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# Emergence of leadership in complex networks and human groups

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**Abstract**—Leadership is a complex yet fascinating phenomenon crucial in many application where coordination of multi-agent systems is involved. This is particularly true in the case of animal groups and other natural systems including human ensembles where, for example in orchestras, a leader is often selected to steer the group towards some desired coordinated behaviour. In this paper we study the emergence of leadership in networks of heterogeneous Kuramoto oscillators, which offer a good model for capturing and describing coordination in human groups performing a joint oscillatory task. We present a mathematical framework to establish phase leadership in the network and apply the results to inform some preliminary experimental results in human groups.

## I. INTRODUCTION

Investigating and analysing the dynamics of a large number of interacting systems is the subject of much ongoing research [1], [2]. These systems can be represented through the formalism of complex networks where each node is a dynamical agent, and the edges between nodes model their interaction. Through this formalism, different application scenarios can be described such as power grids, decentralised logistics, opinion formation, wireless communication, synchronisation, vehicle formation and platooning, as well as genetic and neuronal networks, see e.g. [3].

In addition to all the aforementioned scenarios, movement coordination within a group of people engaged in a joint task represents a perfect candidate to be studied through complex networks. Examples of human coordination take place in many daily actions, from simple oral communication, or walking together in a crowd [4], [5], to more complex and sophisticated activities as dancing [6], playing team sports [7] or in a musical ensemble [8].

In many natural multiagent systems such as animal groups or human cooperation, leadership plays a fundamental role in determining the success or failure of some desired macroscopic behaviour, e.g. from maximizing profit in economics, to steering opinion dynamics in politics, or playing music and sport [9]–[11]. Indeed, leaders set the duration and pace at which organisational processes should take place, supervise and control coordination among several individuals, raise their attentional level, and can even affect others through social and emotional contagion.

In human groups, leaders are generally selected on the basis of different features that depend on the particular context of interest. However, inspiration, effective speaking, confidence, charisma and friendliness represent some of the characteristics that define a good leader [11], together with their ability to make the collective goal of the ensemble, rather than the individual interests of its members, the main source of benefit [12]. If leadership in human ensembles has been mostly investigated in the case of dyadic interactions, only few works have been found in the existing literature that address the problem of leadership emergence in larger human ensembles. Some of the limited results are related to the role of a conductor in a classical orchestra [13], pedestrian walking [14] and group dance [6], but only little quantitative analysis is carried out on how the individual dynamics of the leader or the specific interaction patterns in the group affect leadership emergence.

Another open question in human behaviour is to understand whether it is possible to entrain some or all of the members in the ensemble to promote a desired group behaviour. This is a problem of great importance and application. For example, in the context of Rehabilitation Robotics, where one or more subjects are replaced with virtual agents (avatar or robot), the virtual agent might be required to specifically act as such, or drive another member to act as a leader, in order to help patients recover from their social disabilities [15]–[18].

In this work, we tackle analytically the problem of studying leadership emergence in a network of  $N > 2$  heterogeneous Kuramoto oscillators connected over different topologies by defining leadership in terms of the maximal phase lead between the group of phase locked oscillators. These networks were shown in [19] to offer a good model to capture and describe this phenomenon in a group of human players performing a joint oscillatory task. Specifically, we first find mathematical conditions that allow to predict given random initial conditions what the node is in the network which will end leading in phase all the remaining oscillators. Then we explore the effect of changing the topological structure on the network in determining what node will be the leader. Finally, we present some preliminary experimental results showing how the addition of a virtual leader driven by our mathematical model in a human group can steer and enhance



group coordination.

## II. PROBLEM STATEMENT

We consider an undirected unweighed network of  $N$  heterogeneous Kuramoto oscillators [20] with adjacency matrix  $A = \{a_{ij}\}$ :

$$\dot{\theta}_i(t) = \omega_i + \frac{c}{N} \sum_{j=1}^N a_{ij} \sin(\theta_j(t) - \theta_i(t)), \quad i = 1, \dots, N \quad (1)$$

where  $\theta_i$  and  $\omega_i$  represent phase and oscillation frequency of the  $i$ th node, respectively,  $c > 0$  the coupling strength among the oscillators, and  $a_{ij} = 1$  if nodes  $i$  and  $j$  are neighbours (i.e., they are connected), whereas  $a_{ij} = 0$  if they are not.

Letting  $\phi_{ij}(t) := \theta_i(t) - \theta_j(t)$  and assuming all the phases and their differences belong to the interval  $[-\pi, \pi]$  we define *phase leadership* as follows

**Definition 1 (Phase leadership).** We say that node  $i$  is the *group leader* at time  $t \iff \phi_{ij}(t) > 0 \quad \forall j \neq i$ .

Now, we denote  $q(t)$  the average phase of the group and define  $\phi_i(t) := \theta_i(t) - q(t)$ , and  $\phi(t) := [\phi_1(t), \phi_2(t), \dots, \phi_N(t)]^T$ . Thanks to Definition 1, it is possible to define a *leader hierarchy* in the network. Specifically, assuming that:

$$\phi_h(t) < \phi_k(t) < \dots < \phi_j(t) < \phi_i(t) \quad (2)$$

node  $i$  is defined as the *first leader* (the only node followed by all the others), node  $j$  is defined as the *second leader*, and analogously for all the remaining nodes.

We define the following complex order parameter for each node  $i$  in the network:

$$r_i(t) e^{j\psi_i(t)} = \frac{1}{N} \sum_{k=1}^N a_{ik} e^{j\theta_k(t)} \quad (3)$$

where  $0 \leq r_i(t) \leq 1$  measures the synchronization level of node  $i$  with respect to its neighbours (the closer to 1, the better the coordination among them). It follows that:

$$r_i(t) e^{j(\psi_i(t) - \theta_i(t))} = \frac{1}{N} \sum_{k=1}^N a_{ik} e^{j(\theta_k(t) - \theta_i(t))} \quad (4)$$

so that, by comparing the respective imaginary parts, Eq. 1 can be rewritten as:

$$\dot{\theta}_i(t) = \omega_i + cr_i(t) \sin(\psi_i(t) - \theta_i(t)) \quad (5)$$

In [21] the authors present a sufficient condition for frequency synchronization (i.e., *phase-locking*) of an undirected network of heterogeneous Kuramoto oscillators connected over a generic topology:

$$c \geq \frac{N \|\omega\|_{\mathcal{E}, \infty} \sin(\gamma)}{\lambda_2}, |\theta_{i_0} - \theta_{j_0}| \leq \gamma \implies \begin{cases} \dot{\theta}_i(t) \rightarrow \Omega \\ |\theta_i^*(t) - \theta_j^*(t)| \leq \gamma \end{cases} \quad \forall i, j \in [1, N] \quad (6)$$

where  $\theta_{i_0}$  is the initial value of the phase of the  $i$ th oscillator,  $\Omega$  is the mean value of the oscillation frequencies  $\omega_i$ ,  $\theta_i^*(t)$

represents the phase of the  $i$ th oscillator once all the nodes phase-lock,  $\lambda_2 > 0$  is the second eigenvalue of the Laplacian of the network,  $\omega = [\omega_1, \omega_2, \dots, \omega_N]^T$  is the stack vector of all the frequencies,  $\|x\|_{\mathcal{E}, \infty} := \max_{(i,j) \in \mathcal{E}} |x_i - x_j|$ , with  $\mathcal{E}$  being the set of the network edges, and  $0 < \gamma < \frac{\pi}{2}$ . In what follows, we will assume that such condition is always verified.

Once all the oscillators phase-lock, it is possible to study their dynamics in a moving frame which rotates with angular velocity  $\Omega$ . In such frame, each phase  $\tilde{\theta}_i := \theta_i + \Omega t$  is constant over time, and as a consequence so are  $\psi_i$  and  $r_i$ , from Eq. 3. The constant value of the modulus of the order parameter is then given by:

$$r_{i_\infty} = \frac{1}{N} \sum_{k=1}^N a_{ik} e^{j(\tilde{\theta}_k - \psi_i)} \quad (7)$$

It is worth pointing out that studying the dynamics of the Kuramoto oscillators in the rotating frame corresponds also to replacing  $\tilde{\omega}_i := \omega_i - \Omega$  for each  $i$ th node in the network. Since in such frame all the oscillators are still after achieving phase-locking, equation Eq. 5 can be rewritten as

$$0 = \tilde{\omega}_i + cr_{i_\infty} \sin(\psi_i - \tilde{\theta}_i) \quad (8)$$

which leads to

$$\tilde{\theta}_i = \sin^{-1} \left( \frac{\tilde{\omega}_i}{cr_{i_\infty}} \right) + \psi_i \quad \forall i \in [1, N] \quad (9)$$

and, according to Definition 1, to the conclusion that

$$\text{node } i \text{ leads node } j \iff \tilde{\theta}_i > \tilde{\theta}_j \quad (10)$$

## III. LEADERSHIP ANALYSIS

Next, we provide a closed form solution to predict leader hierarchy in a network of Kuramoto oscillators by simply knowing the values of the frequencies of the oscillators and the underlying network topology. We focus on the following network structures: all-to-all, ring, path and star graph.

### A. Complete graph

In the case of a complete graph, Eq. 1 can be simplified as follows:

$$\dot{\theta}_i(t) = \omega_i + \frac{c}{N} \sum_{j=1}^N \sin(\theta_j(t) - \theta_i(t)), \quad i = 1, \dots, N \quad (11)$$

and Eq. 3 leads to the definition of a unique complex parameter for all the nodes (i.e., it represents the centroid of their phases):

$$r(t) e^{j\psi(t)} = \frac{1}{N} \sum_{k=1}^N e^{j\theta_k(t)} \quad (12)$$

In the phase-locking regime it is possible to set  $\psi = 0$  by appropriately choosing the origin of the rotating frame, so that equation Eq. 9 can be rewritten as

$$\tilde{\theta}_i = \sin^{-1} \left( \frac{\tilde{\omega}_i}{cr_{i_\infty}} \right) \quad \forall i \in [1, N] \quad (13)$$

Therefore, by defining the *angular velocity mismatch* as:

$$\zeta_i := \tilde{\omega}_i = \omega_i - \Omega \quad \forall i \in [1, N] \quad (14)$$

it follows that

$$\text{node } l \text{ is the emerging leader} \iff \omega_l = \max_{i \in [1, N]} \omega_i$$

or equivalently

$$\text{node } l \text{ is the emerging leader} \iff \zeta_l = \max_{i \in [1, N]} \zeta_i$$

### B. Star Graph

In the case of a star graph, assuming without loss of generality that node 1 is the central one, we find

$$\psi_h = \tilde{\theta}_1 \quad \forall h \in [2, N] \quad (15)$$

which from Eq. 7 yields:

$$r_{h\infty} = \frac{1}{N} e^{j(\tilde{\theta}_1 - \tilde{\theta}_h)} = \frac{1}{N} \quad (16)$$

Therefore, Eq. 9 can be rewritten for all the peripheral nodes as:

$$\tilde{\theta}_h = \sin^{-1}\left(\frac{N\tilde{\omega}_h}{c}\right) + \tilde{\theta}_1 \quad \forall h \in [2, N] \quad (17)$$

This means that a leader hierarchy takes place among the peripheral nodes, where node  $h$  leads node  $m$  if the oscillation frequency of the former is greater than that of the latter,  $\forall h, m \in [2, N]$ , with  $h \neq m$ . As a consequence, we find that the leader in a star graph is either the peripheral node with the highest frequency among them or the central node. Denoting with  $p$  the index of the peripheral node with the highest frequency, we have that:

$$\left\{ \begin{array}{l} \text{the central node is the emerging leader} \\ \iff \zeta_p < 0 \quad (\text{i.e., } \omega_p < \Omega) \\ \text{node } p \text{ is the emerging leader} \\ \iff \zeta_p > 0 \quad (\text{i.e., } \omega_p > \Omega) \end{array} \right.$$

or equivalently

$$\text{node } l \text{ is the emerging leader} \iff \zeta_l = \max_{i \in [1, N]} \zeta_i$$

### C. Path Graph and Ring Graph

The analysis for complete and star graphs can be extended to the case of a path or ring topology but it is not reported here for the sake of brevity.

## IV. NUMERICAL SIMULATIONS

### A. Complete graph

We consider an all-to-all network of  $N = 7$  heterogeneous Kuramoto oscillators starting from random initial conditions between  $-0.8$  and  $0.8$ . Each oscillator is assigned a natural frequency given in the inlay in Fig. 1 where it is shown that at steady state a leadership hierarchy emerges with node 1 (the fastest) leading on all the others in phase. Note that the coupling strength  $c$  was set so that phase-locking could be

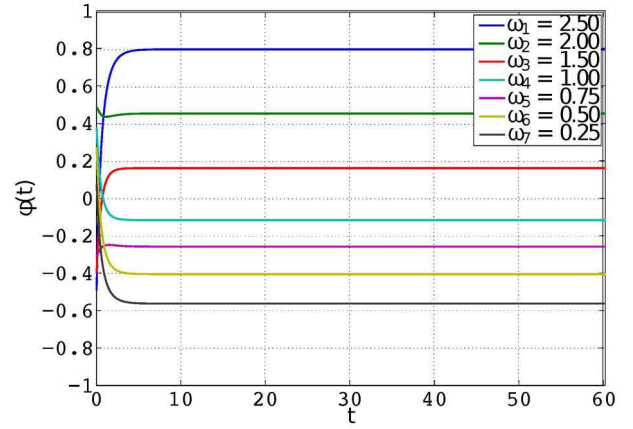


Fig. 1. Relative phases  $\phi(t) = \theta_i(t) - q(t)$  in a network of  $N = 7$  heterogeneous Kuramoto oscillators – undirected unweighted complete graph.  $\omega_i > \omega_j$  with  $i > j$ ,  $\forall i, j \in [1, N]$  –  $c = 2$

achieved. Specifically, given that in a complete graph  $\lambda_2 = N$ , from Eq. 6 it follows that:

$$\begin{aligned} c > |\omega_M - \omega_m| \sin(\gamma), \quad |\theta_{i_0} - \theta_{j_0}| \leq \gamma \\ \implies \begin{cases} \hat{\theta}_i(t) \rightarrow \Omega \\ |\theta_i^*(t) - \theta_j^*(t)| \leq \gamma \end{cases} \quad \forall i, j \in [1, N] \end{aligned} \quad (18)$$

where  $\omega_M$  and  $\omega_m$  represent maximum and minimum value of the oscillation frequencies of the nodes in the network, respectively. Initial conditions and oscillation frequencies were also set such that  $\gamma = 1$  and  $|\omega_M - \omega_m| = 2.25$ , thus leading to  $c = 2 > |\omega_M - \omega_m| \sin(\gamma) \approx 1.89$ .

Following the derivation presented in Sec. III.A we find

$$\begin{aligned} \tilde{\theta}_1 = 0.64, \quad \tilde{\theta}_2 = 0.39, \quad \tilde{\theta}_3 = 0.14, \quad \tilde{\theta}_4 = -0.11, \\ \tilde{\theta}_5 = -0.23, \quad \tilde{\theta}_6 = -0.36, \quad \tilde{\theta}_7 = -0.48 \end{aligned}$$

which confirms that node 1 should be expected to lead asymptotically as observed in the numerical simulations shown in Fig. 1.

### B. Star Graph

Next we consider a star graph where node 1 is assumed to be the central node. We explore two cases in Fig. 2:

- the case where the highest peripheral frequency  $\omega_2$  is greater than the average oscillation frequency  $\Omega$ ;
- the case where the highest peripheral frequency  $\omega_2$  is lower than the average oscillation frequency  $\Omega$ .

Specifically, Fig. 2(a) shows the relative phases between all the oscillators and the phase of the group when setting the values of the oscillation frequencies  $\omega_i$  so that  $2 = \omega_2 > \Omega \approx 1.21$ . Since the initial conditions and the value of the other oscillation frequencies were also set such that  $\gamma = 1$  and  $\|\omega\|_{\mathcal{E}, \infty} = 2.25$ , and given that the second eigenvalue of the Laplacian of a star graph is equal to 1, we set  $c = 14 > \frac{N \|\omega\|_{\mathcal{E}, \infty} \sin(\gamma)}{1} \approx 13.25$  to guarantee phase locking. As predicted by our analytical findings reported in Sec. III.B,



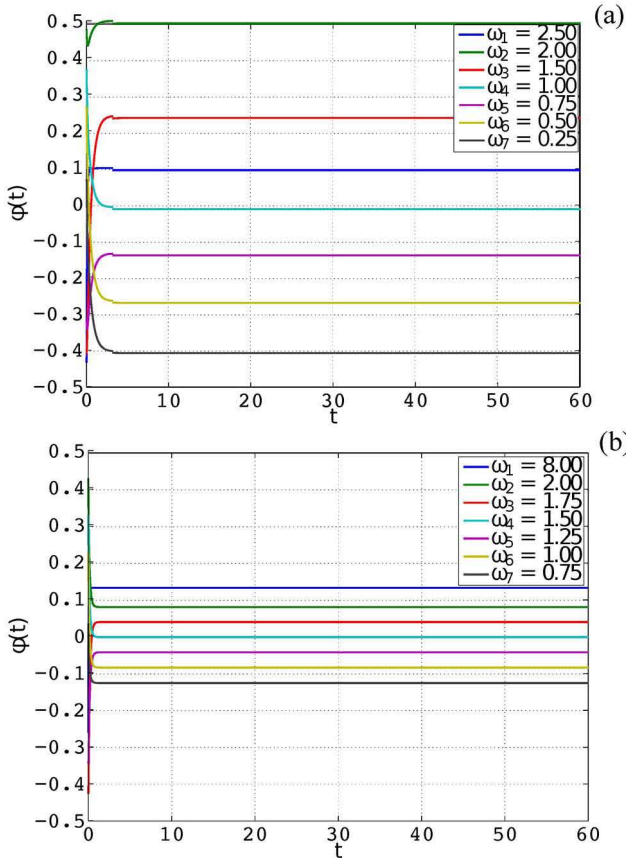


Fig. 2. Relative phases  $\phi(t)$  in a network of  $N = 7$  heterogeneous Kuramoto oscillators – undirected unweighted star graph. Node 1 is the central node, while node 2 is the peripheral with the highest oscillation frequency – (a)  $\omega_2 > \Omega$ ,  $c = 14$ ; (b)  $\omega_2 < \Omega$ ,  $c = 43$  (b).

the emerging leader in the network is the second node, that is the peripheral one with the highest oscillation frequency  $\omega_i$ .

Fig. 2(b) shows numerical results obtained when setting the values of the oscillation frequencies  $\omega_i$  so that  $2 = \omega_2 < \Omega \approx 2.32$ . Since the initial conditions and the values of the parameters were set such that  $\gamma = 1$  and  $\|\omega\|_{\mathcal{E},\infty} = 7.25$ , we set  $c = 43 > \frac{N\|\omega\|_{\mathcal{E},\infty} \sin(\gamma)}{1} \approx 42.7$  to guarantee phase locking. As predicted by our theory, the emerging leader in the network is now the central node (i.e., node 1).

#### V. ADDING A VIRTUAL LEADER IN A HUMAN ENSEMBLE: PRELIMINARY RESULTS

The results shown so far suggest that the structure of the network and the individual dynamics of its nodes can influence the emergence of coordination in the network as recently discussed in [19] where the case of a group of human players performing a joint oscillatory task is proposed as a paradigmatic case of study. Specifically, the case was studied of a group of individuals who were asked to oscillate their hands back and forth at their preferred frequency firstly in isolation and then within the group where synchronization was observed to emerge. Here we explore how the introduction of a virtual agent in the group affects the synchronization level

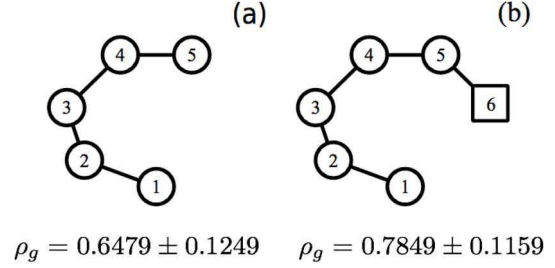


Fig. 3. Network topology implemented without (a) and with (b) virtual player with the relative  $\rho_g := \mu \pm \sigma$ . Each human player is represented by a circle, while the virtual player by a square.

of the ensemble by establishing leadership. All experiments were carried out through Chronos, an innovative software platform to carry out human coordination experiments recently presented in [22].

A path graph, as shown in Fig. 3, was implemented with and without the addition of a virtual player. The synchronization level of the group was quantified using the *group coordination index*  $\rho_g$  which is defined in [19]. Such parameter takes values within the range  $[0, 1]$ , with increasing values representing lower phase mismatches among the members in the group and hence higher coordination among the nodes. Figure 3 shows how the coordination index  $\rho_g$  is higher when a virtual player is introduced to act as leader in the human ensemble, thus suggesting that leadership enhances synchronization. This is also confirmed by ANOVA which was used to test the statistical significance of the results (paired T-test at 90% with confidence level,  $t(6) = -2.125, p = 0.078$ ). Further experimental results are under investigation and will be presented elsewhere.

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#### REFERENCES

- [1] M. Newman, *Networks: an introduction*. Oxford university press, 2010.
- [2] M. E. Newman, “The structure and function of complex networks,” *SIAM review*, vol. 45, no. 2, pp. 167–256, 2003.
- [3] A. Arenas, A. Díaz-Guilera, J. Kurths, Y. Moreno, and C. Zhou, “Synchronization in complex networks,” *Physics reports*, vol. 469, no. 3, pp. 93–153, 2008.
- [4] M. Moussaïd, N. Perozo, S. Garnier, D. Helbing, and G. Theraulaz, “The walking behaviour of pedestrian social groups and its impact on crowd dynamics,” *PLoS one*, vol. 5, no. 4, p. e10047, 2010.
- [5] K. W. Rio, C. K. Rhea, and W. H. Warren, “Follow the leader: Visual control of speed in pedestrian following,” *Journal of vision*, vol. 14, no. 2, pp. 4–4, 2014.
- [6] K. Özçimder, B. Dey, R. J. Lazier, D. Trueman, and N. E. Leonard, “Investigating group behavior in dance: an evolutionary dynamics approach,” in *American Control Conference (ACC), 2016*. IEEE, 2016, pp. 6465–6470.
- [7] P. Silva, D. Chung, T. Carvalho, T. Cardoso, K. Davids, D. Araújo, and J. Garganta, “Practice effects on intra-team synergies in football teams,” *Human movement science*, vol. 46, pp. 39–51, 2016.

- [8] L. Badino, A. D’ausilio, D. Glowinski, A. Camurri, and L. Fadiga, “Sensorimotor communication in professional quartets,” *Neuropsychologia*, vol. 55, pp. 98–104, 2014.
- [9] W. Goebel and C. Palmer, “Synchronization of timing and motion among performing musicians,” *Music Perception: An Interdisciplinary Journal*, vol. 26, no. 5, pp. 427–438, 2009.
- [10] R. Weber, C. Camerer, Y. Rottenstreich, and M. Knez, “The illusion of leadership: Misattribution of cause in coordination games,” *Organization Science*, vol. 12, no. 5, pp. 582–598, 2001.
- [11] R. M. Stogdill, “Personal factors associated with leadership: A survey of the literature,” *The Journal of psychology*, vol. 25, no. 1, pp. 35–71, 1948.
- [12] D. N. Den Hartog, R. J. House, P. J. Hanges, S. A. Ruiz-Quintanilla, P. W. Dorfman, I. A. Abdalla, B. S. Adetoun, R. N. Aditya, H. Agourram, A. Akande *et al.*, “Culture specific and cross-culturally generalizable implicit leadership theories: Are attributes of charismatic/transformational leadership universally endorsed?” *The Leadership Quarterly*, vol. 10, no. 2, pp. 219–256, 1999.
- [13] A. D’Ausilio, L. Badino, Y. Li, S. Tokay, L. Craighero, R. Canto, Y. Aloimonos, and L. Fadiga, “Leadership in orchestra emerges from the causal relationships of movement kinematics,” *PLoS one*, vol. 7, no. 5, p. e35757, 2012.
- [14] K. Rio, S. Bonneaud, and W. H. Warren, “Speed coordination in pedestrian groups: Linking individual locomotion with crowd behavior,” *Journal of Vision*, vol. 12, no. 9, p. 190, 2012.
- [15] C. Zhai, F. Alderisio, K. Tsaneva-Atanasova, and M. di Bernardo, “A novel cognitive architecture for a human-like virtual player in the mirror game,” in *2014 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*. IEEE, 2014, pp. 754–759.
- [16] —, “Adaptive tracking control of a virtual player in the mirror game,” in *53rd IEEE Conference on Decision and Control*. IEEE, 2014, pp. 7005–7010.
- [17] —, “A model predictive approach to control the motion of a virtual player in the mirror game,” in *2015 54th IEEE Conference on Decision and Control (CDC)*. IEEE, 2015, pp. 3175–3180.
- [18] F. Alderisio, D. Antonacci, C. Zhai, and M. di Bernardo, “Comparing different control approaches to implement a human-like virtual player in the mirror game,” in *15th European Control Conference (ECC)*, 2016, pp. 216–221.
- [19] F. Alderisio, G. Fiore, R. N. Salesse, B. G. Bardy, and M. di Bernardo, “Interaction patterns and individual dynamics shape the way we move in synchrony,” *Scientific Reports*, vol. 7, 2017.
- [20] Y. Kuramoto, *Chemical oscillations, waves, and turbulence*. Springer Science & Business Media, 2012, vol. 19.
- [21] F. Dörfler, M. Chertkov, and F. Bullo, “Synchronization in complex oscillator networks and smart grids,” *Proceedings of the National Academy of Sciences*, vol. 110, no. 6, pp. 2005–2010, 2013.
- [22] F. Alderisio, M. Lombardi, G. Fiore, and M. di Bernardo, “A novel computer-based set-up to study movement coordination in human ensembles,” *Frontiers in psychology*, vol. 8, no. 967, 2017.