Comment on "Left-Handed-Media Simulation and Transmission of EM Waves in Subwavelength Split-Ring-Resonator-Loaded Metallic Waveguides"

In a recent Letter, Marques *et al.* [1] observed electromagnetic (EM) wave transmission through a square overcritical waveguide loaded split ring resonators (SRRs), with the position and bandwidth of the passband corresponding approximately to the frequency interval of negative values of efficient magnetic permeability $\mu_{\rm eff}$ of an artificial medium created by such resonators (see [2]). The authors of the Letter explained this unexpected result by the effect of left-handed-media simulation. Their statement, from our point of view, is simply incorrect and does not stand up to elementary criticism as is shown below. We propose a different explanation of the obtained experimental facts.

A dispersion equation for waves in a metallic waveguide filled with an isotropic medium having an electric permittivity ε and magnetic permeability μ (real or artificial) has the following form:

$$h = \sqrt{k_0^2 \varepsilon \mu - \varkappa^2}, \tag{1}$$

where h is the waveguide mode propagation constant, $k_0 = 2\pi f/c$ is the wave number in free space, $\varkappa = 2\pi f_c/c$ is the transverse waveguide number, f denotes frequency, f_c is the waveguide cutoff frequency, and c is light velocity (for the dominant TE_{10} mode $\varkappa = \pi/a$, a is the width of the waveguide). According to Eq. (1), wave transmission through the initially overcritical waveguide ($\varkappa^2 > k_0^2$) can be possible only if the product of ε and μ of the filling medium (left-handed or right-handed) is sufficiently great so that $\varepsilon\mu > \varkappa^2/k_0^2 = f_c^2/f^2$. It is also obvious that a waveguide filling up with a medium having $\varepsilon \approx 1$ and $\mu = \mu_{\rm eff} < 0$ does not improve the situation; just the reverse, it leads to still stronger wave attenuation.

For an explanation of the experimental results we first consider electrodynamic features of an infinite SRR medium formed by regularly arranged resonators whose planes are parallel to each other. We use the following coordinate system: the x axis is normal to the ring plane, and the y and z axes lie in this plane. Our prime interest is plane electromagnetic waves of the so-called TE type, in which the electric field $\mathbf{E}$ is oriented along the y axis: $\mathbf{E} = \mathbf{y}_0 E_y$, and the magnetic field $\mathbf{H}$ and the wave vector $\mathbf{k}$ are in the x, z plane: $\mathbf{H} = \mathbf{x}_0 H_x + \mathbf{z}_0 H_z$, $\mathbf{k} = \mathbf{x}_0 k_x + \mathbf{z}_0 k_z$ ($\mathbf{x_0}$,$\mathbf{y_0}$, $\mathbf{z_0}$ are unit vectors). From the fundamental work [2] it follows, actually, that the vectors of electric inductance $\mathbf{D}$ and magnetic inductance $\mathbf{B}$ in the waves under consideration are expressed in terms of the field intensities $\mathbf{E}$ and $\mathbf{H}$:

$$\mathbf{D} = \mathbf{y}_0 \varepsilon_{yy} E_y, \qquad \mathbf{B} = \mathbf{x}_0 \mu_{xx} H_x + \mathbf{z}_0 \mu_{zz} H_z. \tag{2}$$

Moreover, in the first approximation we can consider values ε_{yy}, μ_{xx}, μ_{zz} in the constitutive relations (2), respectively, to be the following: $\varepsilon_{yy} \simeq 1$, $\mu_{zz} \simeq 1$, and $\mu_{xx} = \mu_{\rm eff}$. The qualitative frequency dependence of $\mu_{\rm eff}$ is shown in Ref. [2]. Subsequent specifications taking into account, in particular, bianisotropy [3] only supplement the initial model but do not introduce principal changes. Note that the choice of $\varepsilon_{yy} \simeq 1$ and $\mu_{zz} \simeq 1$ is not principal either; the only important thing is that they must be positive.

From the Maxwell equations directly it is easy to obtain the relationship for k_x and k_z:

$$k_z^2 = \mu_{\rm eff}(k_0^2 - k_x^2). \tag{3}$$

According to the Brillouin concept, the field of the dominant TE_{10} mode of interest in the waveguide can be represented as a superposition of two plane waves with wave vector components (k_z, k_x) and (k_z, $-k_x$). As a result we have ($k_z = h$, $k_x = \pi/a$) the field components:

$$\begin{aligned} E_y &= \sin(\pi x/a)\exp(i\omega t - ihz), \\ H_x &= -\frac{h}{k_0\mu_{\rm eff}}\sin(\pi x/a)\exp(i\omega t - ihz), \\ H_z &= i\frac{\pi}{k_0 a}\cos(\pi x/a)\exp(i\omega t - ihz), \end{aligned} \tag{4}$$

and the dispersion equation:

$$h = \sqrt{\mu_{\rm eff}\left(k_0^2 - \frac{\pi^2}{a^2}\right)}. \tag{5}$$

As can be seen from Eq. (4), the mode is a backward wave, i.e., the time-averaged Poynting vector is opposite to the direction of its wave vector $h\mathbf{z_0}$. Note that this is absolutely not correlated with the notion of the left-hand medium.

To conclude, we can assert that according to Eq. (5) the transmission of EM waves through the overcritical waveguide really occurs in the frequency range to which the negative values of $\mu_{\rm eff}$ correspond. However, this effect is by no means connected with the left-handed-medium simulation, but is caused by SRR medium anisotropy only.

I. G. Kondrat'ev and A. I. Smirnov
Institute of Applied Physics RAS
Nizhny Novgorod, Russia

Received 1 April 2003; revised manuscript received 30 April 2003; published 8 December 2003
DOI: 10.1103/PhysRevLett.91.249401
PACS numbers: 42.70.Qs, 41.20.Jb, 78.20.Ek

[1] R. Marques, J. Martel, F. Mesa, and F. Medina, Phys. Rev. Lett. **89**, 183901 (2002).
[2] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).
[3] R. Marques, F. Medina, and R. Rafii-El-Idrissi, Phys. Rev. B **65**, 144440 (2002).

