Real option valuation of a decremental regulation service provided by electricity storage

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Abstract

This paper is a quantitative study of a reserve contract for real-time balancing of a power system. Under this contract, the owner of a storage device, such as a battery, helps smooth fluctuations in electricity demand and supply by using the device to increase electricity consumption. The battery owner must be able to provide immediate physical cover, and should therefore have sufficient storage available in the battery before entering the contract. Accordingly, the following problem can be formulated for the battery owner: determine the optimal time to enter the contract and, if necessary, the optimal time to discharge electricity before entering the contract. This problem is formulated as one of optimal stopping, and is solved explicitly in terms of the model parameters and instantaneous values of the power system imbalance. The optimal operational strategies thus obtained ensure that the battery owner has positive expected economic profit from the contract. Furthermore, they provide explicit conditions under which the optimal discharge time is consistent with the overall objective of power system balancing. This paper also carries out a preliminary investigation of the "lifetime value" aggregated from an infinite sequence of these balancing reserve contracts. This lifetime value, which can be viewed as a single project valuation of the battery, is shown to be positive and bounded. Therefore, in the long run such reserve contracts can be beneficial to commercial operators of electricity storage, while reducing some of the financial and operational risks in power system balancing.

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JEL Classification: C61, E22, Q41.

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1 Introduction

1.1 Problem motivation

Electricity supply and demand on a power system must be balanced continuously in real time to ensure its stability. The system operator, an independent entity that manages the transmission system [1, p. 3], may balance the power system by:

- *incremental* actions, such as requesting additional generation or a reduction in demand, when there is a shortfall in supply;

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- *decremental* actions, such as requesting generation to be curtailed or demand increased, when there is an overproduction of energy;

Decremental balancing actions are increasingly relevant for power systems with high levels of intermittent generation from renewable energy sources [2], and electricity storage is widely seen as an important technology for ensuring power system stability in this case [3, 4, 5, 2, 6]. Consequently, this paper analyses a contract in which the owner of a storage device, such as a battery, helps smooth fluctuations in electricity demand and supply by using the device to increase electricity consumption. By analysing the battery owner's decision to enter the contract, this paper investigates whether the battery owner and system operator can benefit from such an arrangement. The analysis uses a model of a market for procuring balancing services that is conceptually similar to those found in European countries such as the United Kingdom, Germany and Denmark. A general discussion of such a market, based on the textbook [1], is presented below.

1.2 Procurement of power system balancing services

Balancing services can be procured by the system operator on either a short-term or long-term basis. Those that are acquired shortly before delivery in real time are sourced from a real-time (alternatively, "spot" or "reserve") market that is managed by the system operator. In Great Britain, for example, the short-term market is known as the *Balancing Mechanism* [7], and in it the system operator trades with eligible participants between *gate closure*, which is one hour ahead of a specific reference time, and delivery (real time) [7]. The short-term approach presents the system operator with considerable risks concerning the amount and cost of balancing services. In order to mitigate such risks, the system operator can negotiate long-term bilateral contracts with third-party balancing service providers in advance [1, p. 60].

In long-term contracts for balancing services, the service provider is first paid a fixed price, referred to as the *contract premium*, to keep available some capacity for altering its generation or demand as required. In accordance with the terminology used previously, this paper associates *incremental balancing reserve* with services that increase generation or reduce demand, and *decremental balancing reserve* with services that decrease generation or increase demand. After receiving the contract premium, a provider of incremental reserve is typically *paid* to increase its generation or decrease its demand for electricity [1, p. 60]. On the other hand, a provider of decremental reserve typically *pays* to withhold its generation or increase its demand for electricity [1, p. 60]. The qualifier "typically" is ascribed here to acknowledge the emergence of negative real-time prices in some markets.

Balancing services are generally categorised according to component of the system imbalance they are meant to address [1, pp. 107–111]. This paper is concerned with *regulation services* which, according to [1, p. 108], are meant to address "rapid fluctuations in load and small unintended changes in generation." Such services help keep the system frequency within a sufficiently small range around its nominal value, for example, and are typically provided by technologies that can rapidly adjust their demand or generation as required [1, p. 108].

Since electricity storage technologies such as batteries, flywheels, superconducting magnetic energy stores, and supercapacitors can provide fast access to power, they have the potential to perform the regulation services that are provided by conventional technologies such as generating units [3, 4, 5]. Accordingly, the recent papers [8, 9] studied an incremental balancing reserve contract between a battery owner and the system operator in which the battery owner provides a regulation service by discharging electricity from the battery. These papers studied the battery owner's decision to enter the contract, and the decision to buy electricity beforehand if necessary. The "lifetime" value aggregated from an infinite sequence of these contracts, which can be viewed as a single project valuation model for an electricity store [10, p. 693], was also studied.

1.3 Aim and scope of the present work

This paper studies a decremental reserve contract between a battery owner and the system operator in which the former uses the battery to provide a regulation service. In this contract, the battery owner first receives a premium to keep some storage available and, when called upon, pays a fixed price to consume one unit of electricity using the battery.

The battery owner must be able to provide immediate physical cover, and is therefore unable to enter the contract when the battery is full. In order to ensure that the battery owner can enter the contract at a future time, we assume that there is a real-time market in which the stored electricity can be sold (see Figure 1 below). Furthermore, we assume that the price per unit of electricity in this market is determined by the instantaneous value of the system imbalance. We study the battery owner's decision to enter a single contract, as well as the lifetime value aggregated from an infinite sequence of these contracts (that is, infinite repetitions of the cycle in Figure 1 below). More precisely, in this paper:

1. When the battery owner is able to provide immediate physical cover, we study its decision to enter the contract via optimal stopping theory. The study of this decision when the battery is full also requires an analysis of the decision to sell electricity prior to entering the contract. As such it is related to a more general optimal stopping problem with two stopping times.

2. We assume a Brownian motion model for the transmission system imbalance, a piecewise linear relationship between the system imbalance and the market price (see (2.1) below), and, like [8], that the system operator uses the contract when the imbalance crosses a fixed threshold value (see equation (2.2) below). A condition (2.7) is also introduced below to disincentivize the battery owner from selling electricity and immediately entering the contract when the system operator intends to use it. Under these assumptions and for specific values of the model parameters, in Theorems 3.1 and 3.2 we provide explicit solutions to these optimal stopping problems.

3. Finally, we prove in Theorem 4.1 that the lifetime value is bounded.

The rest of the paper is organised as follows. Section 2 presents the model and the optimisation problems corresponding to a single contract. Explicit expressions for the contract's value and the battery owner's optimal decisions are derived in Section 3. The lifetime value is studied in Section 4, and afterwards is the conclusion. Proofs of the main results can be found in the electronic supplementary material.

2 A model for the balancing reserve contract

This section describes the model for the contract and the battery owner's optimisation problems, based on the one in [8]. The *random* instantaneous value of the demand-supply imbalance on the power system over time is represented by a standard Brownian motion $X = (X(t))_{t\geq 0}$. A positive (resp. negative) value of $X(t)$ indicates a surplus (resp. shortfall) in the electricity supply at time $t \geq 0$.

Let $x \in \mathbb{R}$ denote the current value of the system imbalance. We suppose there is a deterministic function $x \mapsto f(x)$ that quantifies the relationship between the market price per unit of electricity and the system imbalance. A reasonable assumption is that $x \mapsto f(x)$ decreases in x, so that market prices decrease (resp. increase) as the over-supply (resp. under-supply) of power worsens. A simple model for the market price is provided by the following piecewise linear function:

$$f(x) = \begin{cases} M, & x \le \frac{M-c}{b} \\ c + bx, & \frac{M-c}{b} < x \le \frac{-c}{b} \\ 0, & \frac{-c}{b} < x \end{cases} \tag{2.1}$$

where M, c, b are constants satisfying $0 < c < M$ and $b < 0$. The resulting market prices are non-negative and capped at the level M, which helps compensate for the lack of mean reversion in the imbalance process $(X_t)_{t\geq 0}$. The condition $c < M$ ensures that the upper price cap M is associated with a shortfall in generation.

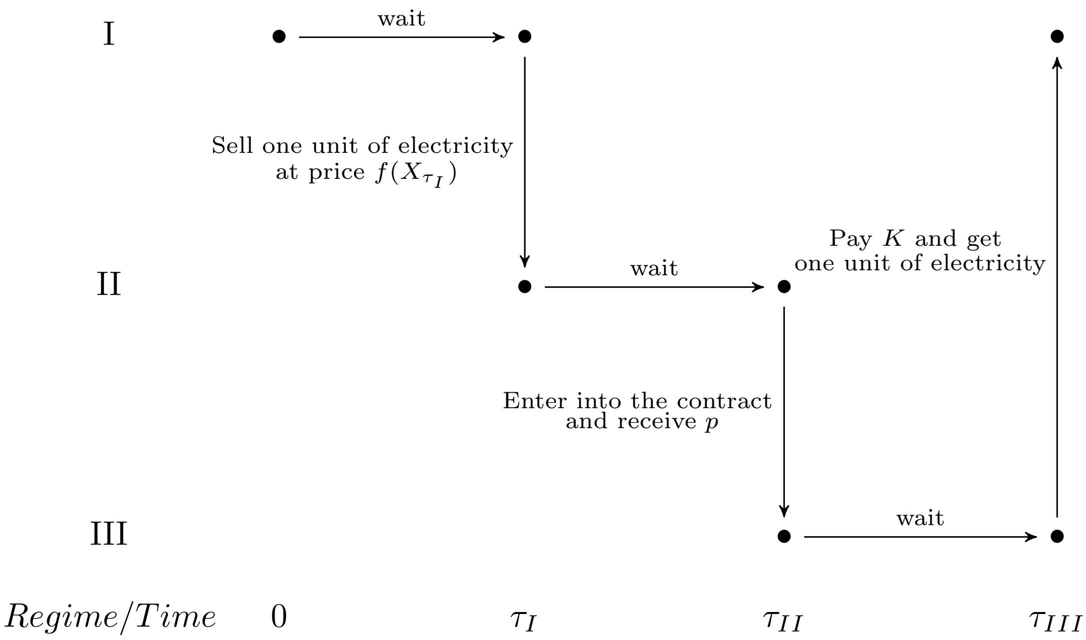

Figure 1: A diagram showing the timeline of the contract in terms of three regimes (I, II, III). The transition from regime I to II, regime II to III, and regime III to I occur at the times τ_I, τ_{II} and τ_{III} respectively.

Figure 1 above illustrates three different *regimes* of operation for the battery owner in relation to the contract. These regimes are described as follows:

1. In regime I, the battery is full and the battery owner has not yet entered into the contract with the system operator. In this regime, the battery owner can sell one unit of electricity on the market at some future time denoted by τ_I. After selling the electricity and being paid the market price $f(X_{\tau_I})$, the battery is no longer full and there is an immediate transition to a new regime II.

2. In regime II, the battery owner can provide immediate physical cover and has not yet entered into the contract with the system operator. The battery owner can enter this contract at some future time τ_{II} and receive a premium $p > 0$. Afterwards, there is an immediate transition to a new regime III.

3. In regime III, the battery owner has entered into the contract with the system operator. It waits until the time τ_{III} that the system operator uses the contract, and then pays K for a unit of electricity that it stores. After this there is an immediate transition to regime I.

Throughout this paper we will use subscripts to help identify cash flows and optimisation problems associated with specific regimes.

Remark 2.1. Without loss of generality, we assume henceforth that the battery's capacity is one unit of electricity. Therefore, the battery is empty when its owner is in regime II.

2.1 The battery owner's valuation problem

We use optimal stopping theory [11, 12] to value the battery owner's decision to enter the contract in accordance with real options analysis [13]. In particular, the valuation is treated as an optimisation problem for the battery owner under the probability measure $\mathbb{P}_x$ under which X has initial value x, $X(0) = x$. Let $\mathbb{E}_x$ denote expectations with respect to $\mathbb{P}_x$, and $\mathcal{T}$ represent the set of stopping times.

We assume that the system operator uses the contract immediately after the imbalance exceeds a fixed threshold $x^* > 0$,

$$\tau_{III} = \inf\{t \geq 0 : X(t) \geq x^*\}. \tag{2.2}$$

Let $r > 0$ represent a constant discount rate and set $a = \sqrt{2r}$. Suppose the battery owner is currently in regime III and the system operator has not yet used the contract. The expected discounted cash flow for the battery owner is:

$$h_{III}(x) = \mathbb{E}_x\{-e^{-r\tau_{III}}(K)\} = \begin{cases} -K, & x > x^* \\ -Ke^{a(x-x^*)}, & x \leq x^* \end{cases} \tag{2.3}$$

In equation (2.3), we used the following formula for the first time the Brownian motion is at a point $y \in \mathbb{R}$, $D_{\{y\}} = \inf\{t \geq 0\colon X(t) = y\}$, (see [14], for example):

$$\mathbb{E}_x\{e^{-rD_{\{y\}}}\} = e^{-a|y-x|}, \quad a = \sqrt{2r}.$$

Suppose now that the battery owner is in regime II and wants to optimise the time it enters the contract. Taking into account the cash flow from the contract premium p, we can formulate the optimisation problem in regime II as one of optimal stopping:

$$V_{II}(x) = \sup_{\tau_{II} \in \mathcal{T}} \mathbb{E}_x\{e^{-r\tau_{II}}(p + h_{III}(X(\tau_{II})))\}, \quad x \in \mathbb{R}. \tag{2.4}$$

In equation (2.4), the *value function in regime II*, $x \mapsto V_{II}(x)$, quantifies the optimal value of the expected discounted cash flow to the battery owner in regime II arising from its decision to enter (or not enter) the contract. According to Chapter 1 of [12], an optimal stopping time $\hat{\tau}_{II} \in \mathcal{T}$ attains the supremum in (2.4) *for all* initial values x of the instantaneous system imbalance. Theorem 3.1 below verifies that the stopping time $\hat{\tau}_{II} = \inf\{t \geq 0\colon X(t) \in S_{II}\}$ is optimal, where S_{II} is the *stopping set* corresponding to (2.4),

$$S_{II} = \{x \in \mathbb{R}\colon V_{II}(x) = p + h_{III}(x)\}.$$

The set S_{II} identifies all values of the system imbalance at which it is optimal to immediately enter the contract.

Suppose now that the battery owner is in regime I and wishes to optimise the time it sells electricity on the market and the time it subsequently enters the contract. We model its strategy in this regime by a pair of times (τ_I, τ_{II}) where $\tau_I, \tau_{II} \in \mathcal{T}$ satisfy $\tau_I \leq \tau_{II}$. The time τ_I determines when it sells the stored electricity in order to transition from regime I to II. The time τ_{II} determines when it enters the contract once in regime II. If we define the set $\mathcal{T}_2$ of such strategies as,

$$\mathcal{T}_2 \coloneqq \{(\tau_I, \tau_{II}) \in \mathcal{T} \times \mathcal{T}\colon \tau_I \leq \tau_{II}\},$$

then the optimisation problem in regime I is given by,

$$V_I(x) = \sup_{(\tau_I,\tau_{II})\in\mathcal{T}_2} \mathbb{E}_x\{e^{-r\tau_I}(f(X(\tau_I))) + e^{-r\tau_{II}}(p + h_{III}(X(\tau_{II})))\}. \tag{2.5}$$

This problem is an *optimal starting and stopping problem*, according to [15], and in this case an optimal strategy $(\hat{\tau}_I, \hat{\tau}_{II}) \in \mathcal{T}_2$ is one that attains the supremum in equation (2.5) *for all* x.

2.1.1 A dynamic programming method for obtaining solutions in regime I

Following the methodology in [15], the solution to the optimisation problem (2.5) in regime I can be obtained recursively using the solution to the optimisation problem (2.4) in regime II as follows:

$$V_I(x) = \sup_{\tau_I\in\mathcal{T}} \mathbb{E}_x\{e^{-r\tau_I}(f(X(\tau_I)) + V_{II}(X(\tau_I)))\}. \tag{2.6}$$

This allows us to determine an optimal strategy $(\hat{\tau}_I, \hat{\tau}_{II})$ for regime I by studying separately the timing decisions for selling electricity in regime I and entering the contract in regime II. Theorem 3.2 below verifies that the stopping time $\hat{\tau}_I = \inf\{t \geq 0\colon X(t) \in S_I\}$ is optimal in (2.6), where S_I is defined by,

$$S_I = \{x \in \mathbb{R}\colon V_I(x) = f(x) + V_{II}(x)\}.$$

The set S_I identifies all values of the system imbalance at which it is optimal to immediately sell the stored electricity in order to enter the contract at a subsequent time.

2.1.2 A condition on the market price at the contract's time of use

In this model, it is possible in regime I for the battery owner to sell electricity and then immediately enter the contract. Furthermore, it is possible in regime II for the battery owner to enter the contract immediately before the system operator uses it at τ_{III}. In short, it is possible to have $\tau_I = \tau_{II} = \tau_{III}$ in Figure 1. Since this action provide no physical benefit to the system operator, the battery owner should not be incentivized to undertake it. If the prevailing market price at τ_{III} is strictly less than the net expenditure $K - p$ of the battery owner, then there is no financial incentive for the aforementioned behaviour. By definition of τ_{III} in (2.2) and as the market price function $x \mapsto f(x)$ is decreasing, this *sustainability condition* is given by,

$$f(x^*) < K - p. \tag{2.7}$$

Note that condition (2.7) also implies $p < K$ since $f(x^*) \geq 0$.

3 Explicit solutions for the timing decisions and optimal values

In this section we provide explicit solutions for the value functions and optimal strategies when the battery owner is in either regime I or regime II for different model parameters. This is accomplished by first solving (2.4) explicitly, then using that explicit representation to characterise the solution to (2.6) (and therefore (2.5)).

3.1 Explicit solutions in regime II

Theorem 3.1. *In regime II, the value function V_{II} (cf. (2.4)) is given explicitly by*

$$V_{II}(x) = \begin{cases} p - Ke^{a(x-x^*)}, & -\infty < x \leq \frac{\ln(\frac{p}{2K})}{a} + x^* \\ e^{a(x^*-x)}\frac{p^2}{4K}, & \frac{\ln(\frac{p}{2K})}{a} + x^* < x. \end{cases} \tag{3.1}$$

Moreover, the set S_{II} of values for the instantaneous imbalance at which it is optimal to immediately enter the contract is defined by:

$$S_{II} = (-\infty, \frac{\ln(\frac{p}{2K})}{a} + x^*] \tag{3.2}$$

According to Theorem 3.1, in regime II there is a point X_{II} defined by,

$$X_{II} = \frac{\ln(\frac{p}{2K})}{a} + x^*, \tag{3.3}$$

below which it is optimal to immediately enter the contract. Since $0 < p < K$ by (2.7) above, we have $\frac{\ln(\frac{p}{2K})}{a} < 0$ and therefore $X_{II} < x^*$. Theorem 3.1 therefore confirms that it is not optimal for the battery owner to offer the contract when the system operator is expected to use it immediately. Figure 2 below illustrates this by plotting the payoff $p + h_{III}$ and value function V_{II} for regime II. The set $S_{II} = (-\infty, X_{II}]$ is highlighted in bold on the curve.

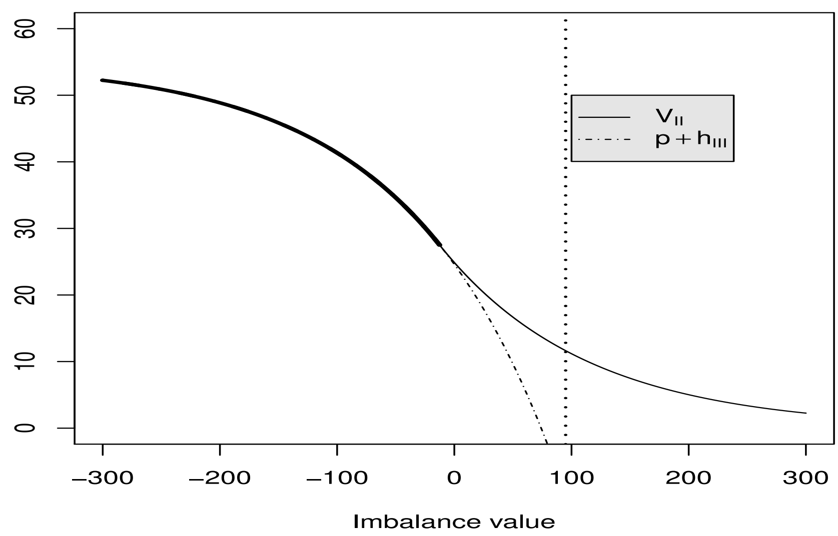

Figure 2: Solution in regime II with parameters $r = 3.2 \cdot 10^{-5}, M = 80, b = -0.5, p = 55, K = 65, c = 50$ and $x^* = 95$. The optimal threshold point in regime II is $X_{II} \approx -13$ and the set of values $x \leq X_{II}$ at which the battery owner should immediately enter the contract is highlighted in bold on the curve. The dotted vertical line denotes the point x^*.

3.2 Explicit solutions in regime I

Let X_{min} and X_{max} denote the points at which the market price attains its minimum value $f(X_{min})$ and maximum value $f(X_{max})$ respectively,

$$X_{min} = \frac{-c}{b}, \quad X_{max} = \frac{M - c}{b}. \tag{3.4}$$

Recall that $X_{max} < X_{min}$ since $x \mapsto f(x)$ is decreasing (cf. (2.1)). Based on Theorem 3.1 above and these price caps, it is instructive to distinguish three main cases:

Case 1. $X_{min} < X_{II}$

Case 2. $X_{max} \leq X_{II} \leq X_{min}$

Case 3. $X_{II} < X_{max}$

All three cases can be analysed using similar techniques. However, we solely present results for **Case 2**, since in the other cases the point X_{II} lies outside the domain of values $[X_{max}, X_{min}]$ where the market price changes (cf. (2.1)). We identify three subcases of **Case 2** for grouping solutions, and these subcases are determined by the values of three points: X_{II} and X_{max}, which are defined in (3.3) and (3.4) respectively, and X_Γ which is the unique solution in $(-\infty, -\frac{c+p}{b}]$ to $\Gamma(x) = 0$ with,

$$\Gamma(x) = \frac{1}{2}e^{-ax}\left(c + p + \frac{b}{a} + bx\right) - Ke^{-ax^*}.$$

For further details see the supplementary material.

Theorem 3.2. *Let S_I denote the set of values for the instantaneous imbalance at which it is optimal to immediately sell the stored electricity. The value function in regime I, V_I, and corresponding set S_I are given explicitly in* **Case 2** *by:*

- **Case 2.1** : *If $X_{max} < X_\Gamma < X_{II}$, then*

$$V_I(x) = \begin{cases} M + p - Ke^{a(x-x^*)}, & -\infty < x \le X_{max} \\ c + bx + p - Ke^{a(x-x^*)}, & X_{max} < x \le X_\Gamma \\ e^{-a(x-X_\Gamma)}(c + bX_\Gamma + p - Ke^{a(X_\Gamma - x^*)}), & X_\Gamma < x \end{cases} \tag{3.5}$$

  and $S_I = (-\infty, X_\Gamma]$.

- **Case 2.2** : *If $X_\Gamma \le X_{max}$, then*

$$V_I(x) = \begin{cases} M + p - Ke^{a(x-x^*)}, & -\infty < x \le X_{max} \\ e^{a(X_{max}-x)}(M + p - Ke^{a(X_{max}-x^*)}), & X_{max} < x \end{cases} \tag{3.6}$$

  and $S_I = (-\infty, X_{max}]$.

- **Case 2.3** : *If $X_{II} \le X_\Gamma$, then*

$$V_I(x) = \begin{cases} M + p - Ke^{a(x-x^*)}, & -\infty < x < X_{max} \\ c + bx + p - Ke^{a(x-x^*)}, & X_{max} \le x \le X_{II} \\ c + bx + e^{a(x^*-x)}\frac{p^2}{4K}, & X_{II} < x \le X_{min} - \frac{1}{a} \\ -\frac{b}{a}e^{a(X_{min}-x)-1} + e^{a(x^*-x)}\frac{p^2}{4K}, & X_{min} - \frac{1}{a} < x \end{cases} \tag{3.7}$$

  and $S_I = (-\infty, X_{min} - \frac{1}{a}]$.

Theorem 3.2 shows that in **Case 2** there is a point X_I below which it is optimal to immediately sell electricity in order to enter the contract at a subsequent time. Figures 3–5 illustrate the solutions for **Case 2** in each of the subcases identified in Theorem 3.2. Each figure shows the payoff function $f + V_{II}$ to optimise in regime I (cf. (2.6)) and the value function V_I from Theorem 3.2. The set $S_I = (-\infty, X_I]$ from Theorem 3.2 is highlighted in bold on the plot of V_I. The set $S_{II} = (-\infty, X_{II}]$ from Theorem 3.1 is highlighted in bold on the horizontal axis. A dotted vertical line is used to denote the point x^*. The model parameters used to generate these figures were chosen to satisfy the sustainability condition (2.7), whilst keeping b, K, c and M fixed for the different cases.

Theorem 3.2 and the illustrations show that there are instances in which it is only optimal to sell electricity at its maximum price (Case 2.2), and others in which the battery owner can act optimally by selling electricity below its maximum price (Cases 2.1 and 2.3). It is important

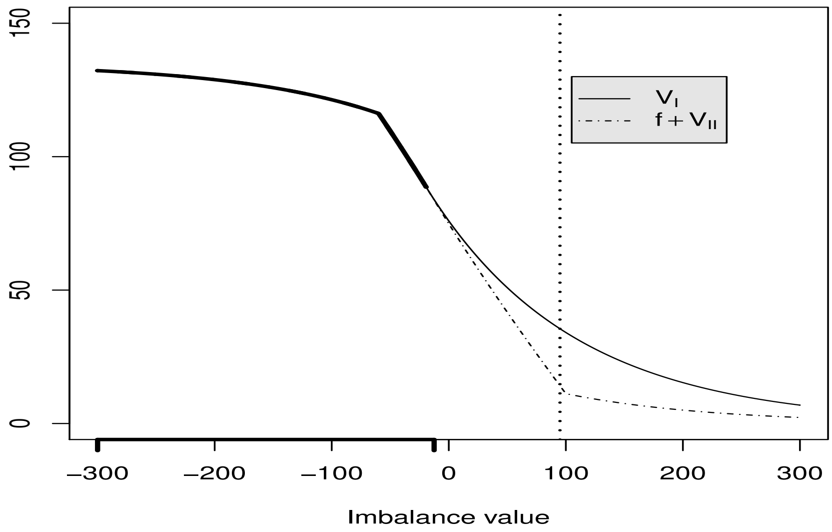

Figure 3: Solution in regime I for Case 2.1 with parameters $r = 3.2 \cdot 10^{-5}, M = 80, b = -0.5, p = 55, K = 65, c = 50, x^* = 95$, $X_{max} = -60$ and $X_{min} = 100$. The set of values $x \leq X_I$ (resp. $x \leq X_{II}$) at which the battery owner should immediately sell the stored electricity (resp. enter the contract) is highlighted in bold on the curve (resp. the x-axis), with $X_I \approx -17$ and $X_{II} \approx -13$. The dotted vertical line denotes the point x^*.

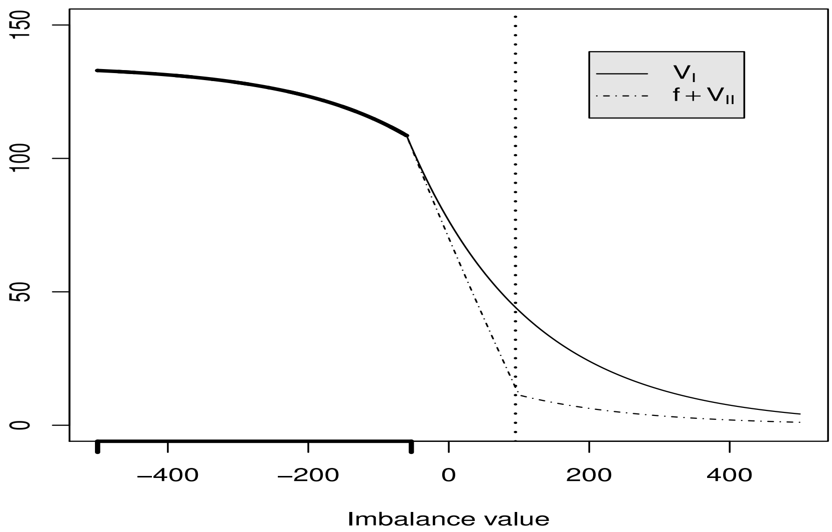

Figure 4: Solution in regime I for Case 2.2 with parameters $r = 1.682 \cdot 10^{-5}, M = 80, b = -0.5, p = 55, K = 65, c = 50, x^* = 95$, $X_{max} = -60$ and $X_{min} = 100$. The set of values $x \leq X_I$ (resp. $x \leq X_{II}$) at which the battery owner should immediately sell the stored electricity (resp. enter the contract) is highlighted in bold on the curve (resp. the x-axis), with $X_I \approx -60$ and $X_{II} \approx -53$. The dotted vertical line denotes the point x^*.

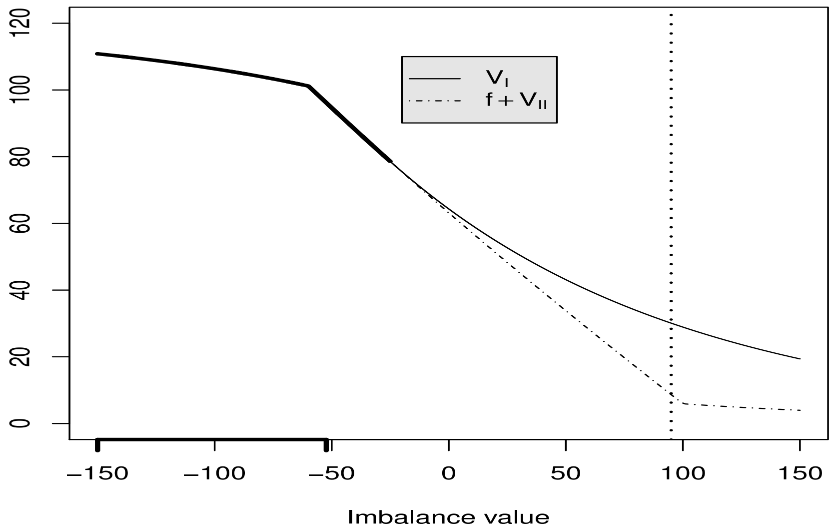

Figure 5: Solution in regime I for Case 2.3 with parameters $r = 3.2 \cdot 10^{-5}, M = 80, b = -0.5, p = 40, K = 65, c = 50, x^* = 95$, $X_{max} = -60$ and $X_{min} = 100$. The set of values $x \leq X_I$ (resp. $x \leq X_{II}$) at which the battery owner should immediately sell the stored electricity (resp. enter the contract) is highlighted in bold on the curve (resp. the x-axis), with $X_I \approx -25$ and $X_{II} \approx -52$. The dotted vertical line denotes the point x^*.

to ensure that this decision rule is consistent with the overall objective of balancing the power system. In Cases 2.1 and 2.2 of Theorem 3.2, the point X_I is sufficiently low to ensure that this is the case. However, in Case 2.3 it is possible that for some values of the model parameters x^*, p, K, r, c and b that the point $X_I = X_{min} - \frac{1}{a}$ satisfies $X_I > x^*$, and exacerbates power system imbalance.

We have just highlighted the possibility that the battery owner's "optimal" strategy for selling electricity in Case 2.3 can further unbalance the power system. First, we point out that this possibility is ruled out if the threshold X_I satisfies $X_I \leq 0$. Figure 5 above illustrates a solution that satisfies this property. The condition $X_I \leq 0$ in Case 2.3 is given more precisely by $X_{min} - \frac{1}{a} \leq 0$, which is equivalent to requiring the parameters r, c and b to satisfy,

$$\frac{-c}{b} \leq \frac{1}{\sqrt{2r}}. \tag{3.8}$$

As an alternative to (3.8), it is possible to find conditions for the parameters x^*, p and K that ensure Case 2.3 does not occur. The supplementary material shows that the condition $X_{II} \leq X_\Gamma$ for Case 2.3 is equivalent to $X_{II} \leq X_{min} - \frac{1}{a}$. This shows we have $X_I \geq X_{II}$ in Case 2.3 on the one hand, and $X_I < X_{II}$ in Cases 2.1 and 2.2 on the other hand. Case 2.3 therefore corresponds precisely to the situation in Case 2 in which, with the exception of the boundary case $X_I = X_{II}$, the battery owner should sometimes wait between selling electricity and entering the contract. In terms of the parameters r, c and b, Case 2.3 occurs precisely when the parameters x^*, p and K satisfy,

$$x^* \leq \frac{-c}{b} - \frac{1 + \ln(\frac{p}{2K})}{\sqrt{2r}}. \tag{3.9}$$

4 Analysis of the lifetime value of offering balancing services

In this section we analyse the lifetime value aggregated from infinite repetitions of the cycle illustrated in Figure 1. For this analysis we introduce the following sequence of intermediary problems based on Section 2 above: for $n \geq 1$ let V_I^n and V_{II}^n be the value functions associated with n iterations of the cycle illustrated in Figure 1, starting from regimes I and II respectively. Setting $V_I^0 \equiv 0$, the sequence of functions $\{V_I^n\}_{n \geq 1}$, can be defined inductively as follows:

$$V_I^n(x) = \sup_{(\tau_I, \tau_{II}) \in \mathcal{T}_2} \mathbb{E}_x \Big\{ e^{-r\tau_I} f(X(\tau_I)) + e^{-r\tau_{II}} (p + h_{III}^n(X(\tau_{II}))) \Big\} \tag{4.1}$$

where h_{III}^n is defined by,

$$\begin{aligned} h_{III}^n(x) &:= \mathbb{E}_x \{ e^{-r\tau_{III}} (V_I^{n-1}(X(\tau_{III})) - K) \} \\ &= \begin{cases} V_I^{n-1}(x) - K, & x > x^* \\ (V_I^{n-1}(x^*) - K) e^{a(x - x^*)}, & x \leq x^* \end{cases} \end{aligned} \tag{4.2}$$

Similarly, for $n \geq 1$ we define V_{II}^n in the following manner:

$$V_{II}^n(x) = \sup_{\tau_{II} \in \mathcal{T}} \mathbb{E}_x \{ e^{-r\tau_{II}} (p + h_{III}^n(X(\tau_{II}))) \}. \tag{4.3}$$

Note that $V_{II}^1 = V_{II}$ and $V_I^1 = V_I$, which is in accordance with the definitions given in Section 2 above.

Definition 1. If the limit $V_I^*(x) := \lim_{n \to \infty} V_I^n(x)$ exists for each $x \in \mathbb{R}$, then we refer to $V_I^*(x)$ as the lifetime value at x starting in regime I. If the limit $V_{II}^*(x) := \lim_{n \to \infty} V_{II}^n(x)$ exists for each $x \in \mathbb{R}$, then we refer to $V_{II}^*(x)$ as the lifetime value at x starting in regime II.

Theorem 4.1. *Let V_I^n, V_{II}^n and h_{III}^n be defined by* (4.1)–(4.3)*.*

1. *For $n \geq 1$, the functions V_I^n, V_{II}^n and h_{III}^n are continuous and bounded. Furthermore, the sequences $\{V_I^n\}_{n \geq 1}$, $\{V_{II}^n\}_{n \geq 1}$ and $\{h_{III}^n\}_{n \geq 1}$ are increasing.*
2. *For $n \geq 1$, the function V_I^n satisfies,*

$$V_I^n(x) = \sup_{\tau_I \in \mathcal{T}} \mathbb{E}_x \{ e^{-r\tau_I} (f(X(\tau_I)) + V_{II}^n(X(\tau_I))) \}.$$

3. *The limits $V_I^* = \lim_{n \to \infty} V_I^n$ and $V_{II}^* = \lim_{n \to \infty} V_{II}^n$ exist and are bounded, continuous functions that satisfy,*

$$V_{II}^*(x) = \sup_{\tau_{II} \in \mathcal{T}} \mathbb{E}_x \{ e^{-r\tau_{II}} (p + h_{III}^*(X(\tau_{II}))) \} \tag{4.4}$$

$$V_I^*(x) = \sup_{\tau_I \in \mathcal{T}} \mathbb{E}_x \{ e^{-r\tau_I} (f(X(\tau_I)) + V_{II}^*(X(\tau_I))) \} \tag{4.5}$$

where $h_{III}^* = \lim_{n \to \infty} h_{III}^n$ is given by,

$$h_{III}^*(x) = \begin{cases} V_I^*(x) - K, & x > x^* \\ (V_I^*(x^*) - K) e^{a(x - x^*)}, & x \leq x^* \end{cases} \tag{4.6}$$

4. *The function V_I^* also satisfies,*

$$V_I^*(x) = \sup_{(\tau_I, \tau_{II}) \in \mathcal{T}_2} \mathbb{E}_x \{ e^{-r\tau_I} (f(X(\tau_I))) + e^{-r\tau_{II}} (p + h_{III}^*(X(\tau_{II}))) \}. \tag{4.7}$$

Theorem 4.1, which is proved in the supplementary material, confirms more than the existence of the lifetime value functions V_I^* and V_{II}^*. First, by inspecting the solutions given in Theorems 3.1 and 3.2 above, we know that $V_I^1 > 0$ and $V_{II}^1 > 0$, which shows that the battery owner can get strictly positive expected economic profit from one contract. Since the sequences $\{V_I^n\}_{n\geq 1}$ and $\{V_{II}^n\}_{n\geq 1}$ are increasing, Theorem 4.1 shows that strictly positive expected economic profit can be accrued from these contracts in the long run, which is important for commercial operators of electricity storage.

5 Discussion and conclusion

In this paper we have used a real options approach to analyse a balancing reserve contract between a battery owner and a transmission system operator. Under this contract, the battery owner helps smooth fluctuations in electricity demand and supply, and therefore provides a regulation service [1, p. 108], by charging the battery. The reserve is also *decremental* in the sense that the action is comparable to a reduction in generation. Under the contract's terms, the battery owner must be able to provide immediate physical cover, which means it can only enter the contract if it has sufficient storage available. If this is the case, it only needs to time its decision to enter the contract, and we say that the battery owner is in regime II. Otherwise, it must decide when to sell the stored electricity, and then when to subsequently enter the contract. In this case, we say that the battery owner is in regime I. Our work is an extension of a recent study [8], which analysed the corresponding problem for a battery owner that provides an *incremental* regulation service to the transmission system operator. Our study and its findings are summarised as follows:

1. In general, the system operator faces operational and financial risks when it turns to the short-term market to procure balancing services [1, p. 60]. For the contract studied in this paper, the system operator mitigates some of its operational risk by reserving battery capacity for balancing services in advance. Some of the system operator's price risk is mitigated by the fixed premium p it pays to reserve battery capacity, and the fixed price K it receives per unit of electricity consumed and stored in the battery, according to the contract.

2. Our probabilistic framework uses a Brownian motion to represent the instantaneous values of the system imbalance over time. Positive values indicate a generation surplus, whereas negative values indicate a shortfall in generation. We assume that the system operator calls on the battery owner to provide its regulation service as soon the system imbalance exceeds a threshold $x^* > 0$.

3. The *net discounted value* in regime II, which is the sum of the battery owner's expected discounted cash flows upon entering the contract, is given by,

$$p - Ke^{\sqrt{2r}\min(x-x^*,0)} \tag{5.1}$$

   where $r > 0$ is the discount rate, and x is the present value of the system imbalance. If the battery owner enters the contract when x is sufficiently small, the net discounted value in regime II is positive. This is confirmed by Theorem 3.1, which shows that there is a point $X_{II} < x^*$ such that the battery owner should enter the contract whenever $x \leq X_{II}$, and defer its entry otherwise. The point X_{II} is given explicitly in terms of the model parameters. We note that our observation that the battery owner should sometimes defer entry into the contract differs from the optimal strategy for the *incremental* reserve contract, which was derived in [8].

4. Our model uses a piecewise linear function f to define a market price that decreases as the present value x of the system imbalance increases. This function is assumed to be nonnegative and first attains its maximum and minimum values at the points X_{max} and X_{min} respectively, where $X_{max} < 0 < X_{min}$. Our model ensures that the battery owner is not incentivized to sell electricity and immediately enter the contract when the system operator plans to use it. This is done by ensuring the net cash flow satisfies $f(x)+p-K < 0$ for all $x \geq x^*$, which is simplified to the *sustainability condition* $f(x^*) + p - K < 0$ given in (2.7).

5. Using equation (5.1) for the net discounted value in regime II and assuming the battery owner follows the optimal strategy given in Theorem 3.1, the net discounted value in regime I is given by,

$$f(x) + e^{\sqrt{2r}\min(X_{II}-x,0)}(p - Ke^{\sqrt{2r}(\min(X_{II},x)-x^*)}). \tag{5.2}$$

   The value given by (5.2) is positive since f is nonnegative and the net discounted value in regime II is positive when the battery owner follows the optimal strategy. Equation (5.2) also highlights the possibility of additional discounting to the cash flow for the net discounted value in regime II. This additional discounting occurs when the battery owner does not immediately enter the contract after selling the stored electricity.

6. Theorem 3.2 presents the solution in regime I for the case $X_{max} \leq X_{II} \leq X_{min}$. This result shows there is a point $X_I \leq X_{min}$ such that for $x \leq X_I$ it is optimal for the battery owner to immediately sell the stored electricity. For $x > X_I$ it is optimal for the battery owner to defer its sale of electricity. This threshold X_I and the optimal net discounted value in regime I have explicit expressions that depend on the model parameters.

7. We have also identified condition 3.9 on the model parameters that determines when $X_{II} \leq X_I$ or, in other words, when it may be optimal for the battery owner to defer entry into the contract after selling the stored electricity. Figure 5 displays a solution in regime I with $X_{II} < X_I < 0$, illustrating that the strategy for selling electricity in this case can be consistent with the overall objective of power system balancing.

8. Our results for the net discounted values in regimes I and II indicate the battery owner has positive expected economic profit from the balancing reserve contract. We showed furthermore that the lifetime value aggregated from an infinite sequence of these contract is positive and bounded. This indicates the potential benefit of such contracts to commercial operators of electricity storage.

The analysis in this paper can be extended to the case of a negative lower cap on the market price. In order to get results similar to those obtained here, the assumption $p \leq K$ should be made explicit. This is because the optimal strategy in regime II is not of the same form as in Theorem 3.1 if $p > K$, as shown in the supplementary material. Another interesting extension involves a more general stochastic process for modelling the system imbalance. In the case of diffusion processes, the transformation technique described in [11], which underpins Theorems 3.1 and 3.2 of this paper, can be used to convert the problem into another one for Brownian motion *without* discounting. This has been done recently in [9] for a slightly different model of the incremental reserve contract studied by [8].

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