Article:

Ismail, K.S., Guadagnini, M. orcid.org/0000-0003-2551-2187 and Pilakoutas, K. orcid.org/0000-0001-6672-7665 (2018) Strut-and-Tie Modeling of Reinforced Concrete Deep Beams. Journal of Structural Engineering, 144 (2). ISSN 0733-9445

https://doi.org/10.1061/(ASCE)ST.1943-541X.0001974

Strut-and-Tie Modelling of RC Deep Beams

Kamaran S. Ismail [1], Maurizio Guadagnini [2], Kypros Pilakoutas [3]

[1] Lecturer, Department of Civil Engineering, Salahaddin University-Erbil, Kirkuk Road, Erbil, Iraq E-mail address: ksismail1@sheffield.ac.uk, ksi312ismail@gmail.com

[2] Senior Lecturer, Department of Civil and Structural Engineering, The University of Sheffield, Sir Frederick Mappin Building, Mappin Street, Sheffield S1 3JD, UK

[3] Professor of Construction Innovation and Director of the Centre for Cement and Concrete, Department of Civil and Structural Engineering, The University of Sheffield, Sir Frederick Mappin Building, Mappin Street, Sheffield S1 3JD, UK

ABSTRACT

Strut-and-tie models are often used for the design of shear critical deep members since they can rationalise the shear transfer within discontinuous or disturbed regions in RC structural elements. Most current codes of practice adopt the strut-and-tie method but provide very little guidance on how to select appropriate strut-and-tie layout and dimensions. Furthermore, the effectiveness factors used to account for the biaxial state of stresses in struts of deep beams are not reliable. This paper reviews the application of strut-and-tie models for the design of RC deep beams and evaluates current formulations of the effectiveness factor. Experimental and numerical studies are used to assess how the effectiveness factor is influenced by different parameters including concrete compressive strength, shear span to depth ratio and shear reinforcement ratio and to arrive at a more reliable strain based effectiveness factor. Various effectiveness factors are examined against an extensive database of experimental results on RC deep beams with and without shear reinforcement. The results show that the proposed effectiveness factor yields the most reliable and accurate predictions and can lead to more economic and safe design guidelines.

INTRODUCTION

RC deep beams where behaviour is predominantly controlled by shear are used in a wide range of structures, such as transfer girders in tall buildings and bridges. It is crucial to predict their capacity accurately as the safety of the entire structure relies on their performance. However, the shear behaviour of RC members is a complex phenomenon, which is influenced by a large number of parameters (Tan and Lu 1999, Collins et al. 2008). This complexity is more pronounced in deep beams as the applied load is transferred mainly through the formation of arching action which causes a highly nonlinear strain distribution in the cross section.

Most codes of practice rely on empirical or semi-empirical equations for design; however, these equations are limited by the extent of the experimental results used for their calibration. Although designing RC deep beams based on these empirical approaches is generally very conservative, they can also lead to very unsafe results (Collins et al. 2008). Collins et al. (2008) examined the accuracy of the shear approaches available in codes of practice such as EC2 and ACI, against and extensive database of RC beams, it was found that shear strength prediction of vast number of the beams are unconservative. There are also unsafe results even after application of the safety factors (Collins et al. 2008). Approaches based on finite element analysis can account for the nonlinearities that describe the behaviour of this type of members, and can lead to good results if an accurate concrete material model is used; however, their implementation is not always practical for design purposes. Thus, design approaches based on the implementation of strut-and-tie mechanistic models have been adopted by modern design codes such as EC2, ACI 318-14 and Model Code 2010 since they appear more rational and relatively simple to apply.

The use of strut-and-tie models (STM) dates back to the pioneering work of Wilhelm Ritter (1899) who tried to explain the contribution of shear reinforcement to the shear strength of beams. Ritter’s truss mechanism was later modified by Morsch Morsch (1902) to better represent the shear behaviour of RC beams. The design of RC members by STM relies on the lower bound theory of plasticity and assumes that both concrete and steel are perfectly plastic materials. As this is not true, there is a need to implement modification factors to adjust both dimension and strength of the strut elements. However, existing guidelines do not provide sufficient information on the effect of all important parameters or the size and strength of the strut elements (Park and Kuchma 2007). This paper aims to develop a unified procedure for using the STM for the design of RC deep beams and predict accurately the size and strength of each element.

STRUT-AND-TIE MODEL

Strut-and-tie models attempt to represent the stress field that develops in the D-regions of concrete elements by approximating the flow of internal compression and tension stresses by means of struts and ties, respectively. The selection of an adequate strut-and-tie model is necessary to capture the strength of RC deep beams with acceptable accuracy. It is commonly accepted that the strut-and-tie mechanism is the basic load transfer mechanism in RC deep beams (Tuchscherer et al. 2014); however, in some cases the truss action mechanism is also thought to contribute to the transfer of the applied load (Bakir and Boduroğlu 2005, Brena and Roy 2009). The type of load transfer mechanism that develops in RC deep beams is mainly controlled by the shear span to depth ratio (a/d) and amount of shear reinforcement. For beams with a/d less than 1.0, the applied load is transferred to the support through the formation of one concrete strut regardless of the amount of shear reinforcement. The adoption of the STM (Figure 1-a) is therefore suitable for the design and analysis of such elements.

Beams with a/d between 1.0 and 2.0 and with shear reinforcement, can develop a combination of both tied-arch and truss action mechanism (Brena and Roy 2009). However, estimating the percentage of load transferred by each of these mechanisms is quite challenging as this varies based on a/d and amount and spacing of shear reinforcement (Brena and Roy 2009). For the sake of simplicity, the adoption of a model based on the development of either a single strut-and-tie (Figure 1-a) or a truss (Figure 1-b) is generally adopted. The ability of these models to capture the real structural behaviour of RC deep beams is assessed in this paper with the aim of developing enhanced design equations.

The current codes of practice do not provide adequate guidance on selecting the size of the elements in the STM. ACI 318-14 provides Eq. 1 and 2 for estimating the width of the inclined strut at the top (W_{ST}) and bottom nodes (W_{SB}) (Figure 1-a), respectively. However, there is no guidance on how to estimate the independent parameters (h_{CS}, h_{Tie} and θ) in these equations. Therefore designers are free to choose the size of the elements in the model; however, this could lead to unsafe or over conservative design solutions (Brown and Bayrak 2008, Collins et al. 2008, Sagaseta and Vollum 2010).

$$W_{ST} = l_{PT} \sin\theta + h_{CS} \cos\theta \tag{1}$$

$$W_{SB} = l_{PB} \sin\theta + h_{Tie} \cos\theta \tag{2}$$

In the current research programme the width of the strut in the top compression zone (h_{cs}) is assumed to be equal to the depth of neutral axis as determined by section analysis (Eq. 3) (Park and Kuchma 2007).

$$h_{CS} = \left(\sqrt{(n\rho)^2 + n\rho} - n\rho \right) d \tag{3}$$

where l_{PT} and l_{PB} are the width of the loading and support plates, and θ is the angle of the strut with respect to the horizontal axis of the beam (Eq. 4).

$$\theta = \tan^{-1}\frac{d - h_{CS}/2}{a} \tag{4}$$

where d is the effective depth and a is the shear span of the beam.

The height of the bottom node (h_{Tie}) is taken as twice the distance from the centre of the main longitudinal reinforcement to the outer tensile face of the beam as shown in (Figure 1-c). The width of the strut at the top (W_{ST}) and bottom (W_{SB}) nodes can be determined by the ACI 318-14 Eq.s 1 and 2 respectively.

In the case of the truss model shown in Figure 1-b, the width of the strut in compression (h_{cs}) and the height of the bottom node (h_{Tie}) remain the same for both diagonals. The intersections of strut, ties and applied loads or support reactions are termed nodes and their capacity is critical when assessing a given STM.

CONCRETE EFFECTIVENESS FACTOR

Node Strength Factor

Nodes are generally named according to the type of interconnected members, i.e. C-C-C (Compression-Compression-Compression), C-C-T (Compression-Compression-Tension) and C-T-T (Compression- Tension -Tension), and their strength is a function of the state of stress they are subjected to. C-C-C nodes are located in well confined regions and their strength can generally exceed the uniaxial strength of concrete, but the latter can be conservatively used for design. In this paper, with the exception of EC2, ACI 318-14 and Model Code 2010, which they provide strength factors for the C-C-C nodes, to assess other strut effectiveness factors available in the literature the uniaxial concrete strength is adopted.

Owing to the existence of tension forces in C-C-T and C-T-T nodes the maximum stress that can be developed in such nodes is generally lower that the uniaxial concrete strength and reduction factors are used to take this into account. Based on the test results of isolated C-C-T and C-T-T nodes, Jirsa et al. (1991) concluded that by using 80% of the uniaxial concrete compressive strength, the prediction of the nodal zone strength is conservative. Unless it is provided, a reduction factor of 0.8 is used to determine the strength of all C-C-T and C-T-T nodes in the assessment of STM with different strut effectiveness factor.

Effectiveness Factor for Inclined Strut

The presence of a transverse tensile field within the shear span weakens the resistance of the concrete struts. This is taken into account through the use of a concrete effectiveness factor (v). In 1985, Marti (1985) proposed the use of a simple reduction coefficient (v=0.6) as effectiveness factor, whilst Collins and Mitchell (1986) proposed Eq. 5 for their modified compression field theory (Vecchio and Collins 1986).

$$\mathrm{v} = \frac{1}{0.8 + 170\varepsilon_1} \quad (5)$$

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)/\tan^2\theta \quad (5a)$$

where ε_1 is the principal tensile strain, ε_s is the longitudinal tensile strain at mid-depth of the beam, which can be estimated assuming that plane sections remains plane (Collins et al. 2008).

In 1993, Vecchio and Collins (1993) proposed a refined equation for the concrete effectiveness factor as shown in Eq. 6.

$$\mathrm{v} = \frac{1}{1.0 + \mathrm{K_c K_f}} \quad (6)$$

$$K_c = 0.35\left(\frac{-\varepsilon_1}{\varepsilon_2} - 0.28\right)^{0.8} \geq 1.0 \qquad (6a)$$

$$K_f = 0.1825\sqrt{f_c} \geq 1.0 \qquad (6b)$$

where ε_1 and ε_2 are the principal tensile and compressive strain, respectively, and f_c is the concrete compressive strength.

Foster and Gilbert (1996) argued that concrete compressive strength and shear span to depth ratio (a/d) influence the effectiveness of concrete cracked in tension and modified Collins and Mitchell's equation (Eq. 5) to integrate the effect of these two parameters. This modified equation (Eq. 7) was calibrated against a database of beams with concrete compressive strength ranging from 20 to 100MPa.

$$v = \frac{1}{1.14 + (0.64 + f_c/470)(a/d)^2} \qquad (7)$$

Based on a series of nonlinear finite element analyses, Warwick and Foster (1993) proposed the following concrete effectiveness factor (Eq. 8) for concrete compressive strength ranging from 20 to 100MPa

$$v = 1.25 - \frac{f_c}{500} - 0.72\left(\frac{a}{d}\right) + 0.18\left(\frac{a}{d}\right)^2 \leq 1.0 \qquad (8)$$

EC2 provides Eq. 9 to calculate the effective concrete strength of the inclined concrete strut

$$f_{ce} = 0.6\nu'\, f_{cd} \qquad (9)$$

where ν' can be calculated according to Eq. 9a and f_{cd} is the design concrete compressive strength.

$$v' = 1 - \frac{f_{ck}}{250} \tag{9a}$$

According to ACI 318-14, the effective concrete strength (f_{ce}) can be calculated using Eq. 10

$$f_{ce} = 0.85\beta_s f'_c \tag{10}$$

where β_s is 0.75 for strut with shear reinforcement satisfying Eq. 10a, else β_s is taken as 0.6.

$$\sum \frac{A_{si}}{b_s s_i} \sin \alpha_i \geq 0.003 \tag{10a}$$

where A_{si} is the area of the reinforcement at spacing s_i in the i-th layer of reinforcement crossing a strut at an angle α_i to the axis of the strut.

Model Code 2010 use Eq. 11.

$$f_{ce} = k_c f_{cd} \tag{11}$$

$$k_c = 0.55\left(\frac{30}{f_{ck}}\right)^{1/3} \leq 0.55 \tag{11a}$$

The above effectiveness factor models are assessed in this paper through a parametric investigation to gain additional insight on the role of each of the considered parameters and inform the development of a more accurate model.

ANALYSIS AND DISCUSSIONS

An extensive database of 519 RC deep beam specimens (Clark 1951, Moody et al. 1954, Moody et al. 1955, Morrow and Viest 1957, Chang and Kesler 1958, Watstein and Mathey 1958, Rodriguez et al. 1959, de Cossio and Siess 1960, Mathey and Watstein 1963, Leonhardt and Walther 1964, de Paiva and Siess 1965, Krefeld and Thurston 1966, Kani

1967, Ramakrishnan and Ananthanarayana 1968, Kong et al. 1970, Manuel et al. 1971, Manuel 1974, Niwa et al. 1981, Smith and Vantsiotis 1982, Mphonde and Frantz 1984, Rogowsky et al. 1986, Subedi et al. 1986, Ahmad and Lue 1987, Lehwalter 1988, Walraven and Lehwalter 1994, Xie et al. 1994, Tan et al. 1995, Tan et al. 1997, Foster and Gilbert 1998, Kong and Rangan 1998, Shin et al. 1999, Tan and Lu 1999, Adebar 2000, Pendyala and Mendis 2000, Oh and Shin 2001, Lertsrisakulrat et al. 2002, Yang et al. 2003, Tan et al. 2005, Seliem et al. 2006, Zhang and Tan 2007, Tan et al. 2008) (Table 1) was used to evaluate the performance of the STM, and examine the effectiveness of existing approaches in determining the concrete effectiveness factors.

Suitability of models

As discussed earlier a combination of arch and truss action can develop in beams with shear reinforcement and shear span to depth ratio between 1.0 and 2.0. The specimens within the database that satisfy these conditions (136 RC deep beams) were used to assess the accuracy of the STM (Figure 1a) and Truss Model (TM) (Figure 1b) in predicting shear strength. The strut effectiveness factor was taken as equal to one at this stage of the comparative study. The results (Figure 2) show that the TM yields very conservative results in almost all of the analyzed cases. In addition, the highly scattered results obtained from the implementation of a TM suggest that such an approach cannot be used for the design of RC deep beams. Figure 2 shows that using the STM generally leads to more consistent and accurate results and is more suitable for the design of RC deep beams with and without shear reinforcement. This agrees with the findings of other researchers (Kani 1979, Tuchscherer et al. 2011). However, the result of STM can be further improved if an appropriate effectiveness factor is adopted. Therefore, STM (Figure 1a) will be used for the purpose of evaluating of the existing effectiveness factors and proposing new effectiveness and node factors.

Evaluation of existing effectiveness factors

The eight different formulations for effectiveness factors presented in the previous section earlier are assessed in the following. The results are shown in Figure 3 and Figure 4; and the statistical analyses are summarized in Figure 5 and Figure 6 for RC deep beams with and without shear reinforcement, respectively. Overall, for all eight effectiveness factors the predictions for beams with shear reinforcement are more conservative than those without shear reinforcement. The effectiveness factors proposed by Collins and Mitchell (Eq. 5), Vecchio and Collins (Eq. 7) and Modified Collins and Mitchell (Eq. 10) lead to very conservative results. This is most probably due to the fact that, in these equations, the tensile strain in the concrete needs to be calculated based on the assumption that plane sections remain plane after bending. However, this assumption is far from accurate for deep beams. The effectiveness factor proposed by Marti (i.e. 0.6) (Marti 1985) can lead to very unsafe results for RC deep beams without shear reinforcement, as the single factor proposed cannot account for all parameters. Additionally, experimental and numerical investigations conducted by the authors (Ismail et al. 2015, Ismail et al. 2015) show that in many cases the effectiveness factor is lower than 0.6, especially for RC deep beams without shear reinforcement. Although the effectiveness factor proposed by Warwick and Foster (Eq. 11) accounts for the effect of concrete compressive strength and shear span to depth ratio, the non-uniform performance of this model shows that other parameters affect shear behaviour and their effect should be taken into account.

The models proposed by EC2, ACI 318-14 and Model Code 2010 also lead to very unsafe results especially for RC deep beams without shear reinforcement. This can be attributed again to the fact that these codes do not account for all the important influencing parameters such as shear span to depth ratio and shear reinforcement (EC2 and Model Code 2010); or concrete compressive strength and shear span to depth ratio (ACI 318-14).

The safety of the above models was further checked by introducing the appropriate material partial safety factors (1.5 for concrete and 1.15 for steel) for all models except for ACI 318-14 which is strength reduction factor (0.75). With the exception of the predictions by equations of Collins and Mitchell and Vecchio and Collins for RC deep beams without shear reinforcement, which are over conservative and uneconomic, all other models do not yield an adequate level of safety for all RC deep beams with and without shear reinforcement. The result of the analysis is summarised in Table 2. Therefore, a more sophisticated effectiveness factor model that accounts for all influencing parameters and yields conservative and economic results is required for design purposes. This paper aims to propose new node strength factors and effectiveness factor that account for all influencing parameters and yield more accurate results.

PROPOSED EFFECTIVENESS FACTOR

Equations describing the development of biaxial stress fields, such as those included in the modified compression field theory (Vecchio and Collins 1986) can be used to determine the effective compressive strength of concrete subjected to lateral tensile strain. Bazant and Xiang (1997) derived a simple equation (Eq. 12) based on the theory of fracture mechanics to predict the compressive strength (σ_c) of a concrete specimen subjected to lateral tensile strain.

$$\sigma_c = \sqrt{2EG_f\ h/s}D^{-1/2} \tag{12}$$

where E and G_f are the modulus of elasticity and fracture energy of concrete, respectively; h is the width of the crack band, s is the spacing of cracks in the crack band and D is the width of the specimen.

Equation 12 can be used to estimate the effectiveness factor of an inclined strut. Model Code 2010 equations are used here to determine the modulus of elasticity and fracture energy of concrete and D is taken as the width of the strut (W_S). The effectiveness factor v (Eq. 13) can be expressed as the ratio between Eq. 12 and the uniaxial strength of the concrete (f_c) to obtain:

$$\mathrm{v} = \sqrt{\frac{2EG_f}{W_s}\frac{h}{s}} \Big/ f_c \tag{13}$$

According to Bazant and Xiang (1997), in the crack band the intact concrete between cracks behaves as columns of width s. The strain energy in the crack band releases due to buckling of these columns and failure occurs once the released energy reaches the fracture energy of the concrete. The presence of lateral tensile strain increases the crack width in the crack band which in turn increases the energy release rate and decreases the compressive capacity. This means that the value of h/s is directly affected by lateral tensile strain. Since the value of h/s needs to be determined by calibration of experimental results, the authors propose a more direct approach where h/s in Eq. 13 is replaced by lateral tensile strain and the equation needs to be calibrated by a factor (α) as shown in Eq. 14.

$$\mathrm{v} = \alpha\sqrt{\frac{2EG_f}{W_s\varepsilon_1}} \Big/ f_c \tag{14}$$

Although lateral strain is a more rational quantity to use, it still needs to be quantified either by calculation or calibration of date.

Lateral Tensile Strain in Shear Span

Experimental and numerical data from the finite element model developed and validated by the authors (Ismail et al. 2016a, Ismail et al. 2016b) was used to determine the lateral tensile strain in the shear span of RC deep beams. Figure 7 shows the effect of concrete compressive strength, shear span to depth ratio and effective depth on the lateral tensile strain obtained using finite element analysis for beams with and without shear reinforcement. It can be seen that shear span to depth ratio and effective depth influence the lateral tensile strain whilst concrete compressive strength has almost negligible effect. Therefore, in estimating the lateral tensile strain in the shear span of RC deep beams, shear span to depth ratio and effective depth need to be accounted for. For dimensional purposes, the effective member depth (d) can be normalized by 150mm (based on the experimental results of Walraven and Lehwalter (1994), at the effective depth of 150mm, size effect is effective). Hence, based on best fit analysis, Eq. 15 is proposed to estimate the lateral tensile strains (ε_1) in the shear span of RC deep beams.

$$\varepsilon_1 = 0.02 \frac{(a/d)^{0.5}}{(d/150)^{0.35}} \tag{15}$$

From a direct comparison with the finite element analysis results it can be seen in Figure 8 that this equation leads to a reasonable prediction of lateral tensile strain in the shear span of RC deep beams.

Determination of Factor α

Back analysis was adopted to determine the value of α in Eq. 14 from experimental and numerical data on RC deep beams. The finite element model was used to determine the maximum principal concrete compressive strength in the shear span of the beams (see Table 3 and Table 4 more details of the used beams can be found elsewhere (Ismail et al. 2016-a, Ismail et al. 2016-b, Ismail 2016-c)). The effectiveness factor (v) was calculated as the ratio of the maximum principal compressive strength and uniaxial compressive strength of the concrete. To account for the effect of shear reinforcement, two different values of α need to be adopted as shown in Table 3 and Table 4 for RC deep beams with and without shear reinforcement. An average value of 400 can be used as α for RC deep beams without shear reinforcement or with shear reinforcement ratio less than 0.1%, whilst for RC deep beams with shear reinforcement ratio greater or equal to 0.1% a value of 450 can be used as α. In this context, the shear reinforcement can be taken either as the vertical or horizontal shear reinforcement or a combination thereof.

Node Strength Factor

An accurate estimation of node strengths is also crucial for safe design solutions. For the bottom node which is C-C-T, most codes of practice recommend using a strength which is lower than the uniaxial concrete strength due to presence of a tie in this node. In reality, concrete strength reduces due to the presence of lateral tensile strain and cracks. However, in this region there is no cracking, which means that the tensile stress is always below the concrete tensile strength. Hence, it is still safe to use the uniaxial compressive strength of the concrete without any reduction in estimating the strength of the node.

The strength of the top node (C-C-C) is expected to be higher than the uniaxial concrete strength because it is fully confined when the load is applied through a bearing plate. Therefore, a factor with a value higher than one can be used to account for this confinement. However, for the case when the load is applied through a concrete column, the degree of confinement is lower than applying through bearing plates and the node is under biaxial compression. Therefore, to safely estimate the strength of the C-C-C nodes, the uniaxial concrete strength is used in this paper.

Evaluation of Proposed Model

The shear strength prediction according to the implementation of the STM using the proposed concrete effectiveness factor (including lateral tensile strain predictions) and the factors for estimating the strength of the nodes is shown in Figure 9 and summarized in Figure 5 and

Figure 6 for RC deep beams with and without shear reinforcement, respectively. The use of the proposed model yields overall less conservative predictions with lower standard deviations. This can lead to more economical design solutions, yet maintaining an appropriate level of safety as shown in Table 2 and Figure 10 and 11 show the accuracy of the model for different case scenarios for both beams with and without shear reinforcement respectively.

Figure 12 and Figure 13 show the effect of shear span to depth ratio, concrete compressive strength and member depth (i.e. size effect) on the performance of the three codes of practice discussed in this paper, along with the proposed effectiveness factor for RC deep beams with and without shear reinforcement, respectively. It can be seen that ACI 318-14 which neglects the influence of both shear span to depth ratio, concrete compressive strength and member depth, offer the less reliable predictions. The EC2 and Model Code 2010, though they include the effect of concrete compressive strength, do not sufficiently account for the effect of this parameter and they do not account for the effect of shear span to depth ratio, as evidenced by their variable degree of conservatism. The use of the proposed effectiveness factor accounts for the effect of these parameters more accurately and leads to a more uniform performance level for both RC deep beams with and without shear reinforcement.

CONCLUSIONS

The main conclusions of this research study can be summarized as follows:

1. A tie-arch mechanism is the main resisting mechanism in RC deep beams with and without shear reinforcement and can be best represented by the strut-and-tie model.
2. The selection of an appropriate strut-and-tie model and size of its elements is critical for accurate shear capacity predictions.

3. The effectiveness factor models based on the modified compression field theory show poor correlation against the experimental results, with a large scatter and high coefficients of variation.
4. The STM provision and the effectiveness factors of EC2, ACI 318-14 and Model Code 2010 do not ensure adequate safety levels (after application of safety factors) for RC deep beams without shear reinforcement.
5. A new model which utilises a concrete effectiveness factor based on predicted lateral strain is proposed. The use of the proposed model leads to less conservative yet safe predictions, and can accurately account for the effect of concrete compressive strength, shear span to depth ratio, shear reinforcement and member depth.

ACKNOWLEDGEMENT

The authors acknowledge the financial support of the Human Capacity Development Program (HCDP) of the Ministry of Higher Education and Scientific Research/ Kurdistan Regional Government for the PhD studies of Kamaran S. Ismail.

REFERENCES

ACI Committee, American Concrete Institute, and International Organization for Standardization, (2014). Building Code Requirements for Structural Concrete (ACI 318M-14) and Commentary, American Concrete Institute.

Adebar, P. (2000). "One-way shear strength of large footings." Canadian journal of civil engineering **27**(3): 553-562.

Ahmad, S. H. and D. Lue (1987). "Flexure-shear interaction of reinforced high strength concrete beams." ACI Structural Journal **84**(4).

Bakir, P. and H. Boduroğlu (2005). "Mechanical behaviour and non-linear analysis of short beams using softened truss and direct strut & tie models." Engineering structures **27**(4): 639-651.

Bazant, Z. P. and Y. Xiang (1997). "Size effect in compression fracture: splitting crack band propagation." Journal of Engineering Mechanics **123**(2): 162-172.

Brena, S. F. and N. C. Roy (2009). "Evaluation of load transfer and strut strength of deep beams with short longitudinal bar anchorages." ACI Structural Journal **106**(5).

British Standards Institution, (2004). Eurocode 2: Design of Concrete Structures: Part 1-1: General Rules and Rules for Buildings, British Standards Institution.

Brown, M. D. and O. Bayrak (2008). "Design of deep beams using strut-and-tie models--part I: Evaluating US provisions." ACI Structural Journal **105**(4).

Chang, T. S. and C. E. Kesler (1958). Static and fatigue strength in shear of beams with tensile reinforcement. ACI Journal Proceedings, ACI.

Clark, A. P. (1951). Diagonal tension in reinforced concrete beams. ACI journal proceedings, ACI.

Collins, M. P., E. C. Bentz and E. G. Sherwood (2008). "Where is shear reinforcement required? Review of research results and design procedures." ACI Structural Journal **105**(5).

Collins, M. P. and D. Mitchell (1986). Rational Approach to Shear Design--The 1984 Canadian Code Provisions. ACI Journal Proceedings, ACI.

Collins, M. P., D. Mitchell and E. C. Bentz (2008). "Shear design of concrete structures." The Structural Engineer Journal **86**(10): 32-39.

de Cossio, R. D. and C. P. Siess (1960). Behavior and strength in shear of beams and frames without web reinforcement. ACI Journal Proceedings, ACI.

de Paiva, H. R. and C. P. Siess (1965). "Strength and behavior of deep beams in shear." ASCE Structural Journal **91**(ST5): 22.

Fib (2013). fib Model Code for Concrete Structures 2010, International Federation for Structural Concrete.

Foster, S. J. and R. I. Gilbert (1996). "The design of nonflexural members with normal and high-strength concretes." ACI Structural Journal **93**(1).

Foster, S. J. and R. I. Gilbert (1998). "Experimental studies on high-strength concrete deep beams." ACI Structural Journal **95**(4): 382-390.

Ismail, K. S., M. Guadagnini and K. Pilakoutas (2016a). "Numerical Investigation on the Shear strength of RC Deep Beams Using the Microplane Model." Journal of Structural Engineering. 04016077.

Ismail, K. S., M. Guadagnini and K. Pilakoutas (2016b). "Shear Behaviour of RC Deep Beams." ACI Structural journal. Accepted.

Ismail, K. S. (2016c). Shear Behaviour of Reinforced Concrete Deep Beams. Doctoral thesis, University of Sheffield.

Jirsa, J., J. Breen and K. Bergmeister (1991). Experimental studies of nodes in strut-and-tie models. IABSE Colloquium Stuttgart.

Kani, G. N. J. (1967). "How safe are our large reinforced concrete beams?" ACI journal **64**(3): 128-141.

Kani, M. W., M. W. Huggins, and R. R. Wittkopp (1979). Kani on shear in reinforced concrete, Deptartment of Civil Engineering, University of Toronto.

Kong, F.-K., P. J. Robins and D. F. Cole (1970). Web reinforcement effects on deep beams. ACI Journal Proceedings, ACI.

Kong, P. Y. and B. V. Rangan (1998). "Shear strength of high-performance concrete beams." ACI Structural Journal **95**(6): 677-688.

Krefeld, W. J. and C. W. Thurston (1966). Studies of the shear and diagonal tension strength of simply supported reinforced concrete beams. ACI Journal Proceedings, ACI.

Lehwalter, N. (1988). Bearing Capacity of Concrete Compression Struts in Truss-Systems, Exemplified by the Case of Short Beams, PhD thesis, Darmstadt, 1988.(in German).

Leonhardt, F. and R. Walther (1964). "The Stuttgart Shear Tests, 1961." CACA Translation(111).

Lertsrisakulrat, T., J. Niwa, A. Yanagawa and M. Matsuo (2002). Concept of concrete compressive fracture energy in RC deep beams without transverse reinforcement. Transactions of the Japan Concrete Institute.

Manuel, R. (1974). "Failure of deep beams." ACI Special publication **42**: 15.

Manuel, R. F., B. W. Slight and G. T. Suter (1971). "Deep beam behavior affected by length and shear span variations." Am Concrete Inst Journal & Proceedings **68**(12).

Marti, P. (1985). Basic tools of reinforced concrete beam design. ACI Journal Proceedings, ACI.

Mathey, R. G. and D. Watstein (1963). Shear strength of beams without web reinforcement containing deformed bars of different yield strengths. ACI journal proceedings, ACI.

Moody, K., I. Viest, R. Elstner and E. Hognestad (1954). Shear Strength of Reinforced Concrete Beams Part 1-Tests of Simple Beams. ACI journal proceedings, ACI.

Moody, K., I. Viest, R. Elstner and E. Hognestad (1955). Shear Strength of Reinforced Concrete Beams Part 2-Tests of Restrained Beams Without Web Reinforcement. ACI Journal Proceedings, ACI.

Morrow, J. and I. M. Viest (1957). Shear strength of reinforced concrete frame members without web reinforcement. ACI Journal Proceedings, ACI.

Morsch, E. (1902). "Der eisenbetonbau, seine anwendung und theorie." Wayss and Freytag, AG, Im Selbstverlag der Firma, Neustadt, AD Haardt: 118.

Mphonde, A. G. and G. C. Frantz (1984). Shear tests of high-and low-strength concrete beams without stirrups. ACI Journal Proceedings, ACI.

Niwa, J., K. Maekawa and H. Okamura (1981). "Non-linear Finite Element Analysis of Deep Beams." Advanced Mechanics of Reinforced Concrete, IABSE Colloqium Delft Netherlands: 13.

Oh, J.-K. and S.-W. Shin (2001). "Shear strength of reinforced high-strength concrete deep beams." ACI Structural Journal **98**(2).

Park, J.-w. and D. Kuchma (2007). "Strut-and-tie model analysis for strength prediction of deep beams." ACI Structural Journal **104**(6).

Pendyala, R. S. and P. Mendis (2000). "Experimental study on shear strength of high-strength concrete beams." ACI Structural Journal **97**(4).

Ramakrishnan, V. and Y. Ananthanarayana (1968). Ultimate strength of deep beams in shear. ACI Journal Proceedings, ACI.

Ritter, W. (1899). "Die Bauweise Hennebique (Hennebiques Construction Method)." Schweizerische Bauzeitung **17**: 41-43.

Rodriguez, J. J., A. C. Bianchini, I. M. Viest and C. E. Kesler (1959). Shear Strength of Two-Span Continous Reinforced Concrete Beams. ACI Journal Proceedings, ACI.

Rogowsky, D. M., J. G. MacGregor and S. Y. Ong (1986). Tests of reinforced concrete deep beams. ACI Journal Proceedings, ACI.

Sagaseta, J. and R. Vollum (2010). "Shear design of short-span beams." Magazine of Concrete Research **62**(4): 267-282.

Seliem, H., A. Hosny, H. Dwairi and S. Rizkalla (2006). "Shear behavior of concrete beams reinforced with MMFX steel without web reinforcement." NC State University Final Report, Project No. IS-06-08.

Shin, S.-W., K.-S. Lee, J.-I. Moon and S. Ghosh (1999). "Shear strength of reinforced high-strength concrete beams with shear span-to-depth ratios between 1.5 and 2.5." ACI Structural Journal **96**(4).

Smith, K. and A. Vantsiotis (1982). Shear strength of deep beams. ACI Journal Proceedings, ACI.

Subedi, N., A. E. Vardy and N. Kubotat (1986). "Reinforced concrete deep beams some test results." Magazine of Concrete Research **38**(137): 206-219.

Tan, K.-H., G.-H. Cheng and N. Zhang (2008). "Experiment to mitigate size effect on deep beams." Magazine of Concrete Research **60**(10): 709-723.

Tan, K.-H., F.-K. Kong, S. Teng and L. Guan (1995). "High-strength concrete deep beams with effective span and shear span variations." ACI Structural Journal **92**(4).

Tan, K.-H., F.-K. Kong, S. Teng and L.-W. Weng (1997). "Effect of web reinforcement on high-strength concrete deep beams." ACI Structural Journal **94**(5): 572-581.

Tan, K., G. Cheng and H. Cheong (2005). "Size effect in shear strength of large beams—behaviour and finite element modelling." Magazine of Concrete Research **57**(8): 497-509.

Tan, K. and H. Lu (1999). "Shear behavior of large reinforced concrete deep beams and code comparisons." ACI Structural Journal **96**(5): 836-845.

Tuchscherer, R. G., D. B. Birrcher and O. Bayrak (2011). "Strut-and-tie model design provisions." PCI journal **56**(1): 155-170.

Tuchscherer, R. G., D. B. Birrcher and O. Bayrak (2014). "Experimental Examination of ACI 318 Strut and Tie Modeling Provisions." ACI Special Publication **296**.

Vecchio, F. J. and M. P. Collins (1986). "The modified compression-field theory for reinforced concrete elements subjected to shear." ACI Journal Proceedings **83**(2).

Vecchio, F. J. and M. P. Collins (1993). "Compression response of cracked reinforced concrete." Journal of Structural Engineering **119**(12): 3590-3610.

Walraven, J. and N. Lehwalter (1994). "Size effects in short beams loaded in shear." ACI Structural Journal **91**(5): 585-593.

Warwick, W. and S. J. Foster (1993). Investigation into the efficiency factor used in non-flexural reinforced concrete member design, University of New South Wales.

Watstein, D. and R. G. Mathey (1958). Strains in beams having diagonal cracks. ACI Journal Proceedings, ACI.

Xie, Y., S. H. Ahmad, T. Yu, S. Hino and W. Chung (1994). "Shear ductility of reinforced concrete beams of normal and high-strength concrete." ACI Structural Journal **91**(2).

Yang, K.-H., H.-S. Chung, E.-T. Lee and H.-C. Eun (2003). "Shear characteristics of high-strength concrete deep beams without shear reinforcements." Engineering structures **25**(10): 1343-1352.

Zhang, N. and K.-H. Tan (2007). "Size effect in RC deep beams: Experimental investigation and STM verification." Engineering structures **29**(12): 3241-3254.

Table 1 Summary of the RC deep beams in the database

	RC deep beams without shear reinforcement	RC deep beams with shear reinforcement
Number of the beams	295	224
Concrete strength (MPa)	11 to 87	14 to 90
Shear span to depth ratio	0.25 to 2.0	0.27 to 2.0
Effective depth (mm)	151 to 1750	160 to 1750
Main reinforcement ratio (%)	0.26 to 6.64	0.16 to 4.25
Vertical shear reinforcement ratio (%)	----	0 to 2.45
Horizontal shear reinforcement ratio (%)	----	0 to 3.17

Table 2-Percent of safe shear strength prediction by STM after application of safety factors

	Beams without shear reinforcement (295 beams)			Beams with shear reinforcement (224 beams)		
	Safe prediction (%)	Mean of safe results	Mean of unsafe results	Safe prediction (%)	Mean of safe results	Mean of unsafe results
Marti 1985	88.3	1.93	0.86	99.6	2.28	0.87
Collins and Mitchell 1986	96.0	2.34	0.90	98.7	1.96	0.92
Vecchio and Collins 1993	100	3.32	---	99.6	2.97	0.87
Warwick and Foster 1993	91.0	1.91	0.88	99.6	2.68	0.87
Modified Collins and Mitchell 1996	97.1	2.53	0.95	99.6	3.21	0.87
EC2	90.9	1.92	0.91	99.6	2.59	0.75
ACI 318-14	79.3	1.76	0.85	93.3	1.95	0.84
Model Code 2010	88.7	1.99	0.88	99.6	2.69	0.79
Proposed	100	1.63	---	100	1.59	---

Table 3 Summary of finite element analysis of RC deep beams with shear reinforcement

Specimen		fc (MPa)	b (mm)	d (mm)	ρ (%)	a/d	Bearing Plate width (mm)	Principal concrete strength (MPa)	α	
Experimental (Ismail et al. 2016b)	A2	85.7	100	330	3.655	1.67	100	28	435	Average = 452
	A3	85.1	100	330	3.655	1.67	100	29	451	
	B2	86.6	100	330	3.655	1.29	100	32	462	
	B3	88.1	100	330	3.655	1.29	100	34	489	
	D2	59.7	100	330	3.655	1.67	100	24	410	
	D3	58.1	100	330	3.655	1.67	100	25	430	
	E2	59.1	100	330	3.655	1.29	100	26	416	
	E3	59.2	100	330	3.655	1.29	100	29	463	
	F2	60.6	100	330	3.655	0.91	100	34	488	
	F3	59.5	100	330	3.655	0.91	100	34	490	
	G1	30.9	100	330	3.655	1.67	100	23	467	
	G2	30.5	100	330	3.655	1.29	100	24	457	
	G3	31.3	100	330	3.655	0.91	100	25	429	
dy (Is	BH-S-30	30	200	710	1.300	0.75	150	25	396	

	BH-S-55	55	200	710	1.300	0.75	150	36	476	
	BH-S-80	80	200	710	1.300	0.75	150	39	466	
	BH-M-30	30	200	710	1.300	1.3	150	23	449	
	BH-M-55	55	200	710	1.300	1.3	150	28	451	
	BH-M-80	80	200	710	1.300	1.3	150	32	471	
	BH-B-30	30	200	710	1.300	2	150	19	407	
	BH-B-55	55	200	710	1.300	2	150	25	472	
	BH-B-80	80	200	710	1.300	2	150	27	458	

Table 4 Summary of finite element analysis of RC deep beams without shear reinforcement

Specimen		fc (MPa)	b(mm)	d (mm)	ρ (%)	a/d	Bearing Plate Width (mm)	Principal concrete strength (MPa)	α	
Experimental (Ismail et al. 2016b)	A1	85.2	100	330	3.655	1.67	100	27	420	Average = 398
	B1	86.9	100	330	3.655	1.29	100	31	447	
	C1	85.7	100	330	3.655	0.91	100	34	444	
	D1	58.8	100	330	3.655	1.67	100	21	360	
	E1	58.2	100	330	3.655	1.29	100	24	385	
	F1	60.5	100	330	3.655	0.91	100	28	402	
	H1	35.8	150	449	1.399	1.67	80	21	356	
	H2	35.8	150	328	1.378	1.65	80	18	307	
	H3	35.8	150	219	1.376	1.64	80	17	290	
Parametric study (Ismail et al. 2016a)	BN-S-30	30	200	710	1.300	0.75	150	23	365	
	BN-S-55	55	200	710	1.300	0.75	150	32	431	
	BN-S-80	80	200	710	1.300	0.75	150	38	451	
	BN-M-30	30	200	710	1.300	1.3	150	22	421	
	BN-M-55	55	200	710	1.300	1.3	150	25	413	

	BN-M-80	80	200	710	1.300	1.3	150	30	434	
	BN-B-30	30	200	710	1.300	2	150	18	385	
	BN-B-55	55	200	710	1.300	2	150	23	423	
	BN-B-80	80	200	710	1.300	2	150	26	436	

List of Tables

List of Figures

Figure 3-Effect of concrete effectiveness factor on shear strength prediction of RC deep beams with shear reinforcement by STM

Figure 4-Effect of concrete effectiveness factor on shear strength prediction of RC deep beams without shear reinforcement by STM

Figure 5-Statistical analysis of shear strength prediction of RC deep beams with shear reinforcement by STM

Figure 6-Statistical analysis of shear strength prediction of RC deep beams without shear reinforcement by STM

Figure 7- Effect of a) concrete strength, b) shear span to depth ratio and c) effective depth on the lateral tensile strain

Figure 8- Estimating the lateral tensile strain by Eq. 15

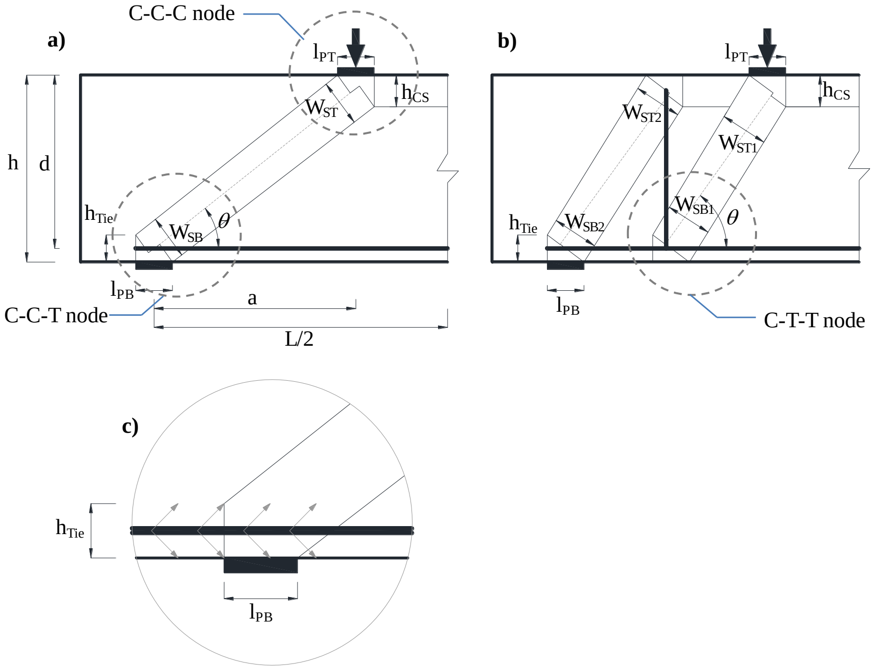
Figure 9-Shear strength prediction by STM with using the proposed concrete effectiveness factor

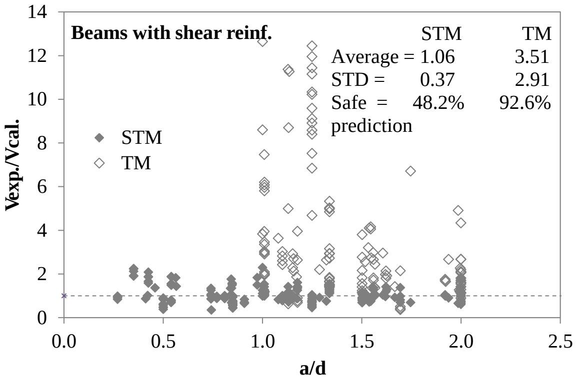
Figure 10-Accuracy of the proposed model for different case scenarios (Beams with shear reinforcement)

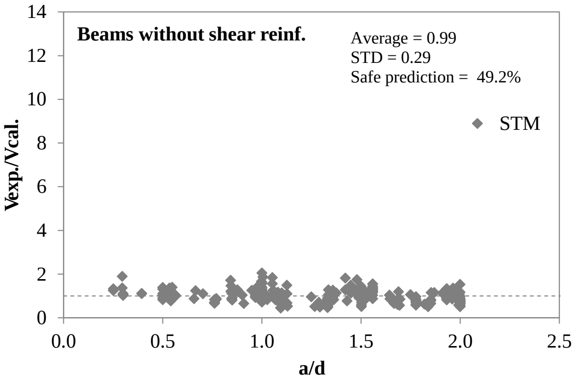
Figure 11-Accuracy of the proposed model for different case scenarios (Beams without shear reinforcement)

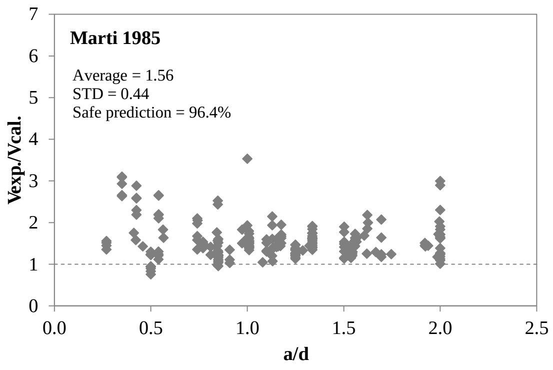
Figure 12- Effect of a) shear span to depth ratio (Oh and Shin 2001) b) concrete strength (Oh and Shin 2001) and c) depth on the performance of different effectiveness factor (Walraven and Lehwalter 1994) (with shear reinforcement)

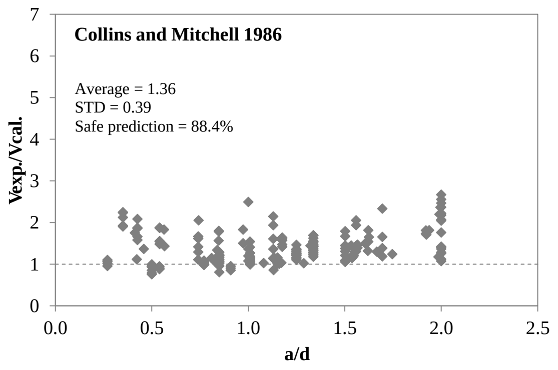
Figure 13- Effect of a) shear span to depth ratio (Oh and Shin 2001) b) concrete strength (Mphonde and Frantz 1984) and c) depth on the performance of different effectiveness factor (Walraven and Lehwalter 1994) (No shear reinforcement)

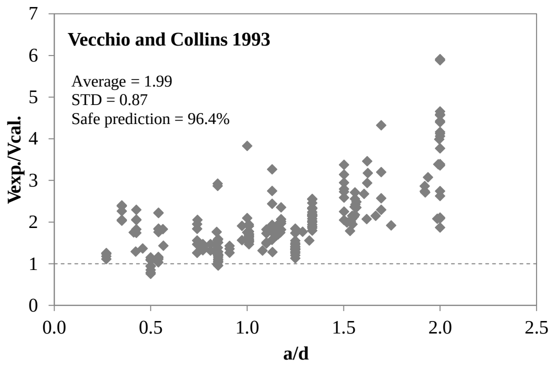

a)
C-C-C node
l_{PT}
h_{CS}
W_{ST}
h
d
h_{Tie}
W_{SB}
θ
l_{PB}
C-C-T node
a
L/2
b)
l_{PT}
h_{CS}
W_{ST2}
W_{ST1}
W_{SB1}
h_{Tie}
W_{SB2}
θ
l_{PB}
C-T-T node
c)
h_{Tie}
l_{PB}

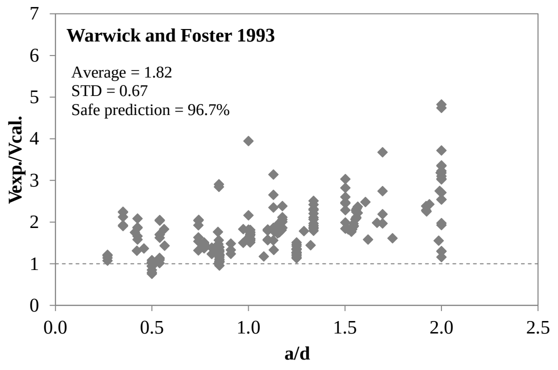

Beams with shear reinf.
STM TM
Average = 1.06 3.51
STD = 0.37 2.91
Safe = 48.2% 92.6%
prediction
STM
TM
Vexp./Vcal.
a/d
14
12
10
8
6
4
2
0
0.0
0.5
1.0
1.5
2.0
2.5

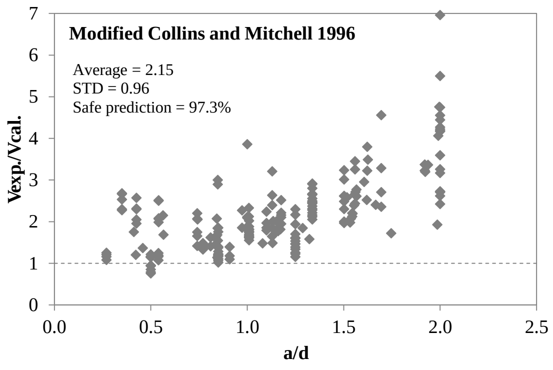

Beams without shear reinf.
Average = 0.99
STD = 0.29
Safe prediction = 49.2%
STM
Vexp./Vcal.
a/d
14
12
10
8
6
4
2
0
0.0
0.5
1.0
1.5
2.0
2.5

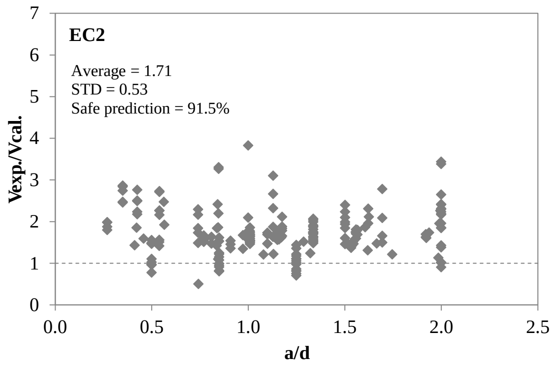

Marti 1985
Average = 1.56
STD = 0.44
Safe prediction = 96.4%
Vexp./Vcal.
a/d
0
1
2
3
4
5
6
7
0.0
0.5
1.0
1.5
2.0
2.5

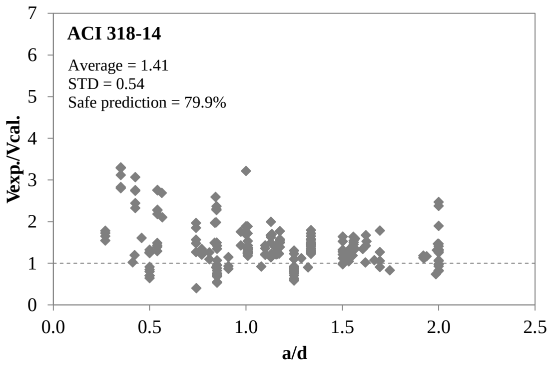

Collins and Mitchell 1986
Average = 1.36
STD = 0.39
Safe prediction = 88.4%
Vexp./Vcal.
a/d
0
1
2
3
4
5
6
7
0.0
0.5
1.0
1.5
2.0
2.5

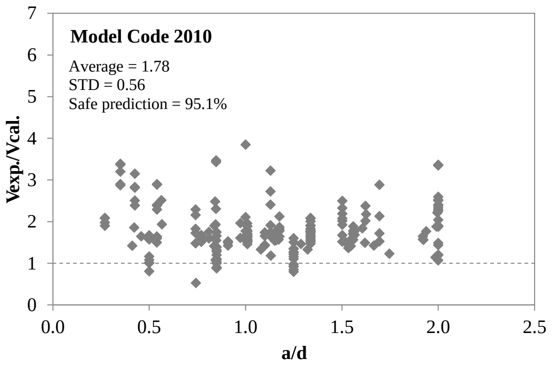

Vecchio and Collins 1993
Average = 1.99
STD = 0.87
Safe prediction = 96.4%
Vexp./Vcal.
a/d
0
1
2
3
4
5
6
7
0.0
0.5
1.0
1.5
2.0
2.5

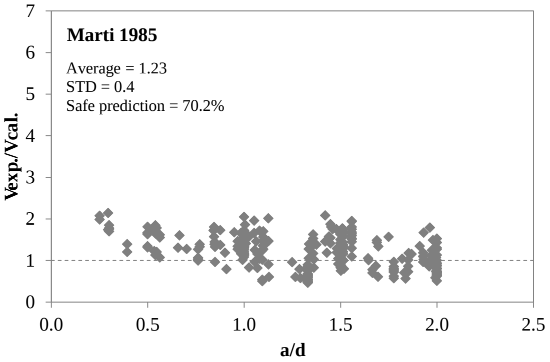

Warwick and Foster 1993
Average = 1.82
STD = 0.67
Safe prediction = 96.7%
Vexp./Vcal.
a/d
0
1
2
3
4
5
6
7
0.0
0.5
1.0
1.5
2.0
2.5

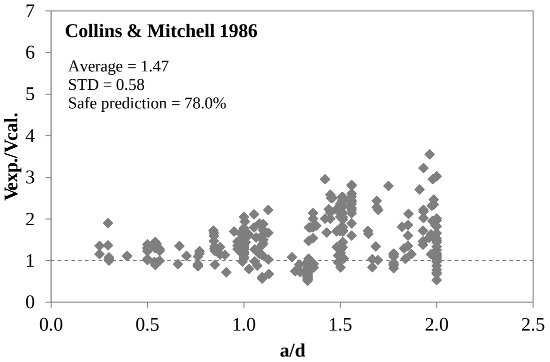

Modified Collins and Mitchell 1996
Average = 2.15
STD = 0.96
Safe prediction = 97.3%
Vexp./Vcal.
a/d
0
1
2
3
4
5
6
7
0.0
0.5
1.0
1.5
2.0
2.5

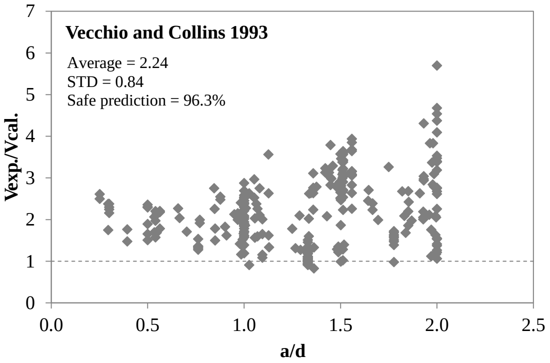

EC2
Average = 1.71
STD = 0.53
Safe prediction = 91.5%
Vexp./Vcal.
a/d
0
1
2
3
4
5
6
7
0.0
0.5
1.0
1.5
2.0
2.5

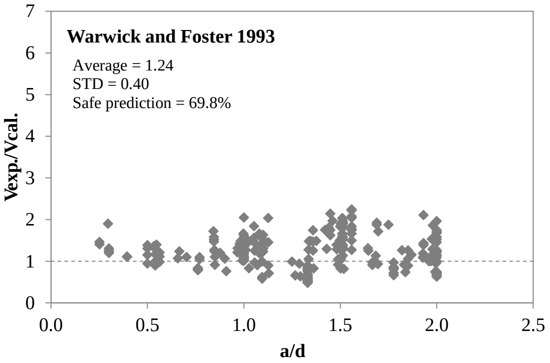

ACI 318-14
Average = 1.41
STD = 0.54
Safe prediction = 79.9%
Vexp./Vcal.
a/d
0
1
2
3
4
5
6
7
0.0
0.5
1.0
1.5
2.0
2.5

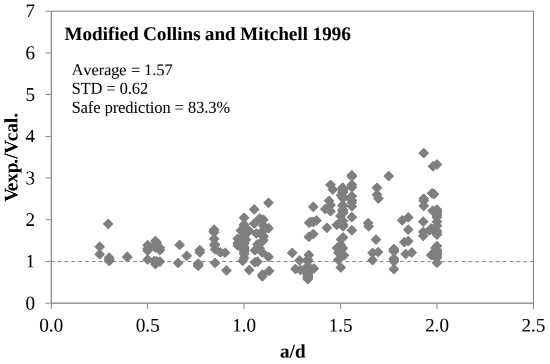

Model Code 2010
Average = 1.78
STD = 0.56
Safe prediction = 95.1%
Vexp./Vcal.
a/d
0
1
2
3
4
5
6
7
0.0
0.5
1.0
1.5
2.0
2.5

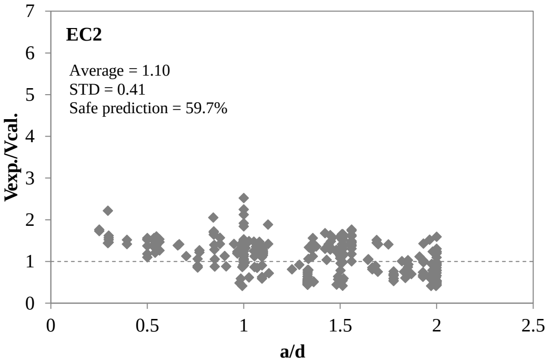

Marti 1985
Average = 1.23
STD = 0.4
Safe prediction = 70.2%
Vexp./Vcal.
a/d
0 1 2 3 4 5 6 7
0.0 0.5 1.0 1.5 2.0 2.5

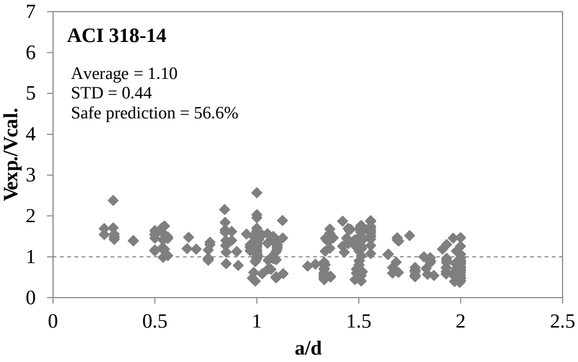

Collins & Mitchell 1986
Average = 1.47
STD = 0.58
Safe prediction = 78.0%
Vexp./Vcal.
a/d
0 1 2 3 4 5 6 7
0.0 0.5 1.0 1.5 2.0 2.5

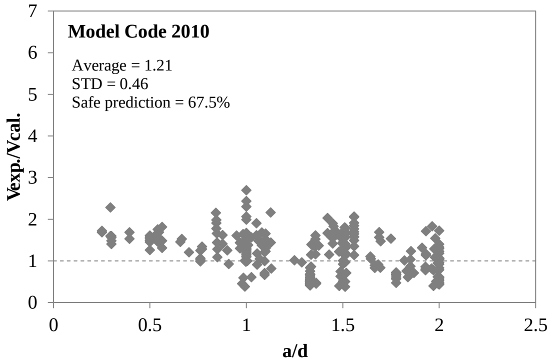

Vecchio and Collins 1993
Average = 2.24
STD = 0.84
Safe prediction = 96.3%
Vexp./Vcal.
a/d
0 1 2 3 4 5 6 7
0.0 0.5 1.0 1.5 2.0 2.5

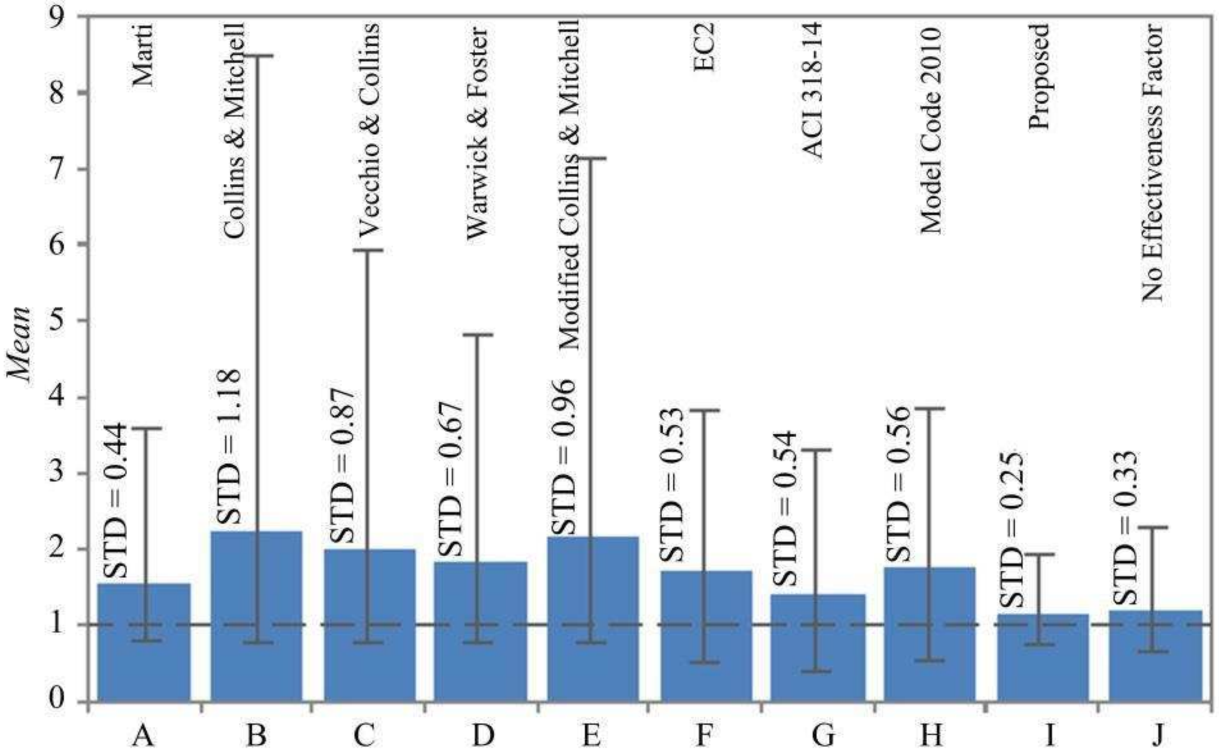

Warwick and Foster 1993
Average = 1.24
STD = 0.40
Safe prediction = 69.8%
Vexp./Vcal.
a/d
0 1 2 3 4 5 6 7
0.0 0.5 1.0 1.5 2.0 2.5

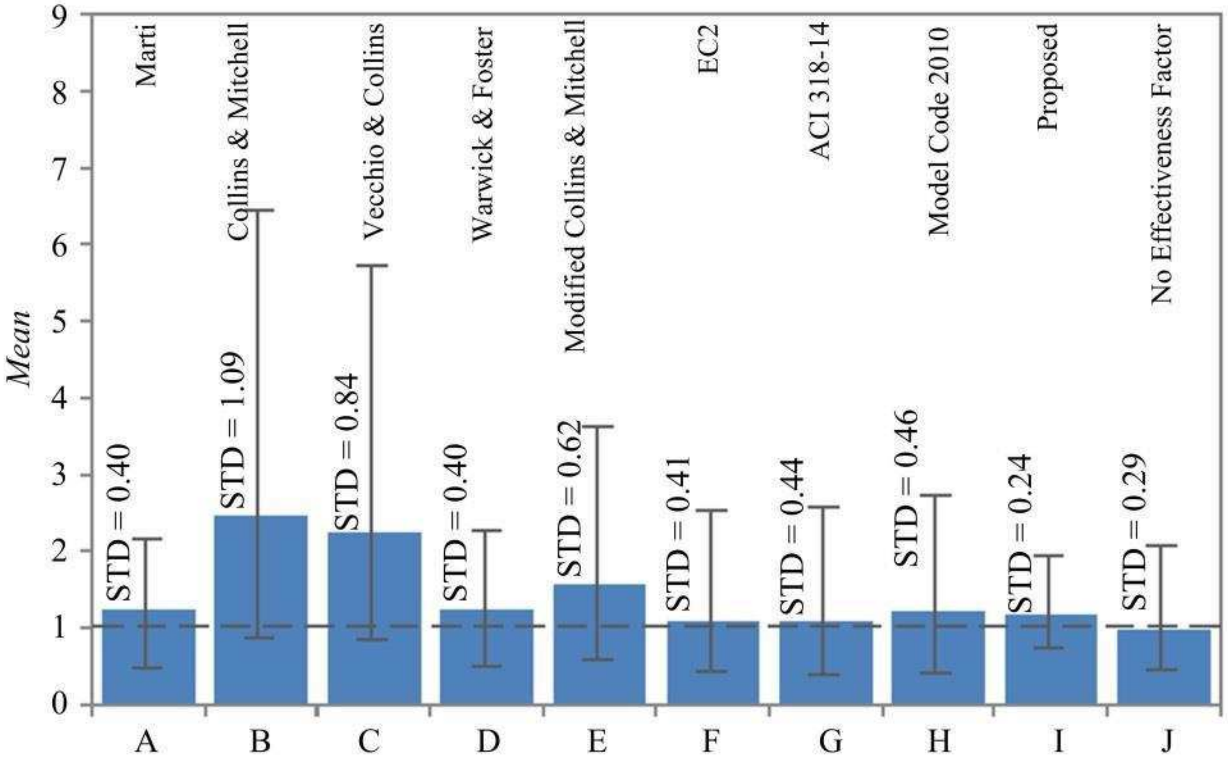

Modified Collins and Mitchell 1996
Average = 1.57
STD = 0.62
Safe prediction = 83.3%
Vexp./Vcal.
a/d
0 1 2 3 4 5 6 7
0.0 0.5 1.0 1.5 2.0 2.5

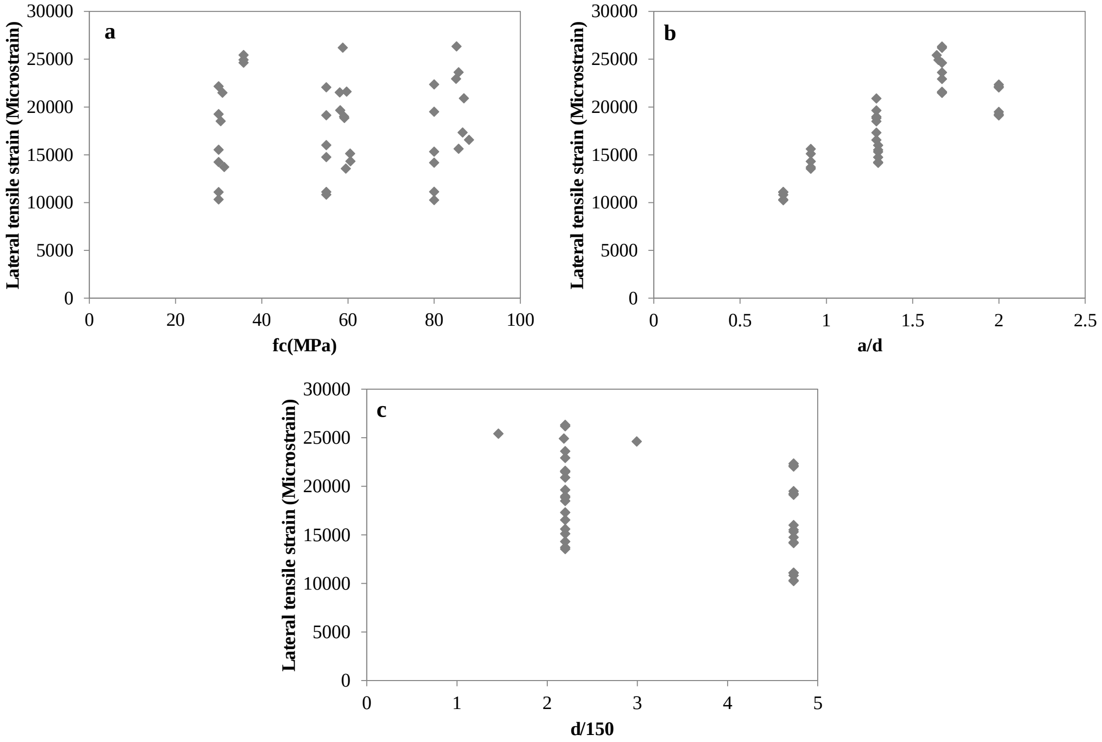

EC2
Average = 1.10
STD = 0.41
Safe prediction = 59.7%
Vexp./Vcal.
a/d
0 1 2 3 4 5 6 7
0 0.5 1 1.5 2 2.5

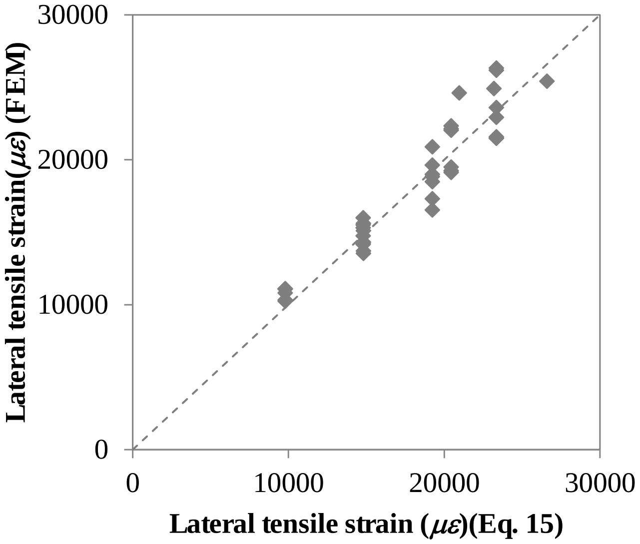

ACI 318-14
Average = 1.10
STD = 0.44
Safe prediction = 56.6%
Vexp./Vcal.
a/d
0 1 2 3 4 5 6 7
0 0.5 1 1.5 2 2.5

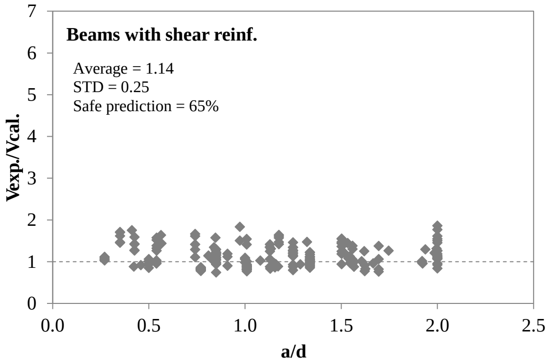

Model Code 2010
Average = 1.21
STD = 0.46
Safe prediction = 67.5%
Vexp./Vcal.
a/d
0 1 2 3 4 5 6 7
0 0.5 1 1.5 2 2.5

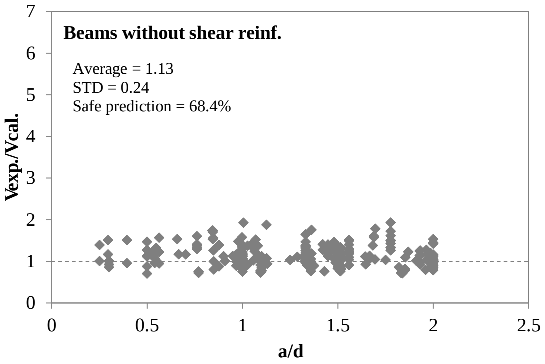

Mean
0
1
2
3
4
5
6
7
8
9
A
B
C
D
E
F
G
H
I
J
Marti
Collins & Mitchell
Vecchio & Collins
Warwick & Foster
Modified Collins & Mitchell
EC2
ACI 318-14
Model Code 2010
Proposed
No Effectiveness Factor
STD = 0.44
STD = 1.18
STD = 0.87
STD = 0.67
STD = 0.96
STD = 0.53
STD = 0.54
STD = 0.56
STD = 0.25
STD = 0.33

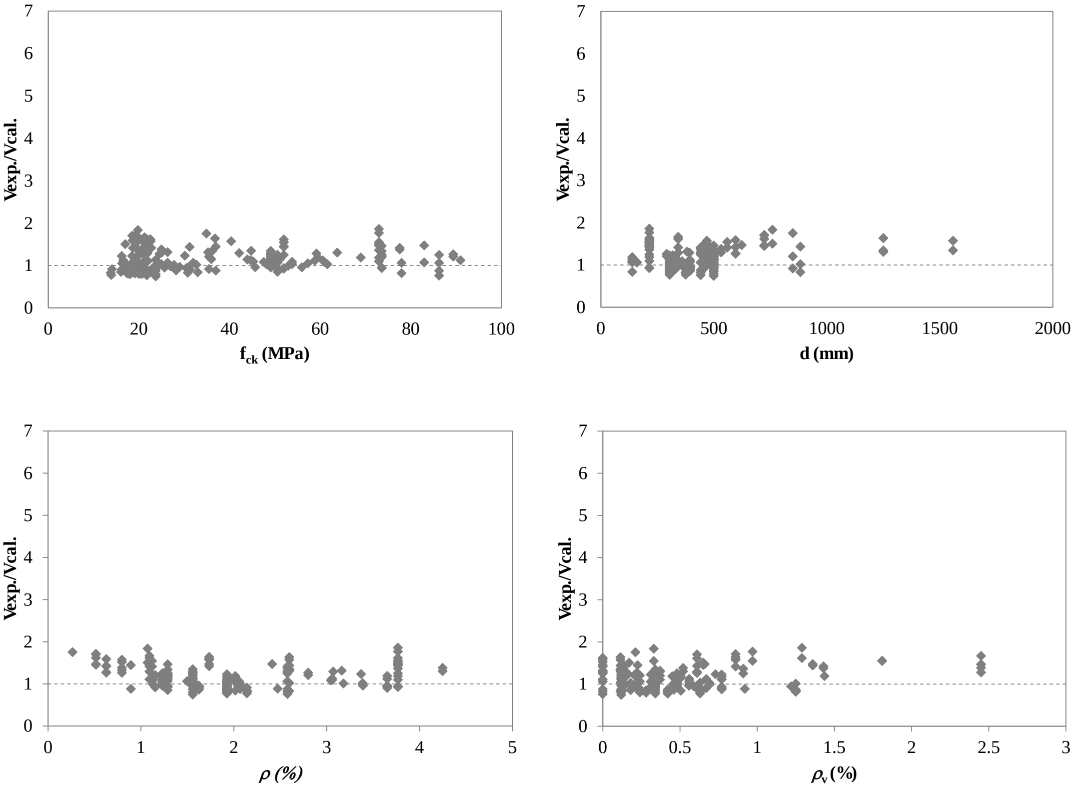

Mean
0
1
2
3
4
5
6
7
8
9
Marti
Collins & Mitchell
Vecchio & Collins
Warwick & Foster
Modified Collins & Mitchell
EC2
ACI 318-14
Model Code 2010
Proposed
No Effectiveness Factor
STD = 0.40
STD = 1.09
STD = 0.84
STD = 0.40
STD = 0.62
STD = 0.41
STD = 0.44
STD = 0.46
STD = 0.24
STD = 0.29
A
B
C
D
E
F
G
H
I
J

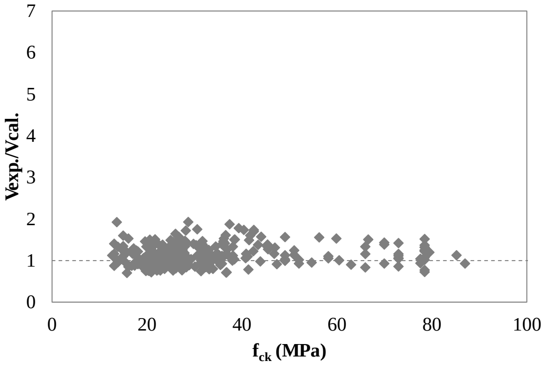

a
Lateral tensile strain (Microstrain)
0
5000
10000
15000
20000
25000
30000
0
20
40
60
80
100
fc(MPa)
b
Lateral tensile strain (Microstrain)
0
5000
10000
15000
20000
25000
30000
0
0.5
1
1.5
2
2.5
a/d
c
Lateral tensile strain (Microstrain)
0
5000
10000
15000
20000
25000
30000
0
1
2
3
4
5
d/150

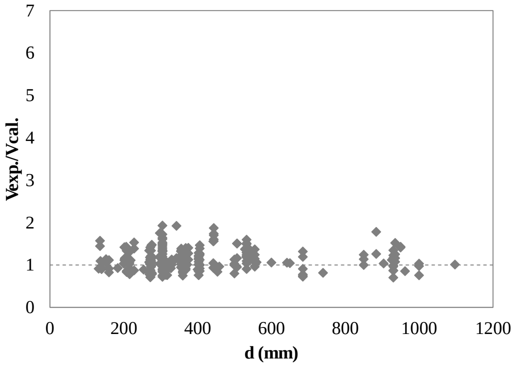

30000
20000
10000
0
0
10000
20000
30000
Lateral tensile strain($\mu\varepsilon$) (FEM)
Lateral tensile strain ($\mu\varepsilon$)(Eq. 15)

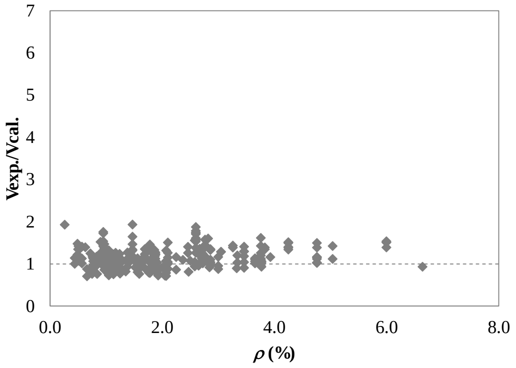

Beams with shear reinf.
Average = 1.14
STD = 0.25
Safe prediction = 65%
Vexp./Vcal.
0
1
2
3
4
5
6
7
0.0
0.5
1.0
1.5
2.0
2.5
a/d

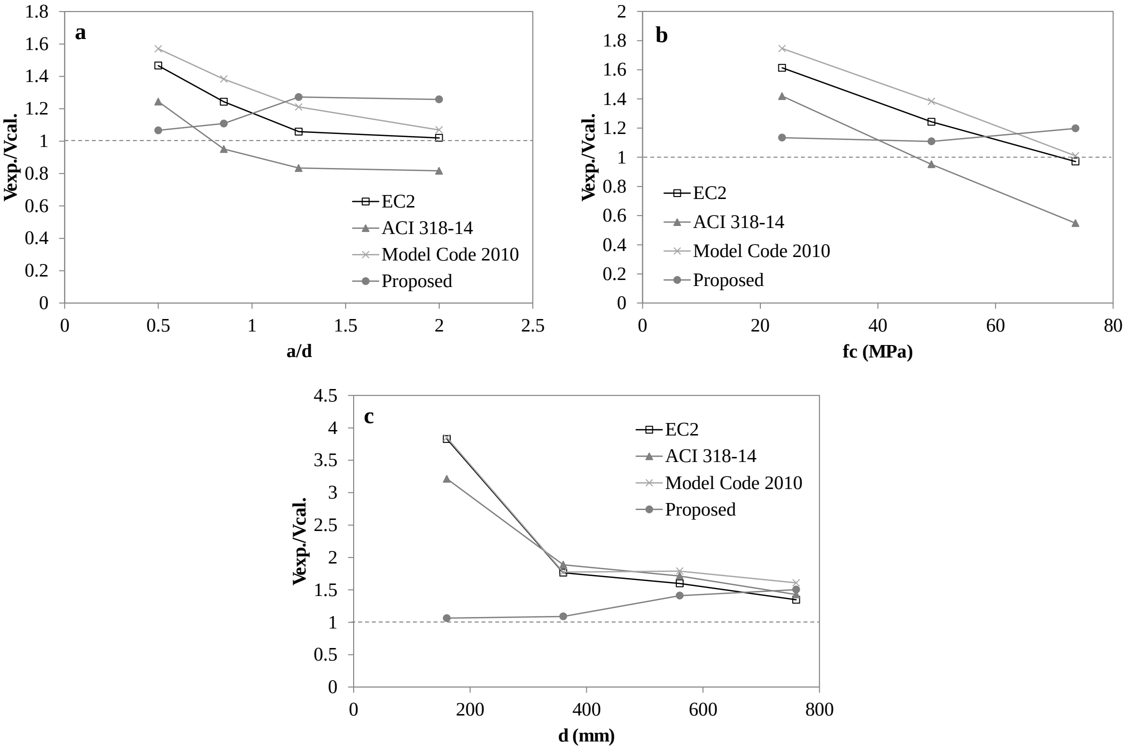

Beams without shear reinf.
Average = 1.13
STD = 0.24
Safe prediction = 68.4%
Vexp./Vcal.
0
1
2
3
4
5
6
7
0
0.5
1
1.5
2
2.5
a/d

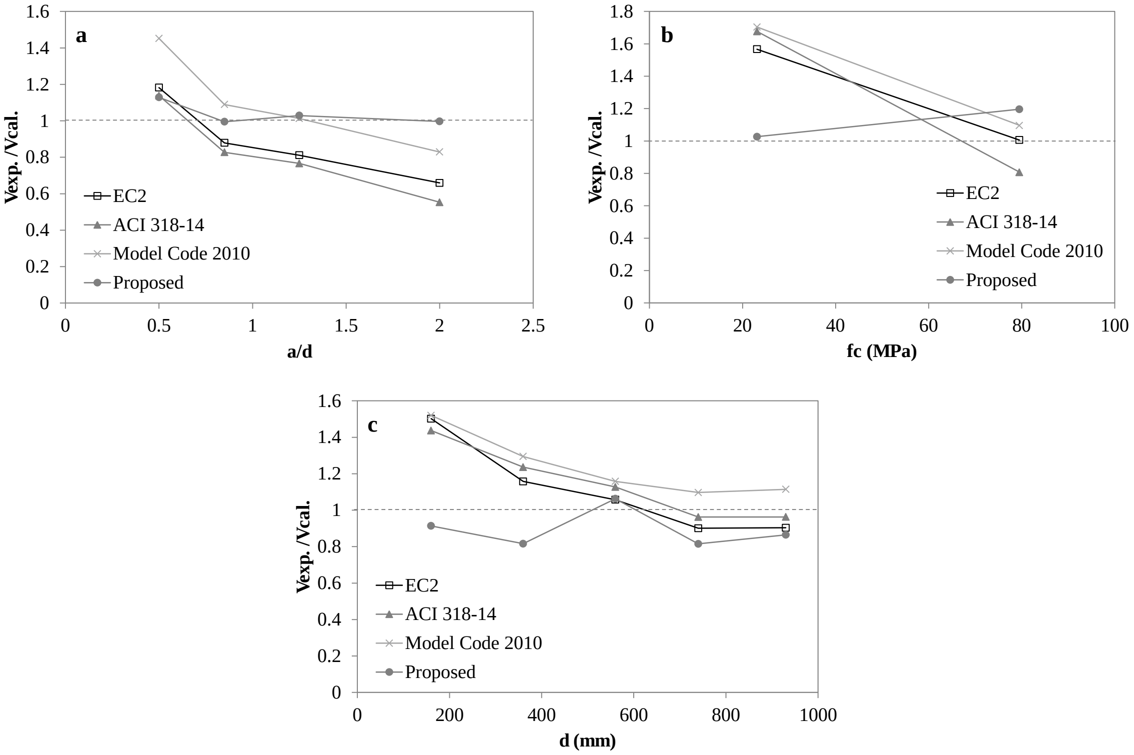

Vexp./Vcal.
0
1
2
3
4
5
6
7
0
20
40
60
80
100
f_{ck} (MPa)
d (mm)
0
500
1000
1500
2000
ρ (%)
0
1
2
3
4
5
ρ_v (%)
0
0.5
1
1.5
2
2.5
3


Vexp./Vcal.
0
1
2
3
4
5
6
7
0
20
40
60
80
100
f_{ck} (MPa)


Vexp./Vcal.
0
1
2
3
4
5
6
7
0
200
400
600
800
1000
1200
d (mm)


Vexp./Vcal.
0
1
2
3
4
5
6
7
0.0
2.0
4.0
6.0
8.0
ρ (%)


a
Vexp./Vcal.
a/d
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
0.5
1
1.5
2
2.5
EC2
ACI 318-14
Model Code 2010
Proposed
b
Vexp./Vcal.
fc (MPa)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
20
40
60
80
EC2
ACI 318-14
Model Code 2010
Proposed
c
Vexp./Vcal.
d (mm)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
0
200
400
600
800
EC2
ACI 318-14
Model Code 2010
Proposed


a
Vexp. /Vcal.
a/d
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
0.5
1
1.5
2
2.5
EC2
ACI 318-14
Model Code 2010
Proposed
b
Vexp. /Vcal.
fc (MPa)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
20
40
60
80
100
EC2
ACI 318-14
Model Code 2010
Proposed
c
Vexp. /Vcal.
d (mm)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
200
400
600
800
1000
EC2
ACI 318-14
Model Code 2010
Proposed