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Deep geothermal single well heat production: critical appraisal under UK conditions

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Abstract

The idea of Deep Geothermal Single Well (DGSW) heat production has existed for many years, but with no consensus regarding its potential applicability: proponents have made claims regarding thermal outputs that appear exaggerated, whereas detractors have stated that the concept can never be economic unless the capital cost of drilling has already been discounted. However, because this technology offers the potential of delivering geothermal heat projects 'off the shelf' with a minimum of site-dependent research, the possibility exists of achieving cost-effective solutions. The present study sets out to investigate this topic subject to environmental and subsidy regimes applicable in the UK; the results might also be useful for other jurisdictions. Under these conditions, the variant of the technology with greatest potential for cost effectiveness is the hcDGSW, or conductive DGSW with heat production via heat pump. Analytic modelling enables the physics of the heat-exchange processes within a hcDGSW to be approximated. It is thus established that this option can indeed be cost-effective under the current UK subsidy regime for deep geothermal heat, provided boreholes are deep enough and in localities where the geothermal gradient is high enough. The environmentally optimum operational mode (optimizing savings in CO_2e emissions) involves heat production at a lower rate than the economically optimum mode (maximizing profit). If such projects are subsidized from public funds, then a particular operational mode might be specified, maybe as a compromise between these optima. After the 20 year duration of the subsidy, the technology might well no longer be economic, but the infrastructure might be easily repurposed for seasonal heat storage, thus offering the potential of making a significant long-term contribution to sustainable future heat supply. These preliminary results indicate that more detailed appraisal of this technology variant is warranted.

Introduction

Most deep geothermal heat production projects are based around the concept of well doublets, in which production of thermal water from one well is balanced by injection of the water (after heat is supplied from it to the heat load) in a second well. The injection of water serves two purposes: it avoids the drawdown in hydrostatic pressure that would result in the geothermal reservoir rocks if only a production well were in operation; it also avoids the need for treatment of the produced water before surface disposal, which in many circumstances would otherwise be necessary. Typically one or both of the injection and production wells are deviated so they reach the geothermal reservoir some distance (up to ~2 km; e.g., Smit, 2012) apart. Over time a project of this type gradually extracts heat from the reservoir rocks surrounding the production well; heat production usually takes place far faster than the associated 'thermal recharge' by upward heat flow from the Earth's interior. The cold thermal front emanating from the injection well will therefore eventually reach the production well, limiting the lifetime of the project (e.g., McDermott et al., 2006; van Wees et al., 2010; Satman, 2011; Smit, 2012). Used in this manner, this technology is thus not indefinitely 'sustainable' but is nonetheless regarded by most authorities as providing 'renewable' energy and therefore in principle eligible for subsidies (e.g., the UK government's Renewable Heat Incentive or RHI) designed to stimulate the development of 'low carbon' heat sources.

Such development requires the identification of reservoir rocks at appropriate temperatures and with suitable physical properties (porosity and permeability, or hydraulic conductivity, associated with fractures or pore spaces) to enable heat production at useful rates; it thus involves exploitation of complex systems that may well not be understood until the development is under way (e.g., Tenzer et al., 2010), and thus requires considerable technical skill. If suitable properties do not exist naturally they can in principle be created by techniques such as hydraulic fracturing, acid injection (to dissolve carbonate cement and increase pore space within rocks), or thermal fracturing (injection of cold water so thermal contraction causes fracturing of the rocks). However, hydraulic fracturing can cause induced seismicity and acid injection might cause environmental pollution; projects using these techniques might thus attract opposition. Commercial deep geothermal projects have nonetheless come on stream in western Europe. Examples include Soultz-sous-Forêts and Rittershoffen in eastern France (e.g., Vidal et al., 2016; Baujard et al., 2017), which have used the aforementioned well stimulation techniques, and many deep geothermal heat projects in the Netherlands (e.g., Bleiswijk; Ramaekers et al., 2006; Simmelink and Geel, 2008; Donselaar et al., 2015). However, each of these examples has depended on prior knowledge of rock properties from petroleum exploration. Since heat is inherently a less valuable commodity than oil or natural gas, it is more problematic whether a project of this type could be implemented on a commercial basis in a region that has not previously been documented in detail for this reason, such as most of the onshore UK, since the cost of the necessary site-dependent research might never be recoverable from the value of the heat that could be produced (cf. King et al., 2015). For both these reasons, the UK will benefit from any alternative approach to geothermal development that is uncontroversial and straightforward to implement 'off the shelf', without detailed site-dependent investigation.

The term Deep Geothermal Single Well (DGSW) denotes any geothermal project design that utilizes a single borehole (rather than a doublet), and which extends into the 'deep geothermal' regime, which under current UK regulations means depth >500 m (e.g., AECOM, 2013); many possible variants exist, including both open- and closed-loop designs (Table 1; Figs. 1, 2, 3). Some of these variants extract heat by conduction from the rocks around any borehole (Fig. 3; see, also, below), making them potentially straightforward to implement (since they do not depend on knowledge of hydraulic transport properties); furthermore, no 'well stimulation', such as hydraulic fracturing, is necessary. This technology is thus potentially suitable for providing 'off the shelf' geothermal heat sources in the UK, provided it can achieve an economic return. In the UK, heat output from a DGSW is eligible, in principle, for RHI subsidy; this is currently (February 2017) £0.0514 kWh^{-1}. Recent works on this topic include those by Law (2014), Law et al. (2015), GEL et al. (2016), and Collins and Law (2017). As will become clear below, some aspects of these publications are open to question, including apparent overestimation of outputs of useable heat and underestimation of system operating costs and impacts of uncertainty in knowledge of rock properties at depth and regulatory issues. The present study seeks to focus discussion on the variant of this technology (the hcDGSW; Table 1 and Fig. 2(b)) that is shown to offer the greatest potential under present UK economic and regulatory conditions; the analysis might also be useful in other jurisdictions. This investigation concentrates on underlying principles; practical details, such as designs of components (e.g., pumps and heat exchangers) and wall thicknesses of pipework (to handle the imposed fluid pressures and maintain the necessary thermal insulation) are beyond the scope of the present study (see, e.g., Rafferty, 2001; Law et al., 2015; Alimonti et al., 2016).

Many workers have published analyses of deep DGSW case studies, for production of either geothermal electricity or geothermal heat (e.g., Nalla et al., 2004; Wang, 2009; Wang et al., 2010; Bu et al., 2012; Cheng et al., 2013; Law et al., 2015, 2016; Alimonti and Soldo, 2016; Alimonti et al., 2016; Cho et al., 2016; Noorollahi et al., 2016; Riahi et al., 2017). However, some of these analyses use numerical modelling techniques that are either not explained or not validated against analytic calculations, others incorporate non-physical assumptions such as constant-temperature boundary conditions at depth or the assumption that these systems can operate under steady state. Furthermore, some of these studies analyse the physics of heat exchange around and within boreholes without considering how this heat transfer interacts with any heat load. However, it is clear from the substantial literature on shallow borehole systems connected to heat pumps (e.g., Bloomquist, 1999; Rees et al., 2004; Orio et al., 2005; O'Neill et al., 2006; Kavanaugh et al., 2012; Liu et al., 2016) that borehole heat exchanger designs and heat loads should be matched, making it essential to consider both in combination, rather than either in isolation, to achieve optimal solutions; this is shown in the present study to also be so for deep borehole systems. In the absence of any established method, this study will present an analytic approach for modelling both subsurface installations and heat loads for deep DGSW installations, those aspects of the underlying physics that appear most important to the author being included. Such an analytic approach incorporates exact solutions for particular aspects of this coupled problem, which approximate the conditions that can be anticipated during the operation of real DGSW installations. This approach might be useful for validating future numerical solutions, but the immediate aim is to permit first-order assessments of recent DGSW investigations in the UK (e.g., Law et al., 2015, 2016; Collins and Law, 2017), for which high heat outputs have been reported. The existing literature on shallow systems (e.g., Rees et al., 2004; Banks, 2012) indicates that the useable heat output increases with borehole depth, so higher values are expected for increasing borehole depth, but the manner in which heat output scales with borehole depth has not hitherto been established. Law et al. (2015, 2016) and Collins and Law (2017) have reported results of numerical analyses that quantify the rates of heat production that they consider feasible for particular borehole depths and bottom hole temperatures, but the software used has not been published, no validation against analytical calculations has been reported, nor any indication of how heat production depends on site conditions in general, or how much of the heat produced can provide output of useable heat, bearing in mind that most designs involve reinjection of water and the heat it contains.

The Southampton geothermal project in southern England, dating from the early 1980s, was the first to produce hot water from a deep borehole, discharging the cooled water into the environment, rather than reinjecting it (e.g., Barker, 1986). As many authors (e.g., Downing et al., 1984; Downing, 1986; Barker et al., 2010) have discussed, the Southampton-1 borehole reaches permeable Triassic sandstone at 76 °C at 1827 m depth. Well-testing established that this aquifer would be unlikely to sustain the high flow rates originally envisaged, which would have provided several megawatts of heat output using a conventional well doublet. It was therefore decided to develop this single borehole as a heat source (wellhead temperature 74 °C) for district heating, discharging the produced water into the sea. In its original form (here designated as wDGSW; Fig. 1(a)) this project had a useable heat output of ~1.15 MW (Barker et al., 2010). It was modified in 1991 with the addition of a heat pump (to the hwDGSW configuration in Fig. 1(b)); by reducing the temperature at which the produced water is rejected to the environment, this increased the output of useable heat to ~2.2 MW (Barker et al., 2010), the flow rate required for this being ~7.5 l s^{-1}.

Although the Southampton-1 produced water is not treated, in general in the UK treatment of produced water from deep geothermal projects will be necessary before discharge into the environment (e.g., Atkins, 2013). For example, the radionuclide ^{226}Ra is one of its more potentially hazardous constituents, since radium is chemically similar to calcium and so can participate in many chemical reactions in the environment. According to Edmunds (1986), the Southampton-1 well produces 39 picocuries of ^{226}Ra per kg of water, equivalent to ~1.4 Bq kg^{-1} or ~1.4 Bq l^{-1}. At ~7.5 l s^{-1}, this well thus discharges ~340 MBq of alpha radiation from ^{226}Ra per year. The relevant regulations (DEFRA, 2011, p. 81) permit discharges with ^{226}Ra activity of up to 100 Bq l^{-1} for this purpose, although in water produced as a by-product of hydrocarbon production any concentration above 1 Bq l^{-1} would require treatment. Edmunds (1986) also reported the concentration of iron in this water as 4.1 mg l^{-1}. For each chemical contaminant, such as this, the regulatory requirement is for concentration in the water body receiving the discharge to not exceed a specified limit, which for iron is 1 mg l^{-1} (e.g., DEFRA, 2014). Since the Southampton discharge is into the sea, the dissolved iron is evidently diluted sufficiently for compliance. However, the present regulatory presumption is that no additional discharge should adversely affect any water body, which effectively means that concentrations in the discharge should not exceed the regulatory limit for the water body. Thus, as Atkins (2013) noted, 'discharge to surface without treatment is unlikely to be a viable option' for future DGSW systems in the UK. The UK has a widespread legacy of mining, which has caused many discharges of groundwater that do not comply with present environmental standards and so now require treatment (e.g., Younger et al., 2005). As Johnston et al. (2008) have explained, this situation arose as a consequence of an ambiguous legal framework concerning responsibility for historical discharges, which was rectified for future discharges by a change to the law in 1999. This situation thus provides no precedent for tolerating untreated discharges from future DGSW systems (see, also, Atkins, 2013, and Abesser et al., 2014); indeed, knowledge of mine water treatment costs can inform discussion of potential treatment costs for water discharged from these systems (see below).

Heat might also be extracted by conduction from the rocks surrounding a borehole heat exchanger, containing a closed loop of heat transfer fluid, a variant of the technology (depicted schematically in Fig. 2(a)) designated here as a conductive DGSW or cDGSW. As far as can be established, this idea was first proposed by Lockett (1986). This author reported that circulation of 'special fluids' within a borehole in rocks at a temperature of 150 °C might produce sufficient heat for electricity generation at 2.5 MW, a claim that was unsupported by calculations and seems exaggerated (cf. Alimonti et al., 2016). A similar concept was independently studied by Rybach et al. (1992); these authors estimated the potential heat output as so low that this technology would in their view never be economic if drilling costs were included, but might be viable when applied to existing boreholes, for example 'dry' hydrocarbon wells or unsuccessful conventional geothermal boreholes. As Westaway (2016) noted, an attempt was made in 2011 to patent the cDGSW concept; it lapsed, presumably because the applicant became aware that the idea was not original. Law et al. (2015) noted that the cDGSW concept offers the potential for 'off the shelf' geothermal heat sources that require minimal site-specific investigation, potentially speeding up the adoption of geothermal energy in the UK. Most recently, the Glasgow-based energy company Geothermal Power Ltd. (http://geothermalpowerltd.com/) has advocated cDSGW use for electricity generation and that they, too, are in the process of patenting the technology. In principle, a cDGSW installation might be combined with a heat pump, to increase the output of useable heat from the resulting hcDGSW (Fig. 2(b)) although (again, as far as can be established) there has been no published analysis of the potential output or economic viability of this DGSW variant.

A further recent development (e.g., Law et al., 2015; GEL et al., 2016) has been the variant (which might be termed the 'dual mode' DGSW, or dDGSW; Fig. 3(a)), with the borehole heat exchanger left open around its base so the heat production that is feasible from a cDGSW can be supplemented by 'bleed flow' of thermal ground water. GEL et al. (2016) have since proposed a further hdDGSW variant with a heat pump (Fig. 3(b)). They deduced that this might be economically viable with RHI subsidy payments, although their analysis omitted water treatment costs. However, the need for permeable bedrock for these variants to function, plus the site-specific nature of the options for treating the produced water, negate the original objective of providing 'off the shelf' geothermal heat. Collins and Law (2017) have discussed two such projects, for a hdDGSW installation in the Aberdeen Granite beneath the Aberdeen Exhibition and Conference Centre (AECC) in the Scottish city of Aberdeen, and for a scheme to heat part of an outdoor swimming pool in Penzance, Cornwall, involving drilling into the Land's End Granite, also mentioning a project in East Ayrshire, Scotland. Planning documentation (Cornwall, 2017; East Ayrshire, 2017; Geon, 2017) indicates that the Penzance scheme has a dDGSW design, whereas the latter project, now known as HALO and located in the town of Kilmarnock (where drilling into Palaeozoic sedimentary rocks is proposed), is intended to function as a hcDGSW (see below).

This proliferation of alternatives (Figs. 1, 2, 3) has created a somewhat confused situation, especially as workers have not always been clear which variant is being described in a given document and some aspects, such as treatment of produced water, have not received sufficient attention. Furthermore, the contrasting claims regarding thermal output and economic viability or otherwise require resolution, especially as not all variants (including the hcDGSW) have been fully analysed. The present study sets out to address these issues. It will, first, present a quantitative evaluation, using analytical calculations, of the potential output of a hcDGSW (Fig. 2(b)) relative to a cDGSW (Fig. 2(a)). Second, two key issues affecting the viability of the alternative dDGSW variant, the low hydraulic conductivity of many lithologies and the potential cost of produced water treatment, will be discussed. Third, the analytical model for a hcDGSW, the technology variant shown to have the most potential, will be used to develop an economic model for assessing effectiveness in terms of both cost and greenhouse gas emissions. Finally, the cDGSW/hcDGSW conceptual model by Law et al. (2015) and some individual DGSW projects will be discussed in the light of the preceding analyses. Given the approximations made in the aforementioned analytical calculations, the potential merits of the hcDGSW variant, thus identified, require confirmation by more detailed numerical modelling, beyond the scope of the present study.

cDGSW Theory

A cDGSW can be envisaged as a closed loop formed by inserting a length of pipe into a cased borehole, as in Fig. 2(a). Water (or, possibly, an alternative heat transfer fluid; e.g., Alimonti and Soldo, 2016) is produced from the borehole at temperature T_o and supplied to a surface heat exchanger that extracts heat to the heat load at its operational temperature T_E. The water (or other circulating heat-transfer fluid) is thus itself cooled to T_E within the heat exchanger before reinjection at temperature T_D. As Law et al. (2015), Alimonti and Soldo (2016), and other workers have discussed, a suitable configuration has the upward flow in a central pipe within the well, with the downward flow in the surrounding annulus (Fig. 2(a)). During its downward flow the circulating fluid absorbs heat from the surrounding rocks; during its upward flow, which might be much faster if the pipe is much narrower than the annulus, it is assumed (for calculation purposes) that the fluid maintains temperature T_o. In principle, a cDGSW (and, indeed, any other variant of DGSW) might operate intermittently, possibly delivering heat on diurnal or seasonal cycles. However, since the thermal processes involved are governed by linear equations, after many such cycles the thermal state around a DGSW will be indistinguishable from that which would exist had it been operated to produce heat at a constant rate equal to the time-averaged rate for the actual pattern. The development of theory will therefore assume heat production at a constant rate.

Law et al. (2014) showed by field testing that in such a configuration the downward flow maintains roughly constant temperature T_D until it reaches a depth z_L where the initial temperature of the surrounding rock T_L equals T_D. Thus, between the Earth's surface and depth z_L the return flow heats this surrounding rock, and therefore only between z_L and the well bottom at depth z_M is heat extracted from the surrounding rock. This turns out to be a significant limitation of the cDGSW concept (see below). To facilitate the analysis, the ratio z_L / z_M is designated as f; the proportion of the borehole that acts as a heat source is thus 1-f.

The general equation governing heat flow in cylindrical polar co-ordinates is

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right), \quad (1)$$

where T is temperature, t is time, κ and k are the thermal diffusivity and conductivity of the rock around a borehole, and r, θ and z are radial distance (from the axis of a borehole), azimuth, and depth. The solutions assume no azimuthal dependence, also that the abrupt radial temperature gradients

($\partial T/\partial r$) that can be expected beyond the borehole radius a are much greater than the vertical geothermal gradient, so the latter is neglected. Equation (1) can thus be simplified to

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \quad . \qquad (2)$$

Starting at time t=0, when radial temperature gradients first develop, heat is assumed to flow inward across the boundary at r=a at a steady rate Λ, per unit surface area of the borehole, or at rate ζ per unit depth, where $\zeta = 2 \pi a \Lambda$ ($2 \pi a$ being the circumference of the borehole). The resulting net heat production is the sum of all these contributions across the depth range from z_L to z_M. Equation (2) has been solved subject to these boundary conditions by many workers (e.g., Carslaw and Jaeger, 1959, pp. 338-341). The solution can be expressed in terms of combinations of a decaying exponential function of time and Bessel functions of radial distance, and has no simple general form. However, Carslaw and Jaeger (1959) found power series approximations that are valid, separately, in the limits of $t \ll r^2/\kappa$ and $t > r^2/\kappa$. The latter solution, valid at (relatively) long timescales after t=0, can be written as

$$\Delta T(r,t) = \frac{-\Lambda a}{2 k} \left(\ln \left(\frac{4 \kappa t}{C r^2} \right) + \frac{a^2}{2 \kappa t} \ln \left(\frac{4 \kappa t}{C r^2} \right) + \frac{1}{4 \kappa t} \left(a^2 + r^2 - 2 a^2 \ln\left(\frac{a}{r} \right) \right) \right) , \qquad (3)$$

where ΔT is the temperature change from the initial conditions and $C \equiv \exp(\gamma)$, γ being Euler's constant (~0.57722….). For $t \gg r^2/\kappa$, this solution can be approximated further and rewritten in terms of ζ as

$$\Delta T = \frac{-\zeta}{4 \pi k} \ln \left(\frac{4 \kappa t}{C r^2} \right) . \qquad (4)$$

Furthermore, if ζ is assumed to be zero at depth z_L and to increase linearly to a maximum value ζ_M at depth z_M, then

$$\zeta_M = \frac{2 Q}{z_M (1 - f)} . \qquad (5)$$

ΔT will thus be proportional to depth beneath z_L. The temperature of the water produced from the borehole, T_O, will therefore equal the initial temperature T_M at depth z_M, minus ΔT calculated using equation (4) for r=a and $z=z_M$ so $\zeta=\zeta_M$. It follows that the rock at r=a at all depths $z_L \leq z \leq z_M$ will cool to the same limiting temperature in the limit of $t \rightarrow \infty$. This clearly approximates reality, as it implies a step change in T at depth z_M; the importance of this approximation is addressed below.

To summarize the underlying physics, the heat production from the borehole, at rate Q, occurs as a result of the warming of the circulating fluid as it flows downward, absorbing heat through the outer wall of the borehole heat exchanger. This fluid then flows upward along the inner pipe of the heat exchanger, maintaining the same flow rate q_C as no 'bleed flow' into the borehole is assumed. For plausible operational modes of cDGSW or hcDGSW installations, the upward flow along this inner pipe (of internal diameter D) will be turbulent (see below), so will approximate a uniform velocity profile V; q_C can thus be approximated as $\pi V D^2 / 4$. If the circulating fluid is injected at temperature T_D and produced at temperature T_O, the rate of heat production will balance the rate of heat gain by the circulating fluid; thus

$$Q = \rho c (T_O - T_D) q_C , \qquad (6)$$

where ρ and c are the density and specific heat capacity of this fluid. The value of q_C that is required for a given cDGSW or hcDGSW installation to operate at a given rate of heat production Q is calculated by rearranging equation (6), thus

$$q_C = \frac{Q}{\rho\, c\, (T_O - T_D)} \,. \qquad (7)$$

Since T_O will decrease over time (as in Fig. 4), operational modes at constant Q require progressive increases in q_C over time, given that ρ, c and T_D are assumed to remain constant.

The workflow for present modelling approach involves specifying z_M, a, and D for the borehole, T_S, T_M, k, and κ for the surrounding rock volume, ρ and c for the circulating fluid, and T_D from the characteristics of the surface heat exchanger installation. For each candidate rate of heat production Q, T_O is calculated for a succession of times t using equations (4) and (5), with the corresponding value of q_C determined using equation (7). This differs, for example, from the numerical modelling approach used by Alimonti and Soldo (2016), for which constant q_C is specified and Q, at rates that decrease progressively over time, calculated from it. The present solution nonetheless incorporates the essential physics governing the energy balance for a cDGSW, whereby the heat produced in the flow through a borehole heat exchanger is equal to the heat lost by conduction from the surrounding rock volume.

Nonetheless, although this analytic solution maintains conservation of energy overall, it does not attempt to balance energy for any part of the model. For example, no attempt is made to explicitly model the progressive warming of the circulating fluid as it flows downward along the outer pipe (Fig. 2) or the cooling of the fluid flowing upward along the inner pipe as heat flows radially outward from it into its lower-temperature surroundings (cf. Alimonti et al., 2016). Furthermore, it is evident that (in addition to the pre-existing geothermal gradient) vertical temperature gradients will develop within the rock volume surrounding the borehole, meaning that the approximation used to justify equation (2) will not be valid. Each of these aspects will cause complexity that is not incorporated into the analytic model.

Differentiating equation (4) with respect to r gives

$$\frac{\partial \Delta T}{\partial r} = \frac{\zeta}{2\, \pi\, k\, r} \,. \qquad (8)$$

As time progresses, the volume of rock cooled thus gradually widens, but equation (8) means that the induced radial geothermal gradient increases inward, so the temperature gradient around the borehole will draw heat in from farther out (Fig. 5). Thus, although the borehole becomes surrounded by an ever-widening volume of cooled rock, this radial geothermal gradient will always direct heat inwards towards it. Nonetheless, the operational lifespan of a cDGSW, the time over which it can produce heat before the temperature at r=a cools below any value that is useful, turns out to be very sensitive to choices of parameter values (see below). It is also noteworthy that although Q is the rate of heat production from the borehole, it will only equal the rate of heat output to the load if f=0, i.e., if the reinjection temperature T_D equals the ambient surface temperature T_S. Otherwise, only a proportion f, equal to $(T_o\text{-}T_L) / (T_o\text{-}T_S)$ of Q will be output to the load and the remainder will be reinjected, contributing (as already noted) to heating the rock volume at $z<z_L$. Thus, as the rock volume at $z>z_L$ gradually cools, T_o will eventually (at time t_L) decrease to T_L, at which point the DGSW will cease to deliver any useable heat output, effectively ending its useful life. At this point $\Delta T(z_M)=T_L\text{-}T_M$; substituting this condition into equation (4) and using equation (5), followed by other algebraic steps including recognizing that $T_M\text{-}T_L$ = (1-f) $(T_M\text{-}T_S)$ and that $T_M\text{-}T_S = u\, z_M$, where u is the geothermal gradient, gives

$$t_L = \frac{C\, a^2}{4\, \kappa} \exp\left(\frac{2\, \pi\, k\, (1-f)^2\, z_M^2\, u}{Q} \right) \,. \qquad (9)$$

The exponential term in equation (9) means that t_L, thus calculated, is indeed very sensitive to choices of parameter values, as is illustrated in Fig. 6.

The algebraic threshold of validity of the above 'long-timescale' solutions, at $t=r^2/\kappa$, corresponds to quite short timescales when r is small; for example, at r=a=0.1 m and with κ=1 mm^2 s^{-1} it is 10000 s or circa 2 hours 47 minutes. This solution is therefore applicable near the borehole on all timescales relevant to cDGSW operation. On the other hand, the use of equation (4) to determine the radius r_C of this cooling effect, is more problematic. Equation (4) implies that $\Delta T=0$ when $r=r_C$, thus

$$r_C = 2 \sqrt{\left(\frac{\kappa t}{C} \right)} . \qquad (10)$$

Plots of the function described by equation (4) indeed show that as r approaches r_C the value of $\partial\Delta T/\partial r$ is significant (Fig. 5), so $\partial\Delta T/\partial r$ has a discontinuity at $r=r_C$ which has no physical basis. The higher-order approximation in equation (3) also breaks down in the vicinity of $r=r_C$; this function indeed never actually satisfies $\Delta T=0$, it instead has a turning point beyond which ΔT starts to diverge away from zero. After several algebraic steps this can be shown to occur at $r=r_C$ where

$$r_C^2 = 4 \kappa t + a^2 . \qquad (11)$$

For operation of any significant duration, $r_C^2 \gg a^2$, thus

$$r_C \approx 2 \sqrt{(\kappa t)} . \qquad (12)$$

The estimate of r_C from equation (12) is thus $\sqrt{C}$ times, or ~1.33 times, that from equation (10).

It is also relevant that the solution for an infinitesimally thin 'line source', emitting or absorbing heat at a rate ζ per unit length, is

$$\Delta T = \frac{\zeta}{4 \pi k} E_1(r^2 / (4 \kappa t)) \qquad (13)$$

(e.g., Carslaw and Jaeger, 1959, p. 261), where $E_1(\chi)$ is the exponential integral function. As r becomes larger this will be a close approximation to the solution for a heat source (or sink) of finite radius. However, since $E_1(0)\rightarrow\infty$ this equation is not physically meaningful at small values of r. Furthermore, although $E_1(\chi)$ decreases to very small values as χ increases, it never reaches precisely zero. $E_1(\chi)$ can be accurately approximated (for $\chi\geq1$) as $\exp(-\chi) \times \ln(1 + 1/\chi)$ (e.g., Abramowitz and Stegun, 1964, p. 229); for example, it is thus ~0.25 for χ=1, ~0.05 for χ=2, ~0.01 for χ=3, and ~0.004 for χ=4. Given that the term $\zeta / (4 \pi k)$ is of the order of 1 °C (see below) and a representative decrease in the temperature adjoining a DGSW can be a few tens of degrees Celsius (say, ~50 °C; see below), χ=1 roughly corresponds to a temperature perturbation ~0.5% as large as that adjoining the borehole and can thus provide an alternative estimate of the effective value of r_C. One thus obtains $r_C^2 / (4 \kappa t) \approx 1$, indicating the same value for r_C as is given by equation (12).

A further complicating factor is that in addition to the radial cooling at $z\leq z_M$, cDGSW operation will cool the rock volume at depth $z>z_M$, which will add to the heat produced. An approximate correction will now be derived for this effect, in which it is assumed that this downward cooling effect extends over a depth range r_C below the well bottom at depth z_M, with its radius r_E and the associated temperature perturbation tapering linearly from r_C and ΔT at depth z_M to zero at depth z_M+r_C. Vertical position w within this cooled zone is measured upwards from zero at $z=z_M+r_C$. The cooling at each value of w can be approximated very crudely as extending over a radial distance out to r=w with the temperature perturbation proportional to $(w - r)^2/w^2$. The heat dE lost by cooling a cylindrical shell at vertical position y and radius r, of infinitesimal thickness dy and dr, can thus be estimated as

$$dE = \Delta T(r{=}a, z{=}r_C)\, \rho\, c \int_{r_C}^{w} \frac{(w - r)^2}{w^2}\, 2\pi r\, dr\, dw \qquad (14)$$

where ρ and c are the density and specific heat capacity of the surrounding rock. Substituting for ΔT from equation (4) and using equations (5) and (10), together with $\kappa\, \rho\, c \equiv k$, one obtains

$$E(t) = \frac{2\, Q \ln(r_C / a)}{z_M\, \kappa\, (1 - f)\, r_C} \int_{w=0}^{r_C} w \int_{r=0}^{w} \frac{(w - r)^2}{w^2}\, r\, dr\, dw \qquad (15)$$

as an estimate for the total heat lost by cooling of this borehole end zone from time zero up to time t. After multiple algebraic steps, one obtains

$$E(t) = \frac{Q}{12\, z_M\, \kappa\, (1 - f)} \ln\left(\frac{4\, \kappa\, t}{C\, a^2}\right) \left(\frac{4\, \kappa\, t}{C}\right)^{1.5} \qquad (16)$$

or

$$E(r_C) = \frac{Q}{6\, z_M\, \kappa\, (1 - f)} \ln\left(\frac{r_C}{a}\right) r_C^3\ . \qquad (17)$$

The effect of this contribution can be illustrated using a representative example with Q=100 kW, z_M=2500 m, κ=1 $mm^2\ s^{-1}$, a=0.1 m, and t=20 years, a realistic representative lifespan for a cDGSW (see below). Over this period of time the uncorrected heat production will thus be 100 kW $\times$ 20 years or ~63.2 TJ. With these values, r_C will be ~37.7 m after 20 years, and the additional heat production, from the borehole 'end correction', will be ~2.1 TJ. Including this correction would thus only change the estimated overall heat production on this timescale by ~3%. Over time scales such as this, the correction is so small that the various approximations made in its derivation are not of major importance. The essential reason for this is that on these time scales the roughly cone-shaped zone at the base of the borehole, whose heat loss is calculated as this correction, is very small compared with the cylindrical volume flanking the borehole, whose heat loss was calculated previously.

A further correction is necessary to incorporate the upward geothermal heat flow k$\times$u. After time t the rock volume cooled, of radius r_C, will present a horizontal cross-sectional area $\pi\, r_C^2$ to this heat flow. The borehole will thus 'capture' a heat flux of

$$Q_G = \pi\, r_C^2\, k\, u\ , \qquad (18)$$

For u=32 °C km^{-1} (the value adopted by Law et al., 2015, representative of geothermal gradients in radiothermal granites in different parts of Britain; e.g., Lee, 1986; Lee et al., 1987; Manning et al., 2007) and k=3.5 W m^{-1} °C^{-1} (likewise, a typical value for granitic rocks; e.g., Lee, 1986; Wheildon and Rollin, 1986; Lee et al., 1987; Manning et al., 2007) after 20 years, when r_C is again 37.7 m, the resulting additional heat flux will be ~500 W; this effect is thus even smaller in magnitude than the borehole 'end correction'. However, if a cDGSW were designed with very low heat production and a very long lifespan, r_C would ultimately become so large that the geothermal heat flux 'captured' might balance the heat production, creating the possibility of a steady state situation. Equating Q_G in equation (18) to Q and using equation (10) for r_C, one obtains

$$t_E = \frac{C\, Q}{4\, \pi\, k\, \kappa\, u}\ . \qquad (19)$$

Taking reasonable estimates (k ~3.5 W m^{-1} °C^{-1}, κ ~1 $mm^2\ s^{-1}$, u ~32 °C km^{-1}, as before), one can estimate that t_E is ~40 years per kilowatt of heat production Q. This steady-state limiting behaviour is thus only relevant for extremely low heat production, being (for example) ~1000 years for Q=25 kW, by which time the cooled volume will have widened to r_C ~270 m. Such limiting behaviour is an interesting scientific curiosity, but does not help with the assessment of how cDGSW technology might be used to produce worthwhile heat outputs on timescales of practical interest.

Discussion

Figure 4 illustrates the predicted variation in output temperature for a cDGSW, for its first ten years of operation at a range of constant heat production rates. Over this timescale, the radius of the cylindrical rock volume cooled by the operation of the DGSW (Fig. 5) is so small that the aforementioned corrections for heat flow and for cooling at greater depths have no significant effect. Figure 4 thus illustrates a progressive decline in output temperature as the rock volume around the cDGSW progressively cools, the decrease on a given timescale being proportional to the heat production, as is to be expected from the linear governing equation (equation (2)). Figure 5 confirms that the thermal state of the rock volume surrounding a cDGSW progressively evolves over time, with no indication that it reaches a steady state on timescales of decades, although (as noted above) a steady state might ultimately be attained after thousands of years. Nonetheless, both Fig. 4 and Fig. 5 demonstrate that after years of operation of a cDGSW the rates of cooling in and widening of the surrounding affected rock volume will have become very low; Alimonti et al. (2016) described this behaviour as an 'almost steady state condition'.

Figure 6 demonstrates that the performance of a cDGSW depends strongly on its mode of use; if fluid is reinjected at a temperature above the value T_S at the Earth's surface (i.e., f>0), the duration over which a given rate of heat production can be maintained is significantly reduced. This means that the performance of any such installation cannot be analysed in isolation, but must be considered in conjunction with the manner in which the heat output is used. The optimum situation is to reinject fluid at T_S (i.e., f=0). However, since any real heat load will operate at a temperature T_E significantly above T_S, reinjection at f=0 will require use of a heat pump (interfacing between the borehole circulation loop and the heat load) to lower the temperature of the fluid circulating back into the borehole below T_E, ideally to T_S. This is the essential reason why the hcDGSW configuration (Fig. 2(b)) is preferable to the simpler cDGSW variant (Fig. 2(a)). It has previously been noted (e.g., Collins and Law, 2017) that DGSW heat outputs should be interfaced through heat pumps, but the essential underlying physical reason has not previously been explained. The cost of the electricity required for powering the heat pump therefore needs to be factored into any economic analysis (see below). The electrical energy thus used will be converted to heat, contributing to the heat output supplied to the load, also influencing the analysis.

Table 2 illustrates in more detail the performance issues that arise when the heat output from a cDGSW supplies a heat load at a temperature T_E that significantly exceeds the surface temperature T_S (i.e., f>0), for various values of z_M and T_M. Heat production from the cDGSW is thus assumed to occur at rate Q, supplying a heat exchanger operating at a constant temperature T_E=30 °C, with no heat pump. In all cases, the output temperature T_o from the cDGSW decreases over time at a rate that is sensitive to Q. The proportion of the heat production that forms useable heat output, Q_U, is estimated as $Q \times (T_o-T_E)/(T_o-T_S)$ and thus becomes ever smaller as T_o decreases, even though Q is assumed constant. These results indicate that optimal cDGSW operation is quite sensitive; if Q is too high relative to z_M and T_M the cDGSW will cool so quickly that Q_U is limited, whereas if Q is too low Q_U will likewise be minimal. For a model cDGSW with z_M=2500 m and T_M=90 °C, with T_E=30 °C, the optimum value of Q to maximise the useable heat output over 5 years turns out to be ~160 kW, decreasing to ~140 kW if the timescale is set to 20 years. This model cDGSW might thus produce a steady output of useable heat over 20 years of ~70 kW or thereabouts, by initially operating at Q ~80 kW then gradually increasing Q to ~140 kW as the surrounding rock volume cools. Table 2 also shows the corresponding values for z_M=2000 m, for which the maximum feasible steady output of useable heat over 20 years would be only ~30 kW, and for z_M=3000 m, for which it would be ~130 kW.

For the three cDGSW configurations discussed above, the outputs of useable heat over 20 years can be estimated, respectively, as ~22.8, ~12.3 and ~5.3 GWh. At £0.03 per kWh, the value of the heat produced would be ~£0.61M, ~£0.37M and ~£0.16M; if a RHI subsidy of £0.0514 per kWh were included, these figures would rise to ~£1.86M, £1.00M and ~£0.43M. The capital costs of these projects might be estimated, respectively, as ~£2.9M, ~£2.3M and ~£1.8M (see below). It is thus apparent (even before any estimation of operating costs) that such projects have no chance of being cost-effective under current UK conditions. The calculations nonetheless exemplify that (over the depth range considered) deeper cDGSW boreholes would be less uneconomic than shallower ones, the increase in output of useable heat outweighing the corresponding increase in drilling costs, an effect that is explored further later in this study.

A further point evident from Fig. 6 is that a small increase in the rate of heat production from a cDGSW or hcDGSW can dramatically shorten its lifespan. This indicates that careful design, taking account of the local geothermal gradient and other site-specific parameters, is essential. Assuming a given borehole depth z_M, rate of heat production Q, and value of f, from equation (9) it is evident that the project lifespan depends exponentially on the geothermal gradient u and on the thermal conductivity k of the rock volume. A hcDGSW design that performs well at one site, with particular values of k and u, might thus have a much shorter lifespan at another site with slightly lower k or u.

The figures for specific performance (i.e., heat output per unit length of borehole) warrant comparison with those for 'ground source heat pump' systems utilising shallow-borehole (depth <~100 m) heat exchangers. The latter typically have specific performance of ~50-200 kWh m^{-1} yr^{-1} over annual operating cycles (e.g., Hepbasli, 2005), or ~6-23 W m^{-1}. The hcDGSW designs being discussed have higher specific performance, for instance 50 W m^{-1} for a 2000 m deep system with Q=100 kW. For comparison, doublet systems might have heat outputs of many megawatts (say 8 MW from two 4000 m boreholes; e.g., van Wees et al., 2010), indicating a specific performance of 1000 W m^{-1}. However, the overall energy balance is very different for shallow-borehole systems compared with DGSW systems: the former can operate sustainably because of influxes of geothermal heat from below and by surface heating from above, whereas (as noted above) for the latter the issue is not sustainable operation but determination of the lifetime over which 'heat mining' is worthwhile. To establish this, it is noted that a 50 m borehole with specific performance 10 W m^{-1} would have annual heat output of ~16 GJ. From Fig. 5, a year of operation will cool the surrounding rock volume to a radial distance of ~28 m, so will cool a cross-sectional area of ~2500 m^2. A representative geothermal heat flow is, say, ~90 mW m^{-2} or ~3 MJ m^{-2} yr^{-1}. Global warming and localized surface heating due to urban development can cause downward heat flows from the surface that can exceed the upward geothermal heat flux (e.g., Bayer et al., 2016; Westaway and Younger, 2016), amounting, say, to ~4 MJ m^{-2} yr^{-1}. The combination of these flows might thus supply ~18 GJ yr^{-1} of heat into the ~2500 m^2 cross-sectional area, sufficient to balance the heat production. This essential difference, between shallow-borehole systems that can operate sustainably and DGSW systems that 'mine heat', has not been recognized before (cf. Collins and Law, 2017).

Factors affecting dDGSW operation

As already discussed, the alternative dDGSW variant (Fig. 3(a)) involves production of thermal ground water to enhance the heat output that is feasible from conduction alone. Law et al. (2015) have indeed suggested that heat production might be supplemented in this manner; for example, 'bleed flow' at rate q_B=2 l s^{-1} at 50 °C above T_D would generate heat output of 2×10^{-3} m^3 s^{-1} $\times$ c $\times$ ρ $\times$ 50 °C (where c=4186 kg m^{-3} and ρ=1000 kg m^{-3} are the specific heat capacity and density of water; cf. equation (6)) or ~400 kW. Law et al. (2015) stated that q_B=2 l s^{-1} 'could be achieved from almost any geological formation'. In the light of this, the hydraulic conductivity of relevant rock formations will now be discussed, along with the handling of the water produced from dDGSW projects, which might include treatment costs.

Effects of hydraulic conductivity

Barker (1986) investigated the transient drawdown effect of a dDGSW, demonstrating that the required timescale is likely to be quite short. This topic can thus be analyzed, to place limits on the 'bleed flow' that might be feasible, using the standard Thiem (1906) solution for the steady-state drawdown ΔH of the phreatic surface in a confined aquifer of transmissivity T_K in the vicinity of a production well:

$$\Delta H = \frac{q_B}{2\,\pi\, T_K} \ln\left(\frac{r_A}{r}\right) . \qquad (20)$$

Here, r_A denotes the 'radius of influence' of the aquifer, and r is again radial distance. Law et al. (2015) envisage placing an open (screened) section in the well casing of substantial vertical extent above the well bottom. The maximum drawdown in this layer, at r=a, can be designated as ΔH_o. At distance from the borehole, the transmissivity of the layer in which this drawdown occurs will equal $K\times\Delta H_o$, where K is its hydraulic conductivity, from which it follows that the minimum value of K that can support the required flow rate under steady-state conditions is

$$K = \frac{q_B}{2\,\pi\,\Delta H_o^2} \ln\left(\frac{r_A}{a}\right) . \qquad (21)$$

Taking ΔH_o=100 m, r_A ~1 km and a=0.1 m, for q_B=2 l s^{-1} one obtains K ~3×10^{-7} m s^{-1}. Although some lithologies have K above this threshold, many in the UK and elsewhere do not (e.g., Allen et al., 1997; Lewis et al., 2006). Abesser et al. (2014) reached a similar conclusion, noting that water yields of ≥1 l s^{-1} are only feasible from boreholes in lithologies classified as 'moderate' or 'good' aquifers. Whether 'bleed flow' can significantly supplement the limited heat output of a cDGSW is thus site-dependent and cannot be presumed in general. With ΔH_o=150 m and q_B=0.9 l s^{-1}, equation (21) gives ~6×10^{-8} m s^{-1} as the hydraulic conductivity threshold for dDGSW operation to be feasible, indicating (as is evident from the form of equation (21)) that this threshold decreases as q_B is reduced or H_o is increased. Coarse sediments might well thus have sufficient hydraulic conductivity for dDGSW operation to be feasible. For example, K has been reported as ~10^{-5} m s^{-1} in Triassic sandstone from southern England (e.g., Smith, 1986); the operation of the aforementioned Southampton geothermal borehole as a wDGSW / hwDGSW indeed provides clear evidence of high K in such rocks.

Nonetheless, given that dDGSW projects have been proposed in granite, it is important to assess whether this lithology might have sufficient hydraulic conductivity for this technology to be feasible. It is indeed well known that intact granite at depth has very low K. For example, Brace et al. (1968) determined the permeability of Westerly Granite at ~50 MPa (roughly equivalent to ~2000 m depth) as ~63 nD (~6.3×10^{-20} m^2), equivalent (at ~70 °C, so water has a viscosity of ~0.4 mPa s) to K ~4×10^{-12} m s^{-1}. At 10 MPa (depth ~400 m), they measured its permeability as ~230 nD, indicating K ~10^{-11} m s^{-1}. Martinez-Landa and Carrera (2005) reported that intact granite at the Grimsel Test Site in Switzerland, at ~450 m depth, has K ~10^{-12} to ~10^{-10} m s^{-1}, in good agreement. Such values are so far below any conceivable threshold required for a dDGSW or hdDGSW to function that this technology stands no chance of success in intact granite.

On the other hand, many workers (e.g., Martinez-Landa and Carrera, 2005; Hamm et al., 2007) have reported hydraulic conductivities orders-of-magnitude higher in fractured granite; however, the values depend on the precise geometry of the fractures at each site, and are thus not readily predictable in general. For example, Martinez-Landa and Carrera (2005) reported that in fractured granite at the Grimsel site K varies between ~10^{-10} and ~2×10^{-8} m s^{-1}, with occasional more conductive fractures. For fractured granite in Korea at depths of <100 m, Hamm et al. (2007) characterized the statistics of variations of K, and how these correlate with length- and aperture-distributions of the fractures; K here thus varies between ~2×10^{-10} and ~3×10^{-6} m s^{-1}. Illman et al. (2009) reported results of tomographic analysis of pumping-test results to constrain variations in K in fractured granite in Japan. Their results vary between ~6×10^{-8} and ~10^{-6} m s^{-1}, the latter value occurring at ~500 m depth. Since fractures in granite (and other rocks) are typically created as a result of the effects of cooling and erosional unloading (cf. Brace et al., 1968; McGarr and Gay, 1978; Bourne and Willemse, 2001), one expects them to open progressively as depth decreases. Furthermore, since permeability and hydraulic conductivity depend on the cube of fracture aperture (e.g., Snow, 1969), one expects these quantities to decrease with increasing depth. However, it is difficult to extrapolate results such as those stated above for the higher pressure at greater depths. Overall, one does not expect K to be low enough at depths of ~2000 m in granite to sustain hdDGSW operation, although exceptions (at the high-K 'tail' of probability distributions) occasionally occur.

To illustrate this point with UK examples, Table 3 lists analyses of the hydraulic transport properties for the Rosemanowes production well (well RH-15) in the Carnmenellis granite, Cornwall (after Richards et al., 1994), and for the Eastgate-1 borehole in the Weardale granite, County Durham (after Manning et al., 2007, and Younger and Manning, 2010). Well RH-15 only has sufficient hydraulic conductivity to be viable as a hdDGSW over restricted vertical extents, none of which has sufficient transmissivity to yield the required flow. Furthermore, the transmissivity and conductivity values are relative to a nearby injection well; if that had not been operating, no fluid would have been produced at this production well. At Eastgate, rapid ingress of water occurred at ~410 m depth when the wellbore crossed a mineral vein, revealing the highest transmissivity ever measured within granite (Younger and Manning, 2010). Setting aside this exceptional discovery, the water ingress into the rest of this borehole at first sight indicates sufficient transmissivity for its use as a hdDGSW. However, as Manning et al. (2007) pointed out, this ingress occurs at points where the wellbore crosses other fractures, the most important of these being at ~730 and ~756 m depths. Once again, the overwhelming majority of the section in this borehole evidently has very low hydraulic conductivity, making it unsuited for use as a hdDGSW. The flow rates in the highly transmissive parts of this borehole are far in excess of what would be needed for hdDGSW operation; as Manning et al. (2007) and Atkins (2013) indeed noted, they might indeed be utilised for a conventional well doublet.

An additional factor affecting the viability of any dDGSW system, evident from previous analyses of shallow systems (e.g., Rees et al., 2004), is the increase in operating costs that will result from the greater drawdown within the borehole, which will accompany increased rates of 'bleed flow', due to the need for pumping to lift the circulating water through a greater height. If significant 'head lift' for produced water is necessary, the electrical power requirement might be large (e.g., Younger, 2014), potentially outweighing the value of the heat produced. To analyse this effect, q_C and q_B may be designated, respectively, as the volume flow rate required to produce heat by conduction (i.e., for the closed-loop circulation within a DGSW borehole heat exchanger) and the rate of bleed flow, with $\Gamma\equiv q_B/q_C$. It is assumed that the heat production utilizes a heat pump with coefficient of performance (COP; the ratio of useable heat output to electrical energy consumed) Ψ, the heat is sold at a price P_H per unit of energy produced, and the electricity used to power the heat pump contributes its own heating effect, then the rate of heat sale can be expressed as $\rho\times c\times\Delta T\times(q_C+q_B)\times((\Psi+1)/\Psi)\times P_H$ where ρ and c are the density and specific heat capacity of the circulating water and ΔT is its temperature above ambient. The rate of expenditure on operating costs can be expressed as $\rho\times g\times H\times(q_C+q_B)\times P_E$ + $\rho\times c\times\Delta T\times(q_C+q_B)\times(1/\Psi)\times P_E$ where g is the acceleration due to gravity, H is the lift height or 'hydraulic head' in the borehole, and P_E is the cost per unit of electrical energy used. The first of these terms represents the 'head lift' pumping and the second the operation of the heat pump, both these processes being assumed 100% efficient. Defining $\beta\equiv P_E/P_C$, the difference in operating surplus (or deficit) between using bleed flow and omitting it from the design (thus assuming a closed loop configuration with H=0, and with Γ=0) can thus be determined. The condition for the alternative open loop design (with H and γ both nonzero) being the more profitable of the two can thus be written, after several algebraic steps, as $H<H_o$ where

$$H_o = \frac{c\,\Delta T}{g}\,\frac{\Gamma}{1+\Gamma}\,\frac{1+\Psi-\beta}{\Psi\,\beta}\,. \qquad (22)$$

Using plausible values for these parameters (ΔT=40 °C, c=4186 J kg^{-1} °C^{-1}, g=9.81 m s^{-2}, β=3 and Ψ=4), H_o is ~250 m for Γ=0.1, increasing to ~1400 m for Γ=1. As another example, Law et al. (2015) discussed a dDGSW scenario in which 200 kW of heat production at ΔT=40 °C was inferred to include a contribution of 50 kW from bleed flow. Thus, Γ=1/3, q_C ~0.9 l s^{-1} and q_B ~0.3 l s^{-1} are required. From equation (22), H_o is ~700 m; equating this value to ΔH_o in equation (21) gives K ~9×10^{-10} m s^{-1}. Even with such extreme drawdown, it is unlikely that K will be low enough in intact granite to make dDGSW operation worthwhile; moreover, this calculation takes no account of capital cost, which might make neither the cDGSW nor the dDGSW option economic relative to other energy sources (see below).

Produced water treatment and associated costs

As already noted, it is a legal requirement in the UK (and other European Union member states) for produced water discharged into the environment from boreholes to comply with regulatory limits for concentrations of contaminants such as dissolved metallic ions, and if necessary to require treatment (e.g., DEFRA, 2014). From previous experience (e.g., Edmunds, 1975, 1986; Younger et al., 2016), water produced from depths of ~2 km or more is expected to be a concentrated brine, potentially containing dissolved metallic ions including radionuclides in significant concentrations. Radium, an example radionuclide, is chemically similar to calcium, so can precipitate as radioactive scale within pipework, thus requiring costly treatment. Setting the latter aspect aside for now and concentrating on the discharge of produced water, treatment technologies are classified as active or passive (e.g., Johnson and Hallberg, 2005): the former typically require inputs of electricity and chemical reactants, making them more expensive; the latter typically involve 'natural' processes such as filtration and biochemical reactions (e.g., sulphate reducing bacteria removing metallic ions from solution as insoluble sulphide 'sludge'), which typically proceed without routine human intervention within artificial wetlands at site. Active treatment options include installing treatment plant at site or transporting produced water by tanker lorry for treatment elsewhere. There is a substantial literature on this topic; contributions relevant to UK issues include the works by Younger (2000) and Younger et al. (2002, 2005). Many workers (e.g., Kiessig et al., 2004; Younger et al., 2005) have noted that cost estimates depend on site-specific details, including discharge rates, so once again 'off the shelf' solutions are not possible. It should also be noted that rather than preventing all pollution from entering the environment, the aim of these treatment options is to reduce pollutant concentrations to 'acceptable' levels that the environment can bear (e.g., Kiessig et al., 2004); adopted strategies thus require regulatory approval.

Since hdDGSW installation in granite is being proposed (GEL et al., 2016; Collins and Law, 2017; see below), the track record of treatment of mine water discharges from this lithology is relevant. The largest mine water treatment scheme in a granitic region of the UK is for the former Wheal Jane tin and copper mine in Cornwall, SW England (e.g., Knight Piesold and partners, 1995, 1998; Coulton et al., 2003; Younger et al., 2005), which adjoins the Variscan age Carnmenellis pluton. Commissioned in the 1990s, this active treatment scheme was originally budgeted with £4.25M capital cost and £1.03M annual operating cost, to treat ~5×10^{6} m^{3} of water per year (e.g., Knight Piesold and partners, 1998) at a typical rate of ~160 litres per second, principally for removal of iron, copper and zinc, also arsenic, cadmium, manganese, nickel and aluminium. The annual operating cost has subsequently been reported as ~£1.5M (e.g., Morris, 2014; Peacock, 2014), or ~£0.3 per cubic metre.

Bailey et al. (2016) describe a more recent active treatment scheme, to remove zinc from discharge from the former lead and zinc mine at Force Crag in Cumbria, NW England, a region underlain by Caledonian age granite plutons. Bailey et al. (2016) reported that the option chosen had a capital cost of €1.92M and annual operating cost of €0.17M to treat discharge at ~6 $l\ s^{-1}$. The latter amount is ~€0.9 or ~£0.8 per cubic metre. Alternative passive treatment options would have been cheaper, but were not preferred because of the proven technology adopted (Bailey et al., 2016).

For comparison, passive treatment is utilized for example at the former Pöhla-Tellerhäuser uranium mine in the Erzgebirge mountains of SE Germany. Uraniferous mineralization occurs here in hydrothermal veins associated with Variscan granite intrusions (e.g., Förster, 1999; Förster et al., 1999). As discussed, for example, by Kunze and Küchler (2003), Kiessig et al. (2004) and Küchler et al. (2005), after uranium mining ceased in 1990 the main concerns at this site have been dissolved iron, radium and arsenic, as well as uranium and manganese, in waters that discharge at ~17 $m^3\ hr^{-1}$ or ~5 $l\ s^{-1}$. The passive treatment system, installed at a capital cost of ~€0.5M, comprises six artificial wetlands covering almost 500 m^2 of land. The operating cost was estimated as ~€2 per cubic metre in the first two years of operation, when its performance was intensively monitored, subsequently decreasing to ~€0.2 or £0.17 per cubic metre. This system superseded an active treatment plant with operating cost ~€4 per cubic metre; it paid for itself in circa one year.

Turning to the possible option of transporting produced water for off-site treatment, it is noted that the 2 $l\ s^{-1}$ discharge rate envisaged for a dDGSW would result in 173 m^3 of produced water per day. Transporting this volume to a treatment plant in 20 m^3 tanker lorries would require 9 movements every 24 hours. The cost of treatment depends on local conditions and potential economies of scale; Dahm and Chapman (2014) reported that in the USA such costs might thus vary between 0.20 and 8.50 per barrel (~£1-£40 per cubic metre). As Collins (2016) has discussed, in the USA water treatment, including transportation by tanker lorry, for a small-scale installation such as this might cost 2.80 per barrel, equivalent to ~£9 per cubic metre or ~£1600 per day. One might break this amount down in terms of the costs of buying and operating the tanker lorries required, the wages for their drivers, and the direct costs of the water treatment, using whichever technology is adopted at the treatment site. Dahm and Chapman (2014) also described modular active treatment units, which can operate automatically at drilling sites without routine operator intervention, and might (subject to environmental permitting) discharge the treated water directly into the environment (e.g., into the sea). The treatment cost was reported as ~0.75 per barrel or ~£3.80 per cubic metre. Each of these calculations assumes continuous operation, whereas a dDGSW might be operated intermittently, at times of peak heat demand. Such intermittency would result in higher unit costs for treatment of produced water, because assets (e.g., tanker lorries) would only see intermittent use.

It is evident that these costs require consideration relative to the value of the heat that is produced. For example, if a dDGSW or hdDGSW produces heat at 400 kW with 2 $l\ s^{-1}$ of 'bleed flow', it implies 18 litres of discharge per kWh. If the water treatment costs as much as £3.80 per cubic metre (see above), it would equate to £0.068 per kWh. If each kWh of heat produced has a notional value of, say, £0.03, then the water treatment cost would exceed the value of the heat produced, calling the economics of the project into question. This is of course not a definitive limit as the value of the heat produced might be influenced by subsidy payments (see below). Nonetheless, even if cheaper water treatment is feasible, its cost might well be a substantial proportion of the value of the heat produced and thus form an essential budget component for any project.

Analysis of hcDGSW economics

The preceding sections highlight several difficulties with the DGSW concept. First, dDGSW or hdDGSW operation requires rocks of relatively high hydraulic conductivity at depth. Second, treatment costs for the produced water will adversely impact economic viability, making dDGSW or hdDGSW operation particularly problematic in granite; intact granite will have too low a hydraulic conductivity and even if the granite is fractured the probability of a given section in it of vertical extent ~100-300 m having sufficient transmissivity is low. Whilst one might conceivably find a suitable site for this technology somewhere, it would require a substantial research (e.g., using electrical prospecting techniques; cf. Beamish, 1990), thus defeating the aim of providing an 'off the shelf' technology. The third issue concerns the cDGSW variant. In its simplest form, without a heat pump (Fig. 2(a)), the COP will be high; for example, Law (2014) reported that a prototype installation had a COP of ~50. However, as already noted, T_D for this variant will significantly exceed the ambient temperature T_S so f will be well above zero, limiting the useable heat output. In contrast, for the hcDGSW variant, with the circulating fluid also fed through a heat pump, to reduce its reinjection temperature much closer to T_S (Fig. 2(b)), significantly higher heat production can be anticipated, in accordance with earlier discussion, outweighing the fact that the COP for the additional heat output supplied via the heat pump will inevitably be less than that extracted via the heat exchanger (see below). On this basis, the hcDGSW is the variant with the greatest potential viability.

A model is thus developed to assess the output performance and economics of a hcDGSW. It is assumed that an electrically powered pump maintains the circulation around the closed loop at the rate necessary for the required rate of heat production by creating a pressure difference ΔP. ΔP will equal the sum of pressure drops along the circulation loop, in the heat exchanger and the associated pipework. Most of this pressure drop will be in the pipe carrying the upward component of the circulation, as a result of its substantial length L and relatively small diameter D. Neglecting other contributions, one may write (e.g., from Lyons et al., 2009, p. 167)

$$\Delta P = \frac{f_D \rho V^2}{2 D} L \qquad (23)$$

where f_D is the Darcy-Weisbach friction factor for the flow regime. The value of f_D itself depends on the Reynolds Number, Re, of the flow, where

$$Re \equiv \frac{V D \rho}{\eta} \qquad (24)$$

η being the (dynamic) viscosity of the fluid. In general, if Re < ~2000, the flow is laminar, whereas at higher Re it is turbulent. For laminar flow, f_D takes standard values as a function of Re (e.g., McKeon et al., 2004). However, for turbulent flow, f_D also depends strongly on the roughness, Ω, of the inner surface of the pipe, being markedly higher the rougher this is. Ω is defined as

$$\Omega \equiv \frac{\varepsilon}{D}, \qquad (25)$$

where ε is the characteristic height of surface irregularities; ε might be ~0.05 mm for steel pipe or ~0.005 mm for composite or plastic pipe (e.g., Enggcyclopedia, 2011). One may look up f_D for the value of Ω corresponding to any choice of ε and D and for any given value of Re on a standard 'Moody Diagram' (Moody, 1944) for input into calculations using equation (23).

Assuming 100% efficiency, the electrical power Q_E used by the pump will equal $\Delta P \times Q$; assuming, also, that all the produced heat is used, the COP, Ψ, will equate to Q / Q_E. These equations can be combined to give

$$\Psi = \frac{\pi^2 \rho^2 c^3 (T_O - T_S)^3 D^5}{8 f L Q^2}. \qquad (26)$$

As a test of this equation, it is noted, once again, that Law (2014) reported Ψ ~50 for a prototype cDGSW installation, which had instantaneous heat production Q=380 kW, L=1800 m, and T_O=63 °C. T_S can be estimated as ~10 °C and ρ, c and η as ~1000 kg m^{-3}, ~4186 J kg^{-1} °C^{-1}, and ~0.45 mPa s, respectively, so q_B was ~1.7 l s^{-1} but D was not specified. Westaway (2016) estimated D as ~40 mm from a photograph of the pipe used in this prototype test, which implies V ~1.4 m s^{-1}, suggesting Re ~120,000. The flow was thus highly turbulent; from Moody (1944) for very smooth pipe f_D would be ~0.018. For this configuration, equation (26) predicts Ψ ~300, although with D ~30 mm it would reduce to ~70, in rough agreement with the Law (2014) estimate. The 'thermosiphon effect', whereby the lower mean density of the fluid in the inner pipe, due to its higher mean temperature, reduces the pressure drop below the value of ΔP calculated using equation (23), and thus reduces the electrical power requirement for pumping (e.g., Alimonti et al., 2016; Spitler et al., 2016) and the associated operating cost, is also neglected; the calculations set out below indicate that this item forms a minimal part of the overall budget of a DGSW project.

The COP for the heat pump, when extracting heat from a source at temperature T to a heat load at temperature T_E, is assumed to vary, after Baster (2011), as

$$\Psi = 6.70 \exp(-0.022 \times (T_E - T)) \quad . \qquad (27)$$

If the heat pump thus cools the circulating fluid from an initial temperature T_I to the ambient surface temperature T_S, its mean COP, Ψ_M, will be

$$\Psi_M = \frac{1}{T_I - T_S} \int_{T_I}^{T_S} \Psi \, dT \qquad (28)$$

or

$$\Psi_M = \frac{6.70}{0.022\,(T_I - T_S)} \left(\exp(-0.022 \times (T_E - T_I)) - \exp(-0.022 \times (T_E - T_S))\right) \quad . \qquad (29)$$

The heat exchanger is presumed to cool the circulating fluid to temperature T_E; the heat pump then cools it further, from T_E to T_S, after which it is reinjected. If T_O<T_E then the heat exchanger is assumed to be bypassed, such that T_I=T_O. Equation (27), for heat pump performance, was derived by Baster (2011) by regression analysis using an ensemble of performance data for air source heat pumps. It is used here as an approximation, in the absence of performance data for any water source heat pump with the required operating characteristics. The analysis proceeds by calculating the variation of T_O with time for a given Q using equations (3), (4) and (5), then calculating the heat outputs from the heat exchanger and heat pump, along with their electricity consumption. The calculations are run for a lifespan of 20 years consistent with the current RHI subsidy regime.

Recent geothermal projects in the UK have involved drilling to ~1 km at Eastgate, County Durham (Manning et al., 2007; Younger and Manning, 2010), with a budget of ~£0.5M, and to ~1.8 km at the Science Central site in Newcastle upon Tyne (e.g., Westaway and Younger, 2016; Younger et al., 2016), with a budget of ~£1.2M. The £1.35M capital cost estimated by GEL et al. (2016) for the proposed 2 km AECC borehole is consistent with these values. In the geothermal project costing model used in the Netherlands, boreholes of depth z_M (in metres) have estimated capital cost Σ (in €)

$$\Sigma = f_K \, (0.2 \times z_m^2 + 700 \times z_M + 250000) \qquad (30)$$

(van Wees et al., 2010), where f_K is a scale factor to incorporate overheads, typically set to 1.5. This predicts much higher costs than would be expected in the UK, reflecting different local conditions (salaries, etc). However, it can be used to estimate the costs of deeper onshore boreholes in the UK, none having been drilled for many years, in proportion to the cost of drilling to 2000 m. One can thus estimate £1.80M, £2.28M and £3.44M as the capital costs of 2500 m, 3000 m and 4000 m boreholes, ~33%, ~69% and ~155% higher, respectively. Again for the AECC project, GEL et al. (2016) have estimated a budget of £2.293M, consisting of £1.350M for drilling and completion of the well, £0.265M for the required equipment, and £0.678M for overheads, including project planning and project management costs. The 'off the shelf' nature of hcDGSW schemes is expected to reduce planning and project management costs; such a project based on a 2 km borehole might thus be delivered for well below £2M, say ~£1.8M, with corresponding costs estimated as ~£2.4M, ~£3.0M and ~£4.6M for 2.5 km, 3 km or 4 km boreholes. It is assumed that the electricity used costs £0.095 per kWh, a typical tariff for a 'large' industrial consumer in the UK (DECC, 2016a), the heat produced is sold at £0.03 per kWh, and a RHI subsidy of £0.0514 per kWh is payable. Once the system is commissioned, it is assumed to operate automatically, so the operating costs equate to the cost of the electricity used.

Figure 7(a) illustrates the predicted revenue surplus (revenue, including RHI subsidy payments, minus operating costs) over a 20 year project lifespan for the 2000 m hcDGSW installation. The optimum configuration has Q=193 kW and yields a revenue surplus of £2.44M. Figures 7(b) and (c) illustrate the corresponding solutions for z_M=2500 m, with the optimum at Q=295 kW, yielding a revenue surplus of £3.71M; z_M=3000 m, with the optimum at Q=416 kW, yielding a revenue surplus of £5.23M; and z_M=4000 m, with the optimum at Q=711 kW, yielding a revenue surplus of £8.97M. It is thus evident that with optimal operation each of these designs can cover its own capital cost, the budget surplus increasing with z_M. More formal economic calculations, for example calculating the Net Present Value at the end of the project life including index-linking, to take account of future price inflation, of subsidy payments, heat sales, and estimates of future costs of electricity purchase, would reach similar conclusions. Figure 9 illustrates the predicted variations over time in T_O for the four solutions in Fig. 7. Overall, these calculations indicate that, over the depth range 2-4 km, the deeper a hcDGSW borehole is, the more heat it can produce on a given timescale, the value of the increased heat production outweighing the increased cost of drilling. The ensemble of solutions obtained for different depths z_M indicate that the optimum heat output and operating surplus over 20 years, Q and S, vary with z_M as:

$$Q = 0.000131\, z_M^{1.87} \qquad (31)$$

and

$$S = 0.00000166\, z_M^{1.87} \qquad (32)$$

with Q in kW and S in M£. Combining equations (31) and (32) gives

$$S = 0.0127\, Q \quad , \qquad (33)$$

indicating proportionality between S and Q, a surprising result given the complexity of the equations whose solution yields the values of S and Q.

Once the twenty-year duration of the RHI subsidy has elapsed, continued heat production at what was previously the economically optimum rate would no longer be economic for z_M=2000 m, 2500 m, or 3000 m, but would be economic for ~5 years longer for z_M=4000 m (Fig. 9). A possible next phase might be to re-purpose such a borehole for seasonal heat storage rather than heat production, as Westaway (2016) has discussed. Based on Westaway's (2016) calculations, the rocks surrounding a 3000 m deep vertical borehole might have a seasonal heat storage capacity of ~2 GWh. Currently the RHI subsidy scheme excludes production of stored heat from any deep borehole, making it uneconomic in the UK to purpose-drill boreholes for this function. However, once the capital cost of a borehole has already been discounted as a result of heat production, it might thus be reused at little additional extra cost. Ultimately, after many such boreholes have been developed, such reuse might contribute significantly towards achieving a sustainable provision of heat supply, which represents ~50% of the energy demand in the UK (e.g., DECC, 2012).

The limited output of useable heat from hcDGSW projects invites economic comparison with other small-scale renewable energy technologies, such as micro hydro schemes (cf. Bracken et al., 2014). Bracken et al. (2014) indeed discussed one such scheme in northern England, with a capital cost of £415,000, producing 0.165 GWh of electricity per year with an estimated 40 year lifespan, so 6.6 GWh in total. Assuming the resulting zero-carbon output substitutes for electricity derived from the UK's current generating mix (in 2015, 337.7 TWh of electricity was generated with 144.1 MT of CO_2e emissions, according to DECC, 2016b, and BEIS, 2017, giving 0.427 kg CO_2e per kWh), the lifetime carbon budget can be calculated. If the project began operating now (November 2017) its electricity sales into the National Grid might attract a feed in tariff (FIT) of 7.78 p per kWh (OFGEM, 2017) in addition to a sale price of maybe 5 p per kWh (e.g., Renewables First, 2015), so 12.78 p per kWh in total. The total revenue from electricity sales over 40 years, at present prices, ~£843,000, would significantly outweigh the capital cost, and would involve an overall saving in CO_2e emissions of 2818 tonnes. As a worst-case scenario, if a small part of the capital cost had been covered by a grant from public funds, making the entire project ineligible for FIT subsidy, the revenue from electricity sales would be only ~£330,000, making the net cost of the project at present prices ~£85,000, so the emissions saving would have been achieved at a cost of ~£30 per tonne. However, DTI (1999) estimated that the potential of micro hydro (and small-scale hydro) projects in the UK is ~35 TWh per year, or roughly 10% of the present generating capacity (and, thus, ~2.5% of the total energy supply). Thus, although this technology is economic under the present subsidy regime, it has insufficient capacity to make a significant difference to the present-day energy mix.

The economically optimum z_M=3000 m hcDGSW solution in Fig. 7(c) and Fig. 9 produces 92.7 GWh of heat and consumes 24.4 GWh of electricity over 20 years, at average annual rates of 4.64 and 1.22 GWh yr^{-1}. If the heat were used to offset burning of natural gas (with emissions equivalent to 0.181 kg CO_2e per kWh according to EIA, 2016) and the electricity were again derived from the UK's current generating mix, the lifetime carbon budget can be calculated. Allowing for the carbon embodied in the construction of the hcDGSW (CO_2e of ~110 tonnes for the drilling activities and ~240 tonnes for the materials used, assuming use of well-casing made of a corrosion-resistant composite material; data from GEL et al., 2016, scaled in proportion to z_M=3000 m), the overall carbon budget (expressed in terms of CO_2e emissions) balances a saving of ~16785 tonnes against ~10422 tonnes for the electricity used plus the embodied ~350 tonnes, giving a net saving of ~6013 tonnes. The overall cost of the RHI subsidy would be ~£4.77M, or ~£793 per tonne of CO_2e emissions saved. Calculated on the same basis, the z_M=4000 m solution in Fig. 7(d) and Fig. 9 would produce 151 GWh of heat and consume 34.7 GWh of electricity over 20 years, achieving a net saving of ~11995 tonnes of CO_2e emissions in return for a RHI subsidy of ~£7.75M or ~£646 per tonne.

The above values can be compared with costs and environmental benefits of other energy supply technologies (such as, micro hydro projects) or energy efficiency measures. The results indicate that hcDGSW projects can be economic in a locality with a geothermal gradient as high as 32 °C km^{-1}. Table 4 lists the towns and cities in Britain where the geothermal gradient is as high as this. The urban areas thus covered represent ~20% of the UK population, so the potential capacity of DGSW projects in these localities might amount to ~20% of the demand for space heating or ~10% of the total energy demand in the UK. Thus, although the cost per unit emission saving is much greater than for, say, micro hydro, the overall potential of this technology is also much greater.

The cost per unit emission saving can be reduced significantly by operating a hcDGSW below its economically optimum heat production rate. For example, for the aforementioned z_M=3000 m design, operation for twenty years at Q=215 kW would consume 4.9 GWh of electricity at cost of ~£0.5M and output 42.6 GWh of heat providing ~£3.5M of revenue, including RHI subsidy, thus just covering the ~£3.0M capital cost, but reducing the cost per unit emission saving to ~£416 per tonne of CO_2e emissions saved. Likewise, for the z_M=4000 m design, operation for twenty years at Q=328 kW would consume 5.2 GWh of electricity costing ~£0.5M and output 62.7 GWh of heat providing ~£5.1M of revenue, thus just covering the ~£4.6M capital cost, but further reducing the cost per unit emission saving to ~£372 per tonne of CO_2e emissions saved. Anticipating future decarbonisation of electricity generation, greater savings in emissions at correspondingly reduced unit cost would result. A future regulatory regime might approve subsidy payments subject to a mode of operation that achieves an approved compromise between the economically optimum and environmentally optimum heat outputs. These economic and environmental considerations confirm that the hcDGSW concept might have substantial potential as a source of low-carbon heat, warranting more detailed investigation.

DGSW Case studies

Arguably the most detailed published numerical model for a cDGSW is that by Alimonti and Soldo (2016), which was applied by these authors and by Alimonti et al. (2016) to assess the geothermal potential of repurposing a disused oil well at Villafortuna in northern Italy. Their analysis makes various assumptions, for instance that the radius of the cooling effect around a cDGSW, given by equation (12), is exact rather than an approximation. Their method, analyzing heat flow using the thermal resistance of each concentric component of the borehole heat exchanger and surrounding rock volume, is valid under steady-state conditions (e.g., Carslaw and Jaeger, 1959, pp. 189-193), and is thus also an approximation under the non-steady-state conditions during DGSW operation. Furthermore, as already noted, their analysis assumed operation at constant volume flow rate, with heat production declining over time, rather than at a constant rate of heat production, although after many years of operation their predicted rate of change of the rate of heat production is low. Nonetheless, for the parameters applicable to their case study (z_M=6100 m, a ~0.1 m, and D=0.045 m for the borehole; T_S=25 °C, T_M=170 °C, k=2.5 W m^{-1} $°C^{-1}$, and κ=1.2 mm^2 s^{-1} for the surrounding rock volume; and T_D=40 °C for the surface heat exchanger installation) Alimonti et al. (2016) determined that the optimum operational mode required q_B=20 m^3 hr^{-1}, resulting in Q=1200 kW at T_O=100 °C after 10 years of operation. For comparison, using the present analytic model with the same input parameters, T_O=100 °C after 10 years of operation would require output at a steady Q=530 kW. However, it is unclear whether this means that the present analytic approach has a tendency to underestimate the output achievable from cDGSW installations, or whether the difference in operational mode or the approximations made in the Alimonti et al. (2016) analysis account for the difference. Comparison will now be made between the results of the present analysis and the Law et al. (2015) DGSW conceptual model. The economics of two proposed DGSW projects, in Aberdeen and Kilmarnock, Scotland, and a hypothetical hcDGSW project in Darlington, northern England (cf. Table 4), will also be discussed.

Law et al. (2015) conceptual model

Law et al. (2015) presented numerical modelling results for a model DGSW operating in either cDGSW or dDGSW modes. This was assumed to have z_M 2500 m, u 32 °C km^{-1}, and T_M 80 °C, implying T_S=0 °C; other parameters (including T_L, a, k, and κ) were not specified. Law et al. (2015) stated that their analysis used the U.S. Geological Survey finite element code SUTRA (Voss and Provost, 2010), their mesh being defined for a volume of square cross-section with sides extending 250 m away from the borehole, representing a single 90° quadrant (the other three quadrants being equivalent, given the symmetry of the problem). They did not specify the spacing of elements within their mesh or indicate the nature of the boundary conditions applied at its vertical and horizontal surfaces. Although SUTRA is primarily intended for solving subsurface fluid flow problems, it can also calculate heat conduction in the absence of fluid flow. As Law et al. (2015) explained, their method incorporates calculations using the thermal resistance of each concentric component of the borehole heat exchanger and surrounding rock volume. However, as already noted, this method is valid under steady-state conditions (e.g., Carslaw and Jaeger, 1959, pp. 189-193), and thus serves only as an approximation under the non-steady-state conditions that develop during DGSW operation.

The outputs of this model in cDGSW mode, for steady heat production of 50, 100, 200 and 400 kW, over ten years of operation, are summarized in Fig. 4. Thus, after ten years, T_O is predicted to be ~70, ~64, ~51 and ~27 °C, respectively, indicating cooling in each of these cases by ~10, ~16, ~29 and ~53 °C at the bottom hole depth. The cooling on this timescale per kilowatt of heat production is thus ~0.20, ~0.16, ~0.15, and ~0.13 °C kW^{-1} for these four configurations. Their model is therefore non-linear, despite the governing equations (e.g., equation (2)) being linear, which is difficult to understand. Law et al. (2015) also inferred that for Q=50 or 100 kW their model cDGSW reaches a steady state within ten years, which is untenable given that heat is being produced much faster than it is being replenished by conduction from surrounding parts of the subsurface rock volume; however, this behaviour can be described as an 'almost steady state condition' (see earlier discussion).

Although Fig. 4 indicates that the cooling curves predicted in the present study have similar general shape to those reported by Law et al. (2015), they do not match closely, and cannot be made to agree for any single set of model parameters. This latter aspect is illustrated in Fig. 8 in which the present analytic solutions are converted into dimensionless form, using the dimensionless variables $y=-4 \pi k \Delta T / q_M$ and $x=\ln(4 \kappa t / a^2)$. Transformed thus, these solutions all plot as the same straight line with gradient -1 and y-intercept γ. However, when transformed in the same way, the Law et al. (2015) solutions plot as curves, which do not overlie each other and diverge as x increases. Different choices of the parameter values required to transform these solutions to dimensionless variables would change the positions of these solutions on this plot but would not affect the predicted shape of the transformed solutions, so incorrect choices of these values are not the cause of these mismatches.

It was initially assumed that the cause of these mismatches was the omission from the analytic model, in its original form, of the borehole 'end-correction' and the effect of geothermal heat flow; hence, the derivation of these corrections. It was indeed initially thought possible that these corrections might have a larger effect the greater the subsurface cooling, and might thus act to 'draw in' additional heat from the surroundings to the borehole, either below it or beyond it to the sides. However, the small magnitude of these corrections (noted above), for DGSW operation over timescales relevant to Figs. 4 and 8, mean that this is not the explanation.

It will be recalled that the calculations for cDGSW operation in the present study incorporate a balance between heat production from the borehole heat exchanger and heat lost by conduction due to cooling of the surrounding rock volume. It can also be presumed that the Law et al. (2015) calculations likewise incorporate conservation of energy. Nonetheless, the Law et al. (2015) calculations indicate less cooling of the rock volume surrounding the borehole at its bottom hole depth than the calculations in the present study, despite indicating higher heat outputs. For this to be feasible while maintaining conservation of energy requires the Law et al. (2015) numerical model to produce more cooling than the present model across at least part of the range of depth between the Earth's surface and the well bottom. The resulting creation of a non-linear geothermal gradient along the borehole might account for the non-linear behaviour that has been noted. Nonetheless, the test data provided by Law (2014) indicate a linear temperature gradient at depths where the temperature exceeds the injection temperature T_D, and indeed motivated incorporating the equivalent assumption into the present analytic model. However, these data only relate to eight hours of operation; in the absence of test data documenting the DGSW behaviour on much longer timescales, this discrepancy cannot currently be resolved.

The mismatch between predictions Q_A from the present analytic model and for Q_N from the Law et al. (2015) and Alimonti et al. (2016) numerical models, for heat output rates over ten years of cDGSW operation, is illustrated in Fig. 10. These data points fit the regression equation

$$Q_N = 0.2995\, Q_A^{1.3134} \qquad (Q_A \geq 50 \text{ kW}) . \tag{34}$$

Calculations in the present study incorporate values of Q_A, but might in principle be modified to use values of Q_N from this regression equation. This would result in significantly more favourable assessments for options with Q_A >>50 kW. On the other hand, it is possible that the apparent consistency between these two numerical modelling approaches might reflect their use of similar methods, including applying theory for steady-state behaviour to non-steady-state situations, in which case neither numerical method approach might provide accurate results.

The HALO project, Kilmarnock

Planning documentation (East Ayrshire, 2017) states that this project will produce heat at a rate of 2.1 GWh yr^{-1}, equivalent to a steady rate of 239 kW, from a 2 km deep borehole to be located at British National Grid (BNG) reference NS 42641 38505. The geothermal gradient in this part of SW Scotland can be estimated using data from the Slatehole borehole (British Geological Survey [BGS] identifier NS42SE4; at NS 4907 2343, ~16 km to the SSE), where a bottom hole temperature of 40.0 °C at 1024 m depth and a geothermal gradient of 29.8 °C km^{-1} have been measured (e.g., Gillespie et al., 2013). Extrapolation at a constant geothermal gradient would give a temperature of ~69 °C at 2 km depth. The capital cost of this DGSW installation has been funded through a £1.8M grant from the Scottish Government (Geon, 2017); as a result of this public funding of capital expenditure, the heat output is ineligible for RHI subsidy. Law and Collins (2017) state that a heat pump would be used in any DGSW installation with a basal temperature as low as this, given the need to interface with a heating system operating at ~70 °C. Preliminary documentation (Geon, 2017) depicted a closed circulation loop, implying (in the terminology used in the present study) that the design is for a hcDGSW. Subsequent definitive documentation (East Ayrshire, 2017) indicates that the design will allow for the possibility of bleed flow, but the standard operational mode will involve closed circulation; this project will therefore be analysed as a hcDGSW. The same documentation (East Ayrshire, 2017) also indicates that the design includes a very wide borehole, with a diameter of 0.75 m.

The reported output, equivalent to a steady 239 kW, is well above the optimum value of 194 kW for a 2 km borehole under the high geothermal gradient conditions envisaged for Fig. 7(a), and is thus likely to exceed even more the optimum operational mode for the lower geothermal gradient in the Kilmarnock area. The local outcrop consists of the Scottish Coal Measures Group of Upper Carboniferous (Westphalian) age, underlain by the Clackmannan Group (Namurian), Strathclyde Group (Viséan), Inverclyde Group (Tournaisian), and Stratheden Group (Late Devonian). Like elsewhere in Britain (cf. Westaway and Younger, 2016) such Carboniferous sequences consist of cyclic alternations of lithologies (mudstone, sandstone, limestone, and coal) with diverse thermal properties. For example, Carboniferous sandstone may have k as high as 4.9 W m^{-1} °C^{-1} (England et al., 1980) whereas the value of k for coal will be an order-of-magnitude less (e.g., Westaway and Younger, 2016). The underlying Devonian 'Old Red Sandstone' rocks include tight sandstones for which k as high as 5.2 W m^{-1} °C^{-1} has been measured elsewhere in Europe (Chekhonin et al., 2012). The thermal conductivity and diffusivity applicable for the HALO DGSW will be a harmonic mean of the values for these diverse lithologies in representative proportions, and has not been reported. Values of k=4 W m^{-1} °C^{-1} and κ=1.5 mm^2 s^{-1} will be assumed for the present analysis, although these might well be overestimates; moreover, the high k in the deeper part of the section implies a lower geothermal gradient than has been assumed by extrapolation from the Slatehole borehole dataset, suggesting that the 69 °C bottom-hole temperature is probably also overestimated. This combination of optimistic assumptions has been made to avoid any risk of underestimating the potential of the HALO project, for the quantitative assessment that follows.

Figure 11(a) indicates that if heat were to be produced from the HALO DGSW at a steady rate of 239 kW, the output temperature would rapidly decline from the initial 69 °C, falling to ~23 °C over twenty years. Figure 11(b) depicts the economic analysis of this DGSW for a 20 year assumed lifespan. The economically optimum heat production rate is surprisingly low, ~32.4 kW, despite the optimistic assumptions. This value is highly sensitive to the assumed pricing structures for heat and for electricity, it being evident that the modest budget surplus depicted is due to the very small difference calculated between expenditure on electricity and revenue from heat sales. A less favourable pricing structure, or a lower bottom hole temperature (<58.1 °C), would make operation of this DGSW uneconomic relative to the option of shutting it down and using the electricity saved (by not powering its heat pump) to heat buildings directly. The economics of this project are indeed hampered by the output temperature being always below the assumed 70 °C input temperature of the heat load (after Collins and Law, 2017), so all the heat output has to be transferred through the heat pump, ~13.6 kW of electrical power being required to supplement the ~32.4 kW of heat production to produce an overall output of ~46 kW of useable heat. As Fig. 11(a) shows, with such a low rate of heat production, very little temperature fall occurs within the DGSW over 20 years, so in principle this project could function at this very low rate for a very long time.

Figure 12 shows the equivalent outputs for a project with the same design parameters as for HALO, but with the borehole diameter reduced to 0.2 m. With this narrower borehole, if heat were to be produced at a steady 239 kW, the output temperature would fall more rapidly, reaching ~23 °C within eighteen months and ~11 °C after twenty years. However, the economically optimum heat production rate, ~29.3 kW, producing ~41.5 kW of useable heat, does not differ much compared with the wider-borehole design adopted. Given the lower capital cost and lower CO_2 emissions from drilling, which would result from this alternative narrower-borehole design, the appropriateness of the proposed design is called into question.

Overall, this HALO case study provides a graphic demonstration of the need to make DGSW evaluations site specific, taking account of local thermal regime and mode of operation (cf. Law et al., 2015; Collins and Law, 2017), the critical factor being accurate estimation of the initial bottom hole temperature. Nonetheless, such calculations are much less challenging than the analysis of hydraulic transport properties and groundwater contamination that would be required, from earlier discussion, to quantify the output and economics of a dDGSW.

Aberdeen Exhibition and Conference Centre

A candidate hdDGSW project has been proposed for the Aberdeen Exhibition and Conference Centre (AECC) in the Scottish city of Aberdeen (GEL et al., 2016), a ~2 km deep borehole being envisaged (to be located circa BNG reference NJ 884 105), with a bottom hole temperature estimated by GEL et al. (2016) as ~70 °C. It has been proposed that heat will be extracted via a heat exchanger to maintain the temperature of an anaerobic digester (AD) for processing municipal waste, then the residual heat (below T_E ~40 °C) will be output via heat pumps for space-heating in adjacent buildings, before the circulating water is reinjected. The project description mentions system design to facilitate 'bleed flow', the proposal specifying the bottom 300 m of the borehole uncased; given the preceding calculations it is evident that the high heat output envisaged (up to 400 kW) will only be feasible with bleed flow. Moreover, since the design calls for the AD unit to operate continuously, the proposal envisages continuous (or near-continuous) bleed flow to sustain its operation.

The draft budget (GEL et al., 2016) indicates a ~£1.6M capital cost, ~£1.35M for the borehole, the rest for the ancillary equipment. Estimated annual operating costs would be ~£49k and annual revenue based on sale of heat at £0.03 per kWh would be ~£239k. On this basis the project has been reported as achieving a high net present value of ~£10.7M after 40 years with a high internal rate of return of ~19%; these economic parameters, which favour commercial investment, assume that RHI subsidy payments (currently £0.0514 per kWh) are included. However, this analysis does not include water treatment costs on the basis that 'as the Aberdeen granite is rather deficient in heavy metals it is very unlikely that any fluids from the DGSW will be the cause of significant environmental contamination by heavy metals', although no quantitative data were provided to substantiate this assertion.

Nonetheless, the surface heat flow in this part of Scotland is low, being depicted on BGS maps (e.g., Busby et al., 2011) as ~50 mW m^{-2}. Some 50 km to the SSW, the Montrose (or Charleton-1) borehole (BGS identifier NO76SW12, circa BNG reference NO 715 605) has yielded a heat flow of 46 mW m^{-2} based on measurements of temperature and thermal conductivity k between 301 and 751 m depths (e.g., Burley et al., 1984; Rollin, 1995); the ~20 °C temperature at 500 m depth indicates a geothermal gradient u of ~22.8 °C km^{-1}, given the ~8.6 °C annual mean surface temperature (e.g., Met Office, 2017), which would imply ~54 °C at 2 km depth. However, ~7 km east of the AECC site, the Bridge of Don-3 borehole (BGS identifier NJ91SE3, circa NJ 951 109) was drilled in basement schist to 1465 m (TVD; 1494 m MD) but is now cased to 1433 m for use as a test facility by the oilfield service provider Weatherford. The bottom hole temperature is reported as 32 °C (Groves et al., 2012), indicating u ~16.3 °C km^{-1}, which would imply ~41 °C at 2 km. However, this measurement was made during drilling and thus requires correction for the associated cooling effect. Established correction procedures exist (e.g., Manetti, 1973; Barelli and Palama, 1981); there is also UK experience, such as for the Eskdale-12 borehole (BGS identifier NZ80NE4, NZ 85783 08180) in northern England, drilled in 1963 to 1873 m depth, within which temperature was measured one hour after drilling and later, so the calculated correction could be verified (Burley et al., 1984). Taking all this into account, it is estimated that the Bridge of Don-3 measurement requires upward revision by ~15 °C at most, making u $\leq$26.2 °C km^{-1} and the temperature at 2 km $\leq$61 °C. The Aberdeen granite has low radioactive heat production: McCay (2016) has reported 11 measurements spanning 1.55 to 3.11 μW m^{-3}, with a mean of 2.17 and a median of 2.04 μW m^{-3}, confirming the ~2.2 μW m^{-3} mean value reported by Wheildon and Rollin (1986) from 3 measurements. Basement rocks like those beneath Aberdeen are typically assigned heat production of ~2 μW m^{-3} (e.g., Wheildon and Rollin, 1986); the Aberdeen granite is therefore not expected to create a significant local heat flow 'high' relative to the above values. Part of the rationale behind the GEL et al. (2016) temperature estimate at 2 km depth was their best-estimate measurement for k in the Aberdeen granite of 2.71 W m^{-1} °C^{-1}, on which basis they converted the reported heat flow of ~50 mW m^{-2} back to u ~18.5 °C km^{-1}, giving ~46 °C at 2 km depth. However, Caledonian age granite intrusions in Scotland and northern England typically have k ~3.1-3.5 W m^{-1} °C^{-1} (e.g., Wheildon and Rollin, 1986), which (calculating on the same basis) would imply rather lower temperatures at depth. GEL et al. (2016) also argued that the heat flow data require correction for palaeoclimate, citing Westaway and Younger (2013), as a basis for their ~70 °C temperature estimate at 2 km depth. However, the Westaway and Younger (2013) analysis (see, e.g., their Fig. 3) indicates a negligible correction to u for the Bridge of Don-3 borehole and a ~5 °C km^{-1} upward correction for the Montrose borehole. On the latter basis, if the thermal state of the Earth's crust is the same at the AECC site as at Montrose, one might argue for a temperature as high as ~64 °C at 2 km depth, although the Bridge of Don-3 evidence indicates less than this. The discussions earlier in the present study, which demonstrate high sensitivity of DGSW output to crustal thermal regime, mean that overestimation of the bottom hole temperature can have a fundamental impact on project economics. Nonetheless, $\geq$70 °C is expected at 2 km depth elsewhere in the UK, in more radiothermal granites (e.g., Lee, 1986) and in Mesozoic depocentres such as the Wessex and Lincolnshire basins of England and the Larne Basin of Northern Ireland (e.g., Smith, 1986). It is thus worthwhile to pursue the implications of the GEL et al. (2016) analysis, since DGSW-based project proposals for such localities might emerge in future.

Assuming income of £0.0814 kWh^{-1} the revenue reported by GEL et al. (2016) implies heat sales of ~2.93 GWh per annum, equivalent to a constant rate of ~334 kW. This is so far above what might be feasibly delivered from a cDGSW of the specified dimensions with the proposed T_E that 'bleed flow' will be essential, as already noted. This scenario can be analysed to first order assuming that ~100 kW of this heat supply might be obtained through heat conduction (based on the preceding calculations), the rest being obtained by 'bleed flow'. Setting aside the question of accurate estimation of T_M, two issues thus call this project into question: the required high hydraulic conductivity of the granite that is presumed to provide the groundwater reservoir for this project; and the arguable need to factor in treatment costs for this produced water. Assuming T_M=70 °C and that all the heat is utilized down to T_S=10 °C, the required circulation rate including the bleed flow would be ~334 kW / (4186 kg m^{-3} $\times$ 1000 kg m^{-3} $\times$ 60 °C) or ~1.3 l s^{-1}; ~0.9 l s^{-1} of this would be 'bleed flow' and the parameter Γ would take the value ~234/100 or ~2.34. If the AD heat exchanger rejects heat below T_E=40 °C then this component of its heat supply will be ~334 kW $\times$ (70 °C – 40 °C) / (70 °C – 10 °C) or ~167 kW. Taking ΔT as 70 °C – 10 °C = 60 °C, equation (22) indicates H_o ~3000 m, greater than the assumed borehole depth. However, for 'head lift' to have only a minimal effect on the project budget, the drawdown must be only a small proportion of this, say 5% or ~150 m.

As already noted, Law (2014) reported a high value of ~50 for the COP at a DGSW test site where the produced water was reinjected. As has also already been noted, this is not necessarily so in dDGSW mode; if significant 'head lift' for the produced water is necessary, the electrical energy requirement might outweigh the value of the heat produced. The AECC project design evidently assumes zero or minimal 'head lift', meaning that the groundwater reservoir at depth has been assumed to be under hydrostatic equilibrium with the ground surface which, as already discussed, requires a very high hydraulic conductivity in the granite. Negligible electrical energy would therefore be needed for pumping the produced water against gravity, the cost of which is neglected in these first-order calculations. It is thus inferred that most of the electrical power consumed will be used by the heat pumps, whose COP, Ψ, will be much lower (say, 4). If these cool the circulating water to 10 °C the additional ~167 kW heat loss will require ~167 kW $\times$ 1 / (Ψ – 1) or ~56 kW of electrical power. Assuming once again that the electricity used costs £0.095 per kWh, this heat pump operation will cost of ~£46k per annum, indeed consuming most of the ~£49k estimated operating budget. This is evidently an approximate analysis, but it implies that, per day of operation, typically, ~80 m^3 of produced water (~0.9 l s^{-1} $\times$ 24 hrs) will require treatment. If this costs ~£3.80 per cubic metre (see above) it will imply an additional cost of ~£300 per day or ~£110k per annum. Even with this additional budget item, then provided T_M is indeed ~70 °C, the project would still be predicted to make an annual surplus of ~£80k (~£239k revenue – £49k operating costs – £110k water treatment costs), again with the RHI subsidy included in the revenue, although the ~£1.6M budget surplus thus accumulated over 20 years of operation would barely cover the capital cost of the drilling. Nonetheless, like for the HALO project, this calculated small surplus is the difference between much larger figures for revenue and expenditure, so is sensitive to small changes in these. If the granite has insufficient hydraulic conductivity to maintain the required 'bleed flow', or if significant 'head lift' pumping is required, increasing the electricity consumption with no additional heat output, or if the bottom hole temperature is significantly below 70 °C, this project might well not recover its own capital expenditure and might even operate at a deficit, with revenue unable to cover operating costs.

Hypothetical Darlington hcDGSW project

Table 4 lists Darlington as the town with the highest temperature at 1 km depth in Britain. This is based on evidence from the Brafferton-1 borehole (BGS identifier NZ22SE105, located at BNG reference NZ 28432 21493) drilled to 1987 m depth, ~6 km north of Darlington town centre, for which Busby et al. (2011) reported a temperature of 54 °C at 1 km depth. Darlington is located within the Stainmore Trough or Stainmore Basin, one of several sedimentary basins in northern England, which formed as a result of crustal extension during the Early Carboniferous. The base of the Carboniferous succession is reported, from seismic surveys, at ~5700 m depth in the Darlington area (Chadwick et al., 1995). The Brafferton-1 borehole record remains commercially confidential and unpublished, but from the regional context (e.g., Collier, 1991; Chadwick et al., 1995), including deep drilling elsewhere (Johnson et al., 2011), the local Carboniferous succession can be inferred to consist mainly of mudstone with interbedded sandstone, limestone, and coal seams. A similar succession farther north, beneath Gateshead, has k ~1.7 W m^{-1} °C^{-1} and κ ~0.9 mm^2 s^{-1} (Westaway and Younger, 2016); these values will be adopted for the present analysis. With this value of k, and a surface temperature of ~9 °C from meteorological data, the ~45 °C km^{-1} geothermal gradient implies ~77 mW m^{-2} of heat flow. Other localities beyond the northern margin of the Stainmore Basin have much higher heat flow, due to heat production in granite intrusions, but lower temperatures at depth due to the higher thermal conductivity of the rock column (e.g., Busby et al., 2011). The borehole cost model in use (equation (30)) gives the capital cost of a 6000 m deep borehole, reaching the base of this sedimentary succession, as ~£8.6M.

It is assumed that this hcDGSW scheme will form part of a future district heating scheme for the urban area of Darlington. In Denmark, such schemes often take as their input water at a temperature as low as ~50 °C (DEA, 2017); they achieve satisfactory space heating using radiators rather larger than are customary in the UK. Even lower input temperatures, as low as ~35 °C, are feasible if underfloor heating is used (e.g., Joule, 2017), rather than radiators of any size. It is assumed that the hypothetical Darlington district heating scheme will be designed on this basis, with T_E=35 °C. It is also assumed that heat is sold, RHI subsidy applies, and electricity purchased, all at the same rates as before. The electricity purchased is again assumed to reflect the present UK generation mix and the heat output is again assumed to substitute for burning natural gas.

Figure 13 illustrates results of analysis of this hcDGSW scheme, developed, costed, and operated for 20 years on this basis. The economically optimum operational mode (Fig. 13(b)) has Q=1095 kW. This generates net revenue of £14.4M, including a surplus of £5.8M after taking account of drilling costs, outputting 211.9 GWh of heat and consuming 30.2 GWh of electricity, saving 24769 tonnes of CO_2e emissions at a cost of £440 per tonne. The environmentally optimum operational mode has Q=490 kW. This produces 89.1 GWh of heat output and consumes 3.1 GWh of electricity, saving 14098 tonnes of CO_2e emissions at a cost of £325 per tonne. The net revenue is £7.0M, indicating a shortfall of £1.6M relative to the drilling costs. Any rate of heat production between 23 and 1208 kW results in a net saving of CO_2e emissions: below 23 kW, the emissions embodied in the borehole drilling and completion are not recouped; whereas, above 1208 kW, T_O falls so low (Fig. 13(a)) that much of the heat extraction is via the heat pump and the associated consumption of electricity is so high that the emissions associated with its generation outweigh those saved by the production of geothermal heat. Despite the higher geothermal gradient and bottom hole temperature, and the greater length of the borehole heat exchanger, the economically optimum heat production rate of 1095 kW is not much greater than for the configuration in Fig. 7(d)). The essential reason for this is the smaller values of k and κ that have been assumed (~1.7 W m^{-1} °C^{-1} and ~0.9 mm^2 s^{-1} compared with ~3.5 W m^{-1} °C^{-1} and ~1.2 mm^2 s^{-1}), reflecting the exponential dependence on k (cf. equation (9)). Nonetheless, once again, the economically optimum output scenario lies very close to the maximum output feasible, again demonstrating the need for careful calculations to determine this optimum mode. This exercise nonetheless demonstrates that deployment of a hcDGSW in a locality of high geothermal gradient, used in an optimal manner, can produce worthwhile savings in CO_2e emissions.

Conclusions

A critical appraisal of the DGSW concept has been presented, driven by the apparent contradiction between the long-standing view (Rybach et al., 1992) that this technology has no potential, other than for repurposing existing boreholes, and claims by commercial developers. The present simplified analytical modelling, which approximates the operational state of a DGSW installation under real conditions, enables some misconceptions about DGSW technology, which have emerged through previous work, to be corrected. It is thus evident, first, that although a cDGSW or hcDGSW might look like an enlarged version of a shallow borehole heat exchanger used for a conventional GSHP installation (and the governing equations, such as equation (4) [cf. Banks, 2012], are similar), its operational principles are different: conventional GSHP installations can function sustainably, whereas a cDGSW or hcDGSW is instead a form of 'heat mining'. Operation of a cDGSW will indeed not attain a steady state over timescales of practical projects (i.e., timescales of decades); the associated 'heat mining' will instead progressively cool an ever-widening volume of surrounding rock, although each part of this rock volume will cool at ever-decreasing rates. The governing thermal physics is linear, the cooling at any point on any particular timescale being proportional to the rate of heat production. Second, when analysing a cDGSW one must distinguish between the heat produced (the heat that reaches the surface) and useable heat output, given that a proportion of the heat production (this proportion depending on the input temperature of the surface heat exchanger) will be reinjected as the heat transfer fluid is circulated. Analysis of the potential of any DGSW installation thus requires consideration of the site-specific combination of geological properties (such as thermal conductivity/diffusivity and geothermal gradient) and mode of operation, not the geological properties alone. The heat reinjected during operation of a cDGSW makes this a less favourable technological variant compared with the hcDGSW, notwithstanding the lower overall COP of the latter due to the electricity consumed by its heat pump. The electrical energy thus consumed will be converted to heat, providing another reason why the heat output from the system (including its heat pump) will differ from the heat produced from the borehole. Furthermore, the heat production from DGSW boreholes is sensitive to the site-specific geological properties; DGSW design must therefore consider such properties on a site-by-site basis, rather than assuming nominal values or that an analysis for one site is applicable to another. Moreover, economically optimal operation of a hcDGSW involves a rate of heat production that is close to the maximum that can be sustained over the lifetime of a project. This means that if a project has been 'under-engineered' (i.e., its heat output capacity has been overestimated, even by a small margin), the surrounding rock volume will cool so rapidly that the actual lifetime of the project is significantly reduced. The environmentally optimum operational mode (optimizing savings in CO_2e emissions) involves heat production at a lower rate than the economically optimum mode (maximizing profit). If such projects are subsidized from public funds, then a particular operational mode might be specified, maybe as a compromise between these optima. Additional issues also affect dDGSW or hdDGSW installations. First, the produced water might well require decontamination treatment, especially if the installation is in granite, which will add significantly to operating costs and might cause regulatory difficulties. Second, these DGSW variants can only function in rocks of relatively high permeability and hydraulic conductivity, and the cost of pumping to maintain 'head lift' can further impact upon the economics of operation.

The present analytic modelling, which approximates the operational state of a DGSW installation under real conditions, indicates that the cDGSW variant has only limited potential; for 2 km deep boreholes, outputs over 20 year timescales of at most ~100 kW are feasible, the value of which (under current UK conditions) is unlikely to cover the capital costs. hdDGSW operation can produce higher heat outputs, and can in principle be economic under current UK conditions with RHI subsidy payments included in revenue, but it is debateable whether this is a justifiable technology for public subsidy given its potential for environmental pollution, even after the produced water undergoes treatment. Furthermore, it requires site-dependent investigations that negate the original aim of providing an 'off the shelf' geothermal energy source. The hcDGSW variant, with a heat pump used to supplement the heat output of a cDGSW and to lower the reinjection temperature of the circulating fluid as close as possible to the ambient surface temperature (Fig. 2(b)), is shown to have the most potential. The analytical solutions have been used to develop an economic model for this variant, which indicates that the optimal heat output and operating surplus increase with borehole depth to the power of 1.87. This increase is faster than the corresponding increase in drilling costs, indicating that optimal hcDGSW designs will involve boreholes rather deeper than the ~2 km depths considered hitherto. When RHI subsidy is included, the hcDGSW variant is indeed shown to have the potential for economic viability, assuming that a heat pump with the specified performance characteristics can be developed. Moreover, after the timescale for RHI subsidy eligibility has expired, this infrastructure can be easily repurposed for seasonal heat storage, thus offering the potential of making a significant long-term contribution to sustainable future heat supply.

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Glossary

Algebraic symbols used in equations in this study are defined here.

Symbol	Units	Quantity represented
C	none	Algebraic constant, defined as equal to exp(γ)
D	m	Internal diameter of a pipe
E	J	Heat loss
K	m s^{-1}	Hydraulic conductivity
L	m	Length of a pipe
P_E	£ kWh^{-1}	Unit cost of electrical energy used
P_H	£ kWh^{-1}	Unit cost of heat energy sold
Q	W	Rate of production of heat energy from a DGSW borehole
Q_A	W	Estimate of Q using analytic calculation
Q_G	W	Rate of upward heat flow from below into cylindrical rock volume of radius r_C
Q_N	W	Estimate of Q using numerical modelling
Q_U	W	Rate of production of heat energy that is useable by the associated heat load
Re	none	Reynolds number quantifying the vigour of fluid flow
S	£	Operating surplus
T	°C	Temperature
T_D	°C	Temperature at which fluid is injected into a DGSW borehole
T_E	°C	Output temperature from a surface heat exchanger or heat pump
T_I	°C	Temperature at which circulating fluid enters a heat pump
T_K	m^2 s^{-1}	Hydraulic transmissivity
T_κ	D m	Hydraulic transmissivity
$\%T_K$	none	Percentage of the total transmissivity of a borehole interval
$\%T_\kappa$	none	Percentage of the total transmissivity of a borehole interval
T_L	°C	Subsurface temperature at depth z_L
T_o	°C	Temperature at which fluid is produced from a DGSW borehole
T_S	°C	Temperature at the Earth's surface
V	m s^{-1}	Velocity of flow along a pipe
a	m	Radius of a borehole (internal radius of borehole casing)
c	J kg^{-1} °C^{-1}	Specific heat capacity
f	none	Ratio, z_L/z_M
f_D	none	Darcy-Weisbach friction factor
f_K	none	Scale factor to incorporate overheads onto the capital cost of a borehole
g	m s^{-2}	Acceleration due to gravity
k	W m^{-1} °C^{-1}	Thermal conductivity

q_B	$m^3\ s^{-1}$	Volume flow rate for ‘bleed flow’ into a DGSW borehole
q_C	$m^3\ s^{-1}$	Volume flow rate for closed-loop circulation in a DGSW borehole
r	m	Radial distance from the axis of a borehole
r_A	m	Radius of influence of fluid production from a well on a surrounding aquifer
r_C	m	Radius of influence of the thermal effect surrounding a DGSW borehole
t	s	Time
t_E	s	Timescale for attainment of steady state conditions around a DGSW
t_L	s	Estimated lifespan of a DGSW project
u	°C km^{-1}	Geothermal gradient
w	m	Vertical position below the base of a borehole
x	none	Dimensionless proxy for time
y	none	Dimensionless proxy for temperature variation
z	m	Depth below the Earth’s surface
z_L	m	Shallow depth limit for heat production from a DGSW borehole
z_M	m	Depth of the base of a borehole
z_1	m	Depth of shallow limit of borehole interval
z_2	m	Depth of deep limit of borehole interval
Γ	none	Ratio, q_B/q_C
ΔH	m	Drawdown to the phreatic surface of an aquifer
ΔH_o	m	Maximum value of ΔH, at $r=a$
ΔP	Pa	Pressure drop due to friction at the rim of a pipe
ΔT	°C	Temperature change between radius r_C and radius r_A, outside a borehole
Δz	m	Depth interval, $z_2–z_1$
Λ	$W\ m^{-2}$	Rate of heat production per unit surface area of a borehole
Σ	£	Capital cost of a borehole
Ψ	none	Coefficient of performance
Ψ_M	none	Mean value of Ψ for a heat pump over a specified temperature range
Ω	none	roughness of the inner surface of a pipe
β	none	Ratio, P_E/P_H
γ	none	Euler’s constant
ε	m	Characteristic height of irregularities on the inner surface of a pipe
ζ	$W\ m^{-1}$	Rate of heat production per unit depth of a DGSW borehole
ζ_M	$W\ m^{-1}$	Value of ζ at depth $z=z_M$
η	Pa s	The (dynamic) viscosity of a fluid
θ	°	Azimuth around a borehole
κ	$mm^2\ s^{-1}$	Thermal diffusivity
κ_P	m^2	Hydraulic permeability
ρ	$kg\ m^{-3}$	Density
χ	none	Dimensionless variable used to specify algebraic functions

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Table 1 – on a separate sheet.

Table 2: Estimates of cDGSW performance

Q (kW)	T_o(5 yrs) (°C)	Q_U(5 yrs) (kW)	T_o(10 yrs) (°C)	Q_U(10 yrs) (kW)	T_o(20 yrs) (°C)	Q_U(20 yrs) (kW)
z_M=2000 m; T_L=30 °C; T_M=74 °C ($f\approx0.31$)						
0	74.0	0.0	74.0	0.0	74.0	0.0
20	66.9	13.0	66.5	12.9	66.0	12.9
40	59.9	24.0	59.0	23.7	58.0	23.3
60	52.8	32.0	51.5	31.1	50.0	30.0
80	45.8	35.3	44.0	32.9	42.0	30.0
100	38.3	29.3	36.6	24.8	34.0	16.7
120	31.7	9.4	29.1	0.0	26.0	0.0
z_M=2500 m; T_L=30 °C; T_M=90 °C (f=0.25)						
0	90.0	0.0	90.0	0.0	90.0	0.0
20	84.8	14.7	84.5	14.6	84.1	14.6
40	79.7	28.5	79.0	28.4	78.3	28.3
60	74.5	41.4	73.5	41.1	72.4	40.8
80	69.3	53.0	68.0	52.4	66.6	51.7
100	64.2	63.1	62.6	62.0	60.7	60.6
120	59.0	71.0	57.0	68.9	54.9	66.5
140	53.8	76.1	51.5	72.5	49.0	68.2
160	48.6	77.1	46.0	71.1	43.2	63.6
180	43.5	72.5	40.6	62.4	37.3	48.1
200	38.3	58.7	35.1	40.6	31.4	13.1
220	33.2	30.3	29.6	0.0	25.5	0.0
z_M=3000 m; T_L=30 °C; T_M=106 °C ($f\approx0.21$)						
0	106.0	0.0	106.0	0.0	106.0	0.0
20	101.9	15.6	101.7	15.6	101.4	15.6
40	97.9	30.9	97.3	30.8	96.8	30.8
60	93.8	45.7	93.0	45.5	92.1	45.4
80	89.7	59.9	88.7	59.7	87.5	59.4
100	85.6	73.5	84.3	73.1	82.9	72.6
120	81.5	86.4	80.0	85.7	78.3	84.9
140	77.5	98.5	75.6	97.3	73.6	96.0
160	73.4	109.5	71.3	107.8	69.0	105.8
180	69.3	119.3	66.9	116.7	64.4	113.8
200	65.2	127.5	62.6	124.0	59.7	119.5
220	61.1	133.9	58.3	128.9	55.1	122.4
240	57.0	137.9	53.9	130.7	50.5	121.5
260	52.9	138.8	49.6	128.7	45.9	115.2
280	48.9	136.0	45.3	121.4	41.3	101.1
300	44.8	127.6	41.0	106.5	36.6	74.4
320	40.7	111.5	36.6	79.4	32.0	29.1
340	36.6	84.4	32.3	35.1	27.4	0.0
360	32.6	41.4	28.0	0.0	22.8	0.0

Calculations, on the basis explained in the main text, demonstrating the performance issues that result for cDGSW installations operated at nonzero values of f. Values of useable heat output Q_U and output temperature T_O after 5, 10 and 20 years of operation are listed. The calculations assume a=0.1 m, T_S=10 °C, u=32 °C km^{-1}, k= 3.5 W m^{-1} °C^{-1}, and κ=1.2 mm^2 s^{-1} (cf. Fig. 4).

Table 3: Hydraulic transport properties of granites: on a separate sheet.

Table 4: Temperatures at 1 km depth: on a separate sheet.

Figure Captions

Figure 1. Schematic diagrams depicting groundwater extraction DGSWs. Thin solid arrows indicate directions of fluid flow; the fluid is shaded to convey an impression of its temperature and/or whether it is warming or cooling at each point in the model. Temperatures at key points are labelled for comparison with the main text. (a) A simple wDGSW in which hot water is pumped out of a permeable aquifer, is cooled by transferring heat to working fluid (which supplies heat to a heat load), then the resulting warm water is discharged into the environment. (b) A hwDGSW, in which after passing through the heat exchanger the ground water is cooled further using a heat pump. The Southampton geothermal project, discussed in the text, operated as in (a) from 1988 to 1991 then was modified as in (b). It does not incorporate treatment of the discharged water but this is depicted schematically as it will arguably be necessary for any future projects of this type.

Figure 2. Schematic diagrams depicting conductive DGSWs, using the same notation as in Fig. 1 plus broad open arrows indicate directions of heat conduction in the subsurface. **(a)** A cDGSW, in which the borehole contains part of a closed circulation loop. Note that with this variant much of the heat produced (at temperature T_O) is not used, because the output temperature of the heat exchanger, T_E, and thus the reinjection temperature T_D of the circulating fluid, significantly exceeds the ambient surface temperature T_S. This heat is reinjected and warms the subsurface down to depth z_L where the initial rock temperature T_L equals the fluid temperature T_E (=T_D). The circulating fluid is thus warmed between depth z_L and the well bottom at z_M, not throughout the full vertical extent of the borehole (the parameter f, which appears in equations defined in the text, being the ratio z_L/z_M). To the best of my knowledge, this system design was first proposed by Rybach et al. (1992). **(b)** A hcDGSW, in which a heat pump is added to the closed-loop configuration in (a), to supplement the useful heat output by cooling the circulating fluid to the ambient temperature T_S. This also increases the proportion of the borehole available for heat production. This configuration, not previously analysed in detail, is favoured in the present study as the preferred DGSW variant.

Figure 3. Schematic diagrams depicting 'dual mode' DGSWs, combining the option of water extraction (with the deeper part of the borehole open to its surroundings) with heat production by conduction, using the same notation as in Fig. 2. **(a)** A dDGSW, in which ground water is drawn into the borehole heat exchanger at depth and discharged at the surface, potentially increasing the thermal output. This variant suffers from the principal disadvantages in Fig, 2(a), that reinjection of warm water reduces the output of useful heat and warms the surroundings of the shallow part of the borehole, limiting the proportion of it available for heat production. It also introduces other disadvantages, including the need for relatively permeable bedrock at depth, scaling of the pipe loop due to precipitation of substances dissolved in the circulating ground water, the need for treatment of the discharged water, and associated regulatory issues. **(b)** A hdDGSW, in which a heat pump is added to the configuration in (a), to supplement the useful heat output by cooling the circulating fluid to the ambient temperature T_S. This also increases the proportion of the borehole available for heat production. However, the difficulties remain over identification of permeable bedrock and disposal of the ground water that is discharged into the environment. A system of this type is proposed at the AECC (GEL et al., 2016).

Figure 4. Calculated variations in output temperature T_O for a cDGSW operating for 10 years at constant rates of heat production Q 50, 80, 140 and 260 kW, assuming z_M=2500 m, T_M=80 °C, T_S=0 °C, a=0.1 m, k=3.5 W m^{-1} °C^{-1}, κ=1.2 mm^2 s^{-1}, and f=0, based on equations (4) and (5) (open symbols). For comparison, solid and dashed lines indicate variations in T_O for a cDGSW with z_M=2500 m, T_M=80 °C, and T_S=0 °C, operating for 10 years with heat production at Q=50, 100, 200 and 400 kW, according to Fig. 5 of Law et al. (2015). See text for discussion.

Figure 5. Predicted temperature variations at depth z_M=2500 m as a function of radial distance r, calculated using equations (4) and (5) for Q=50 kW at different times after the start of cDGSW operation. The other model parameters are as described for Fig. 4; linear ((a)) and logarithmic ((b)) scales for radial distance are used. Note that, for reasons discussed in the text, the calculated values of $r=r_C$ at which the cooling effect reaches zero (respectively, 2.2, 4.6, 9.2, 18.5, 29.2 and 41.3 m) are approximate and underestimate the actual radial distance at which this effect becomes infinitesimally small by a factor of $\sqrt{C}$ or ~1.33. Note, also, that despite the low rate of heat production assumed, the borehole does not reach a thermal steady state on any of the timescales depicted (cf. Law et al., 2015).

Figure 6. Graphs of the notional project lifespan t_L (calculated using equation (9)) for cDGSWs with a=0.1 m in a region with T_S=10 °C, u=32 °C km^{-1}, k= 3.5 W m^{-1} °C^{-1}, and κ=1.2 mm^2 s^{-1}. **(a)** For z_M=2000 m, so T_M=74 °C; the values of f=0.0, 0.2, 0.4 and 0.6 correspond to T_L=10.0, 22.8, 35.6, and 48.4 °C. For f=0.6, t_L=20 years corresponds to Q=43 kW. **(b)** For z_M=2500 m, so T_M=90 °C; the values of f=0.0, 0.15, 0.3 and 0.45 correspond to T_L=10.0, 22.0, 34.0, and 46.0 °C. For f=0.45, t_L=20 years corresponds to Q=125 kW. **(c)** For z_M=3000 m, so T_M=106 °C; the values of f=0.0, 0.1, 0.2 and 0.3 correspond to T_L=10.0, 19.6, 29.2, and 48.4 °C. For f=0.3, t_L=20 years corresponds to Q=286 kW.

Figure 7. Depictions of the potential cost-effectiveness of hcDGSW boreholes for 20 years of operation with different values of z_M as a function of rates of heat supply to the associated heat load. **(a)** 2000 m; **(b)** 2500 m; **(c)** 3000 m; **(d)** 4000 m. Calculations assume T_S=10 °C, u=32 °C km^{-1}, a=0.1 m, k=3.5 W m^{-1} °C^{-1}, κ=1.2 mm^2 s^{-1}, T_E=T_D=40 °C, L=z_M (i.e., boreholes are vertical), ρ=1000 kg m^{-3}, c=4186 J kg^{-1} °C^{-1}, D=0.04 m, and Ω=5×10^{-5}, with heat and electricity pricing, subsidy payments, and other input parameters as described in the text. Dashed horizontal lines denote estimated capital costs; dashed vertical lines denote optimum solutions, at 193 kW in (a), 295 kW in (b), 416 kW in (c), and 711 kW in (d). Note that at power outputs of ≥213 kW in (a), ≥326 kW in (b), ≥465 kW in (c), and 808 kW in (d), greater financial returns would result from ending operation before 20 years have elapsed, because in the later stages of operation T_o is predicted to fall so low (<15.7 °C in (a), <18.2 °C in (b), <20.9 °C in (c), and <27.1 °C in (d)) that Ψ declines sufficiently for the cost of the electricity used to exceed the value of the heat produced, even with RHI subsidy included.

Figure 8. Comparison between calculations of heat production from a cDGSW calculated using equations (4) and (5) with the outputs in Fig. 5 of Law et al. (2015) (cf. Fig. 4). Both sets of outputs have been converted to dimensionless variables x and y (defined in the text), where x is a proxy for time and y is a proxy for the fall over time in the output temperature. The calculations of $\Delta T(z_M)$, using equations (4) and (5), and the transformation to dimensionless variables utilise the following parameter values: z_M=2500 m, T_M=80 °C, and T_S=0 °C (all specified by Law et al., 2015), along with a=0.1 m, k=3.5 W m^{-1} °C^{-1}, κ=1.2 mm^2 s^{-1}, and f=0. See text for discussion.

Figure 9. Variations over time in T_O for the optimum hcDGSW configurations from Fig. 7. After T_O falls below T_E it is assumed that all the heat output is via the heat pump, with the heat exchanger bypassed (cf. Fig. 2(b)). Crosses mark the times when the three setups become uneconomic in the absence of RHI subsidy (i.e., when the operating cost first exceeds the revenue from sale of heat). These are: for T_O≤21.8 °C (after ~17 years) for z_M=2000 m; T_O≤25.7 °C (after ~18.5 years) for z_M=2500; T_O≤29.9 °C (after 20 years) for z_M=3000 m; and T_O≤38.8 °C (after ~25 years) for z_M=4000 m.

Figure 10. Comparison of rates of heat output, for 10 years of operation, of cDGSW installations, predicted by the present analytic model (Q_A) and by numerical models. The present analytic models are adjusted to give the same values of T_O as the extant numerical models. Data points (Q_A, Q_N, in kW) are (50, 50), (80, 100), (140, 200), and (260, 400), based on comparison with Law et al. (2015), and (530, 1200), based on comparison with Alimonti et al. (2016). Values for all model parameters are noted in the text. The trend line defined in equation (34) is also depicted.

Figure 11. Summary estimates for the technical and economic performance of the HALO hcDGSW project in Kilmarnock, Scotland. **(a)** Graphs of predicted temperature decline, calculated using the same procedure as for Fig. 4. The calculations assume different values of Q, with T_M=69 °C, T_S=T_D=9 °C, T_E=70 °C, f=0, a=0.375 m, u=26 °C km^{-1}, k=4 W m^{-1} °C^{-1}, and κ=1.5 mm^2 s^{-1}. **(b)** Graphs of predicted economic performance, calculated using the same operational parameters as for part (a) and the same economic model as for Fig. 7. See text for discussion.

Figure 12. Revision to Fig. 11 for a=0.1 m instead of 0.375 m.

Figure 13. Summary estimates for the technical and economic performance of a hypothetical hcDGSW project in Darlington, northeast England. **(a)** Graphs of predicted temperature decline, calculated using the same procedure as for Fig. 4. The calculations assume different values of Q, with z_M=6000 m, T_M=279 °C, T_S=T_D=9 °C, T_E=35 °C, f=0, a=0.1 m, u=45 °C km^{-1}, k=1.7 W m^{-1} °C^{-1}, and κ=0.9 mm^2 s^{-1}. **(b)** Graphs of predicted economic performance, calculated using the same operational parameters as for part (a) and the same economic model as for Fig. 7. See text for discussion.

Table 1: Summary of DGSW technology variants

Notation	Illustration	Description
wDGSW	Fig. 1(a)	Open-loop design in which hot water is produced, flows through a heat exchanger, and is then discharged into the environment circa the rejection temperature of the heat exchanger. Corresponds to the Southampton project in its original form. Requires permeable bedrock; in the UK requires regulatory approval for the discharge, which will limit future applicability.
hwDGSW	Fig. 1(b)	Open-loop design in which hot water is produced and flows through a heat exchanger, before being cooled further to near-ambient temperature using a heat pump, then discharged into the environment. Corresponds to the Southampton project in its present, modified, form. Requires permeable bedrock; in the UK requires regulatory approval for the discharge, which will limit future applicability. In the USA, shallow versions of this design are known as open loop groundwater heat pump systems.
cDGSW	Fig. 2(a)	Closed-loop design in which water circulates through a borehole, passing through a heat exchanger at the surface, re-entering the borehole circa the rejection temperature of the heat exchanger. Subsurface heat flow to and from the borehole is by conduction only, so the design imposes no constraints on bedrock permeability. However, the reinjection above ambient temperature means that some of the heat produced contributes to heating the bedrock at shallow depths, limiting the usefulness of this design (and favouring the hcDGSW variant, discussed below, instead). I am not aware of any deep geothermal project that uses this variant, although it has featured in desk studies (e.g., by Law et al., 2015).
hcDGSW	Fig. 2(b)	Closed-loop design in which water circulates through a borehole, passing through a heat exchanger then a heat pump at the surface, re-entering the borehole near ambient temperature. The surface heat exchanger is bypassed if the produced water is below its reject temperature. Subsurface heat flow to and from the borehole is by conduction only, so the design imposes no constraints on bedrock permeability. I am not aware of any deep geothermal project that uses this variant, which is investigated in detail in the present study given its future potential. Excluding the surface heat exchanger, this design is equivalent to an upscaled (deep geothermal) version of what is known in the UK a ground source heat pump system and in the USA a closed loop ground coupled heat pump system.
dDGSW	Fig. 3(a)	Open-loop design in which water circulates through a borehole, passing through a heat exchanger at the surface, re-entering the borehole circa the rejection temperature of the heat exchanger, supplemented by flow bled from groundwater then discharged into the environment. Requires permeable bedrock; in the UK requires regulatory approval for the discharge, which will limit future applicability. I am not aware of any deep geothermal project that uses this variant, although it has featured in desk studies (e.g., by Law et al., 2015).
hdDGSW	Fig. 3(b)	Open-loop design in which water circulates through a borehole, passing through a heat exchanger then a heat pump at the surface, re-entering the borehole near ambient temperature, supplemented by flow bled from groundwater then discharged into the environment. The surface heat exchanger is bypassed if the produced water is below its reject temperature. Requires permeable bedrock; in the UK requires regulatory approval for the discharge, which will limit future applicability. I am not aware of any deep geothermal project that uses this variant, although it has featured in desk studies (e.g., by GEL et al., 2016). Excluding the surface heat exchanger, this design is equivalent to an upscaled (deep geothermal) version of what is known in the USA a standing column well groundwater heat pump system.

Details summarized here are discussed at length in the text. U.S. terminology is from Deng et al. (2005).

Table 3: Hydraulic transport properties of granites

Depth (MD)		Depth (TVD)										
z_1 (m)	z_2 (m)	z_1 (m)	z_2 (m)	Δz (m)	T (°C)	η (mPa s)	$\%T_\kappa$	$\%T_K$	T_κ (D m)	T_K ($m^2\ s^{-1}$)	κ_P (m^2)	K ($m\ s^{-1}$)
Rosemanowes RH-15												
2130	2374	2054	2273	219	55	0.507	12	16.20	0.113	2.18×10^{-6}	5.16×10^{-16}	9.99×10^{-9}
2374	2395	2273	2292	19	65	0.436	59	68.48	0.477	1.07×10^{-5}	2.51×10^{-14}	5.65×10^{-7}
2395	2415	2292	2310	18	68	0.417	12	11.76	0.093	2.18×10^{-6}	5.16×10^{-15}	1.21×10^{-7}
2415	2466	2310	2356	46	78	0.365	0	0.00	-	-	-	-
2466	2490	2356	2376	20	80	0.355	12	11.34	0.079	2.18×10^{-6}	3.95×10^{-15}	1.09×10^{-7}
2490	2700	2376	2565	189	95	0.299	5	3.98	0.028	9.10×10^{-7}	1.47×10^{-16}	4.81×10^{-9}
				511			100	100	0.790	1.82×10^{-5}		
Eastgate-1												
410	432	410	432	22	26	0.880	99	99.34	3920	4.37×10^{-2}	1.78×10^{-10}	1.99×10^{-3}
432	995	432	995	563	27	0.859	1	0.66	25.6	2.92×10^{-4}	4.55×10^{-14}	5.19×10^{-7}
				585			100	100	3925	4.40×10^{-2}		

Here, z_1 and z_2 denote the depth limits above and below each interval of each borehole, in terms of both Measured Depth (MD) and True Vertical Depth (TVD). The Eastgate-1 borehole is vertical so these measures of depth are equivalent. Δz is the difference between z_2 and z_1 measured as TVD. T is a representative temperature of the water in each interval. η is the viscosity of water at temperature T. T_κ and $\%T_\kappa$ are the transmissivity of each interval and its percentage of the total transmissivity, expressed as $\kappa_P\times\Delta z$, where κ_P is the permeability of the interval. T_K and $\%T_K$ are the transmissivity of each interval and its percentage of the total transmissivity, expressed as $K\times\Delta z$, where K is the hydraulic conductivity of the interval. Data (i.e., the stated values for z_1, z_2, T, $\%T_\kappa$, and T_κ) are from Richards et al. (1994) for the Carnmenellis Granite (in the Rosemanowes RH-15 borehole, Cornwall), and from Manning et al. (2007) and Younger and Manning (2010) for the Weardale Granite (in the Eastgate-1 borehole, County Durham). The other parameters are calculated as part of the present study, using standard formulae for the inter-relationships between the quantities listed. See text for discussion.

Table 4. Subsurface temperatures

Locality	Population	T (°C)	Ref.	Note
Northern and North-central England				
Darlington	105,600	54	B	
Harrogate	156,300	52	B	
Leeds	781,700	50	B	
Wakefield	336,800	50	B	
Bury	188,700	50	B	
Rochdale	216,200	49	B	
Newcastle upon Tyne	296,500	48	Y	
Barnsley	241,200	48	B	
Oldham	232,700	48	B	
Gateshead	201,600	48	Y	
Stockton-on-Tees	195,700	48	B	
Sheffield	575,400	46	B	
Manchester	541,300	46	B	
Bradford	534,300	46	B	
Huddersfield	437,000	46	B	1
Hull	260,200	46	B	
Ashton-under-Lyne	223,200	46	B	2
Middlesbrough	140,400	46	B	
Lichfield	103,100	46	B	
Stockport	290,600	45	B	
Chesterfield	104,400	44	B	
Nottingham	325,300	42	B	
Bolton	283,100	42	B	
Walsall	278,700	42	B	
Rotherham	261,900	42	B	
Derby	256,200	42	B	
Salford	248,700	42	B	
Trafford	234,700	42	B	3
York	208,400	42	B	
Grimsby	159,100	42	B	4
Mansfield	107,400	42	B	
Wessex Basin and surroundings				
Southampton	254,300	48	B	
Eastleigh	129,600	48	B	
Winchester	122,000	48	B	
Chichester	118,200	48	B	
Tunbridge Wells	117,100	48	B	
Eastbourne	103,100	48	B	
Bournemouth	197,700	46	B	
Maidstone	166,400	46	B	
Poole	151,500	46	B	
Tonbridge	127,300	46	B	
Sevenoaks	119,100	46	B	
Fareham	115,400	46	B	
Crawley	111,400	46	B	
Basingstoke	174,600	44	B	
Havant	123,600	44	B	
Worthing	108,600	44	B	

Croydon	382,300	42	B	5
Bromley	326,900	42	B	5
Brighton	289,200	42	B	6
Greenwich	279,800	42	B	5
Medway	278,500	42	B	
Plymouth	264,200	42	B	
Bexley	244,800	42	B	5
Portsmouth	214,800	42	B	
Sutton	202,200	42	B	5
Thurrock	167,000	42	B	
Horsham	138,000	42	B	
Torbay	133,900	42	B	
Gravesend	106,800	42	B	7
Dartford	105,500	42	B	5
Lewes	101,400	42	B	

Values of the temperature T, measured or estimated at 1 km depth, from Busby et al. (2011) (B) or Younger et al. (2016) (Y), are plotted for towns and cities in Britain with populations >100,000 (population data from ONS, 2017). The values reported were measured by Younger et al. (2016) beneath Newcastle upon Tyne and are assumed beneath neighbouring Gateshead. These relatively high temperatures, listed, arise due to combinations of relatively high heat flow and/or relatively low thermal conductivity sediments, including thick sequences of Carboniferous mudstone in sedimentary basins in northern and central England. Sites are grouped geographically; no localities in Scotland or Wales have high enough subsurface temperatures for inclusion, whereas Northern Ireland has not been assessed. Annual mean surface temperatures are within ~±1 °C of 10 °C at almost all localities listed, so geothermal gradients in the uppermost 1 km beneath the Earth's surface can be readily calculated approximately. Notes: 1, Kirklees Metropolitan Borough (MB); 2, Tameside MB; 3, Trafford MB; 4, North East Lincolnshire Unitary local Authority (UA); 5, London Borough; 6, Brighton & Hove UA; 7, Gravesham District.

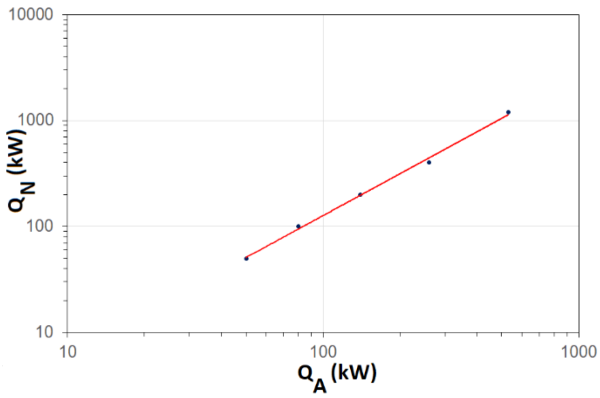


Figure 10

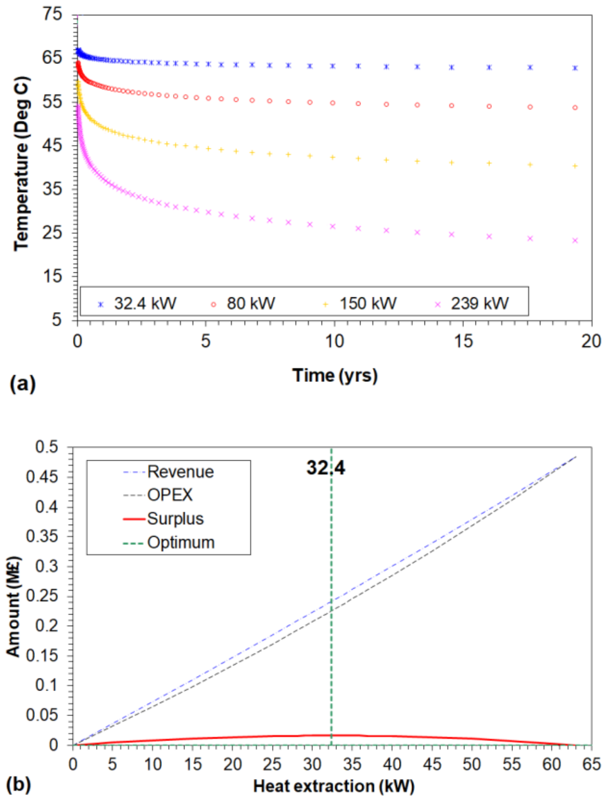
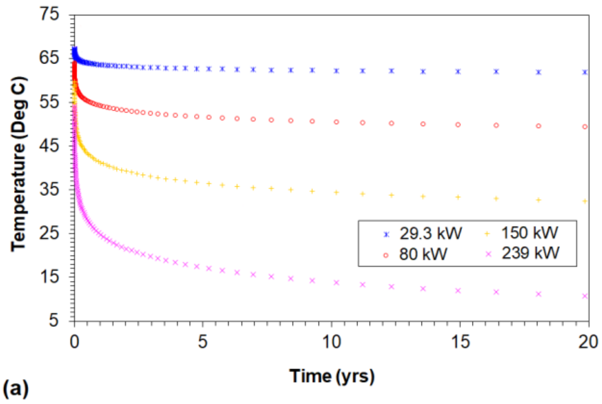
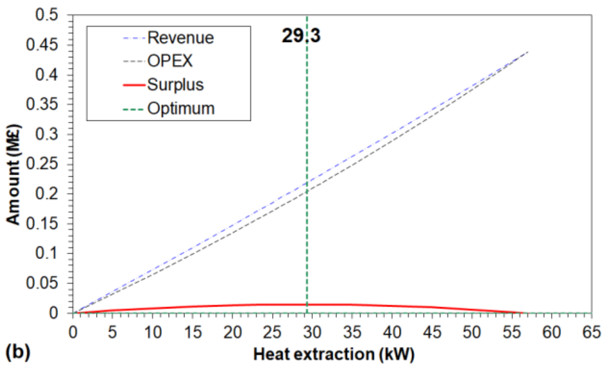

75
65
55
45
35
25
15
5
Temperature (Deg C)
0
5
10
15
20
Time (yrs)
32.4 kW
80 kW
150 kW
239 kW
(a)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Amount (M£)
Revenue
OPEX
Surplus
Optimum
32.4
0
5
10
15
20
25
30
35
40
45
50
55
60
65
Heat extraction (kW)
(b)

Figure 11



Temperature (Deg C)
Time (yrs)
29.3 kW
80 kW
150 kW
239 kW

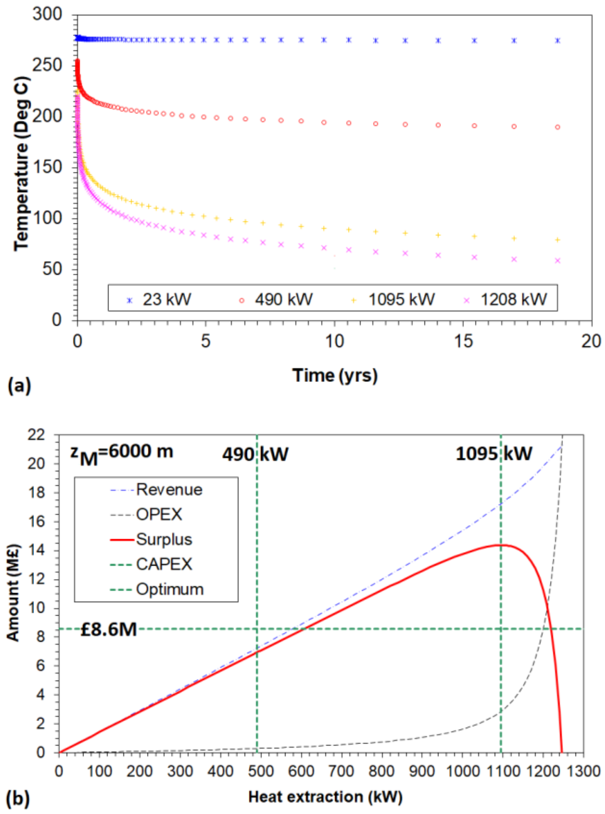
(a)



Amount (M£)
Heat extraction (kW)
Revenue
OPEX
Surplus
Optimum
29.3

(b)

Figure 12



300
250
200
150
100
50
0
Temperature (Deg C)
0
5
10
15
20
Time (yrs)
23 kW
490 kW
1095 kW
1208 kW
(a)
22
20
18
16
14
12
10
8
6
4
2
0
Amount (M£)
z_M=6000 m
490 kW
1095 kW
Revenue
OPEX
Surplus
CAPEX
Optimum
£8.6M
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
Heat extraction (kW)
(b)

Figure 13