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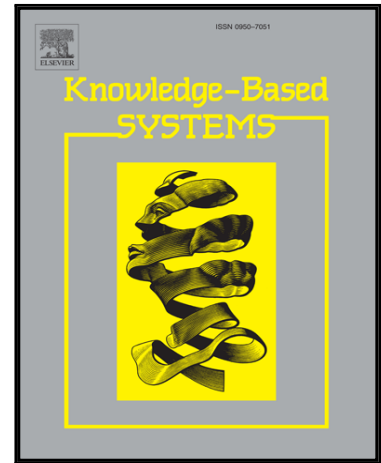
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Highlights

We proposed a novel approach to support consensus reaching processes in multi-attribute large-group decision making, based on comprehensive behavior classification and adaptive weight updating. The main contributions are as follows:

- Comprehensively classify modification behaviors of decision makers into 3 categories using the constructed cooperative index and non-cooperative index.

- Uninorm aggregation operator is used for decision weight updating, laying either award or penalty on decision makers in accordance with their modification behavior.

- In order to lay stricter behavior supervision on highly-weighted clusters, the adaptive weight updating scheme is formulated based on a uninorm aggregation operator with floating neutral element.

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A novel consensus model for multi-attribute large-scale group decision making based on comprehensive behavior Classification and adaptive weight updating

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Abstract

Consensus reaching process (CRP) has received increasing attention in recent years, as the demand for decision results with mutual agreement has greatly grown. With the current tendency to introduce e-democracy and public participation into decision making for public issues, decision makers from various backgrounds are more likely to encounter conflict when attempting to reach a consensus, especially under a multi-attribute large-scale group decision making framework. In order to improve the efficiency of the CRPs, different consensus models have been proposed. Specific patterns of behaviors presented by decision makers, such as non-cooperative behaviors and minority opinions, are also strictly supervised in these models. However, not every type of behaviors is specifically defined and given directed treatment, this includes the behavior of highly-weighted clusters, which may seriously bias group consensus. In this paper, we present a novel CRP model named uninorm-based comprehensive behavior classification (UBCBC) model with enhanced efficiency and rationality. First, a behavior classification model based on the calculation of a cooperative index and a non-cooperative index is proposed to classify three kinds of modification behaviors.

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Second, decision weights in the next iteration of the CRP are updated using a uninorm aggregation operator to reward or penalize the behaviors of clusters. Furthermore, a floating neutral element is introduced into the uninorm aggregation operator to lay stricter supervision upon highly-weighted clusters. Finally, an illustrative example and a numerical simulation are implemented to prove that this model is of high efficiency and feasibility.

Keywords: Multi-attribute large-scale group decision making; Consensus reaching process; Comprehensive behavior classification; Adaptive weight updating

1. Introduction

Group decision making (GDM) is an indispensable component in our daily lives. As the decision object and information involved getting more and more complex nowadays, an integrated group knowledge will be more comprehensive and insightful for us to get the decision done, and GDM has been developed according to the ever-changing environments. Multi-attribute large-scale GDM (MALSGDM) is a combination of the basic theory and procedure of GDM with an application context in which large number of decision makers (DMs) from multiple backgrounds make their evaluations toward a set of alternatives over multiple attributes and select the best alternative after the decision results are aggregated [1-3]. Given the wide variety DMs' different backgrounds and expertise, DMs will have different evaluations on attributes or ordering of alternatives and conflict can easily arises [4]. Recent research has paid great attention to the consensus degree reached after decision results are collected from DMs, especially regarding decision making for public issues such as environmental problems [5][6], policy making [7][8], and infrastructure planning [9][10].

Consensus reaching processes (CRPs) are widely utilized in the GDM and the MALSGDM problems.

The objective of the CRP is to reach a final decision which can satisfy most DMs, instead of giving some DMs an impression that their opinions are not considered seriously [11-13]. In a CRP, DMs or clusters of them [14][15] are asked to modify their decision results towards the aggregated group preference under the supervision of a moderator [16]. Under more complex circumstances where DMs are concerned with different aspects of the evaluation objects and thus every individual has a unique set of attributes and alternatives, suggestions for DMs to change their attributes and alternative set are also generated [17]. Many studies have investigated how to improve the efficiency of the CRPs, and the most widely used concept in the CRP is the co-called "soft consensus" [18], during which a threshold of group consensus degree is predefined, and the CRP is conducted over DMs' or clusters' evaluation information to improve the group consensus degree of the decision result iteratively [19][20].

However, certain modification behaviors of individuals or clusters may prevent the group from reaching a consensus and these behaviors can be generalized as non-cooperative behaviors. In this paper, evaluation modification behavior, or simply modification behavior, refers to the conduct that (clusters of) DMs present regarding their willingness to bring their preferences closer to the rest of the group. According to the study by R. Yager [21], driven by selfish intentions, DMs may strategically manipulate the GDM by the information they provide towards a direction that can benefit themselves, while at the same time disregard the intention of the majority. Indicated by [22], this strategic manipulation of GDM can also be achieved by assigning biased weights to different attributes. Palomares [23] proposed a quantitative model to detect and penalize the non-cooperative behaviors of DMs, and used Self-Organizing Maps to visualize the CRP. Xu [3] designed a comprehensive adjustment coefficient by combining both subjective and objective information to obtain modified preference matrices and identify non-cooperative behaviors. Quesada [24] proposed that a modification behavior should be quantified by the number of assessments that move closer to consensus, and

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that stricter restrictions should be laid upon DMs as the CRP proceeds in order to stimulate cooperative behaviors. This methodology demonstrated that a properly evolving regulation on DMs can enhance the efficiency of the CRP compared with a steady regulation system. In [25], three types of non-cooperative behaviors were proposed: (i) non-cooperative behaviors owing to a DM's refusal to change their preferences; (ii) non-cooperative behaviors owing to dishonest expression; and (iii) non-cooperative behaviors owing to significantly distinct preferences. After identifying different types of non-cooperative behaviors that the DMs present, dynamic multi-attribute mutual evaluation matrices (MMEMs) which evaluate DMs' professional skills, cooperation as well as fairness are updated and experts' weights are derived from MMEMs iteratively. This precise quantified description of different kinds of non-cooperative behaviors shows high efficiency and can help us get the real intentions of DMs which are behind the information they provide, making it possible to manage non-cooperative behaviors according to its sources. While under more complex environment where DMs are not familiar with each other, generating MMEMs may be hard or costly and a more direct way for weight updating may be needed.

It is worth noticing that for some DMs, after each round of modification behaviors, some assessments in the evaluation information may come closer to consensus, while other assessments may evolve in the opposite direction. This indicates that every modification behavior consists of both cooperative components and non-cooperative components, but previous studies categorize the overall behavior of a DM as either cooperative or non-cooperative exclusively. To date, studies concerning identification of modification behaviors have focused on only one of the two aspects, seldom taking both into account. Importantly, it should be noticed that for those who are deemed as non-cooperative, there may also exist certain degree of cooperativeness in their modified preferences, which should be also deemed as the presence of a cooperative behavior to some extent. Namely, the impact of a modification behavior on CRPs should actually be a fusion

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of its consisting two (cooperative and non-cooperative) components. Based on this analysis, it is more rational to depict and define every modification behavior through the comprehensive consideration of both cooperative and non-cooperative components, thus giving every DM a more tailored and personalized treatment.

In a CRP, aside from non-cooperative behaviors, other behaviors presented by individuals or clusters may also need to be specifically treated. For example, in [3], it was stressed that minority opinions are influential during the CRP, while owing to its comparatively low decision weight these minority positions can be easily overlooked. Considering the principle that every DM's preference should be taken into account, minority opinions need special attention. However, no study has yet investigated the influence of highly-weighted cluster opinions, namely majority opinions, on the efficiency of CRPs. In [26], it was pointed out that a highly-weighted cluster tends to be more aggressive in enforcing their own will against others, through its large number of members and high decision weight. For most circumstances, allocating a higher decision weight to a certain entity is deemed as a reward for its contribution to reaching a consensus, which also contains an expectation that they can keep up this cooperative behavior and contribute more. If the later modification behavior presented by a highly-weighted cluster evolves towards a rather non-cooperative one, it will seriously bias the group consensus, as it is much easier for a highly-weighted cluster to strategically manipulate group preference. In other words, the non-cooperative component of a modification behavior can be amplified by its high decision weight, thereby causing drastic conflict in the group.

Meanwhile, it is entirely possible for a highly-weighted cluster to present non-cooperative behaviors, especially if the cluster is formed by a coalition of DMs with similar interests in mind. In most decision making problems, not all behaviors presented by DMs are consistent. In other words, it is possible for a DM

to be cooperative at the beginning of the process, and then become non-cooperative in later rounds of the CRP. The requirements for a modification behavior to be classified as cooperative should therefore be raised as decision weight increases. At the same time, if non-cooperative behavior is ever identified in highly-weighted clusters, they should be punished more severely compared with clusters with low decision weights, to prevent them from causing further conflict.

Based on the challenges discussed above, we propose a novel consensus model for MALSGDM problems, named as uninorm-based comprehensive behavior classification (UBCBC) model to tackle these gaps. The contributions of the proposed UBCBC model in this paper are summarized as,

(i). In order to depict and define modification behaviors more rationally, the cooperative component and non-cooperative component of a modification behavior are separately quantified. Three types of behaviors, namely cooperative leading behavior, non-cooperative leading behavior and average behavior, are defined according to the constituent components of modification behaviors.

(ii). Based on a uninorm aggregation operator, either reward or penalty is applied on the decision weight of each cluster of DMs according to the classification of its modification behavior to enhance the efficiency of reaching consensus in the CRP.

(iii). In order to lay stricter behavior supervision on highly-weighted clusters in the CRP, a floating neutral element which is positively correlated with decision weight, is also introduced to the proposed UBCBC model.

The remainder of this paper is organized as follows. Section 2 presents some basic concepts and knowledge concerning MALSGDM problems, the CRP procedures, and uninorm aggregation operators, as preparations for further discussion. In Section 3 and Section 4, we give detailed elaboration of the proposed

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UBCBC model. In Section 3, we propose the methodology to quantify the cooperative component and non-cooperative component of a modification behavior. By comprehensive consideration of the cooperative index and the non-cooperative index, modification behaviors are classified into three categories and accordingly rewarded or penalized by using a uninorm aggregation operator. In Section 4, a uninorm aggregation operator with a floating neutral element is used to lay stricter supervision on highly-weighted clusters and regulate their behavior. Section 5 provides an illustrative example and a numerical simulation to justify the model we propose in this paper. Finally, some conclusions and future research interest are drawn in Section 6.

2. Preliminaries

This section provides preliminary information regarding MALSGDM problems, the CRP, and uninorm aggregation operators, in preparation for further discussion on these topics.

2.1. MALSGDM problems

For simplicity, let $X = \{x_1, x_2, ..., x_m\}$ be a finite set of m alternatives, let $A = \{a_1, a_2, ..., a_n\}$ be a finite set of n attributes, and let $D = \{d_1, d_2, ..., d_l\}$ be a finite set of l DMs that evaluate alternatives. Usually, when the number of DMs exceeds 11, which is $l \geq 11$, it can be treated as a large-scale group decision making problem [3]. For a specific alternative, each DM d_h provides his/her opinion by means of a $m \times n$ matrix. Suppose $E^h = \begin{pmatrix} e_{11}^h & ... & e_{1n}^h \\ \vdots & \ddots & \vdots \\ e_{m1}^h & ... & e_{mn}^h \end{pmatrix}$ is the evaluation matrix provided by DM d_h, where e_{ij}^h denotes the assessment on the attribute a_j of alternative x_i. As application scenario varies, the assessments made by DMs are expressed in different mathematical forms to tackle the uncertainty and complexity, including linguistic information [27], fuzzy preference relation [28], intuitionistic fuzzy set [29]

and etc. Here for brevity, the assessment e_{ij}^h is presented in the form of accurate numerical value under a pre-determined scale, for example [0,10].

Regarding the initial decision weight of a DM, it is common practice in GDM approaches based on behavior management to place equal weights on DMs before the CRP [30-32], so the initial decision weight vector of DMs can be denoted as $W = \{w_1, w_2, \dots, w_l\} = \{\frac{1}{l}, \frac{1}{l}, \dots, \frac{1}{l}\}$.

The procedural framework for a MALSGDM problem is briefly summarized as follows:

Step 1: Normalization of data

In order to remove the effect of magnitude of information, all assessments in evaluation matrices should be normalized into the interval [0,1].

Step 2: Group clustering

Clustering is recognized as an unsupervised machine learning method. The principle of clustering is to separate DMs into comparatively small clusters, in which DMs' evaluation information have higher consistency and a lower degree of conflict. Various clustering methods have been developed. Niknam [33] proposed a hybrid clustering algorithm based on K-means. Liu [34] developed a partial binary tree DEA-DA cyclic classification model to analyze DM's preferences and form clusters. When DM's preferences are illustrated in vector space, the relativity degree can be calculated by a certain measurement, which in essence transforms the relativity degree between two vectors into cosine similarity degree, to form clusters [35]. After clustering, individuals' evaluations are represented by the cluster's evaluations to simplify the CRP. Using the clustering method proposed by Xu [35], the group can be divided into several small clusters according to the evaluation matrices.

Step 3: Aggregation of clusters' preferences

Various aggregation operators have been proposed to aggregate individuals' information, including weighted averaging (WA) operators, ordered weighted averaging (OWA) operators [36], and intuitionistic fuzzy weighted averaging (IFWA) operators [37]. In this paper, we adopt a WA operator to simplify this process. It will be used to aggregate individual evaluation matrices into a cluster evaluation matrix, and to aggregate clusters' evaluation matrices into group evaluation matrix in each iteration of the CRP.

Step 4: Alternative selection

A proper selection methodology is used to order the alternatives and select the most suitable alternative for the decision making problem.

2.2. The CRP framework

In real-world decision making problems, there is a growing demand to choose the alternative with a high consensus degree among DMs, especially in public decision making issues. In order to achieve a soft consensus among DMs, a CRP can be used to improve group consensus degree iteratively. Based on the general scheme of the CRP proposed by Palomares [23], the framework adopted in this paper for undertaking a CRP is illustrated below:

(a) Calculate group consensus degree and determine parameters

Wang classified the consensus measure functions in clustering-based GDM approaches into two types: one is based on the compactness of clusters, and the other is based on the level of separation [38]. In this paper, the group consensus degree is evaluated by the average distance between clusters and aggregated group evaluation. By comparing it with a threshold, the moderator can determine whether the group needs to enter the next iteration of the CRP. The time for conducting the CRP cannot be unlimited, thus a maximum number of iterations needs to be determined as a stopping condition.

(b) Modify preferences and update decision weight

If the previous group consensus degree fails to meet the demand of the threshold, clusters are required to modify their evaluations in response to the specific explanations made by the moderator, to enhance group consensus degree. Actions taken to improve the efficiency of the CRP can be divided into two types: modification of evaluation information under compulsion, such as replacing parts of assessments of evaluation matrices with group evaluation [39]; and penalizing the decision weights of non-cooperative clusters [23]. During this process, various patterns of modification behaviors may be identified, such as non-cooperative behaviors and minority opinions. In this paper, modifications on evaluation matrices are done under no compulsion, while decision weights of clusters will be rewarded or penalized according to the classification of their modification behaviors. Meanwhile, behaviors of highly-weighted clusters are also given special treatment, as discussed later.

(c) Generate feedback information

Following modification behaviors, the group consensus degree is once more calculated and compared with the consensus threshold, until it meets the demand or reaches the maximum number of iterations.

The general CRP framework in this paper is illustrated below as Fig. 1:
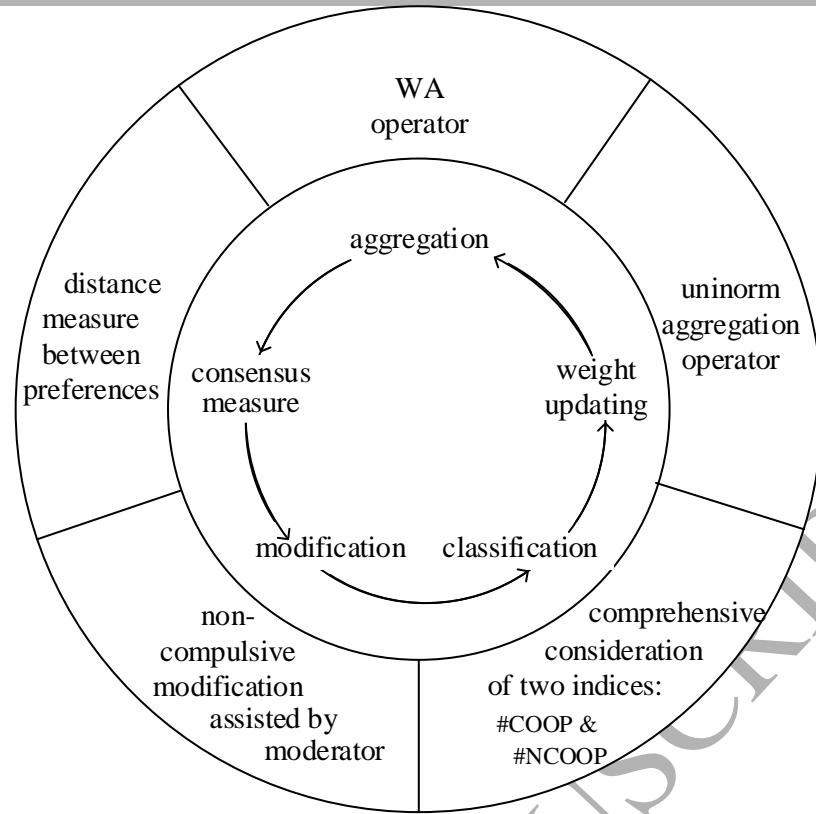
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Fig. 1. Consensus reaching process for MALSGDM problems.

For a better understanding of how the CRP works, here we list two real-world examples to help the readers to gain a deeper insight.

Example 1. A company is considering changing its market strategy next year. The board of directors has come up with 3 strategies to choose from. Each of the strategies is evaluated by the attributes of profitability, feasibility and conformity. They hold a formal meeting to discuss this issue and need to select one strategy by the end of the day. After the initial evaluation is done, a collective evaluation will be gained by summarizing individual evaluations based on each director's influence, which is affected by the proportion of shares he/she is holding. The chairman will act as a moderator to coordinate and ask everybody to modify their evaluations to help them achieve a balance between voicing their opinions and improving group consensus degree. If someone only cares about his/her own opinion and hampers the board from reaching a consensus, the board will consider less of this person's opinion and vice versa.

Multiple rounds of discussion and modification may take place until a considerable consensus is achieved in the board or until the meeting's time is over. A final decision will be made based on the high-consensus evaluations.

Example 2. A series of public hearings will be held by the local government to choose the best location for a waste disposal site from 3 possible locations. The representatives of the government, specialists, citizens from the neighborhood are going to discuss over this issue. The main aspects they are going to discuss will be the proposal's influence on the job market, surrounding environment, community safety etc. It will be hard for people from all walks of life to achieve a consensus at the beginning so there will always be several hearings before the final decision is made. During each hearing, the host will collect opinions from everyone by means of survey and will talk people into devoting to group consensus. If any participator does so, his/her opinion will be more valued by the hearing thus he/she will be given a higher decision weight. Once the local government confirms a preferable consensus is reached in the hearing, the final decision will be made.

2.3. Uninorm aggregation operator

A uninorm aggregation operator is a unification of *t*-norm and *t*-conorm operators, as proposed by Yager and Rybalov [40]. A unique property of the uninorm operator is that it can reinforce two-dimensional samples upwards or downwards, according to the identification of a neutral element. This property has been widely used in group decision making problems, including retrieving historical weight information [24], regulating strategic preference manipulation [41], and constructing gossip-based protocols [42]. The uninorm aggregation operator is defined as follows:

Definition 1. A uninorm is a mapping,

$$U:[0,1] \times [0,1] \rightarrow [0,1],$$

having the following properties for all $a, b, c, d \in [0,1]$

    i).   Commutativity: $U[a,b] = U[b,a]$;

    ii).   Monotonicity: $U[a,b] \geq U[c,d], \; if \; a \geq c \; and \; b \geq d$;

    iii).   Associativity: $U(a, U(b,c)) = U(U(a,b),c)$;

    iv).   Neutral element: $\exists g \in [0,1]: U(a,g) = a$.

The distinction between a uninorm and *t*-norm or *t*-conorm is that a uninorm has a neutral element g in the interval $[0,1]$. If g reaches 1 or 0, the uninorm evolves into a *t*-norm or a *t*-conorm. The uninorm aggregation operator performs differently when the input values vary, as illustrated in Fig. 2.
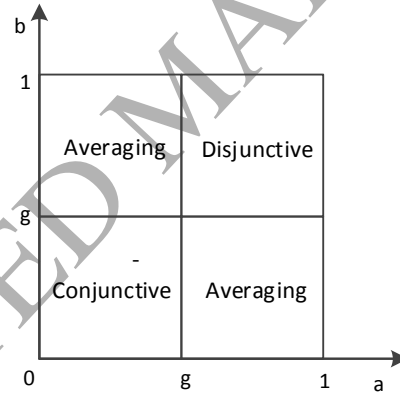
Fig. 2. Behaviors of a uninorm aggregation operator for different input values (adapted from [43]).

A general form of the uninorm aggregation operator was given by Fodor [44] as:

$$U(a,b) = \begin{cases} g \cdot T_u\left(\frac{x}{g}, \frac{y}{g}\right) & if \; 0 \leq a, b \leq g, \\ g + (1-g)S_u\left(\frac{x-g}{1-g}, \frac{y-g}{1-g}\right) & if \; g \leq a, b \leq 1, \\ A_u(a,b) & if \; \min(a,b) \leq g \leq \max(a,b). \end{cases} \tag{1}$$

where T_u and S_u can be any *t*-norm and any *t*-conorm operator, and A_u is an averaging operator. It can be seen from the model that when both input values are greater than the neutral element, an upward

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reinforcement is realized, and the aggregated value is higher than the inputs. When input values are below

the neutral element, the opposite effect occurs. The uninorm aggregation operator used in this paper was

introduced by Fodor [44], as illustrated below:

$$U(a,b) = \begin{cases} \frac{ab}{g} & if\ 0 \leq a,b \leq g, \\ \frac{a+b-ab-g}{1-g} & if\ g \leq a,b \leq 1, \\ A_u(a,b) & if\ \min(a,b) \leq g \leq \max(a,b). \end{cases} \quad (2)$$

3. Classification of modification behaviors and uninorm-based decision weight updating scheme

In this section, the initial group consensus degree is calculated and compared with the pre-defined

consensus threshold. If the group consensus degree does not meet the threshold, a CRP is required to be

conducted iteratively in the group to enhance the group consensus degree to the expected level. During the

CRP, clusters of DMs are required to modify their evaluation information. The modification behaviors can

be deemed as consisting of both cooperative components and non-cooperative components. By

quantification of both components, the characteristics of modification behaviors can be identified in detail

and these behaviors can be further classified into three categories: cooperative leading behaviors,

non-cooperative leading behaviors, and average behavior. The updated decision weight of each cluster is

obtained according to its modification behavior category, using a uninorm aggregation operator.

3.1. Measurement of group consensus degree

After DMs are divided into clusters by the clustering method proposed by Xu [35], group consensus

degree should be measured as a prerequisite of the CRP. For MALSGDM problems, a clustering method is

applied to analyze and classify DMs' evaluation information. Suppose that all the evaluation matrices are

collected from l DMs and they are divided into K clusters. The number of DMs in the kth cluster is l_k.

Suppose DMs are allocated the same decision weight before the CRP, thus $W = \{w_1, w_2, ..., w_l\} =$

$\{\frac{1}{l}, \frac{1}{l}, ..., \frac{1}{l}\}$. The evaluation matrix of the kth cluster can be illustrated as $C^k = (c_{ij}^k)_{m \times n}$. c_{ij}^k is the kth cluster's assessment towards the jth attribute of the ith alternative and can be denoted as:

$$c_{ij}^k = WA\left(e_{ij}^1, e_{ij}^2, ..., e_{ij}^{l_k}\right) = \sum_{h=1}^{l_k} w_h \cdot e_{ij}^h, \tag{3}$$

where $E^1, E^2, ..., E^{l_k} \in C^k$.

The initial decision weight of a certain cluster is determined by the number of DMs it contains, thus the weight of the kth cluster can be denoted as:

$$\lambda_k = {l_k}/{l}. \tag{4}$$

The group evaluation is given by $G = (g_{ij})_{M \times N}$, where g_{ij} denotes the group's assessment towards the jth attribute of the ith alternative:

$$g_{ij} = WA\left(c_{ij}^1, c_{ij}^2, ..., c_{ij}^K\right) = \sum_{k=1}^K \lambda_k \cdot c_{ij}^k, \tag{5}$$

Conflict is reflected by the distance between clusters and the aggregated group evaluation. By calculating the Manhattan distance between evaluation matrices, the conflict degree can be quantified. Take the conflict degree between cluster evaluation matrix C^k and group evaluation matrix G for example, the conflict is given by:

$$C(C^k, G) = d(C^k, G) = \frac{1}{m \times n} \sum_{i=1}^m \sum_{j=1}^n |c_{ij}^k - g_{ij}|. \tag{6}$$

Thus, the consensus degree between C^k and G can be defined as:

$$CD(C^k, G) = 1 - C(C^k, G). \tag{7}$$

The higher the value of CD, the more favorable the consensus degree that has been reached between entities. Conversely, the lower the value of CD, the greater the disagreement between two entities, hence special attention should be paid to this cluster in the CRP. If $CD = 0$, the kth cluster is in total conflict with

the group evaluation. If $CD = 1$, the kth cluster is in total consensus with the group.

Group consensus degree can then be obtained as:

$$GCD = \sum_{k=1}^{K} \lambda_k \cdot CD(C^k, G). \tag{8}$$

The value of GCD ranges from 0 to 1. If GCD is lower than the consensus threshold ε, CRP is applied to help reach a favorable consensus.

3.2. Measurement of cooperative index and non-cooperative index

In a CRP, DMs or clusters of them are required to modify their evaluation matrices under the guidance of the moderator. The modification of evaluation matrix E^h is composed of modifications on its constituent assessments e_{ij}^h. The modification on each assessment can be undertaken in two possible ways: either movement towards the expected consensus, or movement in the opposite direction. Here, we use the percentage of shrinkage or enlargement of the distance before and after modification to define the cooperative index and the non-cooperative index, instead of using the exact distance they move. The reason is that if two clusters have different original distances from the group evaluation, the distances they move cannot be directly interpreted into the endeavor they make owing to the different original distance basis. This is illustrated in detail in the following example 3.

Example 3. For 2×2 evaluation matrices $C^{1(t)} = \begin{pmatrix} 0.63 & 0.54 \\ 0.40 & 0.28 \end{pmatrix}$ and $C^{2(t)} = \begin{pmatrix} 0.43 & 0.83 \\ 0.45 & 0.56 \end{pmatrix}$, the group matrix they are advised to move towards is $G^{(t)} = \begin{pmatrix} 0.80 & 0.55 \\ 0.50 & 0.34 \end{pmatrix}$. The evaluation matrices after modification are $C^{1(t+1)} = \begin{pmatrix} 0.73 & 0.54 \\ 0.40 & 0.28 \end{pmatrix}$ and $C^{2(t+1)} = \begin{pmatrix} 0.53 & 0.83 \\ 0.45 & 0.56 \end{pmatrix}$. Take assessments $c_{11}^1 = 0.63$ and $c_{11}^2 = 0.43$ for example, the exact distances they moved are 0.1 in both cases, but the endeavors they make are different. It is comparatively easier for C^2 to move 0.1 closer to consensus, because this requires covering only 27% of the original distance. For C^1, moving 0.1 closer to consensus means a compromise of

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59% of the original conflict, which is a greater sacrifice of its own interest. Thus, it is fair to evaluate the modification behavior by the shrinkage or enlargement of the original distance.

Thus, we first define the overall distance between cluster C^k and group evaluation matrix G in the tth iteration as:

$$OD^{k(t)} = \sum_{i=1}^{m} \sum_{j=1}^{n} \left| c_{ij}^{k(t)} - g_{ij}^{(t)} \right|. \tag{9}$$

Algorithm 1. Procedures to compute the cooperative index $\#COOP$ of cluster C^k in the tth iteration.

_____

1) Assign $\#COOP^{k(t)} \leftarrow 0$.

2) For each pair of $c_{ij}^{k(t)}$ and $c_{ij}^{k(t+1)}$,

   if $\left| c_{ij}^{k(t+1)} - g_{ij}^{(t)} \right| < \left| c_{ij}^{k(t)} - g_{ij}^{(t)} \right|$,

   then $\#COOP^{k(t)} \leftarrow \#COOP^{k(t)} + \dfrac{\left| c_{ij}^{k(t)} - g_{ij}^{(t)} \right| - \left| c_{ij}^{k(t+1)} - g_{ij}^{(t)} \right|}{OD^{k(t)}}$.

   Until $i = m, j = n$.

3) end if

4) end for

The computation of the cooperative index requires the overall percentage shrinkage of the distance between the kth cluster and the collective evaluation matrix, before and after the tth iteration modifications on $m \times n$ assessments. The index $\#COOP$ ranges from 0 to 1. When the index approaches 1, the kth cluster has a predominantly cooperative attitude. If $\#COOP = 0$, all assessments of the kth cluster have moved away from the group preference or remained unchanged, indicating that cluster C^k has disregarded the

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advice from the moderator, nor has it taken any action to improve consensus. The cooperative index $\#COOP$ is positively correlated with the number of assessments that move nearer to group evaluation and the percentage of shrinkage in the distance between these assessments, thus shows the extent to which the kth cluster wants to cooperate with others in reaching a consensus.

Algorithm 2. Procedures to compute the non-cooperative index $\#NCOOP$ of cluster C^k in the tth iteration.

_____

1) Assign $\#\text{NCOOP}^{k(t)} \leftarrow 0$.

2) For each pair of $c_{ij}^{k(t)}$ and $c_{ij}^{k(t+1)}$,

   if $\left| c_{ij}^{k(t+1)} - g_{ij}^{(t)} \right| > \left| c_{ij}^{k(t)} - g_{ij}^{(t)} \right|$,

   then $\#\text{NCOOP}^{k(t)} \leftarrow \#\text{NCOOP}^{k(t)} + \dfrac{\left| c_{ij}^{k(t+1)} - g_{ij}^{(t)} \right| - \left| c_{ij}^{k(t)} - g_{ij}^{(t)} \right|}{\text{OD}^{k(t)}}$.

   Until $i = m, j = n$

3) end if

4) end for

_____

Computation of the non-cooperative index requires the overall percentage of enlargement in the distance between the kth cluster and the group evaluation matrix, before and after the tth iteration modifications on $m \times n$ assessments. When the index has a value of 0, no evidence of non-cooperative attitude is found. The assessments either come closer to consensus or stay the same. As a prerequisite requirement of uninorm aggregation operator presented in next section, we set 1 as the maximum value of the non-cooperative index $\#NCOOP$. $\#NCOOP$ is positively correlated with the number of assessments that move further from consensus, and with the percentage of enlargement in the distance between these assessments, thus shows

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the extent to which the kth cluster refuses to accept advice from moderator or cooperate to achieve a consensus.

The following example 4 gives a numerical illustration of the calculation of $\#COOP$ and $\#NCOOP$.

Example 4. For 2×2 evaluation matrix $C^{1(t)} = \begin{pmatrix} 0.63 & 0.52 \\ 0.40 & 0.28 \end{pmatrix}$, the group evaluation matrix towards which it is advised to move is $G^{(t)} = \begin{pmatrix} 0.72 & 0.59 \\ 0.50 & 0.34 \end{pmatrix}$. The evaluation matrix after modification is $C^{1(t+1)} = \begin{pmatrix} 0.70 & 0.62 \\ 0.35 & 0.50 \end{pmatrix}$. The overall distance can be calculated as $OD^{1(t)} = |0.63 - 0.72| + |0.52 - 0.59| + |0.40 - 0.50| + |0.28 - 0.34| = 0.32$ using Eq. (9). For assessments $c_{11}^1 = 0.63$ and $c_{12}^1 = 0.59$, their distances from group evaluation diminished, even though $c_{12}^{1(t+1)} = 0.62$ became higher than $g_{12}^{(t)} = 0.59$ after modification. Thus, the cooperative index is calculated as $\#COOP^{1(t)} = \frac{(0.09 - 0.02) + (0.07 - 0.03)}{0.32} = 0.344$. Assessments c_{21}^1 and c_{22}^1 moved further from group evaluation. Even though c_{22}^1 moved towards consensus, the extent of change applied on this assessment was exceedingly large, therefore the distance to consensus still enlarges. Thus, the non-cooperative index is calculated as $\#NCOOP^{1(t)} = \frac{(0.15 - 0.10) + (0.17 - 0.06)}{0.32} = 0.500$.

3.3. Classification of modification behaviors and updating decision weight based on the uninorm aggregation operator

The $\#COOP$ and $\#NCOOP$ indices provide detailed information about a cluster's endeavor to enhance or reduce the consensus degree in the group. By combing both indices with the uninorm aggregation operator, modification behaviors can be specifically classified in a meaningful way. The uninorm aggregation operator has the properties of upward and downward reinforcement. That is, for a two-dimensional uninorm aggregation operator, if values of both dimensions surpass the neutral element g, the aggregated value will be upward reinforced to amplify its positive influence in reaching consensus; and

if both values are below g, the aggregated value will be downward reinforced. This feature of the uninorm aggregation operator can be used in the classification of decision behaviors and the computation of performance value (PV) of each behavior, which can be used to determine the decision weight of the kth cluster in the next round of the CRP.

The different categories of modification behaviors are defined based on the value of $\#COOP$ and $\#NCOOP$ as follows.

1) Cooperative leading behavior (CLB); when $g < \#COOP,\ 1 - \#NCOOP \leq 1$

   A modification behavior is classified as a cooperative leading behavior if this behavior makes a significant contribution to reaching a consensus, and the non-cooperative attitude it shows is negligible by comparison. In traditional models, a modification behavior is defined by evaluation of either cooperative or non-cooperative degree, but never both of them. Thus, such models cannot comprehensively depict the behavior presented by clusters. In our model, we give freedom to clusters to preserve opinions that may interfere with reaching a consensus. As long as a modification behavior is dominated by its cooperative component, its decision weight can still be lifted in the CRP. This behavior will be upward reinforced by utilizing the uninorm aggregation operator to accelerate CRP efficiency. Its performance value (PV) is calculated as an upward enhancement on the basis of $\#COOP$ and $(1 - \#NCOOP)$.

2) Non-cooperative leading behavior (NLB); when $0 \leq \#COOP, 1 - \#NCOOP < g$

   The non-cooperative behavior is dominated by components that enlarge the distance between a cluster and group preference. This type of behavior has negative performance on both the cooperative and non-cooperative dimensions. It seriously damages the efficiency of the CRP and causes further conflict

in the group. When it occurs, the cluster will be penalized by a decrease in its decision weight, using the uninorm aggregation operator to downward reinforce the decision weight. Its PV is calculated as a downward enhancement on the basis of $\#COOP$ and $(1 - \#NCOOP)$.

3) Average behavior (AB); when $\min(\#COOP, 1 - \#NCOOP) \leq g \leq \max(\#COOP, 1 - \#NCOOP)$

The Average behavior has no dominant component. It consists of similar contributions from both the cooperative component and the non-cooperative component. Thus, it cannot be categorized as either cooperative leading behavior or non-cooperative leading behavior. No reinforcement is applied to such behavior. Its PV will be the average of $\#COOP$ and $(1 - \#NCOOP)$.

Therefore, the modification behaviors can be classified into three categories, as shown in Fig. 3.
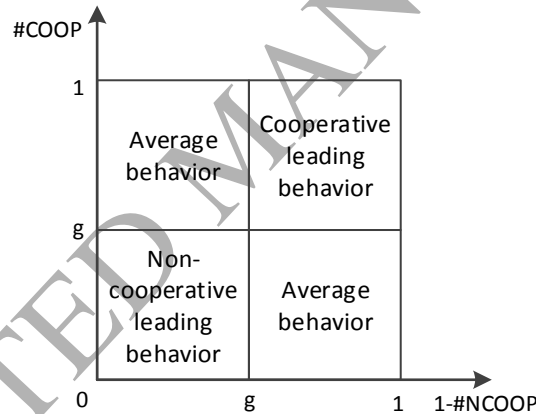

Fig. 3. Classification of modification behaviors.

After the modification behavior presented by each cluster has been identified and classified, the PV of each cluster is calculated using a uninorm aggregation operator, in order to update the decision weight of each cluster in the next iteration of the CRP.

The following uninorm aggregation operator, based on Eq. (2), is defined to calculate the PV of each cluster according to its modification behavior classification in the tth iteration, with A_u being an averaging

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operator.

$$PV^{k(t)} = \begin{cases} \frac{\#COOP \cdot (1 - \#NCOOP)}{g} & 0 \le COOP, 1 - \#NCOOP < g; \\ \frac{\#COOP + (1 - \#NCOOP) - \#COOP \cdot (1 - \#NCOOP) - g}{1 - g} & g < COOP, 1 - \#NCOOP \le 1; \\ A_u(\#COOP, 1 - \#NCOOP) & otherwise. \end{cases} \tag{10}$$

3.4. Weight updating and normalization

The decision weight of each cluster is updated according to the PV calculated in the previous subsection to reward cooperative behaviors and penalize non-cooperative behaviors:

$$\theta_k^{(t+1)} = \lambda_k^{(t)} \cdot PV(C^{k(t)}). \tag{11}$$

In order to meet $\sum_{k=1}^{K} \theta_k^{(t+1)} = 1$, where $\theta_k^{(t+1)}$ represents the non-normalized decision weight, the decision weight of each cluster in the $(t + 1)^{\text{th}}$ iteration is normalized into $\lambda_k^{(t+1)}$ by:

$$\lambda_k^{(t+1)} = \frac{\theta_k^{(t+1)}}{\sum_{k=1}^{K} \theta_k^{(t+1)}}. \tag{12}$$

4. Supervision on the behavior of highly-weighted clusters based on the uninorm aggregation operator with a floating neutral element

The CRP is an iterative process. DMs or clusters are rewarded or penalized according to their modification behaviors in each round of the CRP. In real-world decision making problems, the pattern of modification behaviors presented by a cluster may not always be consistent. It is highly possible for a cluster to cooperate in the first rounds of the CRP in order to gain a heavy decision weight, before becoming non-cooperative in later rounds. Considering the amplification effect of the non-cooperative component caused by a heavy decision weight, the CRP may be seriously sabotaged by non-cooperative behavior presented by highly-weighted clusters.

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By adopting a floating neutral element, which is positively correlated with decision weight, in the behavior classification model, behaviors of heavy-weighted clusters will be supervised under a stricter standard. It will be more difficult for them to be classified as showing cooperative leading behavior and upward reinforced because standard has been raised. This methodology suits for real decision making situations, where enhanced high decision weight also brings about expectations from clients that DMs should do better and further accelerate the CRP. Example 5 illustrates the advantages of applying this method. The second advantage of introducing a floating neutral element is that non-cooperative leading behavior by heavy-weighted cluster will be penalized more severely, as the PV will be calculated lower. An example of this can be found in the illustrative example in Section 5.

Example 5. Suppose we have a group consisting of three clusters, $C^1, C^2,$ and C^3 (detailed information is provided in Table 1). For Cluster C^1, $\lambda_1^{(t)} = 0.3$. The group consensus degree after the tth iteration is $GCD^{(t)} = 0.826$. For better illustration, only cluster C^1 modifies its evaluation matrix, and the other two clusters remain the same in the next round of the CRP. Table 2 illustrates the updated evaluation matrices of the clusters. It can be seen that the cooperative index and the non-cooperative index of C^1 are both 0.5. If g has a stable value and $g = 0.5$, the modification behavior of C^1 will be classified as average behavior, and $PV^{1(t)} = \frac{0.5+0.5}{2} = 0.5$. The group consensus degree in the $(t + 1)^{\text{th}}$ iteration will be $GCD^{(t+1)} = 0.821$, which is higher than that of the tth iteration. On the other hand, if g is given a floating value that is positively correlated with decision weight, the results are as shown in Table 3. Suppose $g^* = 0.6$. The modification behavior of C^1 will be classified as a non-cooperative leading behavior, $PV^{1(t)*} = \frac{0.5*0.5}{0.6} = 0.42$ and its decision weight will be downward reinforced. The group consensus degree can then be calculated as $GCD^{(t+1)*} = 0.833$, which is higher than in the tth iteration. This means the CRP can benefit from laying stricter supervision on highly-weighted clusters. This also shows that, to improve the efficiency

of the CRP, modification behaviors consisting of the same components may be classified into different

categories, as decision weight differs in this model.

Table 1

Evaluation information and decision weights of clusters after the tth iteration.

	$c_{11}^{(t)}$	$c_{12}^{(t)}$	$c_{21}^{(t)}$	$c_{22}^{(t)}$	$\lambda_k^{(t)}$
C^1	0.50	0.80	0.20	0.30	0.30
C^2	0.10	0.30	0.60	1.00	0.10
C^3	0.20	0.40	0.80	0.60	0.60
G	0.28	0.51	0.60	0.55	

Table 2

Evaluation information and updated decision weights of clusters in the $(t+1)^{\text{th}}$ iteration based on a stable g value.

	$c_{11}^{(t+1)}$	$c_{12}^{(t+1)}$	$c_{21}^{(t+1)}$	$c_{22}^{(t+1)}$	PV_k	$\lambda_k^{(t+1)}$
C^1	0.88	0.57	0.55	0.10	0.5	0.3
C^2	0.10	0.30	0.60	1.00	0.5	0.1
C^3	0.20	0.40	0.80	0.60	0.5	0.6
G	0.39	0.44	0.71	0.49		

Table 3

Evaluation information and updated decision weights of clusters in the $(t+1)^{\text{th}}$ iteration based on a floating g value.

	$c_{11}^{(t+1)}$	$c_{12}^{(t+1)}$	$c_{21}^{(t+1)}$	$c_{22}^{(t+1)}$	PV_k	$\lambda_k^{(t+1)}$
C^1	0.88	0.57	0.55	0.10	0.42	0.26
C^2	0.10	0.30	0.60	1.00	0.50	0.11

C^3	0.20	0.40	0.80	0.60	0.50	0.63
G	0.37	0.43	0.71	0.51		

Based on these hypotheses, we propose an approach to dynamically assign a floating value to the neutral element g with the following characteristics:

1) The neutral element g is positively correlated with the cluster's weight.

2) The neutral element g ranges from 0 to 1 to fit the uninorm aggregation model.

3) When the cluster's weight reaches 1, g reaches its maximum value of 1 and no longer goes up, which means $\frac{dg}{d\lambda_k}(\lambda_k = 1) = 0$.

4) The floating neutral element g is used to regulate the behavior of highly-weighted clusters. For clusters whose weights are near 0, the value of g shall not be sensitive. Conversely, g shall be sensitive when the decision weight is comparatively high. Thus, we set $\frac{dg}{d\lambda_k}(\lambda_k = 0) = 0$.

To satisfy all the requirements for the floating neutral element, we propose the formula of g as:

$$g = \frac{1+g_{\lambda_k=0}}{2} + \left[\frac{1-g_{\lambda_k=0}}{2} * \cos((\lambda_k - 1) * \pi)\right]. \tag{13}$$
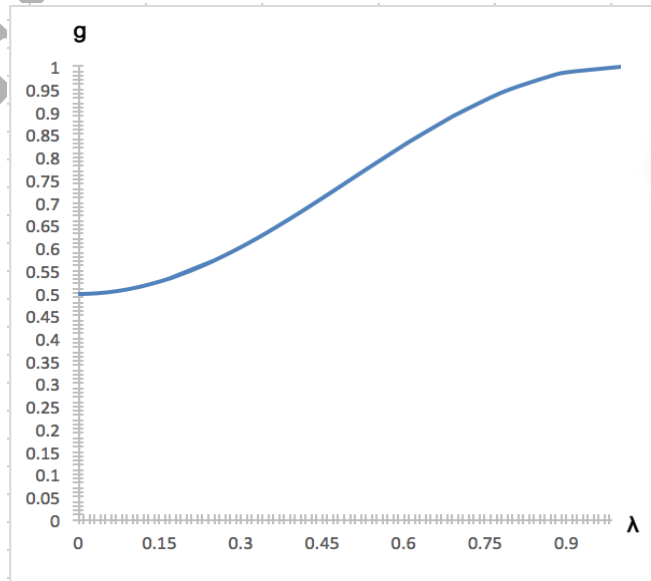
Fig. 4. Graphical illustration of the neutral element ($g_{\lambda_k=0} = 0.5$).

Fig. 4 shows a graphical illustration of g when $g_{\lambda_k=0}$ is set to 0.5. The function graph of g may differ when $g_{\lambda_k=0}$ is given different values, while it must be in the range [0,1]. The feasibility and performance of Eq. (13) is justified through an illustrative example in Section 5. Other formulations of g may also fit this model, but only the formulation in Eq. (13) is illustrated in detail here without any loss of generality.

5. Illustrative example and numerical simulations

In this section, we present an illustrative example to justify the UBCBC CRP model for MALSGDM problem that we propose in this paper. A MALSGDM problem consisting of 30 DMs is formulated. DMs are first clustered into several clusters by the clustering method proposed by Xu [35]. Initial group consensus degree is measured between clusters and aggregated group evaluation. If consensus degree fails to meet a pre-set threshold, a CRP is required to be complemented. During the CRP, every cluster is requested to present a modified evaluation matrix under the guidance of moderator. The decision weight of each cluster is updated in view of the modification behavior it presents. Once the group consensus degree meets with the threshold, a soft consensus is achieved in the group. The threshold of consensus degree is set as 0.9 and the acceptable maximum number of iterations of CRP is set as 10.

To demonstrate the enhancement of CRP efficiency that results from adopting UBCBC model, three baseline consensus models are presented for comparison: (i) The model that neither rewards nor penalizes clusters, with no supervision on modification behaviors; (ii) The model that classifies modification behaviors and updates decision weights of clusters based on a uninorm aggregation operator with a stable neutral element; (iii) Our proposed model, that classifies modification behaviors and updates decision

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weights of clusters based on a uninorm aggregation operator with a floating neutral element, laying stricter supervision on highly-weighted clusters.

5.1. A MALSGDM problem formulation

Suppose the government decides to add a new subway line to the public transportation system in the city of Tianjin, China. A group of 30 DMs collected from different stakeholders must evaluate three construction plans, $x_1, x_2,$ and x_3. Each of the plans is evaluated with respect to 4 attributes: social impact, environmental impact, project budget and technical feasibility.

Each DM's evaluation information is interpreted and standardized into a 3×4 matrix, which is visualized using SOM Toolbox PDA projection (Fig. 5). SOM Toolbox is a plug-in for the well-known software package MATLAB, which can realize the 2-D PCA projection of multi-dimensional information [45]. By combining the clustering method proposed by Xu [35] and this visualization of evaluation information, DMs are divided into 5 clusters. The evaluation information of each cluster is aggregated using Eq. (3). The detailed aggregated evaluation information of each cluster is illustrated in Table 4. The initial decision weight of each cluster is decided by the number of DMs it contains. The group consensus degree before CRP is calculated as 0.759 by using Eq. (6), (7), and (8). As the consensus degree is lower than the threshold 0.9, a CRP is conducted among clusters.
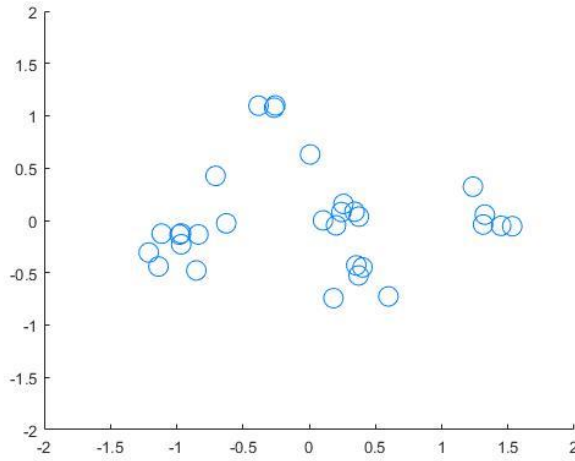
Fig. 5. Visualization of DMs' evaluation information before CRP.

Table 4

Evaluation information and decision weights of clusters before CRP.

	c_{11}^k	c_{12}^k	c_{13}^k	c_{14}^k	c_{21}^k	c_{22}^k	c_{23}^k	c_{24}^k	c_{31}^k	c_{32}^k	c_{33}^k	c_{34}^k	$\lambda_k^{(0)}$
C^1	0.24	0.43	0.26	0.51	0.56	0.31	0.59	0.76	0.46	0.28	0.51	0.56	0.233
C^2	0.91	0.62	0.82	0.06	0.12	0.54	0.49	0.89	0.90	0.08	0.58	0.24	0.333
C^3	0.12	0.80	0.88	0.18	0.93	0.80	0.20	0.53	0.88	0.03	0.90	0.92	0.100
C^4	0.06	0.09	0.18	1.00	0.88	0.22	0.96	0.14	0.16	0.92	0.58	0.87	0.167
C^5	0.50	0.14	0.40	0.44	0.57	0.74	0.89	0.32	0.51	0.74	0.15	0.19	0.167
G	0.46	0.43	0.52	0.40	0.51	0.49	0.63	0.60	0.61	0.37	0.53	0.48	

5.2. Comparison between the UBCBC models and the no-penalizing model

For brevity, only the first two rounds of the CRP are described in detail here. Under the guidance of the moderator, all clusters are advised to move towards group evaluation to enhance the consensus degree. Modified evaluation information of each cluster is illustrated in Table 5. As mentioned at the beginning of Section 5, three models are used to conduct the CRP, namely:

i). M1: The model with no supervision on modification behaviors.

ii). M2: The UBCBC model with a stable neutral element value.

iii). M3: The UBCBC model with a floating neutral element value.

The results of the modification behavior classification and updated decision weights after the first round of the CRP are listed in Table 6.

By conducting:

- M1: No supervision is laid upon clusters' modification behaviors. Decision weights of clusters are neither rewarded nor penalized. Decision weight stays the same, at $\lambda^{(1)} = (0.233, 0.333, 0.100, 0.167, 0.167)$. Consensus degree is calculated as $GCD^{(1)} = 0.767$ after the first round of the CRP.

- M2: C^1 and C^2 are detected as presenting non-cooperative leading behaviors, in which the values of $\#COOP$ and $(1 - \#NCOOP)$ are both lower than the neutral element g. The endeavors of C^1 and C^2 have limited contributions to improving group consensus degree on some assessments, while at the same time cause severe conflict on other assessments. Thus, the decision weights of C^1 and C^2 are downward reinforced to prevent them causing higher levels of conflict. The modification behavior of C^4 has high scores for both $\#COOP$ and $(1 - \#NCOOP)$. Thus, it is classified as cooperative leading behavior, which is highly beneficial for reaching a consensus. $PV^{4(1)} = 0.913$, which is much higher than those of other clusters. The modification behavior of C^4 in the first round of the CRP helps it to be allocated the highest decision weight in the next round. The behaviors of C^3 and C^5 are typically mixed, either $\#COOP$ is high and $(1 - \#NCOOP)$ is low, or vice versa. They are classified as showing average behavior and neither rewarded nor penalized. The updated decision weights after

normalizing are $\lambda^{(1)} = (0.177, 0.162, 0.133, 0.350, 0.179)$ according to Eq. (12). Group consensus

degree is calculated as $GCD^{(1)} = 0.787$, which is still below the threshold of 0.9.

- M3: The classification of modification behaviors in M3 is the same as M2. The difference appears in the

  PVs of the modification behaviors. C^1 and C^2 are classified by both M2 and M3 as having

  non-cooperative leading behaviors. While as two comparatively highly-weighted clusters, they have

  high neutral element values. Based on Eq. (10), more severe penalties are laid upon them. The PVs of

  these behaviors are decreased by 27% and 20% respectively compared to M2. The PVs of other clusters

  are almost unchanged. The updated decision weight is changed to

  $\lambda^{(1)} = (0.166, 0.137, 0.141, 0.367, 0.190)$. Consensus degree is updated to $GCD^{(1)} = 0.793$, which is

  higher than the value obtained by applying M1 or M2.

Table 5

Evaluation information of clusters after the first round of the CRP.

	$c_{11}^{k(1)}$	$c_{12}^{k(1)}$	$c_{13}^{k(1)}$	$c_{14}^{k(1)}$	$c_{21}^{k(1)}$	$c_{22}^{k(1)}$	$c_{23}^{k(1)}$	$c_{24}^{k(1)}$	$c_{31}^{k(1)}$	$c_{32}^{k(1)}$	$c_{33}^{k(1)}$	$c_{34}^{k(1)}$
C^1	0.41	0.55	0.49	0.71	0.72	0.20	0.49	0.71	0.48	0.20	0.55	0.49
C^2	0.95	0.91	1.00	0.25	0.00	0.90	0.63	0.77	0.70	0.05	0.20	0.00
C^3	0.12	1.00	0.92	0.18	0.80	0.74	0.39	0.73	0.90	0.33	0.95	0.60
C^4	0.40	0.34	0.44	0.61	1.00	0.42	0.80	0.42	0.40	0.53	1.00	0.49
C^5	0.25	0.29	0.18	0.24	0.30	0.52	0.81	0.35	0.40	0.50	0.11	0.11

Table 6

Classification of modification behaviors and calculation of updated weights in the first round of the CRP.

			PV	Updated weight
#COOP	$1 - $#NCOOP	Category	(M2/M3)	(M2/M3)

C^1	0.396	0.417	NLB	0.330/0.293	0.177/0.166
C^2	0.216	0.491	NLB	0.212/0.169	0.162/0.137
C^3	0.252	0.908	AB	0.580/0.580	0.133/0.141
C^4	0.634	0.882	CLB	0.913/0.907	0.350/0.367
C^5	0.292	0.645	AB	0.468/0.468	0.179/0.190

In the second round of the CRP, the group evaluation that clusters are recommended to move towards is calculated first. Detailed modified evaluation information for the second round of the CRP is provided in Table 7. Updated weights are different when applying different CRP methods, and the aggregated group evaluation in this round is also slightly different. These differences also lead to the differences in the values of $\#COOP$ and $(1 - \#NCOOP)$, which are given in Table 8.

- M1: No supervision is laid upon clusters' modification behaviors. Decision weight stays the same at $\lambda^{(2)} = (0.233, 0.333, 0.100, 0.167, 0.167)$. The group consensus degree is calculated as $GCD^{(2)} = 0.803$ after the second round of the CRP.

- M2: All clusters except for C^5 present average behavior. C^2 and C^3 have comparatively higher values of PV than C^1 and C^4, which means the modification behaviors of C^2 and C^3 consist of higher percentages of cooperative components. C^5 presents cooperative leading behavior, $\#COOP^{5(2)} = 0.594$ and $(1 - \#NCOOP^{5(2)}) = 0.989$. Only the decision weight of C^5 is upward reinforced, as a reward for its contribution to reaching a consensus. The updated decision weight after the second round of the CRP is $\lambda^{(2)} = (0.125, 0.159, 0.145, 0.249, 0.323)$. The updated group consensus degree is $GCD^{(2)} = 0.819$.

- M3: In the second round of the CRP, the biggest difference between M2 and M3 is in the classification of the behavior of C^4. As the cluster with the highest decision weight in this round, stricter supervision should be laid upon C^4. By applying M3, the neutral element, which is positively correlated with decision weight, can be calculated as $g = 0.75 + 0.25 * \cos\big((0.367 - 1) * \pi\big) = 0.649$, according to Eq. (13). As the values of $\#COOP$ and $(1 - \#NCOOP)$ are both below g, the behavior of C^4 is classified as non-cooperative behavior and downward reinforced. The PV of C^4 in this round is 0.189, much lower than the PV of 0.390 which was obtained by applying M2. For the other clusters, PVs are almost the same regardless of which method is applied. The updated decision weight is $\lambda^{(2)} = (0.131, 0.154, 0.176, 0.146, 0.393)$. The updated group consensus degree is 0.832, which is significantly higher than the value obtained by applying M2.

Table 7

Evaluation information of clusters after the second round of the CRP.

|	$c_{11}^{k(1)}$	$c_{12}^{k(1)}$	$c_{13}^{k(1)}$	$c_{14}^{k(1)}$	$c_{21}^{k(1)}$	$c_{22}^{k(1)}$	$c_{23}^{k(1)}$	$c_{24}^{k(1)}$	$c_{31}^{k(1)}$	$c_{32}^{k(1)}$	$c_{33}^{k(1)}$	$c_{34}^{k(1)}$
C^1	0.410	0.640	0.570	0.640	0.700	0.260	0.580	0.800	0.450	0.150	0.340	0.650
C^2	0.760	0.850	0.880	0.100	0.210	0.750	0.850	0.870	0.950	0.100	0.310	0.080
C^3	0.200	0.920	0.850	0.240	0.740	0.600	0.220	0.700	0.780	0.280	0.900	0.450
C^4	0.250	0.260	0.250	0.820	0.900	0.310	0.870	0.340	0.450	0.400	0.950	0.380
C^5	0.360	0.450	0.380	0.350	0.500	0.550	0.720	0.550	0.510	0.330	0.350	0.210

Table 8

Classification of modification behaviors and calculation of updated weights in the second round of the CRP.

	#COOP	$1 - \#NCOOP$	Category	PV	Updated weight

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	(M2/M3)	(M2/M3)	(M2/M3)	(M2/M3)	(M2/M3)
C^1	0.196/0.179	0.580/0.576	AB/AB	0.388/0.378	0.125/0.131
C^2	0.239/0.232	0.840/0.847	AB/AB	0.539/0.539	0.159/0.154
C^3	0.263/0.263	0.931/0.931	AB/AB	0.597/0.597	0.145/0.176
C^4	0.214/0.228	0.567/0.538	AB/NLB	0.390/0.189	0.249/0.146
C^5	0.594/0.587	0.989/0.989	CLB/CLB	0.991/0.990	0.323/0.393

Fig. 6 illustrates the evolution of the CRP results by visualizing the clusters' and the group's evaluation information when M2 is applied. The behavior patterns of clusters can be easily identified. For example, between (a) and (b) in Fig. 6, we can see that C^4 moved a large distance to the group evaluation, making a significant contribution to reaching consensus. Thus, it was classified as showing cooperative leading behavior and upward reinforced. Both C^1 and C^2 moved further from the first round's group evaluation (even though C^1 was closer to the second round's center); thus, they were classified as showing non-cooperative leading behavior and downward reinforced. C^3 and C^5 had mixed behaviors, with contributions from both the cooperative component and the non-cooperative component being equal. It is also difficult to tell whether they moved nearer to or further from the group evaluation in (a); thus, they are classified as showing average behavior.
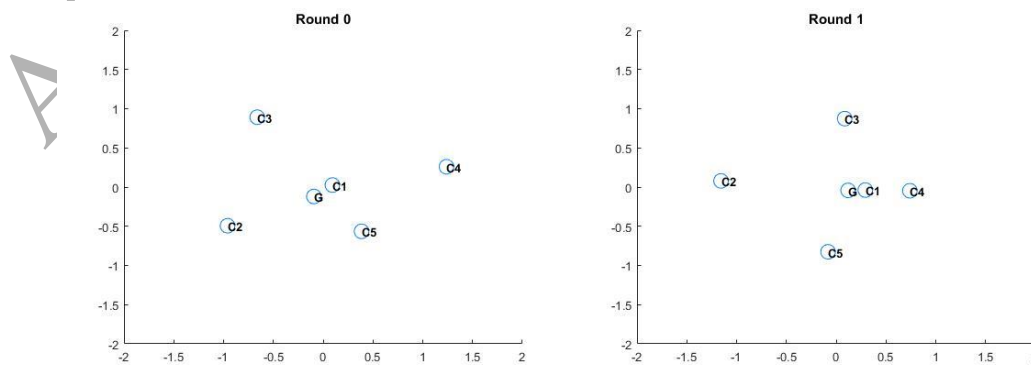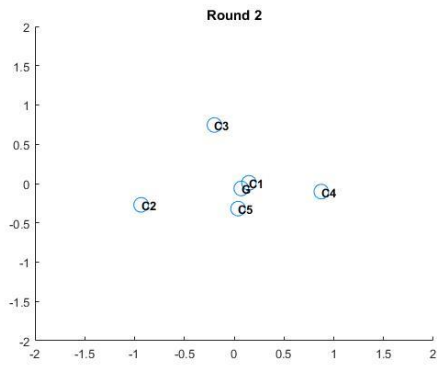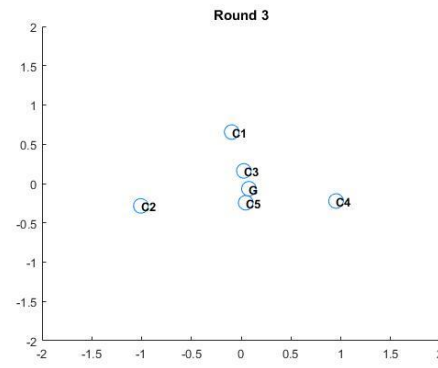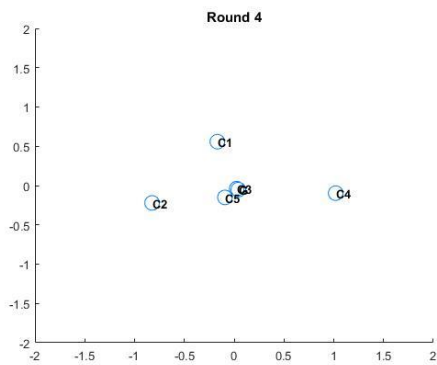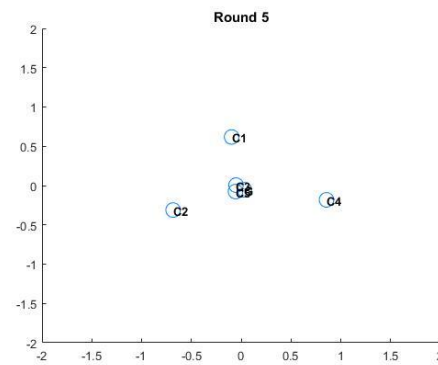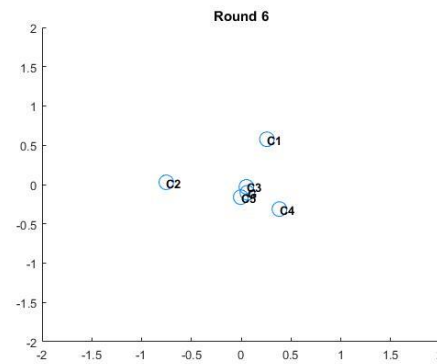
(a) Before CRP (GCD=0.759).   (b) Round 1 (GCD=0.787).

(c) Round 2 (GCD=0.819).   (d) Round 3 (GCD=0.833).

(e) Round 4 (GCD=0.855).   (f) Round 5 (GCD=0.879).

(g) Round 6 (GCD=0.908).

Fig. 6. Visualization of clusters and group evaluation information during CRP, based on the uninorm aggregation

operator with a stable neutral element.

After six rounds of the CRP, as illustrated in Fig. 6, the group has reached a group consensus degree of

0.908, which is higher than the threshold, and therefore a soft consensus is achieved.

For better comparison of the three different CRP models, Fig. 7 shows the evolution of group consensus degree when different CRP models are applied. When M1 is utilized, during which no supervision is laid upon clusters' behaviors, soft consensus is far from being reached after six rounds of the CRP. M1 also cannot promote consensus when non-cooperative behavior happens, as the group consensus degree decreases after the third round of modification. M2, which classifies modification behaviors and either rewards or penalizes them using a uninorm aggregation operator, shows high efficiency in reaching consensus. Non-cooperative behavior is penalized, and cooperative behavior is rewarded. After six rounds of the CRP, the group consensus degree is 0.908 and the threshold is reached. With M3, further improvements are made. Stricter supervision is laid upon highly-weighted clusters by introducing a neutral element with a floating value, which is positively correlated with decision weight. The efficiency of M3 is even higher than that of M2, reaching consensus after only five rounds of CRP. In specific situations, the efficiency of M3 may be higher still.
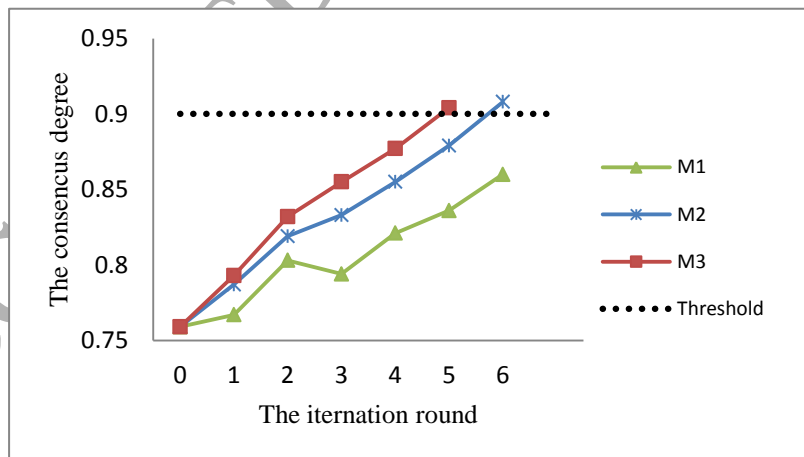

Fig. 7. Evolution of group consensus degree when different CRP methods are applied.

5.3. Further numerical simulation

In order to get a deeper insight into the advantages of applying the proposed UBCBC CRP model, a

further numerical simulation is conducted to compare the efficiency of M1, M2 and M3. For simplicity, we still use the settings in the fore-mentioned illustrative example, starting at the point where 5 clusters are going through the CRP. There are 3 alternatives and each alternative has 4 attributes to be evaluated. M1, M2 and M3 are parallelly applied on one round of CRP for 10000 times. The reason why we focus on only one round of the CRP is that the overall performance of it can be considered as a cumulation of independent rounds. The evaluation matrices and decision weights of each clusters before this round of the CRP is generated by random number generator in MATLAB. Especially, one of the five cluster's decision weight is set to a comparatively higher value (0.3 in this simulation). We track this cluster down to demonstrate the advantages of applying floating neutral element scheme (M3) when highly-weighted cluster presents non-cooperative behaviors. The modifications made to evaluation matrices are simulated by scaled normal-distribution numbers. The reason to scale them is to better simulate DM's behaviors in the CRP because each assessment in the evaluation matrices are not expected to go through huge modifications during the CRP. For example, it is hardly seen the scenario that one assessment changes from 0.2 to 0.9 but it is more possible for it to change to 0.1 or 0.3 in CRP. Another restriction is that the assessments in evaluation matrices after this round of CRP must range from [0,1].

The results of the simulation are summarized in Table 9. If the group consensus degree after this round of CRP is higher when methodology A is adopted compared to methodology B, we say A out-performs B. 6 different sets of data are tested on and they differ from each other by the degree to which the normal distributed modification behaviors are scaled. For each set of data, M1, M2 and M3 are implemented parallelly for 10000 times each with the randomly generated evaluation matrices and decision weights of DMs. For example, when the modification behaviors are scaled to 0.3, it means that all the modifications made to the assessments of the evaluation matrices will fall into a normal distribution in the scaled range of

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[-0.3,0.3]. Here we present some explanations for the results when the normal distributed modification behaviors are scaled to a specific level, for instance 0.3.

- The out-performance rate of M2 over M1 is 83.57%. This rate is calculated by the number of simulations in which M2 out-performs no penalty scheme M1 divided by the number of simulations, which is 10000. A high value of this rate indicates that by applying uninorm based weight updating scheme M2, we can achieve higher efficiency of improving consensus.

- The out-performance rate of M3 over M2 is 69.65%. This rate is calculated by the number of simulations in which M3 out-performs M2 and M2 outperforms M1, divided by the number of simulations in which M2 out-performs M1 in 10000 simulation times. This rate indicates that, under the circumstances where M2 can have prominent effect in improving consensus, the possibility of achieving an even better consensus result by adopting M3 is approximately 70%.

- The percentage of the highly-weighted cluster presents NLB when M3 out-performs M2 is 61.00%. This rate indicates that when M3 out-performs M2, the highly-weighted cluster is presenting NLB in 61.00% out of 10000 simulations. We can say that the reason why M3 can achieve a higher efficiency is related to the NLB presented by highly-weighted cluster.

Table 9

Statistical results of the numerical simulation.

To what degree the modifications are scaled	0.20	0.25	0.30	0.35	0.40
Out-performance rate of M2 over M1	73.71%	79.85%	83.57%	86.79%	88.78%
Out-performance rate of M3 over M2	63.44%	70.18%	69.65%	65.70%	62.19%
Percentage of the highly-weighted cluster presents NLB when M3 out-performs M2	47.84%	54.41%	61.00%	60.29%	58.80%

To have a more direct vision of the comparison between different methodologies, we recorded 500 simulation results when modifications are scaled to 0.3 and put them into a graph as shown in Fig.8. The vertical axis shows the group consensus degree after one round of CRP. A higher value in the vertical axis can be interpreted as having a higher efficiency in achieving consensus, as the group consensus degrees before applying the CRP models in each simulation are the same. The green line of M1 is predominantly below other lines in the simulations. The blue line of M2 predominantly outperforms M1 but is still covered or conquered by the red line of M3. The red line of M3 is predominantly above all other lines. This indicates by applying M3 in the CRP can harvest the most favorable results of improving consensus.
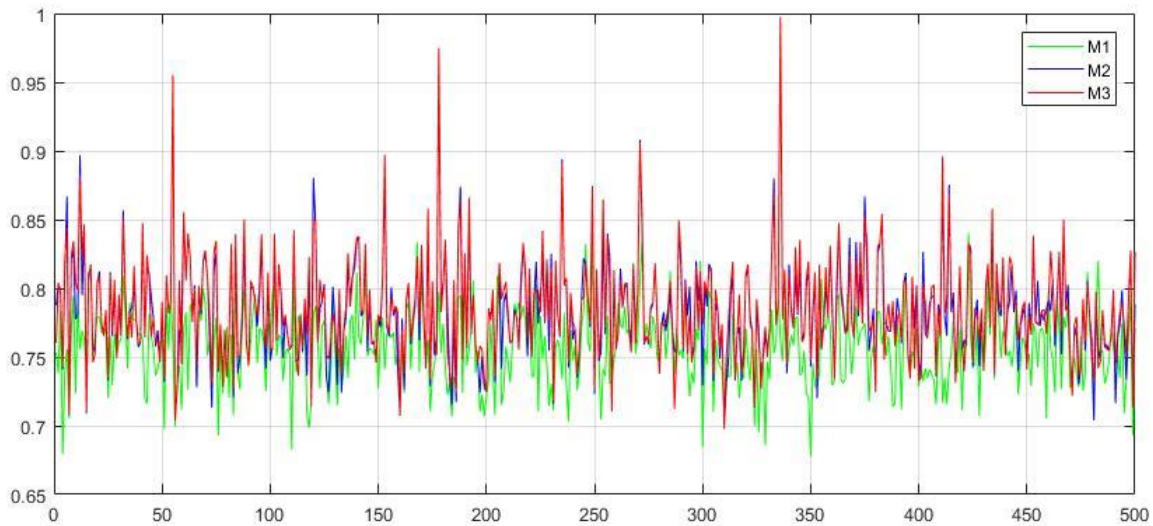

Fig. 8. The group consensus degree after one round of the CRP with three different methodologies.

5.4. Complexity analysis of proposed methodologies

For a convenient narrative, we let T be the limit on the number of iterations in the CRP, l be the number of DMs, and m, n be the number of alternatives and the corresponding attributes, respectively. The proposed consensus model can be divided into three phases: the clustering process, calculation of group

consensus degree and modification process, as well as the procedures to compute the cooperative and non-cooperative indices of each cluster. In the CRP, the clustering process is with the computational complexity $O(l * (l - 1))$ and is implemented as an off-line process. Therefore, the computational complexity of the clustering phase is usually ignored in practical applications. The computational complexity of the calculation and modification process is $O(T)$. In the third phase, there are two algorithms in the former phase with the same computational complexity, which is $O(m * n)$. As the third phase is nested in the calculation and modification phase, the computational complexity of second phase cannot be ignored. The alternative selection process is also an off-line process. In consequence, the proposed consensus model has the computational complexity $O(T * m * n)$.

5.5. Conclusions

In conclusion, the advantages of the proposed UBCBC CRP model can be summarized as follows.

- The cooperative component and the non-cooperative component of a modification behavior presented by a certain DM or cluster can be quantified separately. By comparing the cooperative index and non-cooperative index with a neutral element, modification behaviors can be classified into three categories. Non-cooperative behaviors can be strictly supervised.

- Decision weights are rewarded or penalized according to behavior classification. The enhancement of the CRP efficiency resulting from applying the UBCBC model (in terms of convergence towards the desired level of consensus) is clearly illustrated by the illustrative example and numerical simulation.

- By adopting a floating neutral element in the uninorm aggregation operator, the efficiency of the CRP is further improved for MALSGDM problem. On one hand, stricter supervision is laid upon highly-weighted clusters, which means they have to make more contributions to reaching a consensus,

and behavior once deemed as cooperative may be deemed as non-cooperative when decision weight increases. On the other hand, the non-cooperative behavior of highly-weighted clusters is more severely penalized.

6. Concluding remarks

A growing demand for reaching consensus in multi-attribute large-scale group decision making (MALSGDM) has made improvement of consensus reaching process (CRP) efficiency increasingly necessary. At the same time, in terms of the rationality of the CRP, specific behavior patterns should also be described and supervised as comprehensively as possible throughout the whole process. In this paper, a novel CRP model named as UBCBC model is proposed. First, the cooperative and the non-cooperative component of a modification behavior are measured separately for each cluster of DMs. By comparing the cooperative and non-cooperative indices with a neutral element, behaviors of clusters can be classified into three categories. A cluster is rewarded or penalized according to its behavior category, by increasing or decreasing its decision weight. During this process, not only are non-cooperative behaviors specifically identified, but all modification behaviors are given directed treatment, and the convergence efficiency of the CRP for MALSGDM problems is improved. Moreover, by introducing a floating neutral element into the UBCBC model, highly-weighted clusters can be restricted by stricter supervision, and the negative impact of their non-cooperative behaviors can be hugely relieved. An illustrative example along with a numerical simulation is presented to provide a detailed insight into the potential improvements in efficiency and rationality by comparison of three models, showing that the UBCBC model proposed in this paper has great potential for applications in real-world decision making problems.

One of the hottest topics in the future will be investigating the influence of trust information over GDM problems. The basic idea towards this combination is that, DMs in the group are prone to be

influenced by the ones they trust. Social Network Analysis, which takes a group of individuals as a network and studies the relationship between them, combined with trust evaluation information, has been adopted by researchers on the derivation of the decision weights of DMs [46][47] and evolution of missing evaluation information [48] in GDM problems. Another future interest of research is the GDM problems under emergency scenarios. The frequent occurrences of natural disasters have posed great challenge towards human society. At the same time, a collection of group wisdom will provide us with advantages in these situations. An overview of the development of emergency decision making (EDM) problems was given in [49] and the construction of a emergency decision support system was also illustrated. In [50], considering the ever-changing characteristics of the EDMs, a dynamic attributes' weights determination scheme based on Bayesian network along with a fuzzy decision making framework was proposed. Future research related to the combination of GDM and EDM problems can be further investigated under the guidance of these pioneer works.

Acknowledgements

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