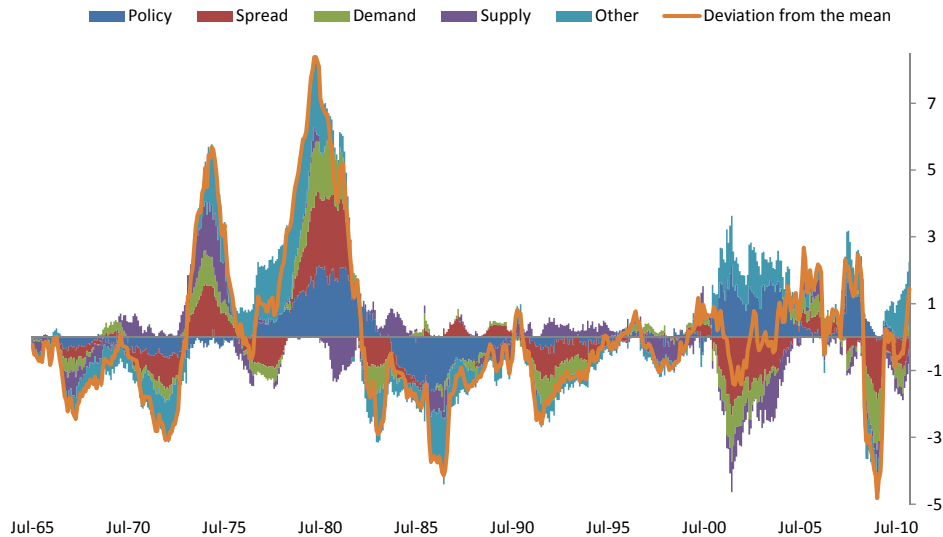
Figure 9: Historical shock decomposition: unemployment rate deviation from the mean



1970s around the time of the 1973-75 recession when oil prices quadrupled, following the embargo imposed by the Organization of Arab Petroleum Exporting Countries. To a lesser extent, negative supply shocks also contributed to the peak of inflation in the early 1980s. From the second regime onwards, we find a muted impact of monetary policy shocks on inflation, a finding that is consistent with the forecast error decomposition. Spread and demand shocks were the key contributors to the brief period in 2009 when inflation fell below zero.

Overall, the analysis shows that the contributions of shocks to movements in the variables is different across regimes. Some interesting patterns emerge. For example, the spread shocks play a relevant contribution in movements in the unemployment rate, stock price growth, and bond-yield spread. Similarly, demand and supply shocks explain a sizeable part of fluctuations in unemployment, inflation, and stock price growth, and their relevance changes across regimes. Finally, the historical contribution of the yield-spread shock to unemployment and inflation has substantially increased from early 2008 onwards, suggesting that this shock became more powerful in influencing movements in these variables.

Figure 10: Historical shock decomposition: inflation deviation from the mean



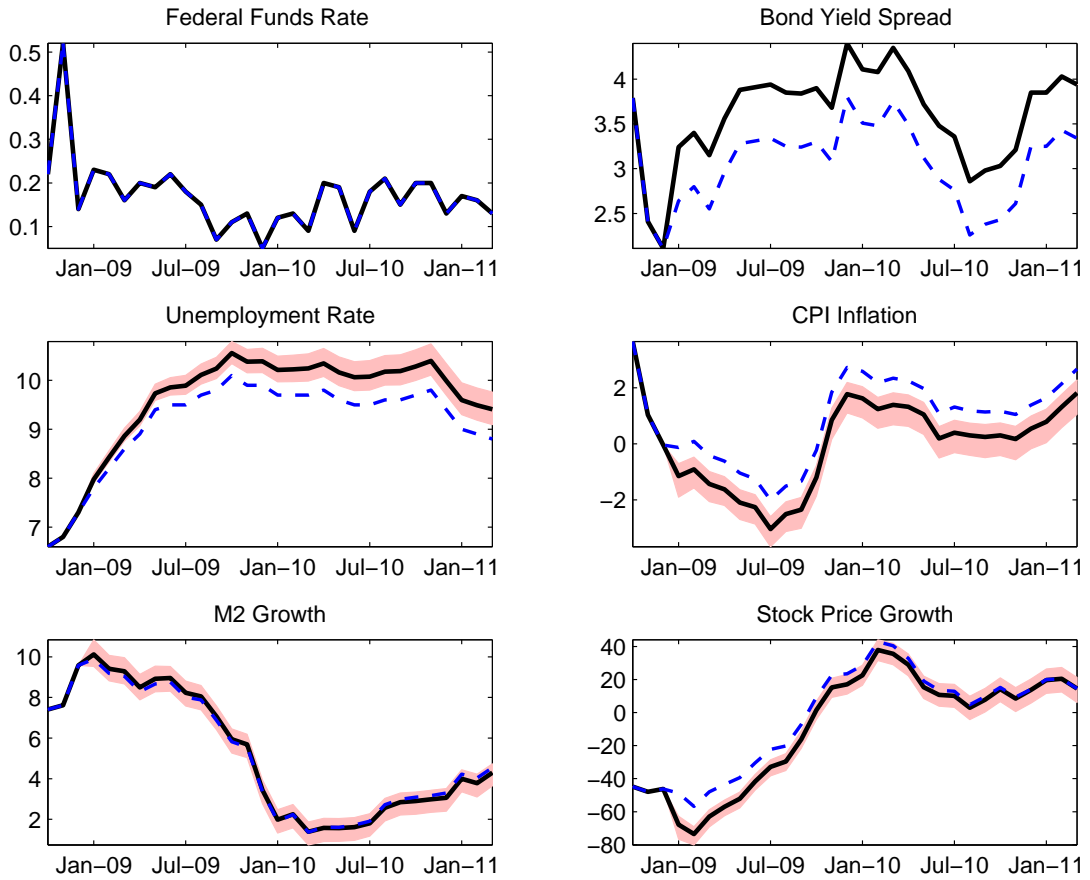
#### 4.4 Counterfactual Scenario of 60 Basis Points Higher Bond-Yield Spread

To evaluate the importance of spread shocks over the crisis period, we run a counterfactual exercise on how the economic outlook would have looked had the yield bond spread been 60 basis points higher.<sup>24</sup> Baumeister and Benati (2013) present a similar counterfactual exercise using an estimated time-varying VAR. However, our methodology is different, and the estimation includes stock price growth and unemployment as indicators of real activity whereas the above-mentioned study focuses on output growth. As outlined below, we find that these differences lead to a different interpretation of the effectiveness of the LSAP.

Figure 11 plots the evolution of the model's six endogenous variables in the data (dashed blue line) and in the counterfactual scenario (solid black line) with the 68% confidence bands. The figure shows no difference between the actual and the counterfactual scenario for the short-term interest rate since the interest rate is assumed not to respond to the spread shock.

<sup>24</sup>Gagnon et al. (2010) and Meyer and Bomfim (2012) estimate that the overall impact of the Fed's LSAP was to compress the 10-year spread of 60 basis points. An alternative counterfactual scenario is to remove the credit spread shock and simulate the model with the remaining shocks, to evaluate the contribution of the spread shock. The findings show that the credit shock had an adverse effect on the economic outlook, consistent with the worldwide narrative. The effect of the shock remains severe despite the actions of the monetary authorities to stimulate demand by lowering the end of the yield curve (QE). An appendix that details the results of this counterfactual exercise is available on request to the authors.

Figure 11: Counterfactual scenario 60 basis points higher bond-yield spread



Notes: The blue dashed line is the actual data, and the solid black line is the counterfactual scenario that imposes 60 basis points higher bond-yield spread than its historical value. The shaded area represents the 68 percent confidence band.

Similar to the results in Baumeister and Benati (2013), we observe a significant impact on economic activity, measured by the unemployment rate.<sup>25</sup> However, the dynamics are quite different. Our estimate at the time of the LSAP announcement (2009Q4) is only 0.2 percentage points, and our counterfactual exercise suggests the spread shock starts to exert significant downward pressure on the unemployment rate from the second half of 2009 onwards, reflecting lag responses of unemployment to the 10-year spread.<sup>26</sup> The results indicate that with a higher bond-yield spread, the unemployment rate would have peaked at 10.6 percent in December 2009 rather than around 10 percent. At the end of the sample, unemployment would have been 0.6 percentage points higher than 8.9 percent. This estimate is in line with the simulation results produced by Chung et al. (2012) using the FRB/US model. Their scenario includes additional purchases announced in November 2011, which is outside our data sample. A closer inspection of their results reveals that based on the initial \$1.75 trillion purchases, the unemployment rate would have been approximately one percentage point lower by the end of 2011. While confirming Baumeister and Benati's results that the LSAP help supported real economic activity during the crisis period, quantitatively, our estimate (measured by the reduction in the unemployment rate) is smaller, and its peak impact is much later, falling after the LSAP announcement.

In general, we find that inflation would have been lower over the crisis period if bond-yield spreads were higher. The decline in inflation from falling commodity prices in late 2008 and early 2009 would have been faster, but the trough reached in July 2009 was similar, around 2 percent. Our results suggest that, on average, inflation would have been one percentage point lower.

In contrast to real economic indicators, we find the spread shock had the largest impact in 2008 prior to the announcement of the LSAP program. With the spread shock, the growth in stock prices would have been about 20 percentage points lower in 2008. This finding is most likely related to the Fed's liquidity support program, the TAF. After announcement of the LSAP in November 2008, the impact on stock prices were generally positive but much less persistent, compared with other

---

<sup>25</sup>Baumeister and Benati finds the Fed's LSAP helped stimulate economic activity by increasing (based on median estimates) annualized real GDP growth by 1.3 percent on impact and it peaked at 1.9 percent three quarters after the impact, rapidly fading over subsequent quarters.

<sup>26</sup>The difference in the results could also reflect the fact that movements in the unemployment rate usually lag GDP growth.

real economy indicators. Finally, the higher-yield spread has a minimal and insignificant impact on money supply growth.

## 5 Conclusion

This paper proposes a novel approach to empirically evaluate the effect of the ZLB using a flexible change-point VAR model that identifies shocks from the theoretical restrictions of an innovative general equilibrium model. The empirical model identifies three break points (four regimes) over the sample period from 1965 to 2011. The fourth regime, which begins in October 2007, coincides with the crisis period. The analysis discloses a range of important changes in the statistical and dynamic properties of key macroeconomic variables over the sample period. Statistical properties such as persistence and volatility of fluctuations in key macroeconomic variables and the volatility of the reduced-form errors have changed throughout the different regimes, with the crisis period being characterized by higher volatility. In addition, although quantitative changes are recorded throughout the whole period, the crisis period is characterized by the relevance of yield spread shocks to generate movements in unemployment and inflation.

We use the model to evaluate the macroeconomic impact of the Fed's LSAP program by constructing a counterfactual scenario that imposes 60 basis points higher than the bond-yield spread. We find this strategy resulted in significant impact in supporting economic activity and help lower the unemployment rate by about 0.6 percentage points while raising inflation by approximately 1 percentage point.

The paper assumes that the Fed's unconventional monetary policy actions over the crisis period are sufficiently represented by the behavior of the 10-year interest rate spread. However, other indicators such as OIS spreads, corporate spreads, and spreads on agency debt also may help characterize the Fed's actions during this period. An alternative approach would be to study the effect of the ZLB using a large number of "spread" indicators within a factor VAR framework. Another interesting extension would be to include indicators on the behavior of banks such as loan approvals or different mortgage yields, which are regarded as important during the crisis. Their

inclusion would enable the analysis to better explain the role of the banking system. In addition, our model does not investigate the role of the agent's expectations that could be particularly important at the ZLB. Finally, another alternative approach would be to model the different regimes within a fully specified dynamic, stochastic, general equilibrium model. This approach, however, would be difficult, requiring an allowance for time-varying parameters within a general equilibrium framework. All of these extensions are outstanding avenues for future research.



# Appendix

## A The Theoretical Model

The theoretical model comprises a continuum of household, a representative finished-goods-producing firm, a continuum of intermediate-goods-producing firms, the government and a central bank. The description of the problem of each agent and the implication for the model's variables is described in turn.

### A.1 The Representative Household

The household comprises a continuum of members represented by the unit square and indexed by the pair  $(i, j) \in [0, 1] \times [0, 1]$ . The first dimension, indexed by  $i \in [0, 1]$ , represents the specific labor service type of each household member. The second dimension, indexed by  $j \in [0, 1]$ , determines the household's disutility from work. The latter is given by  $j$  if he is employed and zero otherwise. The households perceive the long-period bonds,  $B_{L,t}$ , as riskier due to their loss of liquidity, relative to one-period bonds,  $B_t$ . We follow the formulation in Woodford (2001) and long-term bonds are perpetuities that cost  $p_{L,t}$  at time  $t$  and pay an exponentially decaying coupon  $\varpi^s$  at time  $t + s + 1$  where  $0 \leq \varpi^s < 1$ . As it is explained in Woodford (2001) and Chen et al. (2012), the advantage of this formulation is that the price in period  $t$  of a bond issued  $s$  periods ago,  $p_{L-s,t}$ , is a function of the coupon current price,  $p_{L,t}$ , such that:  $p_{L-s,t} = \varpi^s p_{L,t}$ . This idea is encapsulated by the adjustment cost on portfolio decisions, as in Andres et al. (2004) and Harrison (2012), which has the form

$$\frac{1}{2} \left[ \frac{B_t}{p_{L,t} B_{L,t}} \kappa - 1 \right]^2,$$

where the parameter  $\kappa$  denotes the inverse of the steady-state one-period to  $L$ -period bonds such that the portfolio adjustment cost is zero in the steady state and  $p_{L,t} = 1 / (r_{L,t} - \varpi)$ .

The household period utility corresponds to the integral of its members' utilities and is thus given by

$$E_0 \sum_{t=0}^{\infty} \beta^t a_t \left\{ \ln C_t + \ln(M_t/P_t) - \int_0^1 n_t(i) di - \frac{\nu}{2} \left[ \frac{B_t}{B_{L,t}} \kappa - 1 \right]^2 \right\}, \quad (9)$$

where  $C_t$  is aggregate consumption,  $n_t(i)$  is the fraction of members specialized in type  $i$  labor who are employed in period  $t$ ,  $\int_0^1 n_t(i) di = \int_0^1 \int_0^{n_t(i)} j dj di$ ,  $M_t/P_t$  is real money holdings,  $\beta$  is the discount factor  $0 < \beta < 1$  and the parameter  $\nu$  represents the household's preferences for liquidity. The preference shock  $a_t$  follows the autoregressive processes

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at}, \quad (10)$$

where  $0 < \rho_a < 1$ . The zero-mean, serially uncorrelated innovations  $\varepsilon_{at}$  are normally distributed with standard deviation  $\sigma_a$ .

Since each household member  $i$  provides a specific labor service,  $n_t(i)$ , the aggregate labor index accounts for the different labor services according to the constant-returns-to-scale technology  $n_t = \left[ \int_0^1 n_t(i)^{\frac{\theta_w - 1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w - 1}}$ , where  $\theta_w > 0$  is the elasticity of substitution among labor services. Thus, during each period the demand for each labor service is:

$$n_t(i) = [W_t(i)/W_t]^{-\theta_w} n_t, \quad (11)$$

and the aggregate wage is  $W_t = \left[ \int_0^1 W_t(i)^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}}$ .

The representative household enters period  $t$  with money holdings  $M_{t-1}$ , maturing one-period bonds  $B_{t-1}$  and maturing  $L$ -period bonds,  $B_{L,t-1}$ . At the beginning of the period, the household receives a lump-sum nominal transfer,  $T_t$ , from the government, profits  $D_t$  from the intermediate-goods-producing firms, and income from supplying  $n_t(i)$  units of labor at the nominal wage rate,  $W_t(i)$ , to the intermediate-goods-producing firms. Adjusting nominal wages between periods entails some costs,  $G_t^w(i)$ , that are captured by the quadratic adjustment cost function

$$G_t^w(i) = \frac{\phi_w}{2} \left[ \frac{W_{t+1}(i)}{\pi^w W_t(i)} - 1 \right]^2 n_t,$$

where  $\phi_w \geq 0$  governs the magnitude of the nominal wage adjustment cost, and  $\pi^w$  denotes the average, or steady-state, rate of wage inflation.

The household uses its income for consumption  $C_t$ , to purchase  $B_t$ , one-period bonds at the

price of  $1/r_t$  units of money per bond, where  $r_t$  represents the gross, short nominal interest rate between  $t$  and  $t + 1$ , deposits into period  $t + 1$ ,  $B_{L,t}$   $L$ -period bonds at the price  $1/r_{L,t}$  units of money per bond, where  $r_{L,t}$  represents the gross, long nominal interest rate between  $t$  and  $t + L$ . Finally, the household incurs in a time-varying, stochastic, transaction costs  $\xi_t$  for each dollar spent on long-term bonds. Therefore the household's budget constraint is:

$$\begin{aligned} & \left[ M_{t-1} + r_{t-1}B_{t-1} + p_{L,t}r_{L,t}B_{L,t-1} + \int_0^1 W_t(i)n_t(i)di + D_t + T_t \right] / P_t \\ & = C_t + \int_0^1 G_t^w(i)di + [M_t + B_t + (1 + \xi_t)p_{L,t}B_{L,t}] / P_t, \end{aligned} \quad (12)$$

for all  $t = 0, 1, 2, \dots$ . The transaction costs  $\xi_t$  follow the autoregressive processes

$$\ln(\xi_t) = \rho_\xi \ln(\xi_{t-1}) + \varepsilon_{\xi t}, \quad (13)$$

where  $0 < \rho_\xi < 1$ . The zero-mean, serially uncorrelated innovations  $\varepsilon_{\xi t}$  are normally distributed with standard deviation  $\sigma_\xi$ .

Thus the household chooses  $\{C_t, M_t, B_t, B_{L,t}, n_t(i)\}_{t=0}^\infty$  to maximize its utility (9) subject to the budget constraint (12) and demand for each labor service (11) for all  $t = 0, 1, 2, \dots$ . Letting  $\pi_t = P_t/P_{t-1}$ ,  $w_t = W_t/P_t$  and  $m_t = M_t/P_t$  denote the gross price inflation rate, real wages and real money holdings respectively,  $b_t = B_t/P_t$  and  $b_{L,t} = B_{L,t}/P_t$  denote the real one-period and  $L$ -period bonds holdings, respectively, and  $\Lambda_t$  the non-negative Lagrange multiplier on the budget constraint (12), the first-order conditions for this problem are

$$\Lambda_t = a_t/C_t, \quad (14)$$

$$a_t/m_t = \Lambda_t + \beta E_t(\Lambda_{t+1}/\pi_{t+1}), \quad (15)$$

$$\Lambda_t = \beta E_t(r_t \Lambda_{t+1}/\pi_{t+1}) - a_t \nu \left[ \frac{b_t}{p_{L,t} b_{L,t}} \kappa - 1 \right] \frac{\kappa}{p_{L,t} b_{L,t}}, \quad (16)$$

$$(1 + \xi_t)p_{L,t}\Lambda_t = \beta E_t(r_{L,t+1}p_{L,t+1}\Lambda_{t+1}/\pi_{t+1}) + a_t \nu \left[ \frac{b_t}{p_{L,t} b_{L,t}} \kappa - 1 \right] \frac{b_t \kappa}{p_{L,t} b_{L,t}^2}, \quad (17)$$

and

$$(1 - \theta_w) \left[ \frac{W_t(i)}{W_t} \right]^{-\theta_w} n_t = \frac{a_t}{\Lambda_t} \theta_w \left[ \frac{W_t(i)}{W_t} \right]^{-\theta_w - 1} \frac{n_t}{w_t} + G_{t,w}^w(i) + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} G_{t+1,w}^w(i), \quad (18)$$

where  $G_{t,w}^w(i)$  and  $G_{t+1,w}^w(i)$  represent the marginal adjustment cost of a change in current wage for period  $t$  and  $t + 1$ , respectively. Equation (14) states that the Lagrange multiplier must equal the households' marginal utility of consumption. Equation (15) is the standard money demand equation. Equation (16) and (17) represent the demand for one-period and  $L$ -period bonds, respectively. Note that in the absence of the portfolio adjustment cost ( $\nu = 0$ ), one- and long-period bonds have the same interest rate. In addition, as discussed above, the term  $\xi_t$  serves as the exogenous, time-varying change to the long-period interest rate. Finally, equation (18) is the wage Phillips curve equation in its non-linearized form. It equates the marginal benefit to the marginal cost of a change in wages. The marginal benefit of an increase in wage is the higher contribution to labor income (LHS). The marginal cost comprises the disutility from working linked with the change in wage (first term on the RHS), the adjustment cost that the change in current wage entails in period  $t$  (second term on the RHS) and the expected adjustment cost that the change in current wage entails in period  $t + 1$  (third term on the RHS). Note that absent the adjustment cost, the wage equation nests the standard labor supply equation (i.e. the wage equates the marginal rate of substitution between consumption and leisure).

Before concluding the description of the household, we define a measure of the unemployment rate, similar to Gali (2011). We characterize the unemployment rate according to the official definition as the number of unemployed household themselves *actively searching* for a job as a percentage of the labor force,  $l_t(i)$ , such that:

$$u_t(i) = [l_t(i) - n_t(i)] / l_t(i). \quad (19)$$

The labor force is determined by the condition that leads the marginal labor supplier to partic-

ipate in the labor market. In particular, the marginal supplier of the specific labor service of type  $i$ ,  $l_t(i)$  participates in the labor market if the real wage is at least equal to the disutility of work expressed in marginal utility of consumption, which holds when:

$$w_t(i) \geq \frac{a_t}{\Lambda_t} l_t(i), \quad (20)$$

where  $w_t(i) = W_t(i)/P_t$  is the real wage.

In equilibrium, equation (20) holds with equality and provides the labor participation decision for the household.

## A.2 The Representative Finished-Goods-Producing Firm

During each period  $t = 0, 1, 2, \dots$ , the representative finished goods-producing firm uses  $Y_t(f)$  units of each intermediate good  $f \in [0, 1]$ , purchased at nominal price  $P_t(f)$ , to produce  $Y_t$  units of the finished product at constant returns to scale technology  $\left[ \int_0^1 Y_t(f)^{\frac{\theta_p-1}{\theta_p}} df \right]^{\frac{\theta_p}{\theta_p-1}} \geq Y_t$ , where  $\theta_p > 1$  is the elasticity of substitution among goods. Hence, the finished goods-producing firm chooses  $Y_t(f)$  for all  $f \in [0, 1]$  to maximize its profits,  $P_t \left[ \int_0^1 Y_t(f)^{\frac{\theta_p-1}{\theta_p}} df \right]^{\frac{\theta_p}{\theta_p-1}} - \int_0^1 P_t(f) Y_t(f) df$ , for all  $t = 0, 1, 2, \dots$  the first order conditions for this problem are

$$Y_t(f) = [P_t(f)/P_t]^{-\theta_p} Y_t \quad (21)$$

for all  $f \in [0, 1]$ , and  $t = 0, 1, 2, \dots$

Competition drives the finished goods-producing firm's profit to zero at the equilibrium. This zero profit condition implies that  $P_t = \left[ \int_0^1 P_t(f)^{1-\theta_p} df \right]^{\frac{1}{1-\theta_p}}$  for all  $t = 0, 1, 2, \dots$

## A.3 The Representative Intermediate Goods-Producing Firm

During each period  $t = 0, 1, 2, \dots$ , each intermediate goods-producing firm produces a distinct, perishable intermediate good, indexed by  $f \in [0, 1]$ , where firm  $f$  produces good  $f$ . The representative intermediate goods-producing firm hires  $N_t(f)$  units of labor from the representative household, to

produce  $Y_t(f)$  units of intermediate good  $f$  according to the constant return to scale technology

$$Y_t(f) = Z_t n_t(f). \quad (22)$$

The aggregate technology shock,  $Z_t$ , follows a random walk with drift process

$$\ln(Z_t) = \ln(z) + \ln(Z_{t-1}) + \varepsilon_{zt}, \quad (23)$$

where  $0 < \rho_a < 1$ ,  $z > 1$ , and the zero-mean, serially uncorrelated innovation  $\varepsilon_{at}$  are normally distributed with standard deviation  $\sigma_a$ .

Since the intermediate goods are imperfect substitutes in the production of the final goods, the intermediate goods-producing firm faces an imperfectly competitive market. During each period  $t = 0, 1, 2, \dots$  it sets the nominal price  $P_t(f)$  for its output, subject to satisfying the representative finished goods-producing firm's demand. The intermediate goods-producing firm faces a quadratic cost to adjusting nominal prices, measured in terms of the finished goods and given by

$$G_t^p(f) = \frac{\phi_p}{2} \left[ \frac{P_t(f)}{\pi P_{t-1}(f)} - 1 \right]^2 Y_t,$$

where  $\phi_p > 0$  is the degree of adjustment cost, and  $\pi$  is the steady state gross inflation rate. This relationship accounts for the negative effects of price changes on customer-firm relationships. These negative effects increase in magnitude with the size of the price change and with the overall scale of economic activity,  $Y_t$ .

The problem for the firm is to choose  $\{P_t(f), n_t(f)\}_{t=0}^{\infty}$  to maximize its total market value given by

$$E_0 \sum_{t=0}^{\infty} (\beta^t \Lambda_t / P_t) D_t(f), \quad (24)$$

subject to the constraints imposed by (21)-(23). In equation (24),  $\beta^t \Lambda_t / P_t$  measures the marginal utility value to the representative household of an additional dollar in profits received during period

$t$  and

$$D_t(f) = P_t(f)Y_t(f) - n_t(f)W_t - \frac{\phi_p}{2} \left[ \frac{P_t(f)}{\pi P_{t-1}(f)} - 1 \right]^2 Y_t \quad (25)$$

for all  $t = 0, 1, 2, \dots$ . Using equation (22) and equation (21) to substitute for  $n_t(f)$  and  $Y_t(f)$  into equation (25) allows us to write the first order conditions for this problem as

$$(1 - \theta_p) \left[ \frac{P_t(f)}{P_t} \right]^{-\theta_p} = \theta_p \left[ \frac{P_t(f)}{P_t} \right]^{-(1+\theta_p)} \frac{w_t}{Z_t} + G_{t,p}^p(f) + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} G_{t+1,p}^p(f), \quad (26)$$

where  $G_{t,p}^p(f)$  and  $G_{t+1,p}^p(f)$  represent the marginal adjustment cost of a change in current prices for periods  $t$  and  $t + 1$ , respectively. Equation (26) is the standard New Keynesian price Phillips curve in its non-linearized form. It equates the marginal benefit to the marginal cost of changing prices. The marginal benefit is the effect of the price change on the value of production (LHS). The marginal cost comprises the remuneration of labor (first term on the RHS), the adjustment cost that the change in current price entails in period  $t$  (second term on the RHS) and the expected adjustment cost that the change in current price entails in period  $t + 1$  (third term on the RHS).

#### A.4 The Government

The government finances transfer payments minus seigniorage issuing short- and long-term bonds, according to the budget constraint:

$$(M_t + B_t/r_t + B_{L,t}/r_{L,t})/P_t - \left( M_{t-1} + B_{t-1} + \frac{p_{L,t-1}}{p_{L,t}} B_{L,t-1} \right) / P_{t-1} = T_t/P_t. \quad (27)$$

As in Andres et al. (2004), we assume that long-term bonds are taken as exogenous and that short-term bonds are used to finance public financing. To guarantee that prices are determined by monetary policy and that the system is stationary, we assume that transfers are determined by the fiscal rule  $t_t = b_{t-1}^{-\rho_b}$ , where  $0 < \rho_b < 1$ , where  $t_t = T_t/P_t$ .

## A.5 The Central Bank

During each period  $t = 0, 1, 2, \dots$ , the central bank conducts monetary policy by setting the one-period nominal interest rate in reaction to movements in inflation,  $\pi_t$ , and output growth,  $g_t = Y_t/Y_{t-1}$ , according to the Taylor-type rule

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + \rho_\pi \ln(\pi_t/\pi) + \rho_g \ln(g_t/g) + \varepsilon_{rt}, \quad (28)$$

where  $0 < \rho_r < 1$ ,  $\rho_\pi > 1$ ,  $\rho_g > 0$ , and  $r$  and  $g$  are the steady-state values of the nominal interest rate and output growth, respectively. The zero-mean, serially uncorrelated policy shock  $\varepsilon_{rt}$  is normally distributed with a standard deviation of  $\sigma_r^2$ .

## A.6 Symmetric Equilibrium

In a symmetric, dynamic equilibrium, all households and intermediate goods-producing firms make identical decisions, so that  $Y_t(i) = Y_t(f) = Y_t$ ,  $n_t(i) = n_t(f) = n_t$ ,  $D_t(f) = D_t$ ,  $W_t(i) = W_t$ , and  $P_t(f) = P_t$ , for all  $[i, f] \in [0, 1]$  and  $t = 0, 1, 2, \dots$ . These conditions, together with the firm profit conditions (25), the household's budget constraint (12) and the government budget constraint (27), produce the the aggregate resource constraint

$$Y_t = C_t + (\phi_w/2)(w_t/w - 1)^2 n_t + (\phi_p/2)(\pi_t/\pi - 1)^2 Y_t + \xi_t (B_{L,t}/r_{L,t}). \quad (29)$$

Some of the real variables in this model inherit unit roots from the technology shock (23). Hence, we re-write the model in terms of the stationary variables  $y_t = Y_t/Z_t$ ,  $c_t = C_t/Z_t$ ,  $d_t = D_t/(P_t Z_t)$ ,  $\lambda_t = \Lambda_t Z_t$  and  $z_t = Z_t/Z_{t-1}$ . Finally, we follow the finance literature and define asset prices,  $q_t$ , as the expected discounted sum of future dividends, such that:

$$q_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} (d_{t+1} + q_{t+1}).$$



Therefore, the model describes the behavior of the 20 endogenous variables  $\{y_t, c_t, \lambda_t, n_t, u_t, l_t, d_t, q_t, t_t, m_t, r_t, r_{L,t}, b_t, b_{L,t}, \pi_t, w_t, \pi_t^w, a_t, z_t, \xi_t\}$  and the 4 exogenous processes  $\{\varepsilon_{at}, \varepsilon_{zt}, \varepsilon_{\xi t}, \varepsilon_{rt}\}$ . The equilibrium conditions have no analytical solution. Consequently, the system is approximated by loglinearizing its equations around the deterministic steady state. In this way, a linear dynamic system describes the path of the endogenous variables' relative deviations from their steady-state value, accounting for the exogenous shocks.

The model is calibrated on quarterly frequencies using US data. Since we use the theoretical framework to identify the sign of the variables' response to shocks, we need to ensure that the reactions are robust across a broad range of the parameters' calibration. For this reason, as in Pappa (2009), Canova and Paustian (2011) and Mumtaz and Zanetti (2012), among others, we assume that the parameters' values are uniformly and independently distributed over a wide range of plausible values. The range value for each parameter is described below and reported in Table 3. In particular, we allow the real interest rate to vary between 2 and 6.5 percent annually, values that are commonly used in the literature, and they pin down the quarterly discount factor,  $\beta$ , between 0.985 and 0.995. We calibrate the elasticity of substitution among goods,  $\theta_p$ , to vary between 8 and 11, such that the equilibrium price mark-up,  $(\theta_p/(\theta_p - 1))$ , is between 10% and 14.2%, values that are in line with micro- and macro-evidence, as detailed in Rotemberg and Woodford (1999). We use the same values to calibrate the elasticity of substitution among labor inputs,  $\theta_w$ . Consistent with the estimates in Ireland (2011), the degree of nominal price and wage rigidities,  $\phi_p$  and  $\phi_w$ , are allowed to cover the broad range of values between 1 to 90. We need to set values for the magnitude of the household's preferences for liquidity  $\nu$ . Since a precise empirical evidence on this parameter is unavailable, we allow for a range of values between 0 and 0.03, in line with Harrison (2012). The inverse of the steady state ratio of long- and short-term bonds,  $\kappa$ , is allowed to vary between 2 and 4, around the estimated value of 3 in Kuttner (2006). Similarly to Mumtaz and Zanetti (2012), the monetary policy parameters are allowed to vary in the following ranges:  $\rho_r \in [0, 0.99]$ ,  $\rho_g \in [0, 1]$  and  $\rho_\pi \in [1, 5]$ . The autoregressive coefficients of the preference and spread shocks,  $\rho_a$  and  $\rho_\xi$ , are free to vary between 0.75 and 0.99 in line with Ireland (2003).

The steady state value of output growth  $z$  is set equal to 1.0046 to match the average annualized

Table 3: Parameters ranges.

Parameter		Range
$\beta$	Discount factor	[0.985 , 0.995]
$\theta_p$	Elasticity of substitution among goods	[8, 11]
$\theta_w$	Elasticity of substitution among labor inputs	[8, 11]
$\phi_p$	Degree of nominal price rigidities	[1, 90]
$\phi_w$	Degree of nominal wage rigidities	[1, 90]
$\nu$	Household's preferences for liquidity	[0, 0.03]
$\kappa$	Inverse of the steady state ratio of long- and short-term bonds	[2, 4]
$\rho_r$	Interest rate inertia	[0, 0.99]
$\rho_g$	Interest rate reaction to output growth	[0, 1]
$\rho_\pi$	Interest rate reaction to inflation	[1, 5]
$\rho_a$	Autoregressive coefficient, preference shock	[0.75, 0.99]
$\rho_\xi$	Autoregressive coefficient, spread shock	[0.75, 0.99]

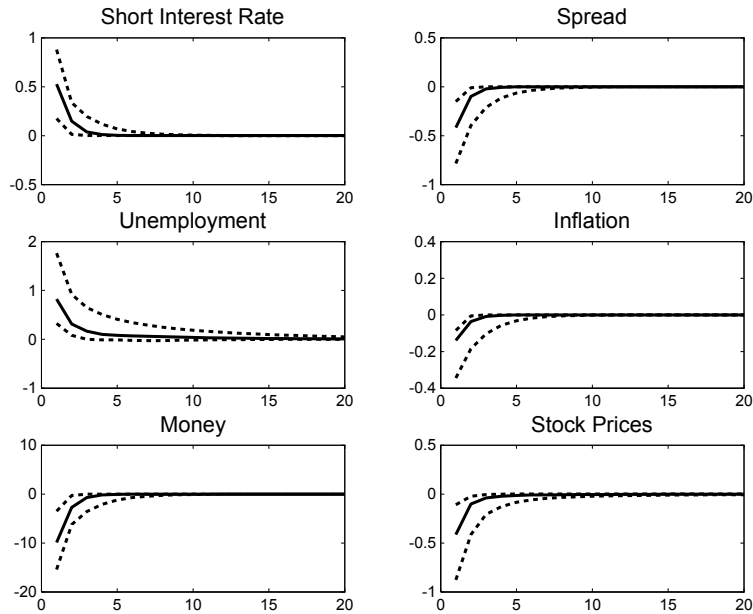
Notes: The table shows the parameters' ranges used to simulate the model.

growth rate of GDP per capital (1.85%), and the steady state values of the preference and spread shocks,  $a$  and  $\xi$ , are conveniently set equal to 1, as they do not affect the dynamics of the system. The standard deviation of technology, monetary policy, preference and spread shocks,  $\sigma_z$ ,  $\sigma_r$ ,  $\sigma_a$  and  $\sigma_\xi$ , are normalized to be equal to 1 percent. Finally, we calibrate the steady state level of gross price and wage inflation,  $\pi$  and  $\pi_w$ , equal to 1, as their values do not affect the dynamics of the system.

By allowing for a wide range of values, we enable the model to produce impulse response functions over a broad range of numerical calibrations, which ideally cover the full spectrum of plausible values in the literature. To illustrate how the variables of the theoretical model react to each shock, Figures 1-4 plot impulse responses of variables to one positive percentage-point deviation of monetary policy shock ( $\varepsilon_{rt}$ ), interest rate spread shock ( $\varepsilon_{\xi t}$ ), demand shock ( $\varepsilon_{at}$ ), and supply shock ( $\varepsilon_{zt}$ ).

Figure 12 shows that in the aftermath of a positive monetary policy shock ( $\varepsilon_{rt}$ ) that raises the nominal interest rate, as implied by the Taylor-type rule, the real money holdings fall due to the higher cost of holding money, as encapsulated by the money-demand equation. The increase in the nominal interest rate leads to a raise in the demand of short-term bonds that generates a fall in real

Figure 12: Impulse Response function to a positive monetary policy shock ( $\varepsilon_{rt}$ )

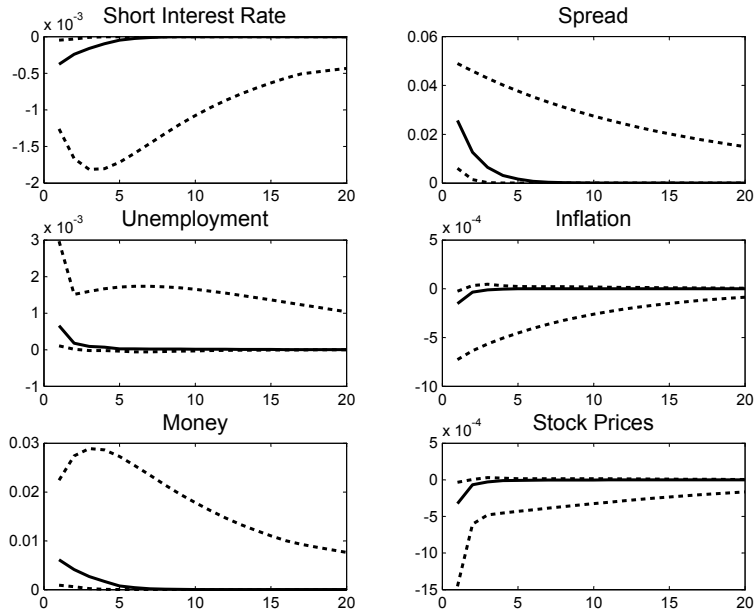


Notes: Each entry shows the percentage-point response of one of the model's variables to a one-percentage-deviation of the shock. The solid line reports the median responses, and the dashed lines report the 2.5 and 97.5 percentiles of responses.

activity and stock prices and a raise in unemployment. The fall in output generates an increase in the long-run nominal interest rate (not reported in the figure), as implied by the long-term bonds demand equation, which is lower than the increase in the short-run interest rate. Therefore the spread between long- and short-run interest rates falls, offsetting some of the original effect of the raise in the short-run nominal interest rate.

Figure 13 shows that the effect of a positive interest rate spread shock ( $\varepsilon_{\xi t}$ ) is to increase the long-run interest rate, which generates a fall in consumption and output, as implied by the demand for  $L$ -period bonds. The contraction in output induces stock prices to fall and unemployment to rise. The fall in output generates a reduction in inflation, as implied by the price Phillips curve, whose effect is to induce a fall in the short-run interest rate through the Taylor rule. Finally, real money holdings increase in reaction to the shock since the money demand equation is negatively

Figure 13: Impulse Response function to a spread shock ( $\varepsilon_{\xi t}$ )



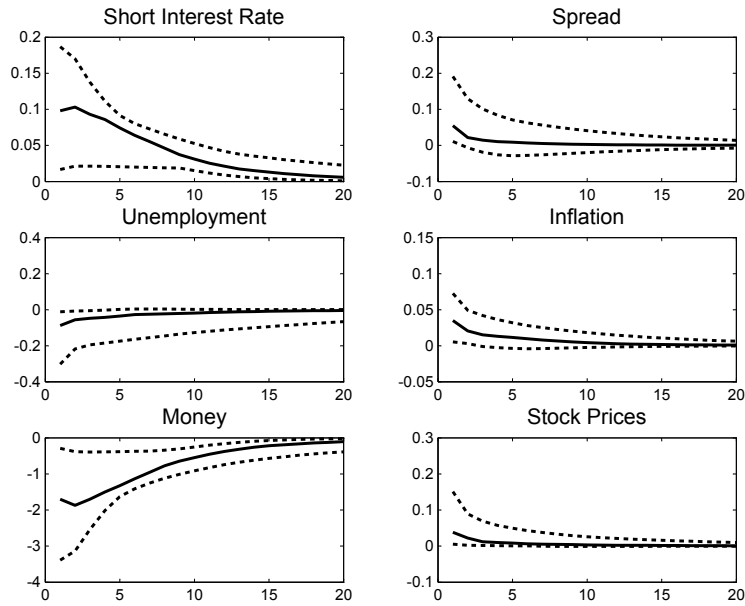
Notes: Each entry shows the percentage-point response of one of the model's variables to a one-percentage-deviation of the shock. The solid line reports the median responses, and the dashed lines report the 2.5 and 97.5 percentiles of responses.

related to the short-run nominal interest rate.

Figure 14 shows the variables' responses to a positive demand shock ( $\varepsilon_{at}$ ). The direct effect of the shock is to increase the marginal utility of consumption for any given level of consumption. Therefore real activity and stock prices increase whereas unemployment falls. The rise in output growth generates an increase in inflation via the price Phillips curve, whose effect is to increase the short-term nominal interest rate, as dictated by the Taylor rule. As a consequence of the increase in the short-term interest rate, real money holdings fall. Finally, the demand for  $L$ -period bonds holdings induces a sharp rise in the long-term nominal interest rate in response to an increase in the marginal utility of consumption. Hence, the interest rate spread increases.

Figure 15 shows the variables' reactions to a positive supply shock ( $\varepsilon_{zt}$ ). The shock produces a permanent increase in output due to the standard random walk specification, which is mirrored

Figure 14: Impulse Response function to a positive demand shock ( $\varepsilon_{at}$ )



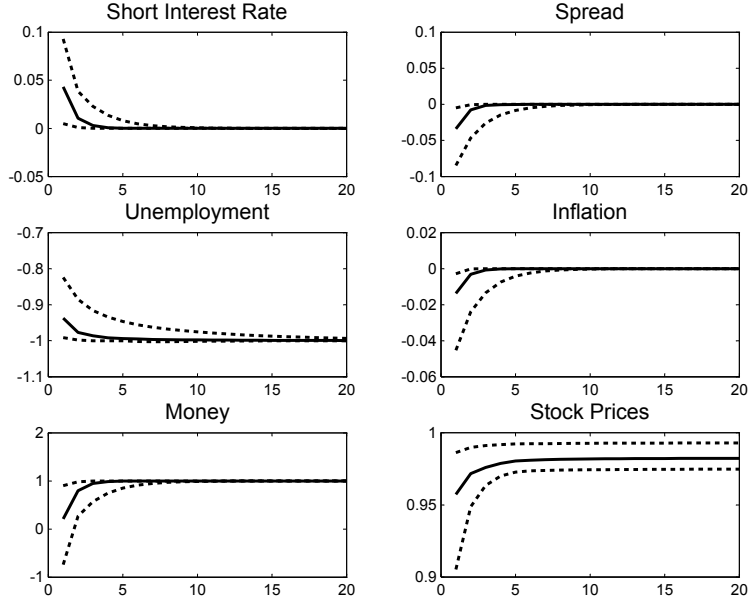
Notes: Each entry shows the percentage-point response of one of the model's variables to a one-percentage-deviation of the shock. The solid line reports the median responses, and the dashed lines report the 2.5 and 97.5 percentiles of responses.

by a permanent increase in stock prices and a decrease in unemployment. Output growth rises sharply and therefore induces the central bank to raise the short-term nominal interest rate, which generates a fall in real money holdings. Finally, the rise in the short-term nominal interest rate generates a fall in the interest rate spread.

## B Description of the Priors

The priors for the VAR( $P$ ) coefficients and the error covariance matrices are set via dummy observations. The normal inverse Wishart prior and is defined as

Figure 15: Impulse Response function to a positive supply shock ( $\varepsilon_{zt}$ )



Notes: Each entry shows the percentage-point response of one of the model's variables to a one-percentage-deviation of the shock. The solid line reports the median responses, and the dashed lines report the 2.5 and 97.5 percentiles of responses.

$$Y_D = \begin{pmatrix} \frac{\text{diag}(\gamma_1 \sigma_1 \dots \gamma_N \sigma_N)}{\tau} \\ 0_{N \times (P-1) \times N} \\ \dots \\ \text{diag}(\sigma_1 \dots \sigma_N) \\ \dots \\ 0_{1 \times N} \end{pmatrix}, \text{ and } X_D = \begin{pmatrix} \frac{J_P \otimes \text{diag}(\sigma_1 \dots \sigma_N)}{\tau} & 0_{NP \times 1} \\ 0_{N \times NP} & 0_{N \times 1} \\ \dots \\ 0_{1 \times NP} & c \end{pmatrix},$$

where  $\sigma_i$  for  $i = 1, 2, \dots, N$  represents scaling factors,  $\gamma_i$  denotes the prior mean for the coefficients on the first lag,  $\tau$  is the tightness of the prior on the VAR coefficients,  $c$  is the tightness of the prior on the constant terms. In order to obtain a value for  $\gamma_i, \sigma_i$ , we estimate an AR(1) model via OLS for each endogenous variable.  $\gamma_i$  is set equal to OLS estimate of the AR(1) coefficient, while  $\sigma_i$  is the standard deviation of the residual. The matrix  $J_P$  is defined as  $\text{diag}(1, 2, \dots, P)$ . We set  $\tau = 0.03$

and  $c = 1$  in our implementation. The value for  $\tau$  implies a relatively high degree of shrinkage (relative to Sims and Zha (1998), for example who employ a value of 0.2 for the VAR coefficients in structural form<sup>27</sup>). While the key results are robust to higher values for  $\tau$ , the precision of the estimates deteriorates in regimes with a few number of observations. Note that in the final regime covering the unconventional monetary policy period, we introduce an additional prior on the VAR coefficients that ensures that lagged coefficients on the non-dependent variables in the interest rate equation are close to zero. This prior is implemented via a prior covariance matrix with the diagonal elements corresponding to the coefficients of interest in the interest rate equation set to small values ( $1e - 12$ ). The remaining diagonal elements are set to 1000.

The prior for the non zero elements of the transition probability matrix  $p_{ij}$  is of the following form

$$p_{ij}^0 = D(u_{ij}), \quad (30)$$

where  $D(\cdot)$  denotes the Dirichlet distribution and  $u_{ij} = 15$  if  $i = j$  and  $u_{ij} = 1$  if  $i \neq j$ . This choice of  $u_{ij}$  implies that the regimes are fairly persistent. The posterior distribution is:

$$p_{ij} = D(u_{ij} + \eta_{ij}), \quad (31)$$

where  $\eta_{ij}$  denotes the number of times regime  $i$  is followed by regime  $j$ .

## C Description of the Gibbs sampling algorithm

The Gibbs sampling algorithm proceeds in the following steps:

### 1. Sampling $S_t$

Following (Kim and Nelson, 1999, Chapter 9), we use Multi-Move Gibbs sampling to draw  $S_t$  from the joint conditional density,  $f(S_t | Z_t, c_S, B_{1,S}, \dots, B_{K,S}, \Omega_S, \tilde{P})$ . Note that we impose the restriction that each regime must have at least  $N \times K + 2$  observations, where  $N$  denotes the number of endogenous variables in the VAR, to ensure sufficient degrees of freedom for

---

<sup>27</sup>Sims and Zha (1998) use a value of 1 for the parameter controlling the prior tightness of the intercept and lag decay.

each regime.

## 2. **Sampling** $c_S, B_{1,S}, \dots, B_{K,S}, \Omega_S$

Conditional on a draw for  $S_t$ , the model is simply a sequence of Bayesian VAR models. The regime-specific VAR coefficients are sampled from a Normal distribution and the covariances are drawn from an inverted Wishart distribution. For the first  $M$  regimes, we use a Normal Inverse Wishart prior (see Kadiyala and Karlsson (1997)). However, as described in detail below, we employ a (Normal diffuse) prior distribution for the VAR coefficients to the final regime, which is compatible with the identification of the shock to the government bond spread. In our sample, the recent financial crisis coincides with the final regime of the estimated VAR model. The prior on the VAR coefficients in this regime implies that the policy rate does not respond to lagged changes in other endogenous variables. This assumption is compatible with restrictions used to identify the shock to the bond-yield spread and reflects the fact that policy rates have reached the ZLB.

## 3. **Sampling** $\tilde{P}$

Given the state variables  $S_t$ , the non-zero elements of the transition probability matrix are independent of  $Z_t$  and the other parameters of the model, and they are drawn from a Dirichlet posterior.

## **D An Application to the Validation of Theoretical Predictions**

The analysis identifies shocks in each regime by letting the rotation matrix  $Q$  to be a function of the different states, which is a standard approach in the literature (see Rubio-Ramrez et al. (2005) and references therein). An interesting application is to impose the identifying restrictions estimated in one regime to infer changes in the responses of the variables in alternative regimes by applying the rotation matrix from a chosen regime to the alternative regimes. This approach is equivalent to identifying shocks in one regime and then using the identifying restrictions from the chosen regime to assess the change in the variables' responses in different regimes.<sup>28</sup> This approach is

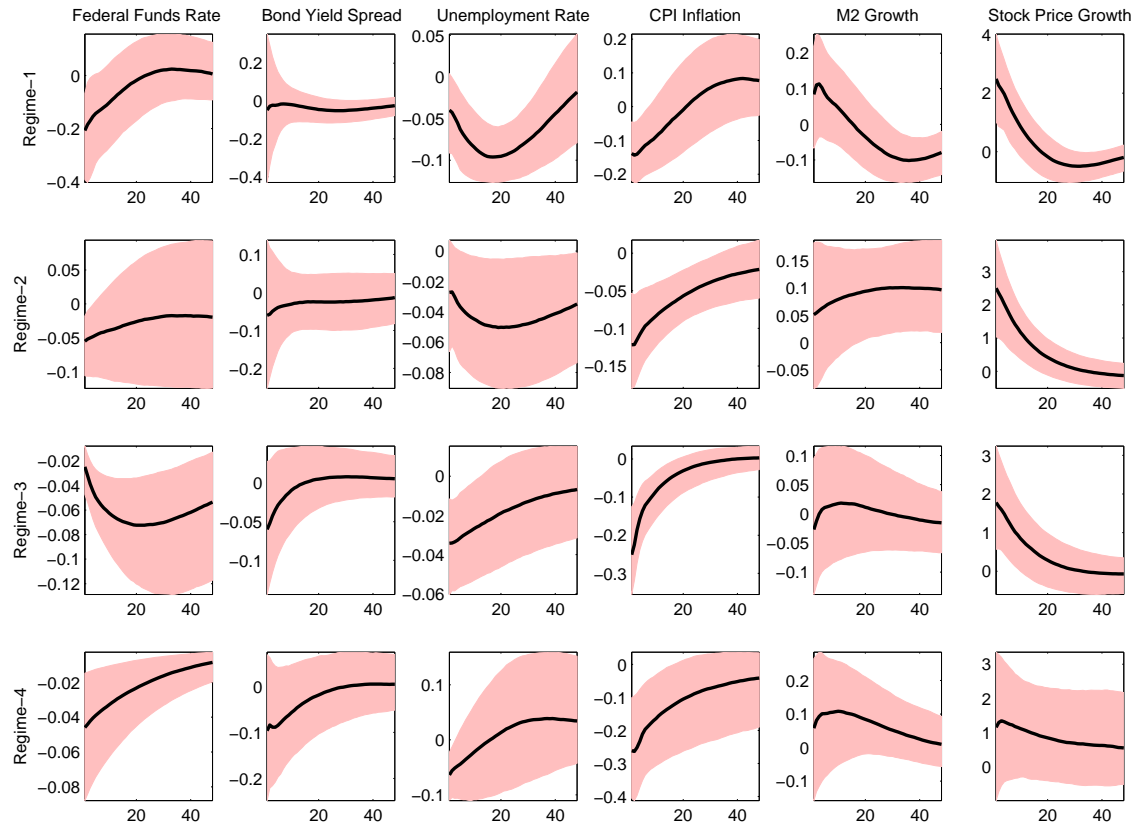
---

<sup>28</sup>We thank an anonymous referee for suggesting this exercise to us. See Faust (1998) for a similar approach on standard VAR models.



useful to validate specific predictions from theoretical models. For instance, the paradox of toil (see Eggertsson (2010)) reveals that an expansionary supply shock leads to an unconventional increase in unemployment when the nominal interest rate is at the ZLB as during the financial crisis, contrary to standard economic consensus. We can assess the empirical support to this theoretical result by applying the rotation from the third regime, which identifies the widely-accepted transmission mechanism of shocks during the Great Moderation period, to the period of the fourth regime that captures the financial crisis. The aim is to establish whether the response of unemployment to an expansionary supply shock reduces unemployment during the fourth regime, as implied by the paradox of toil. Figure 16 shows that the qualitative responses of the variables to the positive supply shock identified imposing the rotation matrix estimated in the third regime on all the other regimes. Important for our analysis, although the response of unemployment to the supply shock diminishes during the financial crisis, the reaction of unemployment is negative in the fourth regime, contrary to the paradox of toil. Although the theoretical results point to an increase in unemployment in response to a positive supply shock when the nominal interest rate is at the zero lower bound, our econometric model does not provide empirical support to this result.

Figure 16: Model validation: impulse response functions to a supply shock identified in the third regime



Notes: The four regimes correspond to the periods of January 1962-January 1992, February 1992-February 2000, March 2000-December 2007 and January 2008-March 2011. The responses in each regime are identified using the rotation matrix  $Q$  estimated in the third regime.

## References

- Andres, J., Lopez-Salido, J.D., Nelson, E., 2004. Tobin's imperfect asset substitution in optimizing general equilibrium. *Journal of Money, Credit and Banking* 36, pp. 665–690.
- Ang, A., Piazzesi, M., 2003. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics* 50, 745–787.
- Arias, J.E., Rubio-Ramirez, J.F., Waggoner, D.F., 2014. Inference Based on SVARs Identified with Sign and Zero Restrictions: Theory and Applications. *International Finance Discussion Papers* 1100. Board of Governors of the Federal Reserve System (U.S.).
- Baumeister, C., Benati, L., 2013. Unconventional monetary policy and the great recession: Estimating the macroeconomic effects of a spread compression at the zero lower bound. *International Journal of Central Banking* 9, 165–212.
- Baumeister, C., Liu, P., Mumtaz, H., 2010. Changes in the transmission of monetary policy: evidence from a time-varying factor-augmented VAR. *Bank of England working papers*. Bank of England.
- Bauwens, L., Rombouts, J.V., 2012. On marginal likelihood computation in change-point models. *Computational Statistics & Data Analysis* 56, 3415–3429.
- Benati, L., Goodhart, C., 2010. Monetary Policy Regimes and Economic Performance: The Historical Record, 1979-2008, in: Friedman, B.M., Woodford, M. (Eds.), *Handbook of Monetary Economics*. Elsevier. volume 3 of *Handbook of Monetary Economics*. chapter 21, pp. 1159–1236.
- Benati, L., Mumtaz, H., 2007. U.S. evolving macroeconomic dynamics: a structural investigation. *Working Paper Series* 0746. European Central Bank.
- Bianchi, F., 2012. Evolving Monetary/Fiscal Policy Mix in the United States. *American Economic Review* 102, 167–172.
- Bianchi, F., 2013. Regime Switches, Agents' Beliefs, and Post-World War II U.S. Macroeconomic Dynamics. *Review of Economic Studies* 80, 463–490.

- Canova, F., Paustian, M., 2011. Business cycle measurement with some theory. *Journal of Monetary Economics* 58, 345–361.
- Chen, H., Curdia, V., Ferrero, A., 2012. The macroeconomic effects of large-scale asset purchase programmes. *Economic Journal* 122, F289–F315.
- Chib, S., 1998. Estimation and comparison of multiple change-point models. *Journal of Econometrics* 86, 221–241.
- Chung, H., Laforte, J.P., Reifschneider, D., Williams, J.C., 2012. Have we underestimated the likelihood and severity of zero lower bound events? *Journal of Money, Credit and Banking* 44, 47–82.
- Cogley, T., Primiceri, G.E., Sargent, T.J., 2010. Inflation-gap persistence in the US. *American Economic Journal: Macroeconomics* 2, 43–69.
- Cogley, T., Sargent, T.J., 2005. Drifts and volatilities: monetary policies and outcomes in the post WWII U.S. *Review of Economic Dynamics* 8, 262–302.
- Curdia, V., Woodford, M., 2010. Credit spreads and monetary policy. *Journal of Money, Credit and Banking* 42, 3–35.
- Del Negro, M., Eggertsson, G., Ferrero, A., Kiyotaki, N., 2016. The great escape? A quantitative evaluation of the Fed’s liquidity facilities. Working Paper 22259. National Bureau of Economic Research.
- Diebold, F., Li, C., 2006. Forecasting the term structure of government bond yields. *Journal of Econometrics* 130, 337–364.
- Diebold, F., Rudebusch, G., Aruoba, S., 2006. The macroeconomy and the yield curve: a dynamic latent factor approach. *Journal of Econometrics* 131, 309–338.
- Eggertsson, G.B., 2010. The paradox of toil. Staff Reports 433. Federal Reserve Bank of New York.

- Faust, J., 1998. The robustness of identified VAR conclusions about money. *International Finance Discussion Papers 610*. Board of Governors of the Federal Reserve System (U.S.).
- Fernandez-Villaverde, J., Rubio-Ramirez, J.F., 2008. How Structural Are Structural Parameters?, in: *NBER Macroeconomics Annual 2007, Volume 22*. National Bureau of Economic Research, Inc. NBER Chapters, pp. 83–137.
- Gagnon, J., Raskin, M., Remache, J., Sack, B., 2010. Large-scale asset purchases by the Federal Reserve: did they work? *Staff Reports*. Federal Reserve Bank of New York.
- Gali, J., 2011. The return of the wage Phillips curve. *Journal of the European Economic Association* 9, 436–461.
- Giannone, D., Lenza, M., Reichlin, L., 2012. Money, credit, monetary policy and the business cycle in the Euro Area. *CEPR Discussion Papers 8944*. C.E.P.R. Discussion Papers.
- Goodfriend, M., McCallum, B.T., 2007. Banking and interest rates in monetary policy analysis: A quantitative exploration. *Journal of Monetary Economics* 54, 1480–1507.
- Harrison, R., 2012. Asset purchase policy at the effective lower bound for interest rates. *Bank of England working papers 444*. Bank of England.
- Ireland, P.N., 2003. Endogenous money or sticky prices? *Journal of Monetary Economics* 50, 1623–1648.
- Ireland, P.N., 2011. A New Keynesian perspective on the Great Recession. *Journal of Money, Credit and Banking* 43, 31–54.
- Kadiyala, K.R., Karlsson, S., 1997. Numerical methods for estimation and inference in bayesian var-models. *Journal of Applied Econometrics* 12, 99–132.
- Kapetanios, G., Mumtaz, H., Stevens, I., Theodoridis, K., 2012. Assessing the economy-wide effects of quantitative easing. *Economic Journal* 122, F316–F347.

- Kim, C.J., Nelson, C.R., 1999. State-space models with regime switching. MIT Press, Cambridge, Massachusetts.
- Kuttner, K., 2006. Can Central Banks Target Bond Prices? NBER Working Papers 12454. National Bureau of Economic Research, Inc.
- Mavroeidis, S., 2010. Monetary policy rules and macroeconomic stability: Some new evidence. *American Economic Review* 100, 491–503.
- McConnell, M., Perez-Quiros, G., 2000. Output fluctuations in the United States: what has changed since the early 1980's. *American Economic Review* 90, 1,464 – 76.
- Meyer, L.H., Bomfim, A.N., 2012. Not Your Father's Yield Curve: Modeling the Impact of QE on Treasury Yields. *Macroeconomic Advisers. Monetary Policy Insights*.
- Mumtaz, H., Surico, P., 2009. Time-varying yield curve dynamics and monetary policy. *Journal of Applied Econometrics* 24, 895–913.
- Mumtaz, H., Zanetti, F., 2012. Neutral technology shocks and the dynamics of labor input: Results from an agnostic identification. *International Economic Review* 53, 235–254.
- Mumtaz, H., Zanetti, F., 2015. Labor Market Dynamics: A Time-Varying Analysis. *Oxford Bulletin of Economics and Statistics* 77, 319–338.
- Nakajima, J., 2011. Monetary policy transmission under zero interest rates: An extended time-varying parameter vector autoregression approach. *The B.E. Journal of Macroeconomics* 11, 1–24.
- Neely, C.J., 2015. Unconventional monetary policy had large international effects. *Journal of Banking & Finance* 52, 101–111.
- Pappa, E., 2009. The effects of fiscal shocks on employment and the real wage. *International Economic Review* 50, 217–244.

- Primiceri, G., 2005. Time varying structural vector autoregressions and monetary policy. *The Review of Economic Studies* 72, 821–852.
- Rotemberg, J.J., Woodford, M., 1999. Interest rate rules in an estimated sticky price model, in: *Monetary Policy Rules*. National Bureau of Economic Research, Inc. NBER Chapters, pp. 57–126.
- Rubio-Ramrez, J.F., Waggoner, D.F., Zha, T., 2005. Markov-switching structural vector autoregressions: theory and application. FRB Atlanta Working Paper 2005-27. Federal Reserve Bank of Atlanta.
- Rudebusch, G.D., Wu, T., 2007. Accounting for a shift in term structure behavior with no-arbitrage and macro-finance models. *Journal of Money, Credit and Banking* 39, 395–422.
- Sims, C., Zha, T., 1998. Bayesian methods for dynamic multivariate models. *International Economic Review* 39, 949–68.
- Sims, C.A., Waggoner, D.F., Zha, T., 2008. Methods for inference in large multiple-equation markov-switching models. *Journal of Econometrics* 146, 255 – 274. Honoring the research contributions of Charles R. Nelson.
- Smets, F., Wouters, R., 2007. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review* 97, 586–606.
- Stock, J.H., Watson, M.W., 2003. Has the business cycle changed and why?, in: *NBER Macroeconomics Annual 2002, Volume 17*. National Bureau of Economic Research, Inc. NBER Chapters, pp. 159–230.
- Strachan, R.W., Dijk, H.K.V., 2013. Evidence On Features Of A Dsge Business Cycle Model From Bayesian Model Averaging. *International Economic Review* 54, 385–402.
- Swanson, E.T., Williams, J.C., 2014a. Measuring the Effect of the Zero Lower Bound on Medium- and Longer-Term Interest Rates. *American Economic Review* 104, 3154–85.
- Swanson, E.T., Williams, J.C., 2014b. Measuring the effect of the zero lower bound on yields and exchange rates in the U.K. and Germany. *Journal of International Economics* 92, S2 – S21.

Woodford, M., 2001. Fiscal Requirements for Price Stability. *Journal of Money, Credit and Banking* 33, 669–728.

Wu, T., 2011. The U.S. money market and the term auction facility in the financial crisis of 2007-2009. *The Review of Economics and Statistics* 93, 617–631.

Zanetti, F., 2007. A non-walrasian labor market in a monetary model of the business cycle. *Journal of Economic Dynamics and Control* 31, 2413–2437.