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USING GEOMETRY TO EVALUATE STRATEGIC ROAD PROPOSALS IN ORBITAL-RADIAL CITIES

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Using Geometry to Evaluate Strategic Road Proposals in Orbital-Radial Cities

Abstract

This paper uses geometry to evaluate major road proposals in cities with road networks consisting of orbital and radial routes. The type of geometry used is a development of the Karlsruhe or Moscow metric after the cities where it was identified, although the results have wider applicability. The paper begins with a detailed consideration of the relationship between route speeds, junction access and service areas. New urban patterns are presented using optimal space filling techniques in which the aim is to maximise drive-time coverage with the minimum number of junctions. The method is then refined to allow for effects such as congestion and interstitial access. The results are then used in a case study to evaluate a well-known strategic road plan for London first proposed in the 1940s. There follows a general discussion about the policy and planning implications for London and further possible developments of the techniques presented.

Introduction

Many cities are based on road networks that radiate outward from the city centre. A large number among them have orbital roads which deflect through traffic away from the centre but also permit local traffic to access suburbs more efficiently (Tripp, 1942). The number of orbital rings in each city varies, but two or more are not uncommon. Ring roads create conditions in which certain services find it advantageous to locate on or near them, especially ones that provide fast and easy access in radial as well as orbital directions. More generally a well-designed transport network confers benefits by improving equity and access, and also the environment.

This paper is concerned with an evaluation of road systems of this type, in particular those exemplified by the classic hub-spoke network. The efficiency and effectiveness of any road network is dependent on a number of factors including route capacity, junction spacing, congestion levels and so forth. Such factors are important in cities with densely populated urban cores, which are usually associated with higher congestion levels - considerations that are especially critical in the hub-spoke case.

The approach used is based on a geometric evaluation of the accessibility properties of such networks taking these factors into account. The role played by junctions is of particular interest as their position and spacing affects access levels by enabling changes of direction, in this case orthogonally. If they are spaced inefficiently then some areas will be less accessible whilst others will, in effect, be 'over-served'. The test of efficiency used is, in this sense, akin to concepts employed in space partitioning studies where the aim to try to fill the urban plane so that no junction is further than a given travel time away. Road junctions thus play a similar pivotal role to, say emergency depots (fire and ambulance services) and so it is argued comparable approaches are appropriate.

It is important to be clear why this is worth doing at all. Many road schemes are forced to adapt to changing traffic conditions, but frequently do so on a piecemeal basis. We choose a road scheme for London, initially published in 1944, namely Abercrombie's influential road plan for Greater London (Abercrombie, 1944), which broke this mould and which happens to be based a hub-spoke system. Whilst Abercrombie's plan was never completed it addresses problems that are just as relevant today, if not more so, and which remain central concerns of any future transport strategy for London. This includes the need to segregate long distance traffic from traffic of a purely local nature, to divert traffic from congested areas by one means or another, and to control parking.

At the heart of Abercrombie's strategic vision is a set of principles for improving communications throughout the urban area to meet the need, for example "….to reduce, as much as possible, time and money spent in diurnal travelling" and so produce an "enormous saving in transport costs" and to keep through traffic away from the centre. In addition, he wanted his plan to create suitable conditions for the decentralisation of population and employment so as to make way for the expected enormous growth in car usage in the post-war period. In this paper we use geometrical methods to revisit these objectives to determine the extent to which London today fulfils that vision, and to provide a test of the methods themselves.

Following a re-cap of the literature, we develop the geometrical methods that are relevant to this kind of network using space-filling optimisation techniques. We then adapt the methodology to take into account the effects of fast routes and congestion before applying the results to Abercrombie's plan and London's network today. We conclude with some general policy implications and suggestions for taking the work forward.

General issues arising

There is a large geographical literature on efficient space-filling techniques, which are primarily designed for the optimisation of settlement patterns or the location of services. Our aim is to adapt them for evaluation transport networks but first we need to consider the problem in general terms. Regular hexagonal areas form the starting point for most discussion of human settlement and service area patterns (Christaller, 1933) as they provide the most efficient way of covering a plane with the minimal number of locations. However, their application inside urban areas is restricted by transport factors such as congestion, street and route geometry. Mayhew & Hyman (1983) show how these can be overcome based on the methods of time surfaces and velocity fields.

Mayhew and Hyman (2000) describe the effects on travel time and routing in cities with fast radial and orbital roads, so introducing another type of geometry. We know that in gridded North American cities movement is constrained to rectilinear steps. It is easily shown that this produces diamond-shaped service areas (for example, see Anjoumani, 1981 and Richardson and Anjoumani, 1978). However, there has been no attempt to investigate equivalent patterns for orbital-radial cities. The first task is to consider the shape and size of service areas in these cases and how they mesh together optimally with no gaps or overlaps.

Initially, it is assumed there is a dense, effectively continuous, fast radial network with a series of rings representing fast orbital roads at suitably spaced intervals. Direct travel is assumed to be possible but it regarded as unusual given the presence of these faster radial and orbital routes. Instead, users are considered to reach their destinations via a series of radial and orbital movements. A variant of this type of metric has been dubbed the Karlsruhe or Moscow metric (Klein, 1988; Okabe, Boots, Sugihara, 1992) after the cities initially associated with the phenomenon (see also Hyman and Mayhew, 2000, forthcoming). This paper is a development of that metric using travel time rather than distance but it also introduces fast routes into the analysis.

A service area is defined in terms of the locus of points that can be reached in a given time from a fixed location or facility, also defined as an isochrone. Initially, we use the term junction or facility interchangeably to define the activity at that location without any loss of generality. The general shape of a service area delineated by an arbitrary isochrone for a fixed position directly ‘on’ the orbital is shown in Figure 1a for the orbital-radial case. A user located on the same radial as the facility reaches it without having to use the orbital road; similarly, an orbital-based user only uses the orbital. All other trips must be accomplished using a combination of radial and orbital legs.

The pointed southern extension is scaled using travel time or speed, so that it just touches the city centre. If radial speeds were any faster or the time allowed any greater then a circular appendage would develop at the centre. Trips originating inside this circle would divert through the city centre because it provides a quicker alternative to the orbital. This effect is shown in Figure 1b. In this case, the service area is the combination of two equal-value isochrones generated by two different routes, a double radial through the centre and the other via the orbital. This condition only arises when radial travel time through the centre is less than the value of the bounding isochrone. Since we are interested in cases that do not divert through the centre we do not consider this case further.

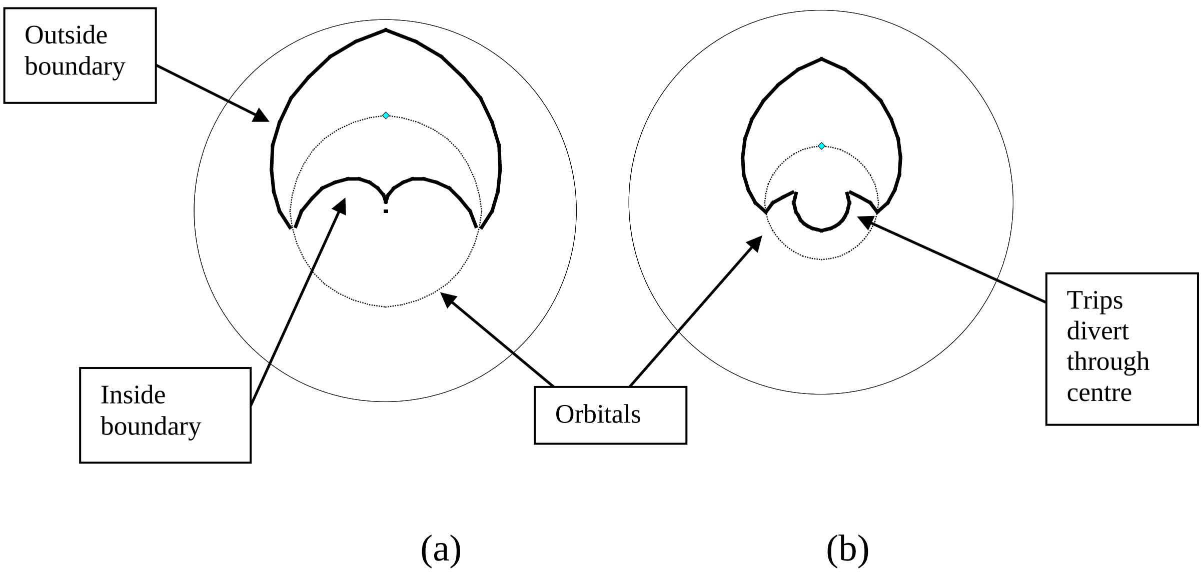


Figure 1. Service area shape for a facility or junction located on an urban orbital: (a) service area in which all trips involve use of the orbital. (b) service area in which some trips divert through the city centre as well as use the orbital.

The bounding isochrones for individual service areas have the following general forms depending on whether the boundary lies inside or outside the orbital. For brevity, only the basic equations for service area boundaries in the clockwise direction are shown ($0 \leq \theta \leq \pi$), in which due north is set to an angle of zero. The equivalent in the counter-clockwise direction is obtained by substituting $2\pi-\theta$ for θ, or through simple symmetry.

Inside orbital (clockwise direction)

$$\bar{r} = R(1+k\theta) - V_R t \qquad 0 \leq \bar{r} \leq R, \qquad 0 \leq \theta \leq \pi \qquad (1)$$

Outside orbital (clockwise direction)

$$\underline{r} = R(1-k\theta) + V_R t \qquad \underline{r} \geq R, \qquad 0 \leq \theta \leq \pi \qquad (2)$$

where r and $\underline{r}$ are the inside and outside boundary radii, R is the radius of the orbital, θ is the angle between the origin and the fixed location on the orbital, V_R is the radial speed, k is the ratio of the radial speed to the orbital speed (defined as V_R/V_O) and t is travel time. It is noted that the derivatives of r with respect to θ are equal except for sign; that is:

$$\frac{d\bar{r}}{d\theta} = -\frac{d\underline{r}}{d\theta} \qquad (3)$$

The orbital tip or apex of a service area makes an angle λ with the city centre given by,

$$\lambda = V_O t / Rk \qquad (4)$$

This result is used for evaluating the size of the service area by integrating the area inside the bounding isochrones and exploiting their symmetry. This yields the following integral equation,

$$A/2 = \int_0^{\lambda} \int_{\underline{r}}^{\bar{r}} r dr d\theta \qquad (5)$$

where A is the required area. As is shown in the annex, equation (5) eventually simplifies to:

$$A = 2V_R V_0 t^2 \qquad (6)$$

This result may be illustrated as follows. Suppose the radial and orbital velocities are 50k/h and 80 k/h respectively and the travel time is 0.25 hours (15 minutes), then the service area is 500 km^2. It is noted that the equation is independent of π as well as the orbital radius, a property which indirectly gives rise to the following limiting restriction. As is evident from Figure 1 the orbital tips must eventually wrap round and meet given a sufficiently large orbital speed or travel time, t. Where there is overlap a heart-shaped area is formed inside the orbital, the tips just touching when the origin to tip distance equals half the orbital circumference, or when

$$t = \pi R / V_O \qquad (7)$$

Of considerable interest is a connection between the formula in equation (6) and its equivalent based on the Manhattan metric. As already noted above the diamond-shaped service areas produced by the Manhattan metric contain straight-line boundaries intersecting the road pattern and creating four identical triangles. The distance from a fixed point to the tips of the diamond are $V_x t$ and $V_y t$ where V_x and V_y are the speeds in the x and y directions. It follows that the total enclosed area must be:

$$A = 2V_x V_y t^2 \qquad (8)$$

which is exactly the same as the result obtained for the orbital-radial case. Thus, service area size is independent of the movement geometry in these cases, a property that is exploited later. If the service population is continuously varying and a function of distance from the city centre then the population contained is found by multiplying $r\, dr d\theta$ by $D(r)$ the density function and integrating over the same limits. This is another useful property that can be employed, for example to evaluate the number of jobs or residences in a given travel time radius.

Meshing together services areas

We now address the problem of optimally meshing together the service areas using the minimum number without gaps or overlaps in order to complete the hub-spoke netwrok. We start by imagining two orbitals with the same number of junctions or facilities equally spaced around them but with the inner facilities ‘staggered’. Applying the bounding isochrone equations to the outer ring yields service areas having concave boundaries on the orbital interior and convex boundaries on the orbital exterior.

The question is whether the exterior boundary of the interior ring ‘fits’ exactly into the interior boundaries of the exterior ring so as to form a non-overlapping pattern. The necessary gradient property follows directly from equation (3), strongly suggesting that with suitable scaling an exact ‘fit’ might be possible. A more detailed comparison of the two equations shows that for this to occur the ratio of the speed on the inner orbital to its radius must equal the ratio of the speed on the outer orbital to its radius.

This means that speeds on outer rings need to be faster than speeds on inner rings - which clearly is not an unreasonable requirement if one examines actual orbital systems. This requirement also means that the time to circumnavigate the city is the same on either orbital.

Figure 2 is based on a system with two equally spaced orbitals and three facilities per ring. Uniform radial speeds are assumed so that inter-ring travel times are identical. In this example the ring radii, numbers of required facilities per ring, and required service time provided the necessary starting conditions. The required speeds are initialised to ensure the tips of each service area just touch and their boundaries mesh. An alternative approach, not considered here, would be to make the speeds endogenous (ensuring they were proportional to the radii) and then to calculate the required number of facilities on each ring. The pattern obtained represents the most “efficient” arrangement from an accessibility standpoint and logically flows from the orbital-radial assumptions

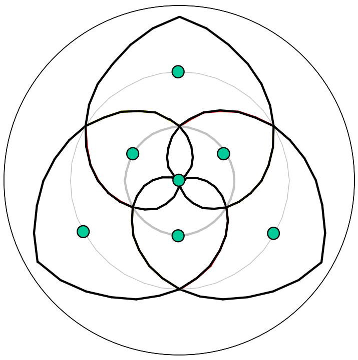


Figure 2. Nested service areas for seven facilities, three each located on the inner and outer ring and one at the centre (see text). The maximum travel time to any facility, 15 minutes, occurs at any point on a service area boundary. The ratio of the speeds on the inner and outer orbital equals the ratio of their radii.

With ring radii assumed to be 12.5kms and 25kms respectively and speeds of 52.35 k/h and 104.7 k/h on the orbitals and 50 k/h on the radials, the 15-minute service area on the outer ring is 654.4 km^2 (2 x 104.7 x 50 x 0.25 x 0.25) and on the inner ring 327.2 km^2 (2 x 52.35 x 50 x 0.25 x 0.25). Tip to tip distance is obtained by doubling the orbital velocity multiplied by the service time or alternatively dividing the circumference by three. Tip to tip distance on the radial axis is obtained by doubling the radial velocity multiplied by the service time. The time to circumnavigate either orbital is 1.5 hours.

It is noted that each service area follows the line of the bounding isochrones so that the maximum travel time applies the length of the boundary. This is in contrast to hexagonal patterns drawn in the uniform plane where bounding isochrones are circles of constant radius. Packing them efficiently means that the time standard applies only at the vertices of each hexagon. It will be noticed that there are three small 'uncovered' areas appearing as spokes in the interior of the inner orbital. These will always appear regardless of scaling. Travel time from these areas is slightly greater that the standard even if trips can divert through the city centre. The obvious solution is to add a seventh facility at the city centre itself.

The extent of the unserved area may be evaluated using the following equation for a single spoke, where S is the required area. This is based on partial integration of the portions of service areas that fall inside the inner orbital and gives,

$$S = \frac{\pi R^2}{3} - V_R V_O t^2 \left(1 - \frac{V_R t}{3R}\right) \qquad (9)$$

Using earlier parameter values the spoke area is 54.56 km^2 or three times this for the whole uncovered area, which in turn equates to 33.35% of the inner orbital area.

It is evident that the number rings is potentially unlimited although in practice the required speeds quickly grow unrealistically high as the urban radius increases. Although it is tempting to draw parallels with hexagonal constructions, it is noted that overlaying a hexagonal tessellation of facilities would not result in a co-incidence of locations as the spacing rules differ. Figure 3 provides a series of examples based on six facilities per ring for a 15-minute travel time standard. The results are shown in table 1. The time to circumnavigate any orbital is now 3 hours, exactly twice the previous example. The area of a spoke is given by the following modified version of equation (9),

$$S = \frac{\pi R^2}{6} - V_R V_O t^2 \left(1 - \frac{V_R t}{3R}\right) \qquad (10)$$

Example	Ring radius (kms)	Radial velocity V_O (k/h)	Orbital velocity V_R (k/h)	Service area km^2
A	12.5	50	26.18	163.59
B	25	50	52.35	327.19
C	37.5	50	78.53	490.78
D	50	50	104.7	654.38
E	62.5	50	130.88	817.97

Table 1: Results for a large urban area with one to six rings and with six facilities per ring. Travel time radius is assumed to be 15 minutes. In the absence of a central facility the ‘uncovered’ area would be 163.69 km^2 or 33.35% of the interior of the inner orbital. The ratio of the ring radius to orbital speed is 0.48. Figure 3 refers. Ring speeds reach unsustainable levels on ring D and E.

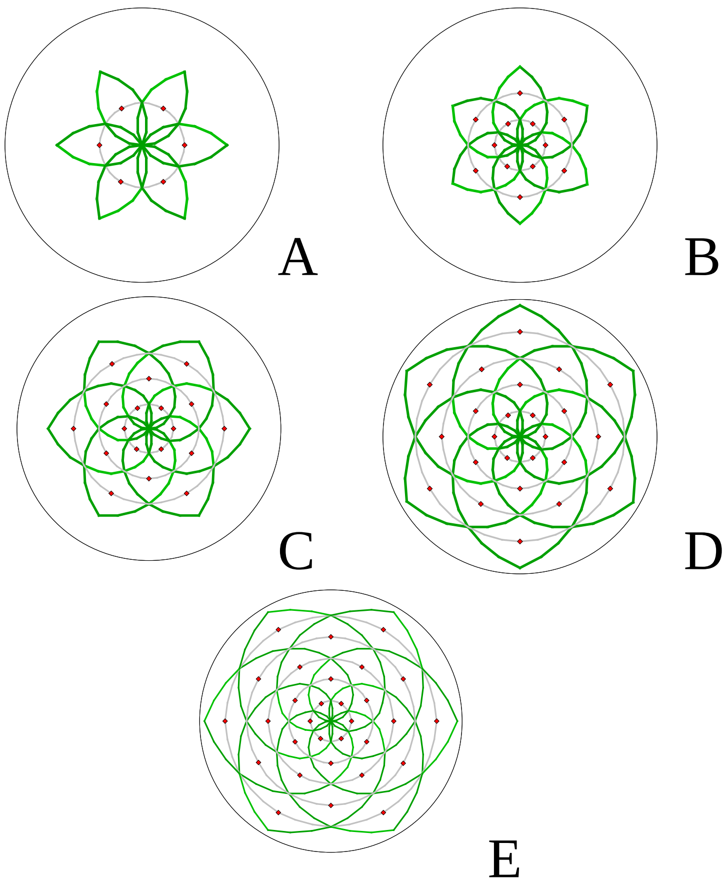


Figure 3: Optimal service area patterns for urban with one to five ring roads based on six facilities per ring. The area of a spoke is 27.27 km^2.

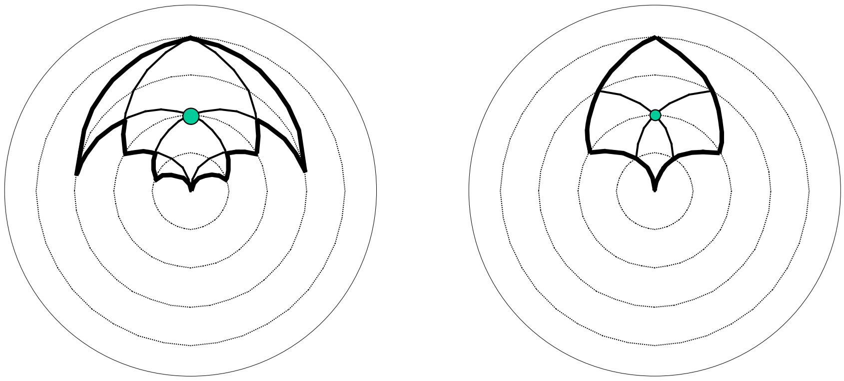
Hierarchical service areas

human settlement theory that in the uniform plane tiered hierarchies of service areas nest together, and that this property is used to explain why some settlements are bigger and offer more services than others. Our aim in this section is to establish the equivalent property for orbital-radial routing. Whilst not directly relevant to what follows it is suggestive of deeper properties about urban structures and transport infrastructure.

Consider Figure 4a, a four-ring example as in Figure 3d but with a 30-minute travel time standard so that the service area boundary is forced to penetrate adjacent orbitals. It is assumed that speeds on the first and third orbitals are no longer in proportion to their radius. The resultant composite service area now has three overlapping components, depending on which orbital route the user chooses to reach the facility. The union of the boundaries farthest from the facility denotes the outer service area boundary for that facility. Notice in particular how a higher assumed speed on ring three creates extensions that wrap farther round the ring.

It means that some areas closer to ring two find it quicker to access the facility via ring three, even though it is farther out. In Figure 3b, orbital speeds proportional orbital radius are re-instated and, as a result, it is observed that the service areas based on each orbital route now nest together exactly. It turns out the total area so delineated is the same shape and size as the service area corresponding to the ring on which the facility is located, that is ring two. A formal proof of this property is straightforward.

The significance of this result is that it allows us to overlay higher order centres onto the previous construction. In doing so the number of facilities in the second or higher tier covering wider areas must be a whole number which means that the number of facilities or junctions per ring in the first level must be at least divisible by two. Figure 5 shows a four-ring city with six facilities and three facilities per ring in levels one and two, the thickly drawn line denoting the boundary of the second tier facilities.



(a) ***(b)***

Figure 4: Service area boundary denoted by thickened line for a facility located on the second of four rings assuming a limiting time of 30 minutes, double the time in Figure 3. In 4a the ring speeds are dis-proportionate to the ring radii whereas in 4b they are assumed to be proportionate. Dotted lines indicate orbital roads.

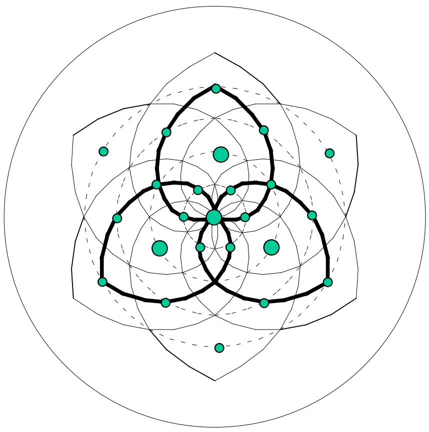


Figure 5: Two-level hierarchy with four rings and six facilities per ring in level one (small circular symbol). Three level two facilities are located on the second ring and one at the city centre (large circular symbol). Dotted lines indicate orbital roads.

Similar considerations apply to higher level as well as to lower level service areas. The unserved area contains spokes that now stretch and touch the second ring. The size of the second tier service area is four times the size of the first tier at 1312.5 km^2 using previous parameters. Clearly, the process of adding additional tiers is subject to simple rules about the number of facilities on each ring. A system with 16 facilities per ring would allow up to four tiers, whereas a system of 12 facilities would allow only three and so on. These results therefore illustrate the potential for evaluating urban structures on a number of different levels, and the possibility for extending orbital radial geometry to address other issues. Our primary task here, however, is an evaluation of the transport properties of such networks, the subject to which we now return.

Inclusion of fast radial routes and congestion effects

Thus far we have assumed a continuum of radial routes as well as constant radial speeds. In order to evaluate Abercrombie's plan we need more explicit tools that focus on access times to major junctions and address the issue speed differences between inner and outer parts of the urban area. Lets us start with the first of our initial assumptions, namely a high density of radial routes. This is not as unreasonable as it seems; much depends on the city, the density of its road network and the distance from the centre.

In this regard Hathaway's (1974) small but finely detailed study of isochrone patterns in London showed isochrones based on car travel to the centre to be a series of concentric circles as far out as 13kms. In other words, in central areas traffic speeds tend to be uniform regardless of class of road. However, as the urban radius increases interstitial gaps appear between fast radial and local routes creating speed differentials which gradually deform the shape of the isochrones. We endeavour to capture this effect in the following way. We do so by assuming interstitial speeds are a function of the local road network and are lower than on fast routes.

Let the interstitial 'all-purpose network' speed be V_A where $V_A < V_R, V_O$. Assume users access nearest junctions on strategic sections of the network using either the nearest fast radial or orbital - whichever minimises their journey time. They do so in two steps, either by a radial and then an arc movement or by an arc and then a radial movement. Depending on whether the journey origin is inside or outside the ring in question and to maximise the range of the service area at a given junction for a given travel time radius, the route that minimises or maximises r *is* selected. Let k_O and k_R be the ratio of the interstitial speed to the orbital speed and radial speed respectively. The bounding isochrone equations of service areas are as follows.

Inside orbital (clockwise direction)

$$\bar{r} = \min\{f_1(\circ), f_2(\circ)\} \qquad 0 \le \bar{r} \le R \qquad (11)$$

Where,

$$f_1(\circ) = R(1 + k_O\theta) - V_A t \qquad (12)$$

and

$$f_2(\circ) = (k_R R - V_A t)(k_R - \theta)^{-1} \qquad (13)$$

Outside orbital (clockwise direction)

$$\underline{r} = \max\{z_1(\circ), z_2(\circ)\} \qquad \underline{r} \ge R \qquad (14)$$

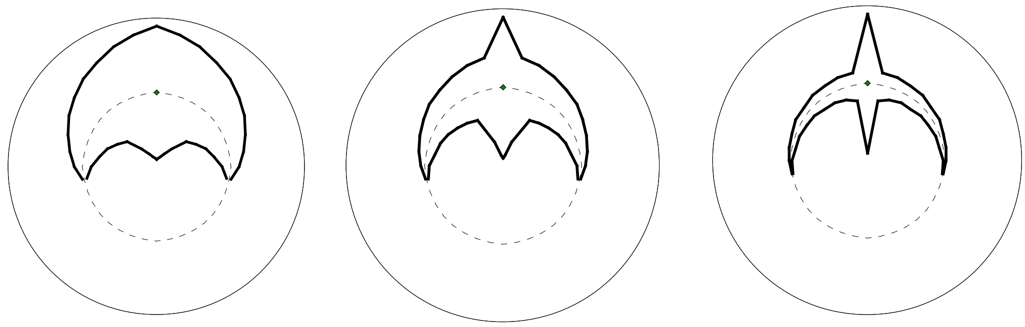
Where,

$$z_1(\circ) = R(1 - k_O\theta) + V_A t \qquad (15)$$

and

$$z_2(\circ) = (k_R R + V_A t)(k_R + \theta)^{-1} \qquad (16)$$

Figure 6 shows the effect of this modification on the basic service area shape for three examples. In 6a the interstitial speed is the same as the fast radial speed in which case we obtain the familiar result already seen in Figure 1. In 6b and 6c the interstitial speeds are reduced (see caption), in which cases the junction service area implodes inwards. As might be expected intuitively it acquires two spikes on the radial axis, whereas the bounding isochrones on the orbital access reduce to a narrow channel either side of the orbital. Again, such effects have an analogy with the Manhattan metric in which the usual diamond shape converts into a four-cornered star.



(a) (b) (c)

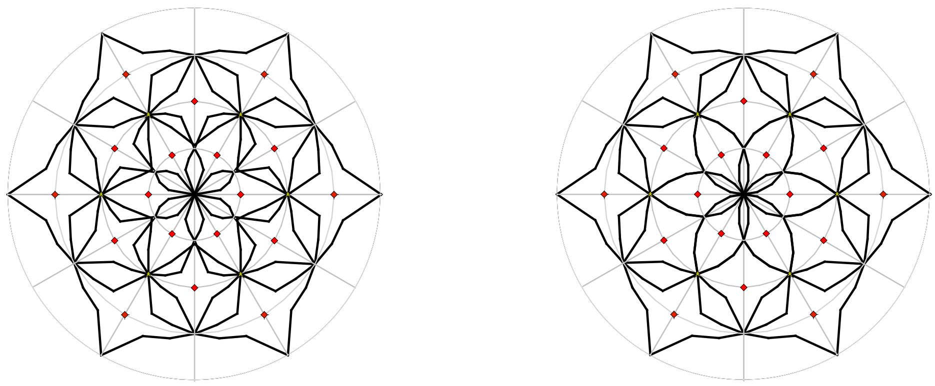
Figure 6: 27- minute service areas for fast orbital radial routes with reduced interstitial speeds on the all-purpose network. Urban ring is 25 kms with orbital and radial speeds of 100 k/h and 50 k/h. The interstitial speeds are (a) 50k/h (b) 30 k/h) and (c) 10 k/h. The service areas are (a) 2205 km^2 (b) 1156 km^2 (c) 504 km^2.

Since the corners of the pointed or 'ragged' service areas in 6 b and c are positioned exactly as they are in 6a they can substitute for one another precisely. Of course, there are now gaps as the boundaries are no longer co-terminus as they were in our previous discussion on service areas. However, if those gaps are underdeveloped and sparsely populated the accessibility loss will be small. The equation used for determining the areas in Figure 6 b and c is based on an extension of the analogy in equation 6 with a rectilinear gridded city. It is,

$$A = 2V_A t^2 \left\{ V_R \frac{1-k_R}{1-k_R k_O} + V_O \left(1 - \frac{k_O(1-k_R)}{1-k_R k_O}\right) \right\} \qquad (16)$$

Where k_R and k_O are the ratios of the interstitial speed to the radial and orbital speeds respectively. As may be verified this equation reduces to equation (6) when the interstitial speed equals the radial speed (i.e. V_A=V_R). The formula is valuable for estimating the approximate population, household and jobs contained within a given travel time radius, although it should be applied with care and only to small areas particularly if local densities and speeds vary significantly. For example, suppose household density is 20 p/ha. Assuming the parameters in the caption to figure 6c this would imply there are around 1m households in a 27-minute travel time radius around a given junction. We discuss this finding in an Abercrombie context below.

Figures 7a and 7b scale these results up to the urban area. They show patterns based on a three-ring urban area, six junctions per ring and 12 radials. The first, 7a, assumes interstitial speeds are the same throughout the urban area whereas the second assumes lower interstitial speeds are only relevant beyond ring two. The effect is as before except the urban boundary now has a 'ragged' edge.



(a) (b)

Figure 7 Tessellated urban areas with 12 fast radials, 3 orbital roads and six facilities per ring (a) slow interstitial speeds everywhere (b) slow interstitial speeds beyond the second orbital.

The second modification concerns our initial assumption that radial speeds are uniform. The problem, as is immediately evident in virtually all cities, is that traffic density exceeds road capacity near the city centre and so average speeds are significantly lower. It follows that the positioning of rings will need to be adjusted if inter-ring travel times are to be equalised and constant time standards between junctions are to be achieved. It is possible to consider almost any continuous function, or piece-wise combination of functions, relating speed to radius to address this problem. The most natural, and one which is often verified empirically (e.g. see Hyman and Mayhew, 2000) is to assume that radial speeds increase in proportion to distance from the city centre. However, it is clear that speeds cannot increase indefinitely and so it is further assumed these must level off at some point.

Interstitial speeds need separate consideration. It is assumed they increase initially at the same rate as radial speeds in the central area where the road network is densest, in line with Hathaway's findings (Hathaway, 1974). This is because, as previously noted, traffic tends to distribute itself so that speeds are fairly uniform regardless of road type. Farther out, as gaps in radials appear it is assumed interstitial speeds level off; then, as local road networks themselves reduce in density, it is assumed interstitial speeds start to fall again. Such assumptions are flexible and do not materially affect the evaluation although they do affect the final patterns obtained.

To determine the equations of the bounding isochrones in the non-linear part of the urban region we proceed as before. Let the function describing radial speeds on the arterials be,

$$V_R = \omega R \qquad (18)$$

Where ω is a constant of proportionality and R is the distance from the city centre. The inter-ring time between R_1 and R_2 is given by

$$t = \int_{R_2}^{R_1} \frac{dr}{\omega r} = \frac{1}{\omega} \left[\ln \frac{R_2}{R_1} \right] \qquad R1 \neq 0 \qquad (19)$$

The associated isochrone equations bounding the service areas are given by,

Inside boundary (clockwise direction)

$$\bar{r} = \mathrm{Re}\, xp(\omega[\frac{R\theta}{V_O} - t]) \qquad 0 \leq \bar{r} \leq R \qquad (20)$$

Outside boundary (clockwise direction)

$$\underline{r} = \mathrm{Re}\, xp(\omega[t - \frac{R\theta}{V_O}]) \qquad \underline{r} \leq R \qquad (21)$$

Let interstitial speeds increase according to

$$V = \phi R \qquad where \qquad \phi < \omega \qquad (22)$$

The bounding isochrones are as follows:

Inside orbital (clockwise direction)

$$\bar{r} = \min\{f_1(\circ), f_2(\circ)\} \qquad 0 \le \bar{r} \le R \qquad (11)$$

Where

$$f_1(\circ) = \operatorname{Re} xp - (\omega(t - \frac{\theta}{\phi})) \qquad (23)$$

and

$$f_2(\circ) = \operatorname{Re} xp - (\phi(t - \frac{R\theta}{V_O})) \qquad (24)$$

Outside orbital (clockwise direction)

$$\underline{r} = \max\{z_1(\circ), z_2(\circ)\} \qquad \bar{r} \ge R \qquad (14)$$

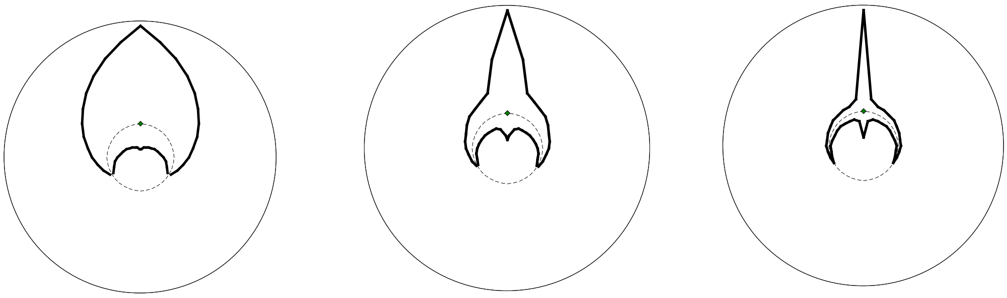
Where

$$z_1(\circ) = \operatorname{Re} xp(\omega(t - \frac{\theta}{\phi})) \qquad (25)$$

and

$$z_2(\circ) = \operatorname{Re} xp(\phi(t - \frac{R\theta}{V_O})) \qquad (26)$$

Figure 8 shows the effect of this refinement on a previous illustration. As might be expected increased speeds cause the service area to stretch along the radials with increasing distance from the city centre.



(a) (b) (c)

Figure 8: 27-minute isochrones with increasing speed gradient from the city centre, where the ring radius is 12.5 kms, and orbital speed is 60km per hour. In (a) ω is 3 and ϕ is 3, (b) 3 and 1.0 (c) 3 and 0.5.

Application to London

This completes the description of the geometric methodology and our attention now turns to the case study based on Abercrombie's plan for London. It is important to be clear on the objectives of the case study; there are four aspects:

- To use the methodology to 'fit', as closely as possible, the theoretical ring-radial structure to the Abercrombie's plan and to note any important differences which might indicate weakness or gaps in the plan;

- To compare the theoretical speeds that would be needed on each element of the network with those currently being achieved to see whether they are realistic or not and to consider the planning implications;

- To check whether those speeds would have the effect of diverting through traffic away from the inner core, as Abercrombie had intended, and if not why;

- To analyse the present distribution of population and employment to see the extent Abercrombie's plan would assist his goal of locating employment near people.

Let us begin with a brief description of the plan itself. Abercrombie envisaged a giant cartwheel of 5 concentric orbital routes (A to E) and 10 major routes, radiating outwards from the second ring, B (see Figure 9a). Junctions in the system would act, in effect, as gateways to different parts of London and also provide local access via minor roads, with new employment appearing in between (subject to land use zoning) which would have the effect of enabling local communities to become economically more self-sufficient.

Abercrombie, basing his ideas on the County of London Plan (LCC, 1943), drew a three-fold distinction between types of major road calling them “express arterials”, essentially motorways in today’s terminology, “arterials” and “sub- arterials.” From the maps he provided proposed ring radii averaged about 2.8, 5.8, 12.5, 20 and 27 kms from the city centre, the second and fourth of these being designated arterial and express arterial, and the others sub-arterial. Abercrombie is imprecise as to the kinds of speeds he expected on each class of road in his plan although his designation of roads into sub-arterial, arterials and express arterials supplies important clues about his intentions and assumptions.

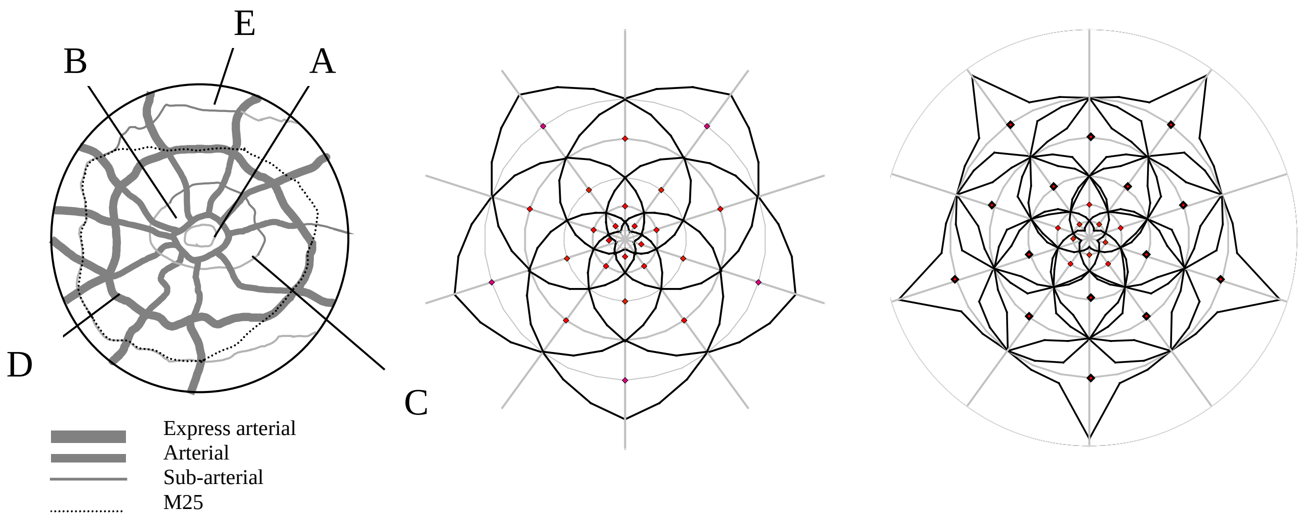
What Abercrombie meant by 'through traffic' is not entirely clear. For the purposes of this analysis, we assume it is a matter of determining, for each ring, whether it would be quicker for a user to use the nearest ring, one farther in or to divert through the centre. Through or external traffic is defined for these purposes as traffic that originates and terminates outside the nearest ring to which it relates. If the quickest route cuts into that ring to access a closer ring then it is deemed to be counter to Abercrombie's aim.

We arbitrarily choose ten minutes as the time standard for junction access. This is judgmental and the result of several iterations to find a set of speeds most closely resembling reality. Statistics on traffic speeds in London, although readily available, have only been collected since 1968. However, it is not particularly material which year one picks as change tends to take place very slowly. Data are based on measurements sampled from over 1,300 miles of roads inside the M25 orbital (DETR, 1998).

Figures are available on peak, average and off-peak conditions on trunk (equivalent to arterial in Abercrombie's terminology) and other roads but not on individual routes except for the M25 orbital (equivalent to express arterial). They indicate that peak and off-peak speeds on the M25 and trunk roads are similar at 82k/h for off-peak and between 58k/h and 60 k/h during the peak. On other roads in outer London the averages range between 27k/h and 37k/h. No detail is available for the speed gradient in central London as only an average is given - which is effectively the same in peak and off-peak conditions at 16k/h.

Figure 9a shows the key features of Abercrombie's plan in terms of the main rings and radials. Fast radials connecting with the fourth ring are of express arterial standard, becoming sub-arterial as far as second ring where they terminate. It is noted that ring E does not connect up on the eastern side of the region and so cannot be considered a completed orbital for these purposes. A dotted line shows the route of today's M25 which, as is seen, is a hybrid of D and ring E.

Figure 9b and c show the tessellated urban area of London using the previously determined speed gradient and 10-minute access standard. There are 10 radials spaced at 36-degree intervals, the same number of radial routes indicated in the Abercrombie plan. The difference between 9b and 9c is that there is no assumed differential interstitial speed in 9b whereas in 9c it is assumed there is - which is why one obtains the pronounced radial extensions. Table 2 gives the speeds and ring spacing, necessary to achieve the selected 10-minute time standard show in these diagrams. As is seen, there are several striking similarities between the model and the plan but also some differences. We now consider each aspect in turn in relation to the original case study objectives.



(a) *(b)* *(c)*

Figure 9: (a) Abercrombie plan and today's M25 (b) tessellations with undifferentiated interstitial speeds (c) tessellations with differentiated interstitial speeds (based on a 10-minute travel time radius).

Ring	Intra-ring radial speed(k/h)	Orbital speed(k/h)	Interstitial Speeds(k/h)	Model rings(kms)	Abercrombie rings(kms)
O	$V_R=\omega R$	7	$V_A=\omega R$	1.85	n.a.
A	$V_R=\omega R$	12.9	$V_A=\omega R$	3.43	2.8
B	$V_R=\omega R$	24.2	$V_A=\phi R$	6.41	5.8
C	45	45.2	30	11.99	12.5
D	45	73.1	20	19.4	20
E	45 (70)=	101.8	15	27	27

$\omega = 3.753$

$\phi = 2.5$

Table 2. Results showing actual and predicted rings, and assumed speeds. As radial and interstitial speeds are assumed to vary continuously in rings O, A, and B only the relevant formulae are shown with the parameters in a footnote.

Ring-radial spacing

As is seen from Table 2, the model replicates Abercrombie's ring spacing quite closely. One very important difference however, is that the model indicates a need for a sixth ring (designated O in the table) near the centre of the city of just over 1.85 km radius - that is a ring inside Abercrombie's innermost ring A. As far as radials are concerned the actual average angle of separation in Abercrombie is 36 degrees, exactly the same as in the model (analysis shows there is no statistically significant difference between them). The main deviant radial is situated in the southeast sector where a maximum separation of 66 degrees is obtained. Examination of Abercrombie's plan shows this is caused primarily by a change of direction between ring C and D in the relevant radial. In fact Abercrombie's more detailed map allows for an existing sub-arterial route to fill the gap but he apparently thought it unnecessary to designate it arterial status.

Orbital-radial speeds

Table 2 shows the predicted orbital speeds needed to be achieved if Abercrombie's plan were to be implemented. They seem plausible when compared with current actual averages, although plainly the predicted speed deemed to be necessary on the outer ring (E) would be very stretching. It is clear Abercrombie regarded the outer ring as mainly recreational, so the comparison is somewhat unfair in this regard. However, ring C is more problematic. Whilst the required speed is realistic, its sub-arterial designation (see also below) would not support the concentrations of traffic experienced today. For ring B, designated 'arterial' the opposite is the case - that is the road seems over designated for the speed deemed to be required.

Predicted radial speeds implied in Abercrombie's plan are also generally reasonable. However, the speed predicted between rings C and E appears to be too low compared even with today's peak averages (45k/h compared with 58-60 k/h). This could suggest these rings are too close to one another although it could also mean the radials have been over-designated. As for the central area itself, it is difficult to reconcile the finely differentiated speeds assumed inside ring C with published figures, which are averaged over a wide area. This suggests a need for further study of this aspect using a more detailed speed profile.

Through traffic

Turning to through traffic, and Abercrombie's aim to divert it, predicted ring speeds indicate there would be no time advantage in using anything besides the first inward ring encountered. This is a simple consequence of the fact that it takes the same time to travel an angle of arc regardless of which orbital one uses. Hence, there is no time saving in using one farther in. Indeed, there would be at least a 20-minute time penalty resulting from travel on the extra radial legs.

For journeys starting midway between two orbitals either orbital is theoretically possible but if the origin were nearer to the outer of the two orbitals then this route would be quicker. Because of the speed gradient applying on radials inside the first three rings there would be an added preference for the outer ring in this situation. These observations apply even on the innermost ring, since it would always be quicker to go round up to half a circumference than to pass through the city centre so keeping the most congested part of the city free of through traffic. We conclude therefore that Abercrombie's plan would meet this particular objective in full.

Taking the first three of Abercrombie's objectives together we can say in summary that the main problems relate to the correct designations of rings. The results appear to indicate that B is over-designated and C is under-designated from the point of view of capacity; secondly that the spacing between rings D and E appears to be insufficient; and thirdly the fact that the analysis has indicated the possible need for another inner ring which we have called for the purposes of this paper ring O. However, his objective to divert through traffic would be met in full. We now turn to the last of Abercrombie's objectives, which is the question of bringing jobs closer to people.

Bringing jobs closer to people

In fact, whether Abercrombie's plan would have resulted in a more even distribution of population and jobs cannot be answered conclusively as it was never completed. The main discrepancies between his plan and today's network of roads relate to the orbitals - that is what was planned and what was actually built. The M25 orbital, finally completed in 1986, is a hybrid of Abercrombie's ring D and E (see figure 9a) and thus appears to address satisfactorily the problem of spacing issue identified above.

Ring C, identified in our analysis as being strategically important but under-designated, corresponds to the North and South Circular roads of today. This route is of high quality only in locally improved sections and many bottlenecks remain especially on the south side. As far as ring B is concerned this was proposed but then abandoned in the 1970s. Ring A, an interconnected series of existing roads, continues to function as a link between the main London rail termini although this is no longer its primary function. Finally, as noted, there is no readily identifiable ring O, although the model suggests one would be desirable.

Despite only a partial correspondence between Abercrombie and today, one can, it is argued, form judgements on the basis present distributions of jobs and households, if only to see how far they are out of kilter. A number of hypotheses are possible. For example, if Abercrombie were correct one would expect the area covered in a given 'drive time' to increase with speed but to what extent would this offset the thinning out of population and employment? Alternately, to what extent has the failure to complete the plan contributed to differences we may observe today between people and jobs?

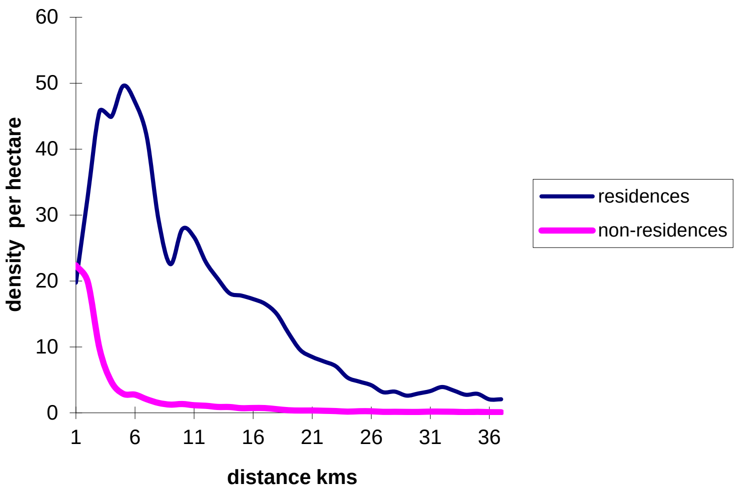


Figure 10: The density of urban households and employers with distance from central London (based on postal addresses).

Figure 10 shows the density of residential and non-residential addresses on distance from the centre of London, based on a ring count at 1km intervals. In using these measures it is implicitly assumed that they are proxies for households (population) and employers (jobs). The graph shows the heaviest concentration of employers at or near the city centre, but a ring of high residential density peaking at just under 6 kms out. Our first reaction therefore would be to suppose that Abercrombie, in its admittedly partially completed form, has had little or no effect as households and employers remain significantly mismatched. However, this view would be based on using distance as a measure of accessibility rather than travel time.

If one assumes the average speeds for each ring as shown in table 2, then the associated service areas may be approximated using equation (17) to give a somewhat different perspective. Table 3, based on this procedure, shows the area, households and employers accessible in a 10-minute drive time whilst the final column shows the ratio of households to employers. Based on the similarity in ratio it appears there is a stronger correspondence than might have been supposed between rings A and E despite the fact that there are huge differences in areas covered (column 2). In practical terms it means that potential employees would have broadly similar access to jobs if they were based in either ring.

On rings B, C and D, with far higher ratios, the picture is quite different. Here, it is evident there is much poorer local access to local jobs than elsewhere. A contributory factor to this, it is suggested, is the absence of a high quality orbital system, although without more evidence it would be overstating the case to say that this situation would not have arisen if Abercrombie had been fully implemented. Finally, ring O, which envelops part of the central business districts stands alone in having a very low ratio, which is the result of a combination of high employer and low residential density.

One of the problems this analysis points to is that, notwithstanding the M25, it appears there has been more investment in radial than orbital routes. For example, there are now over 20 major intersections on the M25 compared with the much smaller number proposed by Abercrombie. Road improvements inside ring C have been limited in scale, comprising a combination essentially of better traffic management schemes, parking restrictions and junction improvements. Circulatory flow has been aided, to a degree, by the introduction of the 'red route' network, initiated in 1989 which has strict parking restrictions and priority public transport lanes. However, a conclusion of this analysis is that the overall impact of these initiatives is small compared with what would have happened if Abercrombie had been implemented.

Ring	Radius Kms	Area Kms^2	Households (000s)	Employers (000s)	Col (4)/Col(5)
O	1.85	2.7	81.2	54.2	1.5
A	3.43	9.26	418.5	66.7	6.27
B	6.41	24.27	116.5	6.8	17.13
C	11.99	90.64	207.6	12.1	17.16
D	19.4	99.18	94.4	6.2	15.23
E	27	122.5	383.4	59.62	6.43

Table 3: Households and employers in a 10-minute radius from locations on each orbital ring in London

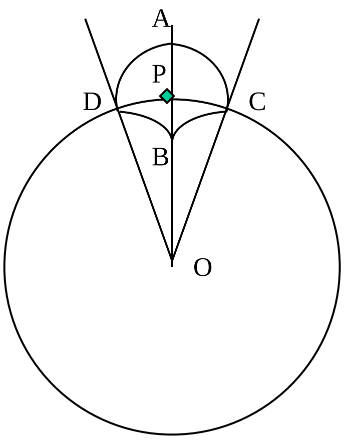
Concluding remarks

The methodology presented here is intended as a tool for evaluating strategic networks in cities and for investigating wider issues to do with relationship between urban form and accessibility. For various reasons grand road schemes on the scale of Abercrombie are out of fashion although the basic concepts themselves remain valid. Nevertheless, the techniques used could be developed to consider any kind orbital-radial movement, even where the issues are only to do with incremental improvement and making better use of existing road capacity. In other words they could provide a benchmark or standard for the strategic evaluation of such schemes, although plainly they would be less useful at a more micro or tactical level.

One thing that Abercrombie failed to address convincingly and which this paper does not address at all was the question of improving public transport infrastructure, either as complementary to his road proposals or as a subject in its own right. For example, he might have analysed, and reached conclusions about how the essentially radially focussed rail network in London could be made to function more effectively by creating more interchanges, cross-London routes and so forth. It is believed these issues are also amenable to the evaluation methodology described in this paper so have the potential to make a contribution to public transport strategy. One possible development could be to evaluate the necessary conditions for integrating services based on population distribution, local access, service frequencies and costs.

There are hence several areas for future research. They include the areas suggested plus, more importantly, developing the methodology to suit other kinds of cities and routing schema. Some progress has already been made in this direction as well as in applying the techniques to other kinds of spatial problems.

<u>Annex:</u> Evaluating the service area of an orbital located facility



This annex derives the service area for a facility located on the orbital (see figure, ABCD). From equations (1) and (2) in the main text,

$$\bar{r} = R - V_R t + Rk\theta$$

$$\underline{r} = R + V_R t - Rk\theta$$

Thus,

$$\bar{r} + \underline{r} = 2R$$

$$\bar{r} - \underline{r} = 2(V_R t - Rk\theta)$$

From equation (4) we note

$$\lambda = V_O t / Rk$$

$$= V_R t / R$$

Let A be the area of ABCD

$$\frac{A}{2} = \int_0^{\lambda}\int_0^{\bar{r}} r dr d\theta - \int_0^{\lambda}\int_0^{\underline{r}} r dr d\theta = \frac{1}{2}\int_0^{\lambda} (\overline{r^2} - \underline{r^2}) d\theta$$

$$= 2R\int_0^{\lambda} (V_R t - Rk\theta) d\theta$$

Thus,

$$= \lambda R V_R t$$

$$A = 2V_R V_O t^2$$

$$= V_R V_O t$$

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