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The Properties of Route Catchments in Orbital-Radial Cities

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Abstract

Orbital roads are an important feature of many cities. They facilitate movement and the flow of traffic linking together peripheral areas whilst avoiding city centres and are used both by local and through traffic. This paper analyses route catchments where routes are either through the city centre or use orbitals for some of the journey, and how they vary with comparative speeds, and journey origin and destination. The paper starts with the 'Karlsruhe metric' and then proceeds with several generalisations. The concepts of hub, rim, core and isovalent points are exemplified. These concepts are potentially useful for exploring transport policy at a strategic level, and for generally improving our understanding of urban structure. The paper concludes with a detailed application to London.

Introduction

Geographers and planners are used to thinking about catchment areas where the aim is to delineate an area influenced or served by a facility or group of facilities. In transportation studies it is also common to talk in terms of catchment areas for railways, roads and other transport corridors serving particular destinations. They are defined as the area in which it is quicker to reach a given destination by a particular class of route than via any other, alternative class of route. For example, suppose it is possible to travel either directly to a destination or to make a detour along a fast highway. From a given start location, the set of locations that are quicker to reach via the fast highway is defined as the fast highway catchment. Whole areas may be divided up in this way as maps depending on the number and class of routes.

We are interested here in route catchments in cities for which a number of ready-made stereotypical network models already exist (e.g. see Haggett and Chorley, 1969 or Vaughan, 1987). In particular, we examine cities with a strong radial orientation of road or rail that are served by one or more fast orbitals. The models and techniques required can be viewed as extensions to those developed for rectangular grid road networks, as typified in North American cities (Anjoumani A, 1981). In a previous paper, we examined routing patterns in cities modelled using a combination of fast routes and slower omni-directional travel in a uniform plane (Mayhew and Hyman, 2000). Models of this form were also employed in O'Kelly 1989 to examine the effects of low cost routes on rents and crop patterns. This paper differs in the following ways: first, we depart from models based on plane geometry and instead concentrate on cities with directional movements either along radials, around orbitals or using some combination of both. Secondly, our methodology is more powerful because it adopts a general analytical treatment. This approach provides more easily programmable results that can be widely applied. We present an application extending our previous London case study and identify new strategic implications of the results.

It is useful to recap briefly on why cities have orbital routes at all. Discussing the principles of road design in his book “Town Planning and Road Traffic (1942) Tripp said: “a circular road of extreme merit must surround the town centre and heart of the city…. their purpose is to promote the amenity of the central area by deflecting from it all vehicles, which do not have to traverse it". Of course, ring roads are also built for safety and speed, ideally having greater capacity and sustaining higher speeds, than the radial routes, which they connect (Ministry of Transport, 1963).

Implied in this is a presumption that some users find it quicker to use the orbital than to go through the city centre. However, the efficacy of the orbital road, as we shall show, is a function not only of its design and capacity but also of its distance from the city centre and of the speeds on the routes that radiate from the centre. It will also depend on where a journey begins and ends, particularly whether one or other end is inside or outside the orbital road. This has important implications for traffic management and road construction. If speeds can regulated it would be theoretically possible to influence the types of trips and therefore volume of traffic that divert through the centre or around the orbital.

Consider an orbital route that is comparatively slow in relation to radial speeds. Traffic passing right across the city would have little incentive to use of the orbital, and would tend to reduce trip length by cutting through the central area. Conversely, if city centre travel were banned, or if there were heavy central area congestion, orbital traffic would be expected to increase significantly. This could have a significant impact on local trading patterns and indeed is one reason why some supermarkets favour out-of-town sites near fast routes. Alternatively, suppose that a cordon charge on vehicles entering the city was imposed just inside the orbital route. Local traffic would now be tempted to cut through the centre. This could result in increased city centre congestion, thereby reversing the policy's intended effect.

Our task is to examine the determinants of the different types of traffic that are likely to make use of orbital and radial routes so that these and other questions may be addressed more systematically. In order to focus on the key issues we adopt a number of simplifying assumptions. For example we assume users are rational and use the route offering the quickest journey time. We also assume that route choices are based on long-run average speeds, so that they are independent of daily fluctuations in travel demand. Both of these assumptions can be relaxed but our aim is to keep the analysis as simple as possible so as to highlight the fundamental analytical results, whilst developing techniques and methods that are of wide general applicability.

Analytical approach

Network models, using nodes and links are typically used to represent transport systems. Such models can incorporate the rich level of detail that is generally needed to appraise transport system improvements. However this very richness of detail can make it difficult to understand the general morphological implications of alternative strategic plans for the development of the network. Additionally, the results so obtained tend be unique to the area being modelled, making it difficult to draw any general conclusions or to make meaningful comparisons between different locations. Detailed network models also make it difficult to make more generic comparisons between types of area, such as between large conurbations, cities, small towns and rural areas.

Smeed (1963) analysed generalised routing systems deriving catchment patterns in the case of direct versus orbital routing, and otherwise laid down some of the basic concepts. Angel and Hyman (1976) analysed the properties of continuous *velocity fields* in which speeds increased with radial distance from the centre**.** Potential applications and extensions to the concept were considered in Mayhew 1981 and Hyman and Mayhew, 1982. A key assumption of velocity field models is that travel is possible in all directions but speeds are independent of the direction of travel.

The approach adopted in this paper is based on quite different assumptions. The basic continuous field approach is retained, but travel is assumed to be restricted to radial and orbital directions. Speeds can be different in orbital and radial directions, and speeds may vary with distance from the city centre. Although the basic concepts have been developed independently, a convenient starting point for our analysis is an abstract model named after the city of Karlsruhe which is a particularly good example of this kind of metric (Klein, 1988; Okabe A, B Boots & K Sugihara, 1992).

The Karlsruhe Metric

We recall that a *route catchment* is defined as the area within which travel to or from a fixed point takes less time via its associated class of routes than by any other class of routes. In the Karlsruhe model routing is either via the centre or via an orbital, so these alternatives define the route catchments of interest.

The city is assumed to be circular and polar co-ordinates are used to represent location. The city centre is taken as the point with a radius of zero. The angular co-ordinate is assumed to vary between $-\pi$ and π, with zero pointing due north.

Let (r,θ) be a varying point and let $(r_1,0)$ be a fixed point. To obtain the shortest route between these points we can our confine attention to:

a) a double radial route, via the city centre,
b) a single radial segment plus a single orbital segment at a radius equal to the lesser of the radii of the fixed and varying points.

For $\theta=0$ the shortest route is a direct radial route. For the purposes of analysis this is best treated as a special case of (b), for which the orbital segment has negligible length.

The Karlsruhe (distance) metric, between two general points, is defined by the equation:

$$KD = \text{Min}\,[r_1 + r, \text{Min}\,[r_1, r]\,|\theta| + |r_1 - r|] \quad (1)$$

The first term in the outer minimisation corresponds to double radial routing, the second term to routing using the orbital. It can be readily verified that the catchment area for orbital routing is given by:

$$|\theta| \leq 2 \quad (2)$$

This is because a circular arc subtending an angle of 2 radians at the city centre has the same length as two radii from the centre to the ends of that arc.

The basic assumptions behind the Karlsruhe model are that trips take *minimum distance routes* and that, at each location both radial and orbital travel is permitted. These restrictions make it difficult to use the model to test the effects of alternative transport plans, such as building a new orbital route or a policy of restricting speeds on radial routes.

In this paper both of these assumptions will be relaxed, leading to a wider class of models which exhibit newly emergent characteristics whilst offering greater realism and improved scope for the analysing the strategic effects of alternative transport policies.

The KT Metric

We now assume that trips take *minimum time routes* and that radial speeds V_R may differ from orbital speeds V_O. The metric defining the minimum time is given by:

$$KT = \text{Min}\,[(r_1 + r)/V_R, \text{Min}\,[r_1, r]\,|\theta|/V_O + |r_1 - r|/V_R] \quad (3)$$

Define the velocity ratio $k = V_R/V_O$. It can be verified that the orbital catchment area is now given by:

$$|\theta| \leq 2 / k \qquad (4)$$

When θ is less than 2/k a radial route would take a longer time than one using an orbital, and when the angle it is greater the radial route takes the least time. The value $\theta = 2/k$ is referred to as the *switching angle*.

Note that if the velocity ratio k is less than $2/\pi$ then the switching angle exceeds π. In this case orbital routes will always take less time than radial routes, irrespective of the value of θ. This condition provides one possible analytical interpretation of Tripp's concept of "a circular road of extreme merit" (see above).

Comparison between KD and KT

It can be noted that in the KT metric minimum time routes may be different from minimum distance routes. For example if $2 < |\theta| < 2/k$ the minimum distance route is a double radial but the minimum time route is radial-orbital. Conversely if $2/k < |\theta| < 2$ the minimum distance route is radial-orbital but the minimum time route is a double radial.

The KT metric relaxes the assumption implicit in the Karlsruhe model that the speed is the same in orbital and radial directions. The next step is to relax the another assumption, namely that orbital travel is available at all locations.

The KT1 Metric

One of the limitations of KT as a model for urban travel is the requirement for there to be a sufficiently dense network of both radial and orbital routes. In most cities whilst there may be many radial routes, there is usually only a single, or at most a very small number of fast orbital routes.

The KT1 metric is designed to reflect the assumption that there is just one orbital route, of radius R. The metric defining the minimum time is given by:

$$KT1 = \text{Min}\,[(r_1 + r)/V_R,\ R|\theta|/V_O + (|R - r_1| + |R - r|)/V_R] \qquad (5)$$

Both the KD and KT metrics gave rise to catchment area boundaries that are independent of distance from the centre (see equations (2) and (4)). This is not the case for the KT1 metric. It is now essential to consider the relationship between the radius of the orbital route and the radii of the fixed and variable locations. Depending on the radius of the fixed location, there is a critical *switching angle* at which the minimum travel time using an orbital route is equal to that on a radial route through the city centre. At angular separations less than the switching angle, the orbital route takes less time, at greater angles, the radial route takes less time.

Table 1 defines travel times on orbital routes and the corresponding switching angles. Different results are quoted, depending on the fixed radius r_1, the variable radius r, and their relationship between the radius R of the orbital.

Domain	Travel Time	Switching Angle
$r_1 \leq R$, $r \leq R$	$(2R - r_1 - r + kR\|\theta\|) / V_R$	$2(r_1+r-R) / kR$
$r_1 \leq R$, $r \geq R$	$(r - r_1 + kR\|\theta\|) / V_R$	$2r_1 / kR$
$r_1 \geq R$, $r \leq R$	$(r_1 - r + kR\|\theta\|) / V_R$	$2r / kR$
$r_1 \geq R$, $r \geq R$	$(r_1 + r - 2R + kR\|\theta\|) / V_R$	$2 / k$

Table 1: Orbital Travel Times and Switching Angles for the KT1 metric

It can be noted that when both fixed (r_1, 0) and variable (r, θ) locations are outside the orbital route R the switching angle is identical to that given in equation (4). For these (external to external) trips the catchment area is identical to that for the KT metric. However, when either location is inside the ring road the switching angle depends on the ratio of its distance from the centre to the radius of the ring road. The resulting shapes of the KT1 catchment areas will be examined in more detail later, but first we need to extend our concept of catchment areas.

The Classification of Catchment Areas in Radial-Orbital Cities

Recall that the catchment area for a given type of route is defined the set of variable locations for which that route type yields the minimum travel time. Consider two locations in a radial-orbital city. We can define the following general sevenfold typology of the routes between these locations:

1 Radial arc	Travel on a single radial arc
2 Radial	Radial travel on 2 radials through the centre
3 Inner Orbital	2 radial arcs + 1 orbital inside both locations
4 Outer Orbital	2 radial arc + 1 orbital outside both locations
5 Cross Orbital	2 radial arcs + 1 orbital between the two locations
6 Complex Orbital	Up to 2 radial arcs + 2 or more orbitals
7 Complex Radial	Any route using 3 or more radial arcs

Route type 1 is only an option when there is no angular separation between the ends of the trip. This only occurs for a negligible proportion of all pairs of trip ends. The size of its catchment area is therefore zero and can either be excluded from the cases to be analysed or treated as a limiting case of one of the other general types. For a wide variety of network conditions it can be shown that route types 6 and 7 do not correspond to minimum time routes and will therefore have no catchment area. So, for the purposes of classifying catchment areas, it is generally be sufficient to restrict attention to route types 2-5. The implied simplifications gives rise to a fourfold *operational* classification of catchment areas:

i.	Radial	Travel through the centre, not using an orbital
ii.	Inner Orbital	Through/strategic travel: inwards/orbital/outwards
iii.	Outer Orbital	Local travel: outwards/orbital/inwards
iv.	Cross Orbital	Arriving or departing travel, partly on an orbital.

Policy measures often aim to reduce traffic of the first type, e.g. by construction or improvement to orbital routes, by reducing speeds on radial routes or by restricting access to the centre at congested times. Strategic orbital routes often tend to be designed to attract inner orbital trips, but may have consequences for local and for cross-orbital trips. In cities with multiple orbital routes the cross-classification of the four operational catchment areas into classes that depend on the specific orbital route used provides an extension to this classification.

Phase Diagrams

In order to compute an accurate map of catchment areas in the polar (r,θ) plane it is helpful to first produce a diagram using a Cartesian coordinate system in which the horizontal axis is the radius r and the vertical axis is the angle θ. A plot of this type is referred to as a *phase diagram*. The boundaries between different regions in the phase diagram yield a precise determination of the edges of the catchment areas.

Often the phase diagram boundaries of catchment areas are *linear*, or close to linear, which greatly simplifies both the analysis and methods for their construction. In this section we will illustrate basic methods for producing phase diagrams. In the next section the construction of maps of catchment areas from these phase diagrams will be discussed.

For the KT1 metric the basic information needed for a phase diagram is given in table 1. The resulting diagrams, for different velocity ratios, are given in figures 1a and 1b, (ignore the vertical dashed line for the moment). Each figure has two distinct parts: the left part applies when the fixed location is inside the orbital route, the right part when it is outside the orbital route. In the latter case the diagram is *independent* of the value of r_1.

It should be noted that the vertical axis on each diagram has a maximum at a value of π. In figure 1a the value of the velocity ratio k has been assumed to be sufficiently high so that the switching angle is less than π. For $r_1<R$ this requires the velocity ratio to exceed $2r_1/\pi R$ and for $r_1>R$, it needs to exceed $2/\pi$. In figure 1b k has been assumed to be less than these critical values. In both parts of the figure, the radius where the sloping line meets the top of the diagram is less than R, so that the catchment boundary does not reach the orbital route at the maximum angular separation of π.

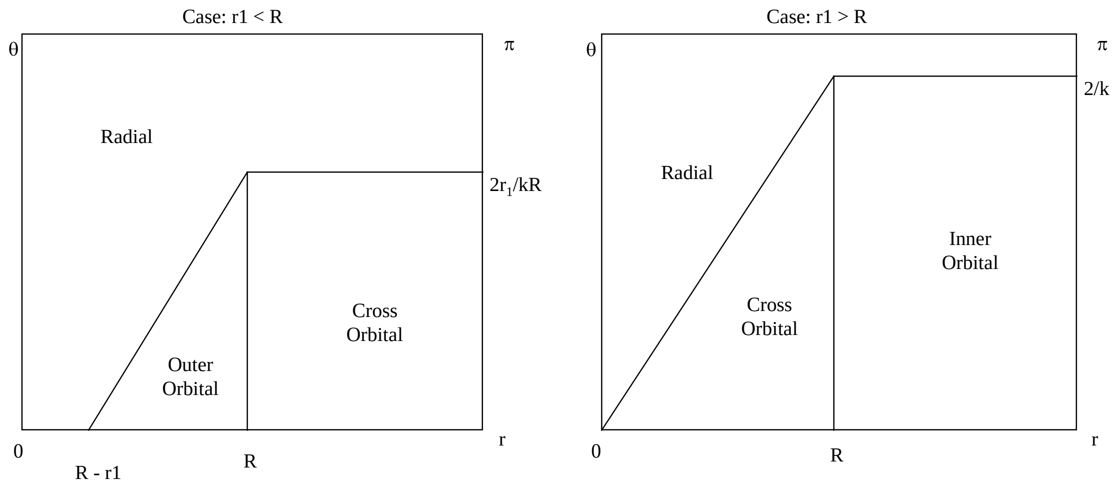

Figure 1a: Phase Diagram for the KT1 metric (High k)

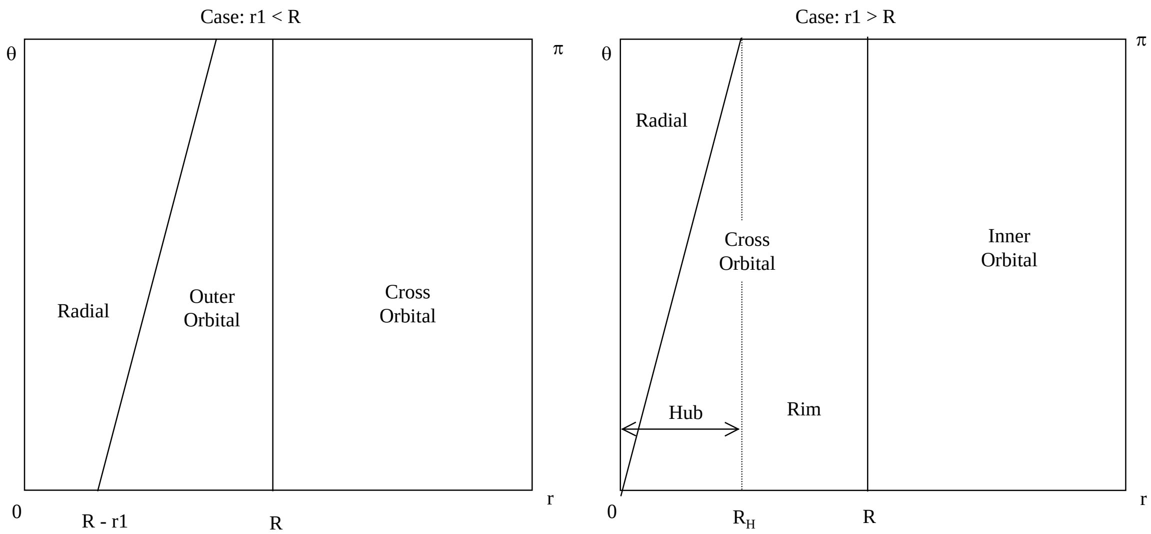

Figure 1b: Phase Diagram for the KT1 metric (Low k)

The Hub and the Rim

In the right-hand part of figure 1b the radius marked by the vertical dashed line is referred to as the *hub radius* and is given by:

$$R_H = \pi k R/2 \qquad \text{for } k<2/\pi \qquad (6)$$

The determination of this radius follows from the equations of the catchment area boundary, to be derived later in this paper. For any starting location outside the orbital, the hub radius represents the maximum possible radius of any finishing location that can best be reached by radial travel. Likewise, for any given finishing location outside the orbital, the hub radius represents the maximum possible radius of starting locations that can best reach it by radial travel. The hub region is thus the union of all radial catchment areas for which the fixed location is outside the orbital.

The annulus between the hub radius and the orbital route is referred to as the *rim*. For start locations outside the orbital, all finish locations within the rim are best reached via a cross-orbital route. For finish locations outside the orbital, all start locations within the rim best reach it via a cross-orbital route. The rim is thus the intersection of all orbital catchment areas for which the fixed location is outside the orbital route.

The Core

Consider a trip between two locations on the orbital route which have an angular separation equal to the switching angle 2/k (cf the last case given in table 1). The travel times using either radial or orbital routing are equal. Draw a chord between the two locations, as shown in figure 2, and define the *core radius* as the distance from the city centre to the closest part of this chord.

Why is this of interest? Consider now a trip between any two locations on the orbital and construct another chord between the two locations. In order to determine the minimum time route between two points on the orbital one merely needs to note whether its chord cuts across the core: if so it is best to take a radial route, if not the orbital route is quickest.

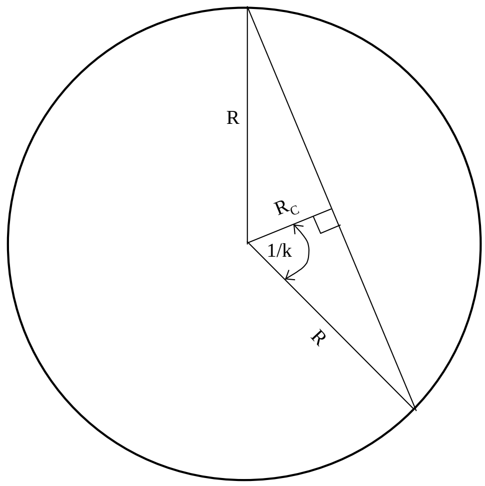

Figure 2: Definition of the Core Radius

It is easy to verify that the core radius is given by:

$$R_C = R\cos(1/k) = R\cos(V_O / V_R) \tag{7}$$

It can be noted that a core radius only exists if $k>2/\pi$, whilst a hub radius only exists if $k<2/\pi$. Hence a core radius only exists when there is no hub radius and vice-versa.

The above constructions for hub and core regions can be contrasted with those developed in Mayhew and Hyman (2000) for cities with a combination of radial and omni-directional routing, where both hub and core radii may co-exist. Whilst it is difficult to make analytical comparisons between models with different routing assumptions, there appears to be very strong grounds for expecting to find hub and core regions in networks that are more general, and hence more complex, than the simple models that we have investigated so far.

Examples

a) Suppose that R=10 kms, r_1=7 kms and k=0.8. The switching angle θ^*, where the spiral arc has a radius of R, is $2r_1/kR$ = 1.75 radians. This is less than π, so the phase diagram is like the first part figure 1a.

b) If R remains at 10 kms, but now r_1=15 kms while k remains at 0.8. The switching angle, is 2/k = 2.5 radians. This is also less than π, so the phase diagram is like the second part of figure 1a. The core radius has a value of 3.2 km.

c) With R=10 kms, r_1=7 kms as in example a), but now with k=0.4 The switching angle equals $2r_1/kR$ = 3.5 radians. This is greater than π, so the phase diagram looks like the first part of figure 1b.

d) Suppose that R=10 kms, but now r_1=15 kms and k=0.6. The switching angle has a value of 2/k = 3.33 radians. This exceeds π, and the phase diagram looks like the second part of figure 1b. The hub radius has a value of 9.4 km, so the rim is 0.6 km wide.

The Construction of Catchment Maps from Phase Diagrams

The boundary of any catchment area in the phase diagram, as in figure 1, is typically a combination of three different types of arc:

Verticals - corresponding to circular arcs in the catchment area map.

Horizontals - corresponding to radial arcs and
Diagonals - corresponding to spiral arcs.

The methods for constructing circular and radial arcs are well known, so we shall confine our attention to the spiral arcs. Graphics facilities in standard spreadsheets often support polar plotting facilities known as “radar” plots. So it is useful to obtain polar co-ordinate equations for the spiral arcs. These can be written in the general form:

$$r^* = a + b\theta \tag{8}$$

where the parameters a and b can be obtained from the phase diagram. For the KT1 metric, when $r_1<R$ we use the upper diagram in figure 1. The intercept a in (8) is the radius corresponding to θ=0 and is given by R- r_1. The slope b is given by the ratio of a horizontal to a vertical section in figure 1a (left-hand part):

$$b = (R-(R-r_1))/(2r_1/kR) = kR/2 \tag{9}$$

Taking these results together we obtain the equation:

$$r^* = R - r_1 + kR\theta/2 \quad \textit{for } 0 \le \theta \le 2/k \tag{10}$$

When $r_1>R$ we use the right-hand diagram in figure 1a and obtain the equation:

$$r^* = kR\theta/2 \tag{10a}$$

It is of interest to note that the slopes in equations (10) and (10a) are identical, both being equal to kR/2. If we set the angular separation equal to π we obtain the hub radius, given in equation (6) above.

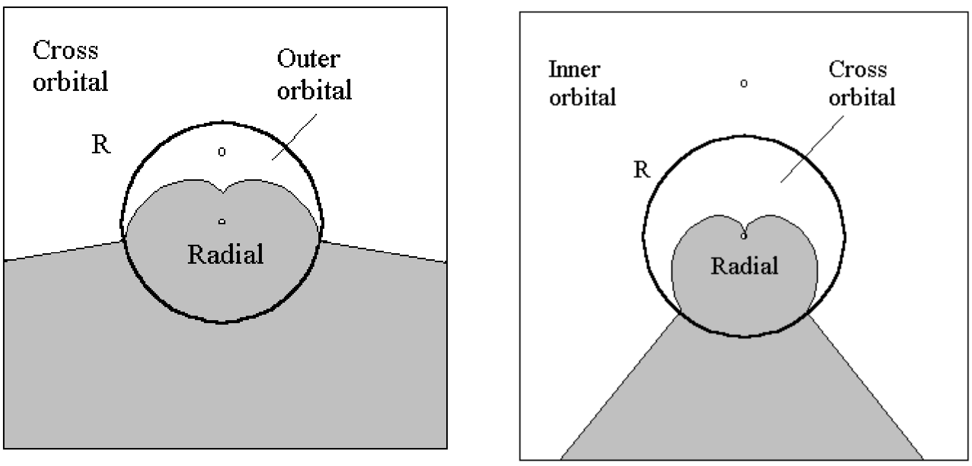

Figure 3a: Catchment Maps for examples a and b (High k)

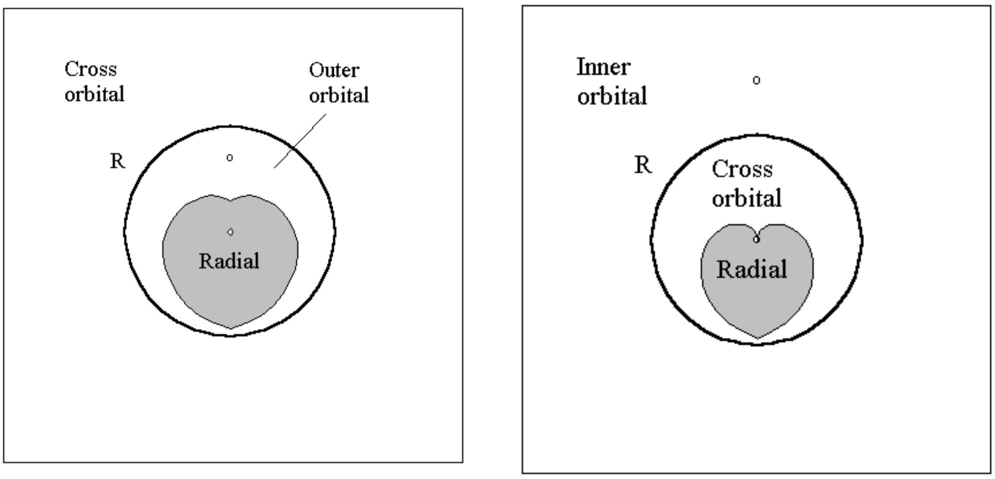

Figure 3b: Catchment Maps for examples c & d (Low k)

<u>Examples</u>

We shall calculate the spiral arc parameters for examples a)-d) above.

a) R=10 kms, r1=7 kms and k=0.8. Equation (10) is the appropriate one and the intercept is R-r1 = 3 kms. The slope is kR/2 = 4. The switching angle is $2r_1/kR$ = 1.75 radians, where the spiral arc of the catchment area boundary cuts the orbital route, as shown in the left part of figure 2a.

b) R=10 kms, r1=15 kms, and k=0.8. Equation (10a) is the appropriate one, which has an intercept of zero. The slope is kR/2 = 4. The switching angle is 2/k = 2.5 radians, as illustrated in the right part of figure 2a.

c) R=10 kms, r1=7 kms and k=0.4. Equation (10) applies and the intercept is 3 kms, while the slope is kR/2 = 2. When $\theta=\pi$ the radius of the spiral arc reaches its maximum value of 9.28 km. This is inside the orbital route, as shown in the left part of figure 2b.

d) R=10 kms, r_1=15 kms, and k=0.6. Equation (10a) applies, with a zero intercept and a slope of kR/2 = 3. When $\theta=\pi$ the spiral arc reaches a radius of 9.42 km. This is also inside the orbital, depicted in the right part of figure 2b.

Figures 3a and 3b illustrate the typical catchment area maps that are obtained for the KT1 metric, with parameters corresponding to the above four examples. Examples c and d are shown in figure 3b, from left to right respectively. In both of these cases the radial catchment areas form complete heart-shaped regions that are entirely confined to the area within the orbital route.

Calculating the Size of the Radial Catchment Area

In this section, we derive the size of the radial catchment area, that is the heart-shaped area in Figures 3a and 3b. (In the cases depicted in figure 3a we shall confine attention to the area enclosed by the orbital route). The magnitude of this area is derived by evaluating the integral:

$$A = 2\int_{0}^{\theta^*}\int_{0}^{r^*(\theta)} r dr d\theta + 2\int_{\theta^*}^{\pi} R d\theta \qquad (11)$$

In equation (11), the limits of integration implicitly depend on both the radius of the fixed location r_1 and k, the ratio of radial to orbital speed. The factor of 2 in the integrals arises from the symmetry between the positive and negative angular co-ordinates. The first term in equation (11) corresponds to the spiral arc of the catchment area boundary. This applies only while the angular co-ordinate is less than the switching angle θ^*. The second term corresponds to the remainder of the angular range, where the radius is R, arising from the confinement of the area of interest to that within the orbital route.

This effect of the above restriction is to remove the through/strategic trips and enable the analysis to concentrate on local, arriving and departing trips. This is done because the analysis of through trips is straightforward, as it can be characterised entirely in terms of the basic switching angle 2/k. Illustrations and examples calculations were presented earlier.

The formulae derived from equation (11) are given in table 2. Four cases, in terms of r_1 and k, need to be distinguished, as indicated in column 1. Column 2 gives the polar equation for the radius r^* of the spiral section of the catchment boundary. Column 3 gives the angular limit of integration θ^*. The fourth column gives the formula obtained for the size of the catchment area.

Domain	**Radius of Spiral**	**Switching Angle**	**Size of Radial Catchment Area**
$r_1<R$, $k>2r_1/\pi R$	$R-r_1+kR\|\theta\|/2$	$2r_1/kR$	$\pi R^2 + 2r_1^2(r_1/3-R)/kR$
$r_1>R$, $k>2/\pi$	$kR\|\theta\|/2$	$2/k$	$R^2(\pi-4/3k)$
$r_1<R$, $k<2r_1/\pi R$	$R-r_1+kR\|\theta\|/2$	π	$\pi^3k^2R^2/12 + \pi(R-r_1)(R-r_1+\pi kR/2)$
$r_1>R$, $k<2/\pi$	$kR\|\theta\|/2$	π	$\pi^3k^2R^2/12$

Table 2: Sizes of radial catchment areas

Figure 4 illustrates typical magnitudes obtained from the equations for the size of the radial catchment area in table 2. The horizontal axis corresponds to the radius of the fixed location r_1. Four curves are drawn, corresponding to the four speed ratios k of 0.5, 0.75, 1.0 and 1.25 and an orbital route of a given radius.

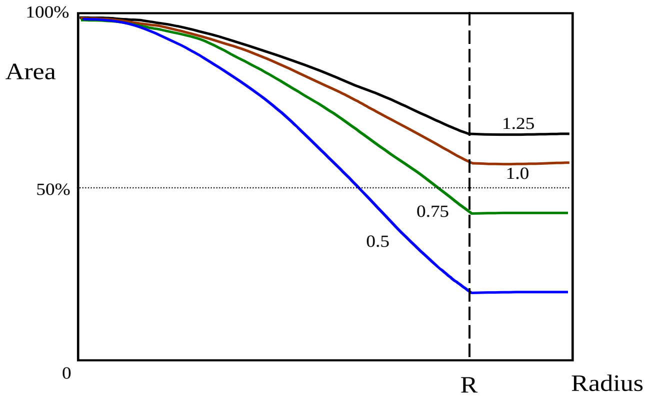

Figure (4). The size of the radial catchment area as a percentage of the total area within the orbital route.

It is noted that all cases intersect the vertical axis at the same point. At this point the size of the radial catchment is a maximum, covering 100% of the area within the orbital. As the fixed location moves away from the city centre the radial catchment declines but then levels off once it reaches the orbital. For example, the curves suggest that the radial catchment area size reaches a minimum of 50% of the area within the ring route when radial speeds are somewhere between 0.8 and 0.95 of the orbital speed**.** Further computations indicate that the speed ratio needs to be 0.85 in order to obtain exactly 50% of the total area.

The general properties of these curves indicate that reductions in the speed ratio have the greatest impact on trips with fixed locations that are comparatively close to the orbital route. In contrast, fixed locations that are less than halfway out towards the orbital route would continue have a substantial propensity to travel via the city centre, even when there are considerable speeds advantages in using the orbital.

This occurs, for the fairly obvious reason that such trips would need to travel for a significant time on the comparatively slow radials in order to access the faster orbital route, which therefore appears to offer few advantages over travel via the city centre. It is therefore of interest to relax the assumption of constant radial speeds, which we examine in the next section.

However, before leaving this issue it is of interest to note that the use made of the orbital for local trips will depend on the radius of the orbital, the ratio between orbital and radial speeds, the density of potential trips near the orbital and the level of junction access. It follows therefore that strategic transport authorities may need to consider, especially if capacity is limited, whether a particular orbital is primarily for diverting local traffic or whether it is aimed at through traffic, and to ensure that there is a full understanding of the implications of the alternatives.

The GKT1 Metric

The KT1 metric assumes that radial speeds are constant, but may differ from orbital speeds. We now permit radial speeds to depend on distance from the city centre. The minimum travel time is then determined by the *GKT1 metric*, defined by:

$$GKT1 = MIN[\tau(0,r_1)+\tau(0,r), R\,|\,\theta\,|\,/V_O+\tau(R,r_1)+\tau(R,r)] \qquad (12)$$

where τ(r,s) is the direct radial travel time between radii r and s, <u>along a single radial</u>. This is given by evaluating the following integral:

$$\tau(r,s)=\left|\int_r^s \frac{dx}{V_R(x)}\right| \qquad (13)$$

and $V_R(x)$ is the radial speed at radius x. Between any pair of radii r and s, it is useful to also define the *mean radial velocity*:

$$U(r,s)=\left|r-s\right|/\,\tau(r,s) \qquad (14)$$

It is also of interest to define the corresponding mean velocity ratios:

$$k(r,s)=U(r,s)/V_O \qquad (15)$$

Table 3 defines the travel times on orbital routes and the corresponding switching angles, generalising the results quoted in table 1. As before, different results are quoted, depending on the relationships between the orbital radius R and the fixed radius r_1 and between the orbital radius and the variable radius r.

Domain	**Travel Time**	**Switching Angle**
$r_1 \leq R$, $r \leq R$	$\tau(r_1,R) + \tau(r,R) + R\|\theta\| / V_O$	$2V_O\,[\tau(0,r_1) + \tau(0,r) - \tau(0,R)] / R$
$r_1 \leq R$, $r \geq R$	$\tau(r_1,r) + R\|\theta\|) / V_O$	$2V_O\,\tau(0,r_1) / R$
$r_1 \geq R$, $r \leq R$	$\tau(r,r_1) + R\|\theta\|) / V_O$	$2V_O\,\tau(0,r) / R$
$r_1 \geq R$, $r \geq R$	$\tau(R,r_1) + \tau(R,r) + R\|\theta\| / V_O$	$2V_O\,\tau(0,R) / R$

Table 3: Orbital Travel Times and Switching Angles for the GKT1 metric

It can be noted that the switching angles are independent of radial speeds at radii in excess of R. Hence the form of the catchment areas do not depend on radial speeds outside the orbital route.

The switching angle given in tables 3 can be used to plot phase diagrams. Their general appearance is similar to figure 3. However the diagonal boundaries (which occur when $r \leq R$) are now curved. If radial speeds increase with r they form concave curves, whilst if radial speeds reduce they are convex. However it can be noted that the switching angles are linear functions of travel time from the city centre. A phase diagram with linear boundaries can therefore be constructed by using travel time from the city centre as the horizontal axis.

When either end of the trip is outside the orbital and the other is at the city centre, the switching angle is zero, so that radial travel is preferred. When both trip ends are close to the city centre the switching angle is negative, so that radial travel is again preferred. As radii of either end of the trip rises the switching angle increases and becomes positive before both ends have reached the orbital route, where it reaches its maximum value of 2/k(0,R).

Travel Times and Mean Velocities for the Power Function

We now consider one specific functional form for the radial velocity that supports simple and realistic models and facilitates easily computable solutions. Suppose that radial speeds vary according to the power function:

$$V_R(r) = ar^p \quad for \;\; r \leq R \qquad (16)$$

Where a and p(<1) are model parameters. It is assumed that the radius r is inside the ring road, since, for the reasons given above, the radial speeds do not need to be defined outside the ring road. The travel time between the centre and radius r is given by:

$$\tau(0,r) = \frac{r^{1-p}}{a(1-p)} \qquad (17)$$

and the average radial velocity between the centre and radius r is given by:

$$U(0,r) = a(1-p)r^p \qquad (18)$$

Mean velocity ratios can be calculated from equation (15).

Catchment Boundaries for the Power Function

The key task in constructing the catchment area is the determination of the polar equations for the spiral arcs on its boundary. This takes the general form:

$$r = (A + B\theta)^q \tag{19}$$

The parameters A, B and q can be derived using table 3 to give:

$$\begin{aligned} A(r_1) &= R^{1-p} - r_1^{\,1-p} && \text{for } r_1 < R \\ &= 0 && \text{for } r_1 \geq R \\ B &= \frac{a(1-p)R}{2V_O} \\ q &= \frac{1}{1-p} \end{aligned} \tag{20}$$

The hub radius can be obtained by setting the angle in equation (19) to be equal to π, and assuming that r_1>R in equation (20). We obtain:

$$R_H = (B\pi)^q \tag{21}$$

The hub/rim distinction will exist as long as equation (21) yields a value less than the orbital radius R.

A GKT1 Model for London

With the aid of a recent popular commercial route finding package, the following measurements were made:

1) The average radius of the M25 orbital route was estimated to be 18 miles (29 km).

2) Times and distances for orbital trips, around the M25, were obtained, resulting in an average orbital speed of 62 miles/hr (104 km/hr), with a standard deviation of about 3 miles/hr (5 km/hr).

3) A series of radial travel times and distances between the centre of London (taken as Fleet Street) and the M25 orbital motorway were obtained from the route-finding package, using a spread of different directions of travel from the centre. The observations are plotted in the first diagram of figure 5 and show an increasing degree of dispersion as the distance increases. When time and distance are plotted on a logarithmic scale this effect is substantially reduced, as shown in the second diagram.

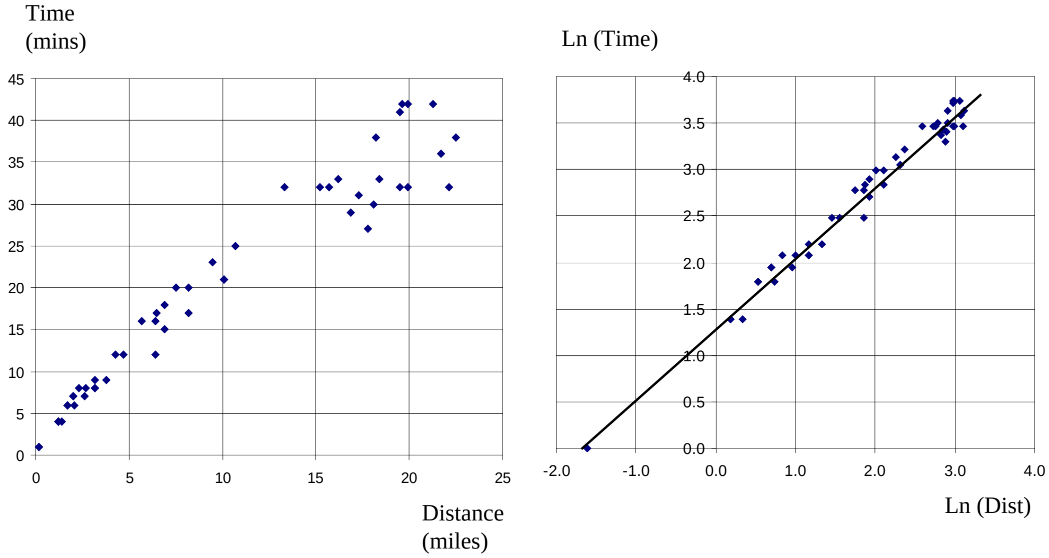

Figure 5: Radial Travel Times in London

The logarithm of the time (in minutes) was regressed against the logarithm of the distance travelled (in miles). This produced the following result:

$$\ln \tau(0,r) = \underset{(0.028)}{1.284} + \underset{(0.013)}{0.775} \ln r \qquad (R^2=0.985) \qquad (22)$$

where standard errors are shown below the estimated coefficients. From equations (18) and (22) we deduce that average speeds from the centre of London are given by:

$$U(0,r) = 16.6 r^{0.225} \qquad (23)$$

From (16) and (23), the radial velocity function can be derived:

$$V_R = 21.4 r^{0.225} \qquad (24)$$

Collecting together the model parameters, we have:

Orbital radius R:	18 miles (29 km)
Orbital velocity V_O:	62 miles/hr (99 km/hr)
Radial factor a:	21.4 miles/hr (34 km/hr)
Radial power p:	0.225

The values of the parameters for the spiral arcs can now be obtained:

$$A(r_1) = R^{1-p} - r_1^{1-p} = 9.39 - r_1^{0.775} \qquad \text{for } r_1<18$$
$$= 0 \qquad \text{for } r_1 \geq 18$$
$$B = a(1-p)R/2V_O = 2.41 \qquad (25)$$

$q = 1/(1-p) = 1.29$
$R_H = (2.41\pi)^{0.775} = 13.6$ miles (22 kms)

The width of the rim is thus 18-13.6=4.4 miles (7 km). The hub therefore extends about three-quarters of the distance from the centre of London to the M25 orbital route.

Figure 6a shows the boundaries of three catchment areas, for three alternative fixed locations, all due north of the centre. The outermost location, marked with a 'plus symbol' is on the orbital route and has a radius of r_1=18 miles (29 km). This has the smallest radial catchment area, which is entirely confined to the hub. The innermost location, marked with a triangle, has a radius of 9 miles (14km). This has the largest catchment area, which extends outside the orbital route in the south-east and south-west directions. The intermediate location is on the boundary between the hub and the rim, at a radius of 13.6 miles (22 km). Its catchment area extends into the southern section of the rim, but is entirely confined to the area within the orbital route.

The dashed circle in figure 6a is the boundary between the hub and the rim.

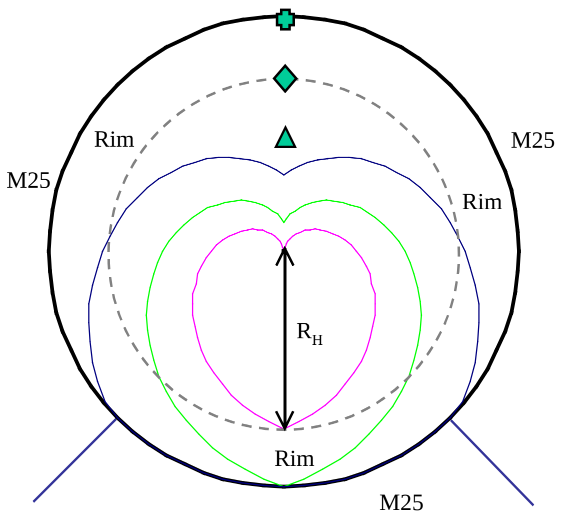

Figure 6a: Radial Catchment Areas for the London GKT1 Model

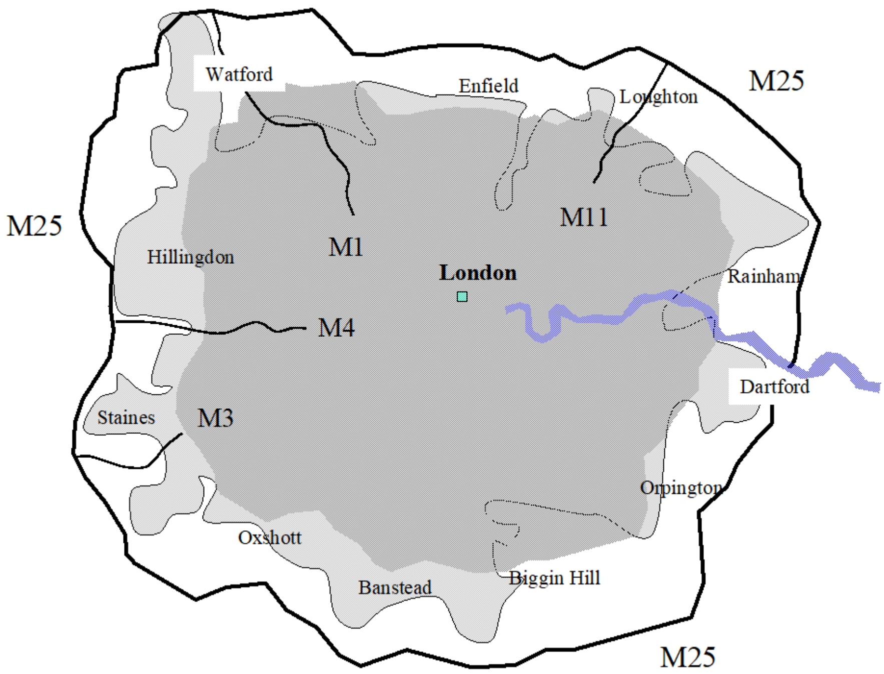

Figure 6b: Hub & Rim for the London GKT1 Model

The hub area of London is mapped in dark grey in Figure 6b. The outer curve shows the M25 orbital motorway. The rim area lies between the M25 and the hub and the names of several locations within the rim are marked. The boundary of London's continuous built-up area (the conurbation) is also shown, with the sections within the rim shaded in light grey.

It can be noted that the entire western edge of the conurbation lies within the rim and that the western half of the southern edge lies within the rim.
The remainder of the boundary of the conurbation, in the north, east and south-east only has isolated fragments within the rim, interspersed with sections that penetrate the hub. There appears to be several sections where the alignment of the M25 hugs the conurbation boundary. This is partly a result of constraints of physical geography, such as the North Downs constraining the alignment of the southern edge or the route.

The extensive development within the western and south-western rim would be expected to have a number of impacts:

- local trips diverting to these sections of the orbital route
- increased lengths of local trips
- increased local economic activity
- increased congestion as a consequence of the above effects

To put this in a contemporary context the UK Government is now planning to make further increases in the capacity of these sections of the M25. It is perhaps of some interest to speculate on what would have happened if these sections of the M25 could have been constructed a few miles further away from the conurbation or if access on to the M25 had been more restricted. The likely effect, would have been a lower level of utilisation by local traffic and, possibly, a lower level of congestion**.** Of course**,** such speculations would be aided by a more detailed knowledge of the extent of local traffic that makes use of these sections, together with the quality and availability of alternative options for local access, but they are beyond the scope of this paper.

The KTN Metric

In this section we relax the earlier assumption of a single orbital route, but return to the assumption of a constant radial velocity V_R. Let use consider a finite set of n=1..N orbital routes with increasing radii R_n, corresponding to orbital velocities V_{On} and an associated metric given by:

$$KTN = \mathrm{Min}\ [(r_1 + r)/V_R,\ \mathrm{Min}_n\ \sigma_n] \qquad (26)$$

where σ_n denotes the travel time using orbital route n:

$$\sigma_n = \tau(R_n, r_1) + \tau(R_n, r) + R_n\ |\theta|\ /\ V_{On} \qquad (27)$$

and τ is the travel time along a single radial segment:

$$\tau(r,s) = |r - s|\ /\ V_R \qquad (28)$$

The travel time for a route using the nth orbital can be expressed as:

$$\sigma_n = (|r_1\text{-}R_n| + |(r\text{-}R_n|)\ /\ V_R + R_n\ |\theta|\ /\ V_{On} \qquad (29)$$

A number of general properties of minimum time routes in the KTN metric can be established, but for the present we shall confine our attention to the case of two orbital routes.

The KT2 Metric

The principal new feature to be studied here is the condition for switching between alternative orbital routes. The critical angle θ^*_{12} at which this occurs is referred to as the *inter-orbital switching angle.* It can be calculated using equation (29) by equating travel times via orbital 1 and orbital 2, giving:

$$|r_1 - R_1| - |r_1 - R_2| + |r - R_1| - |r - R_2| = \Delta\theta^*_{12} \qquad (30)$$

where:

$\Delta = k_2R_2 - k_1R_1$ (31)

and we have defined the velocity ratios $k_n = V_R / V_{On}$, n=1,2.

In general, we need to consider nine different cases, corresponding to whether the radius of end of the trip is inside, between or outside the two orbitals. The resulting switching angles are given in table 4.

θ^*_{12}	$r_1<R_1$	$R_1<r_1<R_2$	$R_2<r_1$
$r<R_1$	$2(R_1 – R_2) / \Delta$	$2(r_1 - R_2) / \Delta$	0
$R_1<r<R_2$	$2(r – R_2) / \Delta$	$2(r_1+ r - R_1- R_2) / \Delta$	$2(r -R_1) / \Delta$
$R_2<r$	0	$2(r_1 - R_1) / \Delta$	$2(R_2 - R_1) / \Delta$

Table 4: Inter-Orbital Switching Angles for the KT2 metric

It should be noted that if $|\theta|>\theta^*_{12}$ then the inner orbital R_1 takes the least time. Conversely, when $|\theta|<\theta^*_{12}$ then the outer orbital R_2 takes the least time.

Isovalent Points

From any given starting locations there may be destinations for which the travel time is equal for all three cases of radial, inner orbital and outer orbital routing. This motivates the definition of something we define as an isovalent point. This is a variable location (r^*,θ^*) where three **(**or more**)** catchment areas meet. On the boundary between two catchment areas travel times are equal for two alternative routes, so if three catchment areas meet at a point, travel times must be the same for all three alternative routes.

It is of interest to determine the isovalent points corresponding to the catchment areas for 1) radial routing, 2) orbital routing via R_1 and 3) orbital routing via R_2. An isovalent point is *robust* if it is unaffected by small changes in the radius of the fixed location. The robustness property is of particular interest to cartographers and geographers because a map describing such points would facilitate route planning for a wide range of alternative trips.

To identify isovalent points the inter-orbital switching angles are equated to the switching angles between radial travel and travel via the inner orbital, for each of the nine cases. It turns out that only one case admits a robust solution, which arises when the fixed location is *outside* the outer orbital and the variable location is *between* the two orbitals.

Figure 7 indicates a typical form for the phase diagram, showing a robust isovalent point (r^*, θ^*). It can be seen that destinations further out than r^*, or with angular separations greater than θ^* would not make use of the inner orbital. Consider locations close to the isovalent point. Locations with a similar angular separation, but with a greater radius, would find it quicker to make use of the outer orbital. Locations with a similar radius, but with a greater angular separation, would find it quicker to use a radial route. The balance between these two alternatives depends on the outer-orbital versus radial switching angle. This has a value of $2r/k_2R_2$, derived from table 1 when $R = R_2$.

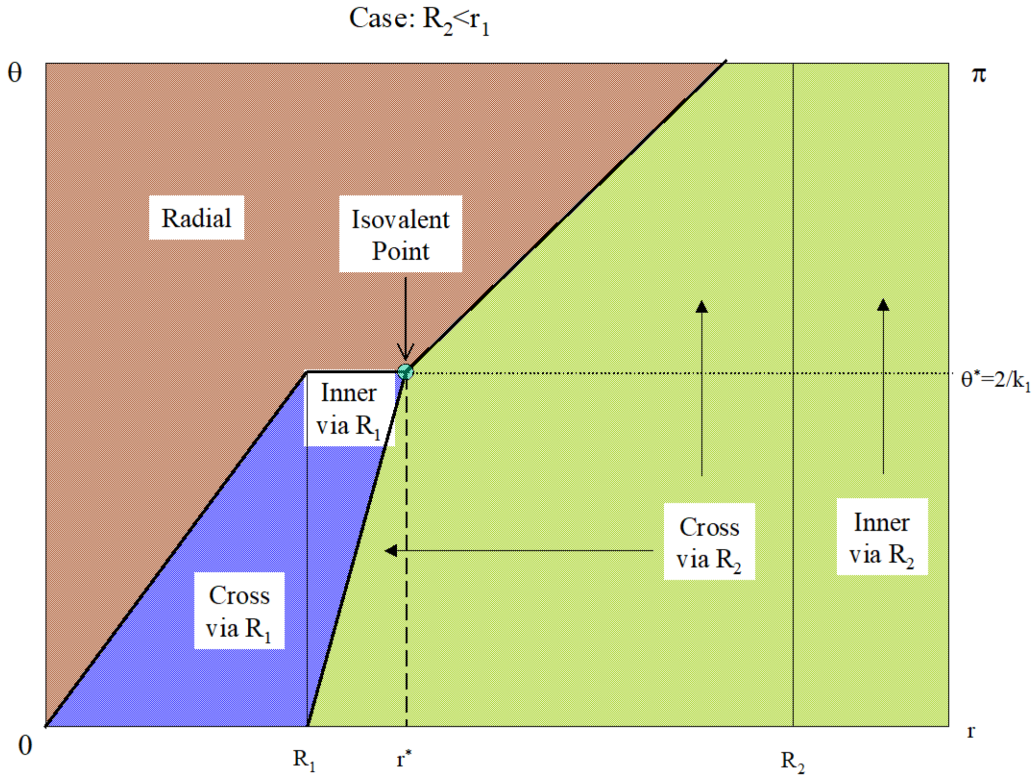

Figure 7: Phase Diagram for the KT2 metric showing an Isovalent Point

Figure 8 shows the form of the three catchment areas. The fixed location, marked with a plus, is due north of the city centre. The isovalent point is also shown (a second point also appears by symmetry).

From table 4 we can determine that the isovalent point must satisfy:

$$r^* = R_1 + \Delta\theta^*/2 \quad (32)$$

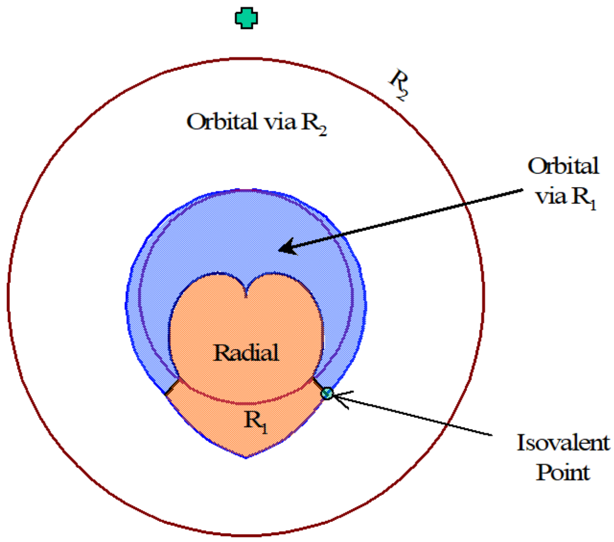

Figure 8: Catchments for the KT2 metric showing an Isovalent Point

The switching angle between radial travel and travel via the inner orbital, is equal to $2/k_1$. Substituting this into equation (32) and simplifying gives us:

$$r^* = R_2\, V_{O1}/V_{O2} \tag{33}$$

Note that equations (32) and (33) are only valid if r^* lies between R_1 and R_2. In order for r^* to be less than R_2 equation (33) implies that $V_{O2}>V_{O1}$, so that speeds on the outer orbital must exceed speeds on the inner orbital. In order for r^* to be greater than R_1 equation (32) implies that $\Delta>0$. This implies that the time to circumnavigate the outer orbital must exceed the time to circumnavigate the inner orbital.

The robust isovalent point can therefore by written as

$$(r^*, \theta^*) = (R_2\, V_{O1}/V_{O2},\ 2\, V_{O1}/V_R) \tag{34}$$

It can be noted that the radial coordinate is independent of the radial speed and the angular coordinate is independent of the speed on the outer orbital.

A KT2 Model for London

The commercial route finding package was used to make the following additional measurements:

1) The average radius of the North and South Circular orbital route was estimated to be 8 miles (13 km).

2) Times and distances for orbital trips, around the North and South Circular route, were obtained, resulting in an average orbital speed of 33 miles/hr (53 km/hr), with a standard deviation of about 4 miles/hr (6 km/hr).

Using the model for radial travel for London, given in equation (23), the average radial speed from the centre to the North and South Circular route was estimated to be 26.5 miles/hr (42 km/hr). We thus have the following measurements:

Radial Velocity V_R:	26.5 miles/hr (42 km/hr)
Inner Orbital radius R_1:	8 miles (13 km)
Inner Orbital velocity V_{O1}:	33 miles/hr (53 km/hr)
Outer Orbital radius R_2:	18 miles (29 km)
Outer Orbital velocity V_{O2}:	62 miles/hr (99 km/hr)

For the inner orbital (N-S circular) route there is no hub radius but there is a core radius given by:

$$R_C = R_1 \cos V_{O1}/V_R = 2.6 \text{ miles (4 km).} \tag{35}$$

This radius is sufficiently small to confine the core area to be within Central London.

The isovalent point is given by:

$$r^* = R_2\ V_{O1}/V_{O2} = 9.6 \text{ miles (15 km)} \tag{36a}$$
$$\theta^* = 2\ V_{O1}/V_R = 143^o \tag{36b}$$

The isovalent radius thus 20% larger than the inner orbital radius and lies at a distance of 1.6 miles (2.6 km) outside the N-S Circular route.

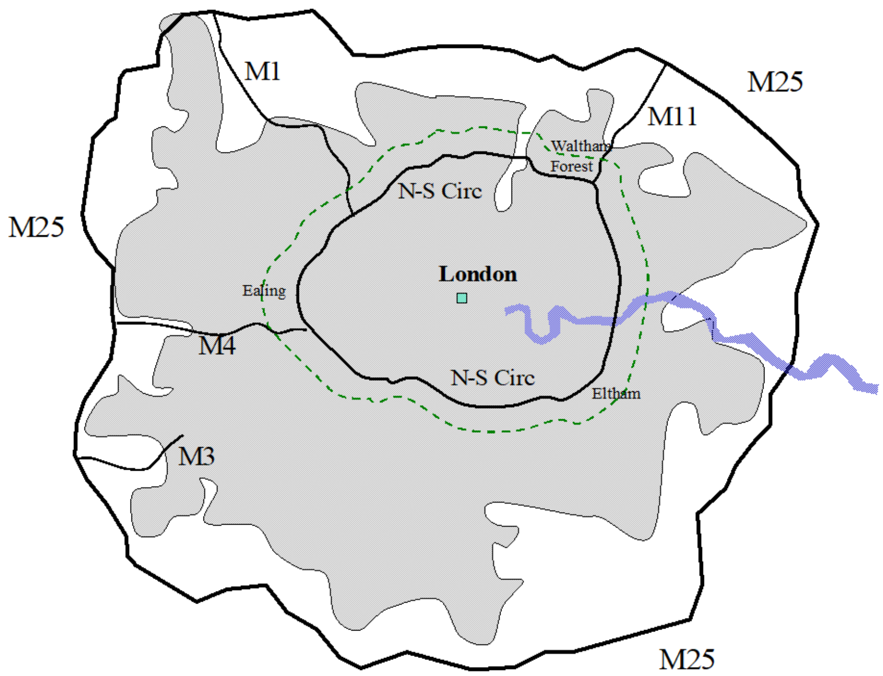

Figure 9: Isovalent Radius for the London KT2 Model

The radius of robust isovalent points for London is shown by the dashed contour in figure 9.

To describe what this means in practical terms we give two illustrations. First, consider a trip starting from a location outside the M25 and consider destinations which can be reached in equal times via the three alternative routes: using the M25, using the N-S circular road and via the centre of London. All such destinations must lie on, or close to, the dashed contour. For the second illustration consider the siting of a commercial facility that serves the south-east region of England and that needs to have a wide choice of routing options available. Locations on the isovalent radius would be, *a priori*, strong candidates.

It can be noted that if one end of a trip is outside the M25 and the other end is outside the isovalent radius then a route using the N-S circular route would take a greater time than using an alternative route. The N-S circular route would therefore be expected to function, to a large extent, as a local rather than a strategic route.

Discussion and Conclusion

In this paper we have concentrated on establishing the basic determinants of route catchments rather than in describing detailed transport system models. However, route catchments have a planning dimension both at a strategic level and in daily route choice decisions. They have implications for the construction of new routes and for measuring the efficiency and effectiveness of traffic management policies. There are also wider potential applications for improving our theoretical understanding of the relationship between city form, accessibility and urban land use and activity patterns.

The paper has examined the basic properties of route catchment areas for orbital radial models of urban travel. We have taken as examples both models for a single orbital route, combined with varying radial speeds, and for two orbitals, with constant radial speeds. Both cases have been illustrated with examples for the London area.

The basic theory predicts the existence of a hub and rim structure when the orbital route is sufficiently fast, and a core structure for slower orbital routes. The existence of a hub/rim structure has been demonstrated emprically in the case of the M25 orbital route and the existence of a core structure has been verified for the North-South Circular route. For cites with more than one orbital route, the basic theory also predicts the existence of isovalent points and again their existence has been verified in the case of the North-South Circular orbital route. Each of these emergent features (hub, rim, core and isovalent points) are believed to be fundamental characteristics of the design and operation of the urban transport system being modelled. These features therefore are elements of a common language for discussing strategic transport planning issues.

The characteristics analysed here have specific implications for urban transport strategies. They provide a basis for a clear identification of the differences between strategic and local traffic and the effects of network configuration and management decisions on different classes of movement. It is interesting to revisit Tripp's thesis that: "a circular road of extreme merit must surround the town centre and heart of the city" and compare the analysis presented here and to the results obtained for London.

If we take as our definition of 'extreme merit' the requirement that it diverts all strategic traffic away from the city centre, it appears that the M25 route is capable of satisfying this need. However if congestion, particularly on the western and south-western sections, continues to rise it may soon cease to provide this function, and strategic movements may begin to cut through the Centre of London. When we attempt to apply Tripp's definition to the North and South circular route it fails the test of 'extreme merit', with some strategic movements already finding it quicker to travel through the centre.

The basic analysis presented here can be extended to deal with cases where there are several orbital routes, giving rise to more complex forms of route catchment. Models can also be constructed that have combinations of orbital, radial and omni-directional routing. For both basic understanding purposes, and to generate simple and valid methods of construction, it is essential to conduct a full analysis of the different cases that can arise. Both GIS software packages and proprietary network modelling packages can greatly aid in the construction of diagrams for more complex applications.

However these packages cannot provide the sole tools for analysis, as the determination of emergent features demands a more analytical approach. The full practical appreciation of the implications of alternative transport polices requires a combination of both analytical and GIS/network approaches, as well as validation of the results in terms of the operation of the actual systems being modelled.

References

Anjoumani A, 1981 “Market area analysis with a rectangular grid network”. *Environment and Planning A*, **13**, 943-954.

Angel S, Hyman G, 1976 *Urban Fields*. London: Pion.

Haggett P, Chorley R J, 1969 *Network Analysis in Geography*. E Arnold, London.

Hyman G, Mayhew L, 1982, “On the geometry of emergency service medical provision in cities”. *Environment and Planning A*, **15** 1669-90.

Klein R. 1988, Abstract Voronoi diagrams and their applications. Lecture notes in Computer Science 333 (International Workshop on Computational Geometry). Berlin Springer-Verlag, 138-154.

Mayhew L, 1981 “ Automated isochrones and the locations of emergency medical facilities in cities”. *Professional Geographer* **33**, 423-8.

Mayhew L, Hyman G (2000) The impact of fast routes on travel geometry. *Environment and Planning B* (forthcoming).

Ministry of Transport, 1963 *“Traffic in Towns: A study of the long-term problems of traffic in urban areas”*. HMSO, London.

Okabe A, B Boots & K Sugihara (1992) *Spatial Tessellations - Concepts and Applications of Voronoi Diagrams*. John Wiley, New York.

O'Kelly M E , 1989 Equilibrium in a two-crop model with a low-cost transportation route. *Environment and Planning A*, **21**, 385-396.

Smeed, R J, 1963 Road development in urban areas. *Jnl of Institute of Highway Engineers*, **10**,5-26.

Tripp H.A., 1942 *"Town Planning and Road Traffic"*. Edward Arnold, UK.

Vaughan R, 1987 *Urban Spatial Traffic Patterns,* Pion: London.