## THE PERILS OF PARSIMONY*

It is widely thought in philosophy and elsewhere that parsimony (or simplicity) is a theoretical virtue in that:

PAR: If $T_{1}$ is more parsimonious than $T_{2}$, then $T_{1}$ is preferable to $T_{2}$ other things being equal.

This admits of many distinct precisifications. I shall focus on the following:
$P A R_{1}$ : If (a) $T_{1}$ is more parsimonious than $T_{2}$ and (b) $T_{1}$ is strictly equal to $T_{2}$ in respects $r_{1}, \ldots, r_{n}$ taken individually, then $\operatorname{Pr}\left(T_{1} \mid E \& B\right)>\operatorname{Pr}\left(T_{2} \mid E \& B\right)$.

Here and throughout $B$ is the background information on hand, $E$ is a piece of evidence, and $T_{1}$ and $T_{2}$ are rival theories each of which potentially explains $E$.
$\mathrm{PAR}_{1}$ itself admits of many distinct precisifications. This is because there are many distinct ways of understanding parsimony, and because, further, there are many distinct ways of specifying exactly what respects are included in $r_{1}, \ldots, r_{n}$. However, I want to set aside for now both the question of how parsimony is to be understood, and the question of exactly what respects are to be included in $r_{1}, \ldots, r_{n}$. ${ }^{1}$

I aim to show that $\mathrm{PAR}_{1}$ is false. If I succeed in this, then it follows that the same is true of any logically stronger parsimony thesis. For example:
$P A R_{2}$ : If (a) $T_{1}$ is more parsimonious than $T_{2}$ and (b) $T_{1}$ is at least roughly equal to $T_{2}$ in
$r_{1}, \ldots, r_{n}$ taken together, then $\operatorname{Pr}\left(T_{1} \mid E \& B\right)>\operatorname{Pr}\left(T_{2} \mid E \& B\right)$.
$P A R_{3}$ : If (a) $T_{1}$ is more parsimonious than $T_{2}$ and (b) $T_{1}$ is strictly equal to $T_{2}$ in $r_{1}, \ldots, r_{n}$ taken individually, then it is rational to believe $T_{1}$ and disbelieve $T_{2}$ given $E$ and $B$.
$\mathrm{PAR}_{2}$ is like $\mathrm{PAR}_{1}$ except that its antecedent is logically weaker than $\mathrm{PAR}_{1}$ 's antecedent. $\mathrm{PAR}_{3}$, in turn, is like $\mathrm{PAR}_{1}$ except that its consequent is logically stronger than $\mathrm{PAR}_{1}$ 's consequent. I am assuming here that it is rational to believe $T_{1}$ and disbelieve $T_{2}$ given $E$ and $B$ only if $\operatorname{Pr}\left(T_{1} \mid E \& B\right)>\operatorname{Pr}\left(T_{2} \mid E \& B\right)$. This strikes me as a very weak assumption. I see absolutely no plausibility in the idea that there can be cases where $\operatorname{Pr}\left(T_{1} \mid E \& B\right) \leq \operatorname{Pr}\left(T_{2} \mid E\right.$

[^0]$\& B)$ but it is rational to believe $T_{1}$ and disbelieve $T_{2}$ given $E$ and $B .{ }^{2}$ So, though my focus shall be on $\mathrm{PAR}_{1}$, my main point about $\mathrm{PAR}_{1}$ - that it is false-carries over to parsimony theses other than $\mathrm{PAR}_{1}$.

It also carries over to Inference to the Best Explanation understood as:
$I B E$ : If, given $B, T_{1}$ is a better potential explanation of $E$ than is $T_{2}$, then it is rational to believe $T_{1}$ and disbelieve $T_{2}$ given $E$ and $B$.

I mean for this to be understood so that whether, given $B, T_{1}$ is a better potential explanation of $E$ than is $T_{2}$ hinges on how $T_{1}$ and $T_{2}$ score in terms of the so-called "explanatory virtues" (or "theoretical virtues"), and so that $T_{1}$ and $T_{2}$ are the only available potential explanations of $E$ in the running. ${ }^{3}$ It follows that IBE's antecedent implies that, given $B, T_{1}$ is the best

[^1]available potential explanation of $E .{ }^{4} \mathrm{PAR}_{1}$ is entailed by $\mathrm{PAR}_{3}$, and the latter, in turn, is entailed by IBE. For, any case where $\mathrm{PAR}_{3}$ 's antecedent holds is a case where, given $B, T_{1}$ is a better potential explanation of $E$ than is $T_{2} .{ }^{5}$ Hence if, as I aim to show, $\mathrm{PAR}_{1}$ is false, then so too is IBE.

The core idea behind IBE can be put as follows:
$I B E^{*}$ : Inferences (at least some of them) should be guided at least in part by explanatory considerations.

This is weaker than IBE. It could be that inferences should be guided at least in part by explanatory considerations, but the way in which this should happen is not the way specified by IBE. Nothing in what I aim to show is meant to undermine IBE*. ${ }^{6}$

It is important to note that many writings on parsimony involve no appeal, explicit or implicit, to $\mathrm{PAR}_{1}$ (or $\mathrm{PAR}_{2}$ or $\mathrm{PAR}_{3}$ ). This is true, for example, of various writings on parsimony and divergence from the truth, ${ }^{7}$ various writings on parsimony and favoring in the sense of likelihoodism, ${ }^{8}$ various writings on parsimony and human cognition,,${ }^{9}$ various

Stathis Psillos, "Simply the Best: A Case for Abduction," in Antonis Kakas and Fariba Sadri, eds., Computational Logic: Logic Programming and Beyond (Berlin: SpringerVerlag, 2002), pp. 605-25; W. V. Quine and J. S. Ullian, The Web of Belief, 2nd ed. (New York: Random House, 1978), chapter 6; Richard Swinburne, Simplicity as Evidence of Truth (Milwaukee: Marquette University Press, 1997), section III; Paul Thagard, "The Best Explanation: Criteria for Theory Choice," Journal of Philosophy, LXXV (1978): 76-92; Timothy Williamson, "Abductive Philosophy," Philosophical Forum, XLVII (2016): 263-80.
${ }^{4}$ It might be that IBE's antecedent should be modified so that it is explicit that, given $B$, $T_{1}$ is a much better potential explanation of $E$ than is $T_{2}$, or so that it is explicit that, given $B$, $T_{1}$ is a satisfactory (or good enough) potential explanation of $E$. For discussion and references, see Douven, "Abduction," op. cit., section 2. But this is unimportant for my purposes.
${ }^{5} \mathrm{I}$ am assuming here that whether, given $B, T_{1}$ is a better potential explanation of $E$ than is $T_{2}$ is fully determined by how $T_{1}$ and $T_{2}$ score in terms of parsimony and $r_{1}, \ldots, r_{n}$.
${ }^{6}$ The same is true of a variant of IBE developed and defended in Frank Cabrera's "Can There be a Bayesian Explanationism? On the Prospects of a Productive Partnership," Synthese, CXCIV (2017): 1245-72, section 5.
${ }^{7}$ See, for example, Tomoji Shogenji, Formal Epistemology and Cartesian Skepticism: In Defense of Belief in the Natural World (New York: Routledge, 2018), chapter 6.
${ }^{8}$ See, for example, Elliott Sober, Ockham's Razors: A User's Manual (Cambridge: Cambridge University Press, 2015).
writings on parsimony and predictive accuracy, ${ }^{10}$ and various writings on parsimony and truth-finding efficiency. ${ }^{11}$ Nothing in what I aim to show is meant to tell against anything in any such writings. ${ }^{12}$

The remainder of the paper is organized as follows. In Section I, I note some different ways of understanding parsimony. Each is an instance of the more general idea that parsimony is a matter of the number of "things" on/in a theory. In Section II, I argue that $\mathrm{PAR}_{1}$ is false. In Section III, I discuss the upshot of this negative result. In Section IV, I conclude.

## I. PARSIMONY AS A MATTER OF THE NUMBER OF THINGS ON/IN A THEORY

There are many distinct ways of understanding parsimony. Here are but two examples:
$P_{1}$ : Parsimony is a matter of the number of entity tokens on a theory.
$P_{2}$ : Parsimony is a matter of the number of concept tokens in a theory.

Other examples are like $\mathrm{P}_{1}$ but where "entity tokens" is replaced by "entity types," "fundamental entity tokens," or "fundamental entity types." Still other examples are like $\mathrm{P}_{2}$ but where "concept tokens" is replaced by "concept types," "primitive concept tokens," or "primitive concept types." ${ }^{13,14}$

[^2]It is important to note the difference between the expressions "on a theory" and "in a theory." If $\mathrm{P}_{1}$ is assumed, then the issue is the number of entity tokens posited by a theory (or, strictly speaking, by a proponent of a theory). If, in contrast, $\mathrm{P}_{2}$ is assumed, then the issue is the number of concept tokens in which a theory is formulated (which might well be different than the number of concept tokens posited by a theory).

I shall remain neutral on how exactly parsimony is to be understood, and shall assume just that:
(*): Parsimony is a matter of the number of things on/in a theory.
This covers $\mathrm{P}_{1}, \mathrm{P}_{2}$, the variants noted above, and much more. ${ }^{15}$
Any way of understanding parsimony on which $\left({ }^{*}\right)$ holds can be used to precisify $\mathrm{PAR}_{1}$. For example, if $\mathrm{P}_{1}$ is assumed, then whether $T_{1}$ is more parsimonious than $T_{2}$ hinges on whether there are fewer entity tokens on $T_{1}$ than on $T_{2}{ }^{16}$

Razor in Science: A Case Study from Biogeography," Biology and Philosophy, XxiI (2007): 193-215, at p. 96; Baker, "Simplicity," op. cit., section 2; Sam Cowling, "Ideological Parsimony," Synthese, CXC (2013): 3889-908, at p. 3899; Jonathan Schaffer, "What Not to Multiply Without Necessity," Australasian Journal of Philosophy, XCIII (2015): 644-64, at p. 646.
${ }^{15}$ It also covers, for example, the idea that parsimony is a matter of the number of fundamental entity types on a theory in a particular area of inquiry (for example, biology).
${ }^{16}$ There is a growing debate on the relative merits of $\mathrm{P}_{1}, \mathrm{P}_{2}$, and variants thereof in the context of theses such as $\mathrm{PAR}_{1}$. See, for example, Alan Baker, "Quantitative Parsimony and Explanatory Power," British Journal for the Philosophy of Science, LIV (2003): 245-59; Baker, "Occam's Razor in Science," op. cit.; Sam Baron and Jonathan Tallant, "Do Not Revise Ockham's Razor Without Necessity," Philosophy and Phenomenological Research, XCVI (2018): 596-619; Ross Cameron, "How to Have a Radically Minimal Ontology," Philosophical Studies, CLI (2010): 249-64; Richard Caves, "Emergence for Nihilists," Pacific Philosophical Quarterly (forthcoming); Cowling, "Ideological Parsimony," op. cit.; Sam Cowling, Abstract Entities (London: Routledge, 2017); Lina Jansson and Jonathan Tallant, "Quantitative Parsimony: Probably for the Better," British Journal for the Philosophy of Science, LXVIII (2017): 781-803; Uriah Kriegel, "The Epistemological Challenge of Revisionary Metaphysics," Philosophers' Imprint, XIII (2013): 1-30, section 3.1; David Lewis, Counterfactuals (Malden: Blackwell, 1973), at p. 87; Daniel Nolan, "Quantitative Parsimony," British Journal for the Philosophy of Science, XLVIII (1997): 329-43; Schaffer, "What Not to Multiply Without Necessity," op. cit.; Theodore Sider, Writing the Book of the World (Oxford: Oxford University Press, 2011); Theordore Sider, "Against Parthood," in Karen Bennett and Dean Zimmerman, eds., Oxford Studies in

Is parsimony sensitive to what information is included in $B$ or how $E$ is understood? It might seem that given (*), the answer is negative. Take $\mathrm{P}_{1}$ for example. It might seem that since the number of entity tokens on $T_{1}$ is internal to $T_{1}$ and hinges not at all on what information is included in $B$ or how $E$ is understood, and since, similarly, the number of entity tokens on $T_{2}$ is internal to $T_{2}$ and hinges not at all on what information is included in $B$ or how $E$ is understood, it follows that whether $T_{1}$ is more parsimonious than $T_{2}$ in the sense of $\mathrm{P}_{1}$ is not sensitive to what information is included in $B$ or how $E$ is understood.

This is a tempting line of thought. But there is a potential problem. Suppose that there is exactly one entity token $e_{1}$ on $T_{1}$. Suppose that there are exactly two entity tokens $e_{2}$ and $e_{3}$ on $T_{2}$. Consider two situations. Situation X is such that $B$ includes the information that $e_{2}$ and $e_{3}$ exist but does not include the information that $e_{1}$ exists. This means that $e_{1}$ is "new" relative to $B$ whereas $e_{2}$ and $e_{3}$ are "old" relative to $B$. Situation Y , in contrast, is such that $B$ does not include the information that $e_{1}$ exists, does not include the information that $e_{2}$ exists, and does not include the information that $e_{3}$ exists. This means that $e_{1}, e_{2}$, and $e_{3}$ are all new relative to $B$. It is not implausible prima facie that $T_{1}$ is less parsimonious than $T_{2}$ in Situation X but more parsimonious than $T_{2}$ in Situation Y. If so, then it is not the case that whether $T_{1}$ is more parsimonious than $T_{2}$ is not sensitive to what information is included in $B$ or how $E$ is understood.

There is another way to look at things however. Perhaps $T_{1}$ is more parsimonious than $T_{2}$ both in Situation X and in Situation Y (because in each situation there is one entity token on $T_{1}$ and two entity tokens on $T_{2}$ ), and perhaps, consistent with this, $T_{1} \& B$ is less parsimonious than $T_{2} \& B$ in Situation X (because there is one more entity token on the former conjunction than on the latter) but more parsimonious than $T_{2} \& B$ in Situation Y (because there is one less entity token on the former conjunction than on the latter).

I want to remain neutral on all this. The important point for my purposes is this:
(**): Take any way of understanding parsimony $\mathrm{P}_{\mathrm{i}}$ on which (*) holds. Let " $\mathrm{P}_{\mathrm{i}}$-things" be the things to be counted when it comes to parsimony on $\mathrm{P}_{\mathrm{i}}$. If (a) there are fewer $\mathrm{P}_{\mathrm{i}}$ things on/in $T_{1}$ than on/in $T_{2}$ and (b) the things in question are all new relative to $B$, then $T_{1}$ is more parsimonious than $T_{2}$ regardless of what information, consistent with (a) and (b), is included in $B$, and regardless of how $E$ is understood.

[^3]Consider $\mathrm{P}_{1}$ and Situation Y for illustration. Given that (a) there is one entity token on $T_{1}$ but two on $T_{2}$, and given that (b) the entity tokens in question are all new relative to $B$, it follows by $\left({ }^{* *}\right)$ that $T_{1}$ is more parsimonious than $T_{2}$ regardless of what information, consistent with (a) and (b), is included in $B$, and regardless of how $E$ is understood. If, for example, $B$ included the negation of an observational consequence of $T_{1}$, or if $E$ were the negation of an observational consequence of $T_{1}$, then though it would follow that $T_{1}$ is false, it would remain the case that $T_{1}$ is more parsimonious than $T_{2}$.

I turn now to my critique of $\mathrm{PAR}_{1}$.

## II. PAR $_{1}$ CRITIQUED

This section is divided into five subsections. In Section II.1, I introduce some terminology. In Section II.2, I raise a yes/no question about how $\mathrm{PAR}_{1}$ is to be understood. In Section II.3, I argue that if the answer is yes, then $\mathrm{PAR}_{1}$ is false. In Section II.4, I argue that if the answer is no, then, still, $\mathrm{PAR}_{1}$ is false. In Section II.5, I conclude that $\mathrm{PAR}_{1}$ is false.
II.1. Terminology. It will help to introduce some terminology. First, $\operatorname{Pr}\left(T_{1} \mid B\right)$ and $\operatorname{Pr}\left(T_{2} \mid\right.$ $B$ ) are $T_{1}$ 's and $T_{2}$ 's "prior probabilities." These are their probabilities given $B$ (the background information on hand) and thus not taking into account $E$. If $B$ is tautological, then $\operatorname{Pr}\left(T_{1} \mid B\right)$ and $\operatorname{Pr}\left(T_{2} \mid B\right)$ are first (or a priori) prior probabilities. But it should not be assumed that $B$ is tautological. The typical case, in fact, is where $B$ is not tautological.
Second, $\operatorname{Pr}\left(E \mid T_{1} \& B\right)$ and $\operatorname{Pr}\left(E \mid T_{2} \& B\right)$ are $T_{1}$ 's and $T_{2}$ 's "likelihoods" (with respect to $E$ ). These are the probabilities that they confer on $E$ given $B$. Third, $\operatorname{Pr}\left(T_{1} \mid E \& B\right)$ and $\operatorname{Pr}\left(T_{2} \mid E\right.$ $\& B$ ) are $T_{1}$ 's and $T_{2}$ 's "posterior probabilities" (with respect to $E$ ). These are their probabilities given $E$ and $B$. They should not be confused with $T_{1}$ 's and $T_{2}$ 's likelihoods (though the terminology, which is standard, can be misleading on that front).

Suppose, to illustrate, that $B$ is the proposition that a card is randomly drawn from a standard (and well-shuffled) deck of cards, and let $R$ be the proposition that the card drawn is a Red, $H$ be the proposition that the card drawn is a Heart, and $J$ be the proposition that the card drawn is the Jack of Hearts. Suppose that $R$ is the evidence, and that $H$ and $J$ are the hypotheses. Then:

$$
\begin{aligned}
& \operatorname{Pr}(H \mid B)=1 / 4>1 / 52=\operatorname{Pr}(J \mid B) \\
& \operatorname{Pr}(R \mid H \& B)=1=\operatorname{Pr}(R \mid J \& B) \\
& \operatorname{Pr}(H \mid R \& B)=1 / 2>1 / 26=\operatorname{Pr}(J \mid R \& B)
\end{aligned}
$$

Here $H$ 's prior probability of $1 / 4$ is greater than $J$ 's prior probability of $1 / 52$, $H$ 's likelihood of 1 is equal to $J$ 's likelihood of 1 , and $H$ 's posterior probability of $1 / 2$ is greater than $J$ 's posterior probability of $1 / 26$.

I noted above that the typical case is where $B$ is not tautological. The card case just given is a case in point, since $B$ includes information about the makeup of the deck and how the card is drawn.
II.2. A Yes/No Question about How to Understand $P A R_{1}$. Is $\mathrm{PAR}_{1}$ to be understood so that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}$ ? This is the yes/no question alluded to above.

There is a natural reading of $\mathrm{PAR}_{1}$ on which the answer is yes. First, suppose that each of $r_{1}, \ldots, r_{n}$ is an explanatory virtue distinct from parsimony. Second, suppose that prior probability and likelihood are explanatory virtues distinct from parsimony. Then it follows that prior probability and likelihood are included in $r_{1}, \ldots, r_{n} .{ }^{17}$

This reading of $\mathrm{PAR}_{1}$ is suggested in part by numerous passages in the literature in which it is claimed in effect (though in different terminology) that prior probability and likelihood are explanatory virtues. Here, first, are some passages in which it is claimed in effect (given the surrounding context) that prior probability is an explanatory virtue:
(A) ... fit with background is also an inferential factor that has an explanatory aspect. One reason this is so is because background beliefs may include beliefs about what sorts of accounts are genuinely explanatory. For example, at given stages of science no appeal to action at a distance or to an irreducibly chance mechanism could count as an adequate explanation, whatever its empirical adequacy. The role of background belief in determining the quality of an explanation shows how explanatory virtue is 'contextual', since the

[^4]same hypothesis may provide a lovely explanation in one theoretical milieu but not be explanatory in another .... ${ }^{18}$
(B) $\quad \mathrm{H}$ will be preferred to $\mathrm{H}^{\prime}$ if H fits better with what we already believe. If this sounds dogmatic or pigheaded, notice again that, inescapably, we never even consider competing hypotheses that would strike us as grossly implausible; the detective would never so much as entertain the hypothesis that the crime was committed by invisible Venusian invaders, nor the mechanic that your car trouble is caused by an infusion of black bile or evil fairy dust. Nor should we consider such hypotheses, even if we could enumerate them all; someone who insisted on doing so would be rightly accused of wasting everyone's time. All inquiry is conducted against a background of existing beliefs, and we have no choice but to rely on some of them while modifying or abandoning others-else how could any such revisions be motivated? ${ }^{19}$

Here, second, are some passages in which it is claimed in effect (given the surrounding context) that likelihood is an explanatory virtue:
(C) On the a posteriori side [of the divide between a priori criteria and a posteriori criteria for determining the best potential explanation] there is, first, the criterion of yielding the data, that is, leading us to expect the events to be explained-either with deductive certainty or with inductive probability. The more data and the more probable some hypothesis renders their occurrence, the more likely it is that-that is, the more probable it is that-that hypothesis is true. ${ }^{20}$
(D) In the best case, $T$ explains $E$ by entailing E. More typically, $T$ must be conjoined with auxiliary hypotheses to entail E. Those auxiliary hypotheses must be evaluated in turn; they may be independently plausible, or the abductive assessment may have to be of their conjunction with T. Obviously the auxiliary hypotheses should not entail E by themselves, otherwise T would be redundant. In still other cases, the connection is irremediably probabilistic: E is probable conditional on T , perhaps in conjunction with auxiliary hypotheses, which as before must themselves be evaluated, and

[^5]should not render T redundant. At a bare minimum, T must be consistent with E. In brief, the closer $T$ comes to entailing $E$, the better (ceteris paribus). ${ }^{21}$

It might be that the data in question in passage (C) go beyond $E$ and include data given in $B$. But that would be okay for my purposes. It would still be the case that at least part of the claim is that whether, given $B, T_{1}$ is a better potential explanation of $E$ than is $T_{2}$ is determined at least in part by $T_{1}$ 's and $T_{2}$ 's likelihoods.

However, $I$ do not want to insist on this way of understanding $\mathrm{PAR}_{1}$. I do not even want to insist on understanding it so that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}$. I aim to show that $\mathrm{PAR}_{1}$ is false regardless of whether it is understood so that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}$.
II.3. PAR $_{1}$ Understood so that Prior Probability and Likelihood are Included in $r_{l}, \ldots, r_{n}$. It is a theorem of the probability calculus that:

$$
\begin{equation*}
\frac{\operatorname{Pr}\left(T_{1} \mid E \& B\right)}{\operatorname{Pr}\left(T_{2} \mid E \& B\right)}=\frac{\operatorname{Pr}\left(T_{1} \mid B\right)}{\operatorname{Pr}\left(T_{2} \mid B\right)} \times \frac{\operatorname{Pr}\left(E \mid T_{1} \& B\right)}{\operatorname{Pr}\left(E \mid T_{2} \& B\right)} \tag{1}
\end{equation*}
$$

This means that the ratio of $T_{1}$ 's posterior probability to $T_{2}$ 's posterior probability equals the product of (a) the ratio of $T_{1}$ 's prior probability to $T_{2}$ 's prior probability and (b) the ratio of $T_{1}$ 's likelihood to $T_{2}$ 's likelihood. ${ }^{22}$

Suppose that $\mathrm{PAR}_{1}$ is understood so that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}$. Take some case where $\mathrm{PAR}_{1}$ 's antecedent is true. It follows that $T_{1}$ is equal to $T_{2}$ both in prior probability and in likelihood:

$$
\begin{align*}
& \operatorname{Pr}\left(T_{1} \mid B\right)=\operatorname{Pr}\left(T_{2} \mid B\right)  \tag{2}\\
& \operatorname{Pr}\left(E \mid T_{1} \& B\right)=\operatorname{Pr}\left(E \mid T_{2} \& B\right)
\end{align*}
$$

But then by (1) it follows that, contra $\mathrm{PAR}_{1}$ 's consequent, $T_{1}$ is equal to $T_{2}$ in posterior probability:

$$
\begin{equation*}
\operatorname{Pr}\left(T_{1} \mid E \& B\right)=\operatorname{Pr}\left(T_{2} \mid E \& B\right) \tag{4}
\end{equation*}
$$

This means that $\mathrm{PAR}_{1}$ is false if it is understood so that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}$.

[^6]This is true whether or not $\mathrm{PAR}_{1}$ is understood so that each of $r_{1}, \ldots, r_{n}$ is an explanatory virtue distinct from parsimony, and whether or not it is understood so that prior probability and likelihood are explanatory virtues distinct from parsimony. If, for whatever reason, $\mathrm{PAR}_{1}$ is understood so that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}$, then $P A R_{1}$ is false.

It is not just that if $\mathrm{PAR}_{1}$ is understood so that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}$, then $\mathrm{PAR}_{1}$ is open to counterexample. If $\mathrm{PAR}_{1}$ is thus understood, then all cases where its antecedent is true are cases where its consequent is false.
II.4. $\mathrm{PAR}_{1}$ Understood so that it is Not the Case that Prior Probability and Likelihood are Included in $r_{1}, \ldots, r_{n}$. Suppose that $\mathrm{PAR}_{1}$ is understood so that it is not the case that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}$. Suppose, that is, that it is understood so that (a) prior probability is not included in $r_{1}, \ldots, r_{n}$ or (b) likelihood is not included in $r_{1}$, $\ldots, r_{n}$ (where "or" here is inclusive). What then?

It will help to consider an example. ${ }^{23}$ Suppose that you know of Species A and Species B , but you are ignorant as to whether the members of either species have wings. Suppose further that you know that Species A and Species B are related as depicted in the graph below:


Species C is the "parent" of Species A and Species B (which are the "children" of Species C). Species D, in turn, is the parent of Species C (which is the child of Species D). You consider two theories. $T_{1}$ is the theory that there is exactly one branch on which there was an evolutionary change (from a parent with no wings to a child with wings, or from a parent with wings to a child with no wings). $T_{2}$ is the theory that there are exactly two branches on

[^7]which there was an evolutionary change. There are fewer branches on which there was an evolutionary change on $T_{1}$ than on $T_{2}$. Given this, and supposing that the branches in question (understood as branches on which there was an evolutionary change) are all new relative to $B$, it follows by $\left({ }^{* *}\right)$ that $T_{1}$ is more parsimonious in the sense of $\mathrm{P}_{1}$ than $T_{2}{ }^{24}$

Let $C_{i}$ be the proposition that there was an evolutionary change on branch $i$ (for any $i=$ $1,2,3)$. There are three ways for $T_{1}$ to be true, and three ways for $T_{2}$ to be true. The former are:
$w_{1}$

$$
C_{1} \& \sim C_{2} \& \sim C_{3}
$$

$$
w_{2} \quad \sim C_{1} \& C_{2} \& \sim C_{3}
$$

$$
w_{3} \quad \sim C_{1} \& \sim C_{2} \& C_{3}
$$

The latter are:

| $w_{4}$ | $C_{1} \& C_{2} \& \sim C_{3}$ |  |
| :--- | :--- | :--- |
| $w_{5}$ |  | $C_{1} \& \sim C_{2} \& C_{3}$ |
| $w_{6}$ |  | $\sim C_{1} \& C_{2} \& C_{3}$ |

Suppose that $B$ includes the information that:
(5) The probability of an evolutionary change on a given branch is 0.5 regardless of whether there were evolutionary changes on other branches, and regardless of whether the members of the parent species (if there is a parent species) had wings.

It follows that $T_{1}$ is equal to $T_{2}$ in prior probability:

$$
\begin{align*}
& \operatorname{Pr}\left(T_{1} \mid B\right)=\operatorname{Pr}\left(w_{1} \mid B\right)+\operatorname{Pr}\left(w_{2} \mid B\right)+\operatorname{Pr}\left(w_{3} \mid B\right)  \tag{6}\\
& \quad=(3) \times(0.5)(0.5)(0.5) \\
& \quad=0.375
\end{align*}
$$

[^8]\[

$$
\begin{align*}
& \operatorname{Pr}\left(T_{2} \mid B\right)=\operatorname{Pr}\left(w_{4} \mid B\right)+\operatorname{Pr}\left(w_{5} \mid B\right)+\operatorname{Pr}\left(w_{6} \mid B\right)  \tag{7}\\
& \quad=(3) \times(0.5)(0.5)(0.5) \\
& \quad=0.375
\end{align*}
$$
\]

Now let $E$ be the proposition that the members of Species A and the members of Species B have wings, and suppose that $B$ also includes the information that:
(8) The members of Species D did not have wings.

It follows that $T_{1}$ is equal to $T_{2}$ in likelihood:

$$
\begin{align*}
& \operatorname{Pr}\left(E \mid T_{1} \& B\right)=\operatorname{Pr}\left(w_{1} \mid T_{1} \& B\right)  \tag{9}\\
& \quad=(0.5)(0.5)(0.5) \\
& \quad=0.125
\end{align*}
$$

$$
\begin{align*}
& \operatorname{Pr}\left(E \mid T_{2} \& B\right)=\operatorname{Pr}\left(w_{6} \mid T_{2} \& B\right)  \tag{10}\\
& \quad=(0.5)(0.5)(0.5) \\
& \quad=0.125
\end{align*}
$$

This is a case, then, where (i) $T_{1}$ is more parsimonious in the sense of $\mathrm{P}_{1}$ than $T_{2}$ and (ii) $T_{1}$ is equal to $T_{2}$ both in prior probability and in likelihood.

It is important to understand why there can be cases like this. The key is $\left({ }^{* *)}\right.$ from Section I (which I repeat below for convenience):
(**): Take any way of understanding parsimony $\mathrm{P}_{\mathrm{i}}$ on which (*) holds. Let " $\mathrm{P}_{\mathrm{i}}$-things" be the things to be counted when it comes to parsimony on $\mathrm{P}_{\mathrm{i}}$. If (a) there are fewer $\mathrm{P}_{\mathrm{i}}$ things on/in $T_{1}$ than on/in $T_{2}$ and (b) the things in question are all new relative to $B$, then $T_{1}$ is more parsimonious than $T_{2}$ regardless of what information, consistent with (a) and (b), is included in $B$, and regardless of how $E$ is understood.

Since (a) there are fewer branches on which there was an evolutionary change on $T_{1}$ than on $T_{2}$ and (b) the branches in question are all new relative to $B$, it follows by $\left({ }^{* *}\right)$ that $T_{1}$ is more parsimonious in the sense of $\mathrm{P}_{1}$ than $T_{2}$ regardless of what information, consistent with (a) and (b), is included in $B$, and regardless of how $E$ is understood. Given this, and given that (5) and (8) are consistent with (a) and (b), it follows that there can be cases where (a) and (b) hold, (5) and (8) are included in $B$, and $E$ is understood as the proposition that the members of Species A and the members of Species B have wings. This means, though, that
there can be case where (i) $T_{1}$ is more parsimonious in the sense of $\mathrm{P}_{1}$ than $T_{2}$ and (ii) $T_{1}$ is equal to $T_{2}$ both in prior probability and in likelihood.

This generalizes to any other way of understanding parsimony $\mathrm{P}_{\mathrm{i}}$ on which $\left({ }^{*}\right)$ holds. Suppose that (a) there are fewer $\mathrm{P}_{\mathrm{i}}$-things on $/$ in $T_{1}$ than on $/$ in $T_{2}$ and (b) the things in question are all new relative to $B$. It follows by $\left({ }^{(* *)}\right.$ that $B$ can be further specified and $E$ can be understood in such a way that (i) $T_{1}$ is more parsimonious in the sense of $\mathrm{P}_{\mathrm{i}}$ than $T_{2}$ and (ii) $T_{1}$ is equal to $T_{2}$ both in prior probability and in likelihood.

This is problematic for $\mathrm{PAR}_{1}$ understood so that it is not the case that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}$. For, since, surely, at least some of the cases in question-cases where (i) $T_{1}$ is more parsimonious in the sense of $\mathrm{P}_{\mathrm{i}}$ than $T_{2}$ and (ii) $T_{1}$ is equal to $T_{2}$ both in prior probability and in likelihood-are cases where $T_{1}$ is strictly equal to $T_{2}$ in $r_{1}, \ldots, r_{n}$ taken individually, it follows that there are cases where $\mathrm{PAR}_{1}$ 's antecedent is true but its consequent is false because $T_{1}$ is equal to $T_{2}$ in posterior probability.

I am assuming here that any proposed specification of $r_{1}, \ldots, r_{n}$ should meet the condition that there are cases where (i) $T_{1}$ is more parsimonious than $T_{2}$ and (ii) $T_{1}$ is strictly equal to $T_{2}$ in $r_{1}, \ldots, r_{n}$ taken individually. For, if $r_{1}, \ldots, r_{n}$ were specified so that there are no such cases, then this would trivialize $\mathrm{PAR}_{1}$ (by making its antecedent inconsistent). ${ }^{25}$

The result, then, is that $\mathrm{PAR}_{1}$ is false even if it is understood so that it is not the case that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}$.
II.5. Summary. In Section II.2, I raised the question of whether $\mathrm{PAR}_{1}$ is to be understood so that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}$. In Section II.3, I argued that if the answer is yes, then $\mathrm{PAR}_{1}$ is false, because then its antecedent requires that $T_{1}$ be equal to $T_{2}$ both in prior probability and in likelihood, and thus, by (1), its antecedent requires that

[^9]$T_{1}$ be equal to $T_{2}$ in posterior probability. In Section II.4, I argued that if, instead, the answer is no, then, still, $\mathrm{PAR}_{1}$ is false, since then its antecedent at least allows that $T_{1}$ be equal to $T_{2}$ both in prior probability and in likelihood, and thus, by (1), its antecedent at least allows that $T_{1}$ be equal to $T_{2}$ in posterior probability. I conclude that $\mathrm{PAR}_{1}$ is false. ${ }^{26}$

## III. DISCUSSION

This section is divided into three subsections. In Section III.1, I consider how, if not by appeal to $\mathrm{PAR}_{1}$, it can be shown that the more parsimonious theory in a given case has the higher posterior probability. In Section III.2, I turn to the idea that central to the aim of science is parsimony understood in terms of the number of brute phenomena on our overall world picture. In Section III.3, I suggest a potential way forward for friends of IBE.
III.1. Higher Posterior Probabilities. $\mathrm{PAR}_{1}$ is false (and so are parsimony theses such as $\mathrm{PAR}_{2}$ and $\mathrm{PAR}_{3}$ ). What now? How, if not by appeal to $\mathrm{PAR}_{1}$, is it to be shown that the more parsimonious theory in a given case has the higher posterior probability?

It follows from (1) in Section II. 3 that each of the following holds without exception:

$$
\begin{align*}
& \text { If (a) } \operatorname{Pr}\left(T_{1} \mid B\right)>p\left(T_{2} \mid B\right) \text { and (b) } \operatorname{Pr}\left(E \mid T_{1} \& B\right) \geq p\left(E \mid T_{2} \& B\right) \text {, then } \operatorname{Pr}\left(T_{1} \mid\right.  \tag{11}\\
& E \& B)>\operatorname{Pr}\left(T_{2} \mid E \& B\right) \text {. }
\end{align*}
$$

If (a) $\operatorname{Pr}\left(T_{1} \mid B\right) \geq \operatorname{Pr}\left(T_{2} \mid B\right)$ and (b) $\operatorname{Pr}\left(E \mid T_{1} \& B\right)>p\left(E \mid T_{2} \& B\right)$, then $\operatorname{Pr}\left(T_{1} \mid\right.$ $E \& B)>\operatorname{Pr}\left(T_{2} \mid E \& B\right)$.

Each of these theses provides a way to show that the more parsimonious theory in a given case has the higher posterior probability. ${ }^{27}$

[^10]Return to the case of Species A and Species B from Section II.4. Suppose that (5) is replaced by:
(13) The probability of an evolutionary change on a given branch is 0.1 regardless of whether there were evolutionary changes on other branches, and regardless of whether the members of the parent species (if there is a parent species) had wings.

It follows that $T_{1}$ 's prior probability is higher than $T_{2}$ 's prior probability:

$$
\begin{align*}
& \operatorname{Pr}\left(T_{1} \mid B\right)=\operatorname{Pr}\left(w_{1} \mid B\right)+\operatorname{Pr}\left(w_{2} \mid B\right)+\operatorname{Pr}\left(w_{3} \mid B\right)  \tag{14}\\
& \quad=(3) \times(0.1)(0.9)(0.9) \\
& \quad=0.243
\end{align*}
$$

$$
\begin{align*}
& \operatorname{Pr}\left(T_{2} \mid B\right)=\operatorname{Pr}\left(w_{4} \mid B\right)+\operatorname{Pr}\left(w_{5} \mid B\right)+\operatorname{Pr}\left(w_{6} \mid B\right)  \tag{15}\\
& \quad=(3) \times(0.1)(0.1)(0.9) \\
& \quad=0.027
\end{align*}
$$

It also follows that $T_{1}$ 's likelihood is higher than $T_{2}$ 's likelihood:

$$
\begin{align*}
& \operatorname{Pr}\left(E \mid T_{1} \& B\right)=\operatorname{Pr}\left(w_{1} \mid T_{1} \& B\right)  \tag{16}\\
& \quad=(0.1)(0.9)(0.9) \\
& \quad=0.081 \\
& \operatorname{Pr}\left(E \mid T_{2} \& B\right)=\operatorname{Pr}\left(w_{6} \mid T_{2} \& B\right)  \tag{17}\\
& \quad=(0.1)(0.1)(0.9) \\
& \quad=0.009
\end{align*}
$$

Hence both by (11) and by (12) it follows that $T_{1}$ 's posterior probability is higher than $T_{2}$ 's posterior probability.

I am not claiming that the only way to show that the more parsimonious theory in a given case has the higher posterior probability is by appeal to (11) or (12). Suppose, for
$\left.T_{1} \& B\right)=1=\operatorname{Pr}\left(E \mid T_{2} \& B\right)$. It can be put as follows: (a) $T_{1}$ entails $E$, (b) $T_{2}$ is a conjunction $T_{3} \& T_{4}$, (c) $T_{2}$ entails $E$ but $T_{3}$ does not, (d) $\operatorname{Pr}\left(T_{1} \mid B\right) \geq \operatorname{Pr}\left(T_{3} \mid B\right)$, and (e) $\operatorname{Pr}\left(T_{4} \mid T_{3} \& B\right)<$ 1. They also consider three scientific cases, and argue that their condition holds in each of them.
example, that you grant for the sake of argument that $T_{1}$ 's prior probability is lower than $T_{2}$ 's prior probability, and yet you show that $T_{1}$ 's prior probability is nonetheless roughly equal to $T_{2}$ in prior probability, and that $T_{1}$ 's likelihood is much higher than $T_{2}$ 's likelihood:

$$
\begin{align*}
& \frac{\operatorname{Pr}\left(T_{1} \mid B\right)}{\operatorname{Pr}\left(T_{2} \mid B\right)} \approx 1  \tag{18}\\
& \frac{\operatorname{Pr}\left(E \mid T_{1} \& B\right)}{\operatorname{Pr}\left(E \mid T_{2} \& B\right)} \gg 1 \tag{19}
\end{align*}
$$

Then, by (1), you thereby show that $T_{1}$ 's posterior probability is higher than $T_{2}$ 's posterior probability.

There are two main lessons here. First, it is true that oftentimes the more parsimonious theory has the higher posterior probability, but this is not because of parsimony in itself. It is because of the background information on hand, and the evidence to be explained. Second, and relatedly, if you want to show that the more parsimonious theory in a given case has the higher posterior probability, then you should look not to $\mathrm{PAR}_{1}$, but instead to theses like (11) and (12).

The idea that background information is all-important in the context of parsimony is not new. Elliott Sober stresses and develops it in numerous writings. ${ }^{28}$ But it has yet to be fully appreciated. It is high time for a change.
III.2. The Aim of Science. Albert Einstein holds that parsimony of a certain kind is central to the aim of science. ${ }^{29}$ Here he states the view:

The aim of science is, on the one hand, a comprehension, as complete as possible, of the connection between the sense experiences in their totality, and, on the other hand, the accomplishment of this aim by the use of a minimum of primary concepts and relations. ${ }^{30}$

Here he elaborates:

[^11]Science uses the totality of the primary concepts, i.e., concepts directly connected with sense experiences, and propositions connecting them. In its first stage of development, science does not contain anything else. Our everyday thinking is satisfied on the whole with this level. Such a state of affairs cannot, however, satisfy a spirit which is really scientifically minded; because the totality of concepts and relations obtained in this manner is utterly lacking in logical unity. In order to supplement this deficiency, one invents a system poorer in concepts and relations, a system retaining the primary concepts and relations of the "first layer" as logically derived concepts and relations. This new "secondary system" pays for its higher logical unity by having elementary concepts (concepts of the second layer), which are no longer directly connected with complexes of sense experiences. Further striving for logical unity brings us to a tertiary system, still poorer in concepts and relations, for the deduction of the concepts and relations of the secondary (and so indirectly of the primary) layer. Thus the story goes on until we have arrived at a system of the greatest conceivable unity, and of the greatest poverty of concepts of the logical foundations, which is still compatible with the observations made by our senses. ${ }^{31}$

If Einstein is right about the aim of science, then there are at least two questions to be answered. Is there a rationale behind the aim of science? Can the aim of science be reached without appeal to $\mathrm{PAR}_{1}$ ?

I have no definite answer to the first of these questions. But I suspect that many theorists would answer it by appeal to the idea that greater parsimony of the kind at issue makes for greater scientific understanding. This idea is nicely articulated by Michael Friedman. ${ }^{32} \mathrm{He}$ writes:

The kinetic theory effects a significant unification in what we have to accept. Where we once had three independent brute facts-that gases approximately obey the BoyleCharles law, that they obey Graham's law, and that they have the specific-heat capacities they do have-we now have only one-that molecules obey the laws of mechanics. Furthermore, the kinetic theory also allows us to integrate the behavior of gases with other phenomena, such as the motions of the planets and of falling bodies near the earth. This is because the laws of mechanics also permit us to derive both the fact that planets obey Kepler's laws and the fact that falling bodies obey Galileo's laws. From the fact that all bodies obey the laws of mechanics it follows that the planets behave as they do, falling bodies behave as they do, and gases behave as they do. Once again, we have

[^12]reduced a multiplicity of unexplained, independent phenomena to one. I claim that this is the crucial property of scientific theories we are looking for; this is the essence of scientific explanation-science increases our understanding of the world by reducing the total number of independent phenomena that we have to accept as ultimate or given. A world with fewer independent phenomena is, other things equal, more comprehensible than one with more. ${ }^{33}$

Perhaps if Einstein is right about the aim of science, then the rationale behind it is that scientists want a maximum of scientific understanding, and that we gain in scientific understanding by gaining in parsimony understood in terms of the number of brute phenomena on our overall world picture. ${ }^{34}$

Friedman goes on to develop a theory of scientific explanation based on the idea that greater parsimony understood in terms of the number of brute phenomena on our overall world picture makes for greater scientific understanding. It is well known, however, that his theory is problematic in its details. Philip Kitcher shows this. ${ }^{35}$ But the idea on which it is based-the idea that greater parsimony understood in terms of the number of brute phenomena on our overall world picture makes for greater scientific understanding-still has wide appeal. ${ }^{36}$ So I want to grant it for the sake of argument.

There is still the second question above. Can the aim of science thus understood be reached without appeal to $\mathrm{PAR}_{1}$ ?

It would be a mistake to think that the answer is negative. Here is a possibility. The inferred theory at the second layer is less parsimonious than its main rival (which was not inferred), and yet the transition from the first layer to the second constitutes an increase in parsimony because there are less brute phenomena on the theory at the second layer than on the theory at the first. Similarly, the inferred theory at the third layer is less parsimonious

[^13]than its main rival (which was not inferred), and yet the transition from the second layer to the third constitutes an increase in parsimony because there are less brute phenomena on the theory at the third layer than on the theory at the second. And so on. So, though $\mathrm{PAR}_{1}$ is false, it might be that Einstein is right about the aim of science and that the aim of science as he describes it can be reached.

There is more. It also might be that it is permissible and perhaps even rational in some sense for scientists to develop and test theories of greater and greater parsimony understood in terms of the number of brute phenomena on our overall world picture. But they could do that without any appeal to $\mathrm{PAR}_{1}$.
III.3. IBE. I have assumed a broadly Bayesian epistemology on which (suppressing reference to background information) each of the following holds:
$N_{S}$ : For any time $t, S$ 's credences at $t$ should be probabilistically coherent (that is, in accord with the probability calculus).
$N_{D}$ : For any times $t_{1}$ and $t_{2}$ such that $t_{1}$ is before $t_{2}$, if (i) $S$ 's conditional credence at $t_{1}$ in $T$ given $E$ is $c$ and (ii) $S$ learns $E$ (and nothing stronger) between $t_{1}$ and $t_{2}$, then $S$ 's unconditional credence at $t_{2}$ in $T$ should be $c$.
$\mathrm{N}_{\mathrm{S}}$ is a synchronic norm (thus the subscript in " $\mathrm{N}_{\mathrm{S}}$ "). It implies, for example, that $S$ should never have a higher credence in a conjunction than in its conjuncts taken individually. For, by the probability calculus, for any propositions $P$ and $Q, \operatorname{Pr}(P \& Q) \leq \operatorname{Pr}(P)$ and $\operatorname{Pr}(P \& Q)$ $\leq \operatorname{Pr}(Q) . \mathrm{N}_{\mathrm{D}}$, in contrast, which is oftentimes called "Strict Conditionalization," is a diachronic norm (thus the subscript in " $\mathrm{N}_{\mathrm{D}}$ "). ${ }^{37}$ Return to the card case from Section II.1. Before the card is drawn, your conditional credence in $H$ given $R$ is $1 / 2$. Suppose that the card is drawn, and that, due to the dealer's sloppiness, you get a glimpse of the card and learn that $R$ (and nothing stronger). Then by $\mathrm{N}_{\mathrm{D}}$ you should come to have an unconditional credence in $H$ of $1 / 2{ }^{38}$

There is much to like in $\mathrm{N}_{\mathrm{S}}, \mathrm{N}_{\mathrm{D}}$, and Bayesian epistemology more generally. It might seem, though, that the proper reaction to my argument against $\mathrm{PAR}_{1}$ and IBE is not to reject

[^14]PAR ${ }_{1}$ and IBE, but to reject $\mathrm{N}_{S}$ or $\mathrm{N}_{\mathrm{D}}$. Consider, for example, the following alternative to $\mathrm{N}_{\mathrm{D}}$ :
$I B E^{* *}$ : For any times $t_{1}$ and $t_{2}$ such that $t_{1}$ is before $t_{2}$, if (i) $S$ 's conditional credence at $t_{1}$ in $T_{1}$ given $E$ is $c$, (ii) $S$ 's conditional credence at $t_{1}$ in $T_{2}$ given $E$ is $c^{*}$, (iii) $S$ learns that $E$ (and nothing stronger) between $t_{1}$ and $t_{2}$, and (iv) $T_{1}$ is a better potential explanation of $E$ than is $T_{2}$, then $S$ 's unconditional credence at $t_{2}$ in $T_{1}$ should be $c+b$, and $S$ 's unconditional credence at $t_{2}$ in $T_{2}$ should be $c^{*}-b^{*} .{ }^{39}$

Here $b$ is a bonus for being a better potential explanation of $E$, and $b^{*}$ is a penalty for being a worse potential explanation of $E$. IBE** is underspecified in several respects. But it will suffice for my purposes. Take some case where $\mathrm{PAR}_{1}$ 's antecedent holds but its consequent does not because $T_{1}$ 's posterior probability is equal to $T_{2}$ 's posterior probability. If friends of IBE rejected $\mathrm{N}_{\mathrm{D}}$ in favor of IBE**, then they could claim that $S$ 's unconditional credence in $T_{1}$ upon learning $E$ should be greater than her unconditional credence in $T_{2}$ upon learning $E$.

I see little plausibility in IBE**. There is simply too much to be said in favor of $\mathrm{N}_{\mathrm{S}}, \mathrm{N}_{\mathrm{D}}$, and Bayesian epistemology more generally, and too much to be said against IBE**. ${ }^{40}$ This is not the place, however, for a general defense of $\mathrm{N}_{\mathrm{S}}, \mathrm{N}_{\mathrm{D}}$, and Bayesian epistemology more generally. I shall instead suggest an alternative way forward for friends of IBE.

I claimed above in the introduction that IBE implies $\mathrm{PAR}_{3}$. I should have claimed, to be more precise, that if, as is standard, IBE is understood so that parsimony is an explanatory virtue, then IBE implies $\mathrm{PAR}_{3}$. It is important to note, however, that there is no necessity in understanding IBE so that parsimony is an explanatory virtue. It can be understood in terms of any proffered list of explanatory virtues. So the fact that $\mathrm{PAR}_{3}$ is false tells not against IBE per se, but against IBE understood so that parsimony is an explanatory virtue.

[^15]It would not be enough for friends of IBE to simply excise parsimony from their list of explanatory virtues. They would need to ensure that their list of explanatory virtues is such that there can be no cases where, given $B, T_{1}$ is a better potential explanation of $E$ than is $T_{2}$, and yet $T_{1}$ is equal to $T_{2}$ both in prior probability and in likelihood. For, otherwise IBE would still be open to counterexample. Is there a way for them to do that?

I have a proposal. It is not new, but it is underdeveloped at this point. The rough idea is to embrace $\mathrm{N}_{\mathrm{S}}, \mathrm{N}_{\mathrm{D}}$, and Bayesian epistemology more generally, and then find a place in Bayesianism for the explanatory virtues. ${ }^{41}$ One way of developing this idea is this. First, designate prior probability and likelihood as "primary" explanatory virtues. Second,

[^16]designate any additional explanatory virtues as "secondary" explanatory virtues. Third, show that the various secondary explanatory virtues can help in determining prior probability and likelihood (the primary explanatory virtues). If IBE were thus reformulated, then no case where $T_{1}$ is equal to $T_{2}$ both in prior probability and in likelihood would be a case where, given $B, T_{1}$ is a better potential explanation of $E$ than is $T_{2}$.

## IV. CONCLUSION

If parsimony (understood in terms of the number of things on/in a theory) is a theoretical virtue, then this is not because $\mathrm{PAR}_{1}$ is true. For, regardless of whether $\mathrm{PAR}_{1}$ is to be understood so that prior probability and likelihood are included in $r_{1}, \ldots, r_{n}, \mathrm{PAR}_{1}$ is false. It follows that the same is true of $\mathrm{PAR}_{2}, \mathrm{PAR}_{3}$, and any other parsimony thesis that is logically stronger than $\mathrm{PAR}_{1}$. It further follows that the same is true of IBE understood so that parsimony is an explanatory virtue. This is the negative side of the story. However, there is also a positive side. First, there are still ways to show that the more parsimonious theory in a given case has the higher posterior probability. Second, it might be that Einstein is right that central to the aim of science is parsimony understood in terms of the number of brute phenomena on our overall world picture, and it might be that the aim of science as he describes it can be reached. Third, it might be that there are ways of understanding IBE on which it is not open to counterexample.

WILLIAM ROCHE

Texas Christian University


[^0]:    * Thanks to Wesley Cray, William Melanson, Michael Roche, Tomoji Shogenji, Elliott Sober, two anonymous referees for this JOURNAL, and the editors for this JOURNAL for helpful comments/discussion.
    ${ }^{1}$ I shall return to the former question in Section I and the latter question in Section II.

[^1]:    ${ }^{2}$ Some theorists distinguish between belief and acceptance, and hold that a subject can accept a given hypothesis without believing it. See, for example, Kevin Elliott and David Willmes, "Cognitive Attitudes and Values in Science," Philosophy of Science, LXXX (2013): 807-17. I leave it open that they are right. I also leave it open that there can be cases where $\operatorname{Pr}\left(T_{1} \mid E \& B\right) \leq \operatorname{Pr}\left(T_{2} \mid E \& B\right)$ but it is rational to accept $T_{1}$ (but not believe it) and reject $T_{2}$ (though not disbelieve it) given $E$ and $B$.
    ${ }^{3}$ Different theorists give different lists of explanatory virtues. See, for example, James Beebe, "The Abductivist Reply to Skepticism," Philosophy and Phenomenological Research, LXXIX (2009): 605-36; Igor Douven, "Abduction," in Edward Zalta, ed., The Stanford Encyclopedia of Philosophy (Summer 2017), URL = [http://plato.stanford.edu/archives/sum2017/entries/abduction/](http://plato.stanford.edu/archives/sum2017/entries/abduction/), section 2; Gilbert Harman, "The Inference to the Best Explanation," Philosophical Review, LXXIV (1965): 88-95; Peter Kosso, Reading the Book of Nature: An Introduction to the Philosophy of Science (Cambridge: Cambridge University Press, 1992), chapter 2; Thomas Kuhn, The Essential Tension: Selected Essays in Scientific Tradition and Change (Chicago: University of Chicago Press, 1977), chapter 13; Peter Lipton, Inference to the Best Explanation, 2nd ed. (London: Routledge, 2004), chapters 7 and 8; William Lycan, "Explanation and Epistemology," in Paul Moser, ed., The Oxford Handbook of Epistemology (Oxford: Oxford University Press, 2002), pp. 408-33, section 3; Kevin McCain, Evidentialism and Epistemic Justification (New York: Routledge, 2014), chapter 6; Ernan McMullin, "The Virtues of Good Theories," in Stathis Psillos and Martin Curd, eds., The Routledge Companion to Philosophy of Science (London: Routledge, 2008), pp. 498-508; Ted Poston, Reason and Explanation: A Defense of Explanatory Coherentism (New York: Palgrave Macmillan, 2014), chapters 2 and 4; Stathis Psillos, "Abduction: Between Conceptual Richness and Computational Complexity," in Peter Flach and Antonis Kakas, eds., Abduction and Induction: Essays on Their Relation and Integration (Dordrecht: Kluwer, 2000), pp. 59-74;

[^2]:    ${ }^{9}$ See, for example, Nick Chater and Paul Vitányi, "Simplicity: A Unifying Principle in Cognitive Science?," Trends in Cognitive Sciences, vii (2003): 19-22.
    ${ }^{10}$ See, for example, Malcolm Forster and Elliott Sober, "How to Tell When Simpler, More Unified, or Less Ad Hoc Theories will Provide More Accurate Predictions," British Journal for the Philosophy of Science, XLV (1994): 1-35.
    ${ }^{11}$ See, for example, Kevin Kelly, "A New Solution to the Puzzle of Simplicity," Philosophy of Science, LXxIV (2007): 561-73.
    ${ }^{12}$ The extant literature on parsimony is vast to say the least. For a helpful overview, and for references, see Alan Baker, "Simplicity," in Edward Zalta, ed., Stanford Encyclopedia of Philosophy (Winter 2016 ed.), URL =
    [https://plato.stanford.edu/archives/win2016/entries/simplicity/](https://plato.stanford.edu/archives/win2016/entries/simplicity/).
    ${ }^{13}$ Since $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ concern tokens as opposed to types, each is a kind of "quantitative" parsimony as opposed to "qualitative" parsimony. But given that $\mathrm{P}_{1}$ concerns entities whereas $\mathrm{P}_{2}$ concerns concepts, the former is a kind of "ontological" parsimony whereas the latter is a kind of "conceptual" or "ideological" parsimony.
    ${ }^{14}$ Is there a non-arbitrary way of specifying the entity/concept types on/in a given theory? There are some difficult issues here. For discussion, see Alan Baker, "Occam's

[^3]:    Metaphysics, Vol. 8 (Oxford: Oxford University Press, 2013), 237-93; Jeroen Smid, "'Identity’ as a Mereological Term," Synthese, CXCIV (2017): 2367-85; Jonathan Tallant, "Quantitative Parsimony and the Metaphysics of Time: Motivating Presentism," Philosophy and Phenomenological Research, LXXXVII (2013): 688-705; William Vanderburgh, "Quantitative Parsimony, Explanatory Power and Dark Matter," Journal for General Philosophy of Science, XLV (2014): 317-27.

[^4]:    ${ }^{17}$ There are probabilistic measures of "explanatory power" on which $T_{1}$ is equal to $T_{2}$ in explanatory power with respect to $E$ given $B$ if $\operatorname{Pr}\left(E \mid T_{1} \& B\right)=\operatorname{Pr}\left(E \mid T_{2} \& B\right)$ and $\operatorname{Pr}\left(T_{1} \mid B\right)$ $\neq \operatorname{Pr}\left(T_{2} \mid B\right)$. See Vincenzo Crupi and Katya Tentori, "A Second Look at the Logic of Explanatory Power (with Two Novel Representation Theorems)," Philosophy of Science, LXXIX (2012): 365-85; Jonah Schupbach, "Comparing Probabilistic Measures of Explanatory Power," Philosophy of Science, LXXVIII (2011): 813-29; Jonah Schupbach and Jan Sprenger, "The Logic of Explanatory Power," Philosophy of Science, LXXVIII (2011): 105-27. If explanatory virtue is a matter of explanatory power thus understood, then prior probability is not an explanatory virtue. But there is a wider sense of "explanatory virtue" on which an explanatory virtue is simply a "theoretical virtue." This is the sense at issue here (in the reading of $\mathrm{PAR}_{1}$ under consideration) and throughout.

[^5]:    ${ }^{18}$ Lipton, Inference to the Best Explanation, op. cit., pp. 122-23, emphasis added.
    ${ }^{19}$ Lycan, "Explanation and Epistemology," op. cit., p. 416, emphasis added.
    ${ }^{20}$ Swinburne, Simplicity as Evidence of Truth, op. cit., p. 18, emphasis added.

[^6]:    ${ }^{21}$ Williamson, "Abductive Philosophy," op. cit., p. 266, emphasis added.
    ${ }^{22}(1)$ is a simple consequence of Bayes's theorem.

[^7]:    ${ }^{23}$ This example is adapted from Elliott Sober's "Let's Razor Ockham's Razor," in Dudley Knowles, ed., Explanation and Its Limits (Cambridge: Cambridge University Press, 1990), section 3.

[^8]:    ${ }^{24}$ I mean for the expression "entity tokens" in $\mathrm{P}_{1}$ to be understood broadly so as to include object tokens, event tokens, fact tokens, process tokens, and property tokens.

[^9]:    ${ }^{25}$ It would not be enough to save $\mathrm{PAR}_{1}$ (on the current way of understanding it) to insist that though there are cases where (i) $T_{1}$ is more parsimonious than $T_{2}$ and (ii) $T_{1}$ is strictly equal to $T_{2}$ in $r_{1}, \ldots, r_{n}$ taken individually, all such cases are cases where $T_{1}$ is not equal to $T_{2}$ in prior probability or likelihood. It would need to be insisted further that all such cases are cases where $T_{1}$ is superior to $T_{2}$ in prior probability or likelihood. For, given (1), if at least some of the cases in question are cases where, say, $T_{1}$ is equal to $T_{2}$ in prior probability and inferior to it in likelihood, then there are cases where $\mathrm{PAR}_{1}$ 's antecedent is true but its consequent is false because $T_{1}$ is inferior to $T_{2}$ in posterior probability. But, given (**), I see absolutely no plausibility in the claim that all cases where (i) $T_{1}$ is more parsimonious than $T_{2}$ and (ii) $T_{1}$ is strictly equal to $T_{2}$ in $r_{1}, \ldots, r_{n}$ taken individually are cases where $T_{1}$ is superior to $T_{2}$ in prior probability or likelihood. If friends of $\mathrm{PAR}_{1}$ believe otherwise, then I urge them to explain themselves. What exactly is included in $r_{1}, \ldots, r_{n}$ in $\mathrm{PAR}_{1}$, and how is it that despite ( ${ }^{(* *}$ ), all cases where (i) $T_{1}$ is more parsimonious than $T_{2}$ and (ii) $T_{1}$ is strictly equal to $T_{2}$ in $r_{1}, \ldots, r_{n}$ taken individually are cases where $T_{1}$ is superior to $T_{2}$ in prior probability or likelihood?

[^10]:    ${ }^{26} \mathrm{PAR}_{1}$ is false regardless of the subject matter. It is not the case, for example, that $\mathrm{PAR}_{1}$ is false in philosophy but true in science. It is false across the board. For relevant discussion, see Baron and Tallant, "Do Not Revise Ockham's Razor Without Necessity," op. cit., section 3.4; Andrew Brenner, "Simplicity as a Criterion of Theory Choice in Metaphysics," Philosophical Studies (forthcoming); Michael Huemer, "When is Parsimony a Virtue?," Philosophical Quarterly, LIX (2009): 216-36; Kriegel, "The Epistemological Challenge of Revisionary Metaphysics," op. cit.; L. A. Paul, "Metaphysics as Modeling: The Handmaiden's Tale," Philosophical Studies, CLX (2012): 1-29; Sober, Ockham's Razors: A User's Manual, op. cit., chapter 5; M. B. Willard, "Against Simplicity," Philosophical Studies, CLXVII (2014): 165-81.
    ${ }^{27}$ Jansson and Tallant, "Quantitative Parsimony: Probably for the Better," op. cit., section 3, give in effect a special case of (11). They give a general condition, where $T_{1}$ is more parsimonious than $T_{2}$ in the sense of $\mathrm{P}_{1}$, under which $\operatorname{Pr}\left(T_{1} \mid B\right)>\operatorname{Pr}\left(T_{2} \mid B\right)$ and $\operatorname{Pr}(E \mid$

[^11]:    ${ }^{28}$ See Elliott Sober, Reconstructing the Past: Parsimony, Evolution, and Inference (Cambridge, MA: MIT Press, 1988); Sober, "Let's Razor Ockham's Razor," op. cit.; Sober, Ockham's Razors: A User's Manual, op. cit. See also Mike Dacey, "The Varieties of Parsimony in Psychology," Mind \& Language, xxxi (2016): 414-37; Anya Plutynski, "Parsimony and the Fisher-Wright Debate," Biology and Philosophy, xx (2005): 697-713.
    ${ }^{29}$ Albert Einstein, Ideas and Opinions (New York: Three Rivers, 1995).
    ${ }^{30}$ Einstein, Ideas and Opinions, op. cit., p. 293, emphasis added.

[^12]:    ${ }^{31}$ Einstein, Ideas and Opinions, op. cit., pp. 293-94, emphasis added.
    ${ }^{32}$ Michael Friedman, "Explanation and Scientific Understanding," Journal of Philosophy, LXXI (1974): 5-19.

[^13]:    ${ }^{33}$ Friedman, "Explanation and Scientific Understanding," op. cit., pp. 14-15, emphasis added.
    ${ }^{34}$ Andrew Brenner, "Mereological Nihilism and Theoretical Unification," Analytic Philosophy, LVI (2015): 318-37 discusses parsimony thus understood. He calls it "theoretical simplicity."
    ${ }^{35}$ Philip Kitcher, "Explanation, Conjunction, and Unification," Journal of Philosophy, LXXIII (1976): 207-12. William Roche and Elliott Sober, "Explanation = Unification? A New Criticism of Friedman's Theory and a Reply to an Old One," Philosophy of Science, LXXXIV (2017): 391-413 argue that Friedman's theory can be modified so that it is immune to Kitcher's objection. They also argue, though, that Friedman's theory is problematic nonetheless.
    ${ }^{36}$ See, for example, Stathis Psillos, Causation and Explanation (Montreal: Mc-GillQueen's University Press, 2002), pp. 271-72.

[^14]:    ${ }^{37}$ Strict Conditionalization stands in contrast to so-called "Jeffrey Conditionalization." See Richard Jeffrey, The Logic of Decision (New York: McGraw-Hill, 1965), chapter 11; Richard Jeffrey, Subjective Probability: The Real Thing (Cambridge: Cambridge University Press, 2004), chapter 3.
    ${ }^{38}$ For further discussion of Bayesian epistemology, see William Talbott, "Bayesian epistemology," in Edward Zalta, ed., The Stanford Encyclopedia of Philosophy (Winter 2016), URL = [https://plato.stanford.edu/archives/win2016/entries/epistemologybayesian/](https://plato.stanford.edu/archives/win2016/entries/epistemologybayesian/).

[^15]:    ${ }^{39}$ This is adapted from Bas van Fraassen, Laws and Symmetry (Oxford: Oxford University Press, 1989), chapter 7.
    ${ }^{40}$ For a recent critical discussion of IBE** (and related theses), see Nevin Climenhaga, "Inference to the Best Explanation made Incoherent," Journal of Philosophy, CXIV (2017): 251-73. I am not claiming that there is nothing to be said in favor of IBE**. See Igor Douven, "Inference to the Best Explanation made Coherent," Philosophy of Science, LXVI (1999): S424-S435; Igor Douven, "Inference to the Best Explanation, Dutch Books, and Inaccuracy Minimisation," Philosophical Quarterly, LXIII (2013): 428-44; Igor Douven and Jonah Schupbach, "The Role of Explanatory Considerations in Updating," Cognition, CxLII (2015): 299-311; Igor Douven and Sylvia Wenmackers, "Inference to the Best Explanation versus Bayes's Rule in a Social Setting," British Journal for the Philosophy of Science, LXVIII (2017): 535-70. The point, to repeat, is just that there's little plausibility in IBE**.

[^16]:    ${ }^{41}$ See Cabrera, "Can There be a Bayesian Explanationism? On the Prospects of a Productive Partnership," op. cit.; Nevin Climenhaga, "How Explanation Guides Confirmation," Philosophy of Science, LXxxIV (2017): 359-68; Climenhaga, "Inference to the Best Explanation made Incoherent," op. cit.; Leah Henderson, "Bayesianism and Inference to the Best Explanation," British Journal for the Philosophy of Science, LXV (2014): 687-715; Michael Huemer, "Explanationist Aid for the Theory of Inductive Logic," British Journal for the Philosophy of Science, Lx (2009): 345-75; Lipton, Inference to the Best Explanation, op. cit., chapter 7; Kevin McCain and Ted Poston, "Why Explanatoriness is Evidentially Relevant," Thought, III (2014): 145-53; Kevin McCain and Ted Poston, "The Evidential Impact of Explanatory Considerations," in Kevin McCain and Ted Poston, eds., Best Explanations: New Essays on Inference to the Best Explanation (Oxford: Oxford University Press, 2018), pp. 121-29; Samir Okasha, "Van Fraassen’s Critique of Inference to the Best Explanation," Studies in the History and Philosophy of Science, XXXI (2000): 691-710; Poston, Reason and Explanation: A Defense of Explanatory Coherentism, op. cit.; Stathis Psillos, "Inference to the Best Explanation and Bayesianism: Comments on Ilkka Niiniluoto's 'Truth-Seeking by Abduction'," in Friedrich Stadler, ed., Induction and deduction in the sciences (Dordrecht: Kluwer, 2004), pp. 83-91; William Roche, "Explanation, Confirmation, and Hempel's Paradox," in Kevin McCain and Ted Poston, eds., Best Explanations: New Essays on Inference to the Best Explanation (Oxford: Oxford University Press, 2018), pp. 219-241; William Roche and Elliott Sober, "Explanatoriness is Evidentially Irrelevant, or Inference to the Best Explanation meets Bayesian Confirmation Theory," Analysis, LXXIII (2013): 659-68; William Roche and Elliott Sober, "Explanatoriness and Evidence: A Reply to McCain and Poston," Thought, III (2014): 193-99; William Roche and Elliott Sober, "Is Explanatoriness a Guide to Confirmation? A Reply to Climenhaga," Journal for General Philosophy of Science, XLVIII (2017): 581-90; Robert Smithson, "The Principle of Indifference and Inductive Scepticism," British Journal for the Philosophy of Science, LXVIII (2017): 253-72; Jonathan Weisberg, "Locating IBE in the Bayesian Framework," Synthese, CLXVII (2009): 125-43.

