Simulation of the Dynamics of Pulsed Pumped Lasers Based on Cellular Automata

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Abstract. Laser dynamics is traditionally modeled using differential equations. Recently, a new approach has been introduced in which laser dynamics is modeled using two-dimensional Cellular Automata (CA). In this work, we study a modified version of this model in order to simulate the dynamics of pulsed pumped lasers. The results of the CA approach are in qualitative agreement with the outcome of the numerical integration of the laser rate equations.

1 Introduction

A laser device generates or amplifies electromagnetic radiation based on the phenomenon of stimulated emission. It is basically composed of:

1. A *laser medium*: an appropriate collection of atoms, molecules, ions or a semiconductor crystal (we will refer to these elements in general as "atoms").
2. A *pumping process* that excites electrons from those atoms to upper energy levels, due to some external electrical, optical or chemical energy source.
3. *Optical feedback elements* that reflects repeatedly the radiation beam into the laser medium (in a laser oscillator), or allow it to pass only once through it (in a laser amplifier).

The working principle of laser is *stimulated emission*: an excited atom can decay to a lower state stimulated by the presence of a photon with energy equal to the difference between the two energy levels, emitting a second photon with the same frequency and propagating in the same direction. The process of absorption has the same probability, so stimulated emission dominates only when a *population inversion* is induced in the material by some pumping mechanism.

A simplified but still realistic model of many real lasers is the four-level laser system shown in Fig. 1. The population dynamics of a laser (the variation with

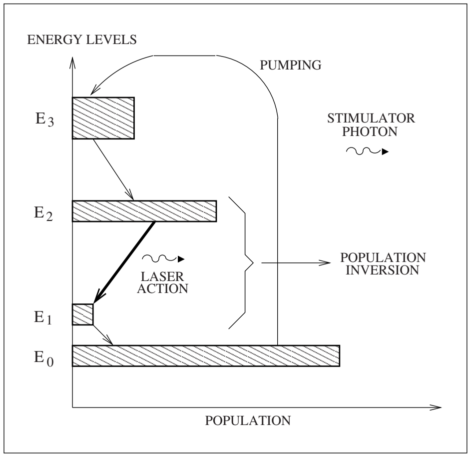

Fig. 1. Schematic view of a four-level laser system.

time in the number of laser photons and in the population inversion, or number of electrons in the upper laser level minus the number of electrons in the lower laser level) is usually described as a system of coupled differential equations called *laser rate equations.*

The rate equations can be put in its simplest form in the case in which the life times of electrons in levels E_1 and E_3 are negligible compared to the life time of electrons in level E_2. Then the electron population of level E_1 is $N_1 \simeq 0$, so the population inversion is approximately equal to the upper laser level population $N(t) = N_2(t) - N_1(t) \simeq N_2(t)$; and the absorption of laser photons by electrons at level E_1 is negligible. Additionally, the pumping into level E_2 can be described by a simple pumping rate R, which is constant for many lasers. For a pulsed laser, the pumping is time dependent, $R \equiv R(t)$.

Under these assumptions, we can take into account only levels E_0 and E_2, and write a simplified form of the laser rate equations including the spontaneous emission process, which is still realistic to describe some dynamic laser behavior -such as relaxation oscillations or gain switching-, as [1,2]:

$$\frac{dn(t)}{dt} = K\,N(t)\,n(t) - \frac{n(t)}{\tau_c} + \varepsilon\frac{N(t)}{\tau_a} \tag{1}$$

$$\frac{dN(t)}{dt} = R(t) - \frac{N(t)}{\tau_a} - K\,N(t)\,n(t) \tag{2}$$

Equation (1) gives the variation with time on the number of laser photons, $n(t)$, which is related to the laser beam intensity. The first term on its right hand side, $+KN(t)n(t)$, represents the increasing on the number of photons by stimulated emission (K is the coupling constant between radiation and the population inversion). The next term, $-n(t)/\tau_c$, introduces the decaying process of laser photons inside the cavity and τ_c is the decay time of the laser photons. The term $+\varepsilon N(t)/\tau_a$ represents the spontaneous emission process, where ε is the fraction of inversion decay processes which are radiative and for which the emited photon has the same frequency and direction of the laser radiation and τ_a is the decay time of the population inversion. This contribution was not taken into account in our previous study [3]. Here, this term has been taken into account in order to get more precise results and to be able to integrate the differential equations (1) and (2) with the initial conditions $n(0) = N(0) = 0$, because otherwise the solution for the number of photons is always $n(t) = 0$.

Equation (2) represents the temporal variation of the population inversion, $N(t)$. The term $+R(t)$ introduces the pumping of electrons with a pumping rate R to the upper laser level. The term $-N(t)/\tau_a$ represents the decaying process of the population inversion with the characteristic time τ_a. Finally, the product term $-KN(t)n(t)$ accounts for the decreasing of the population inversion by stimulated emission.

In this work, the dynamics of a pulsed pumped laser is explored. We suppose that the pumping has a pulsed form of width $2t_p$ which is greater than the life time τ_a of the population inversion. The time dependence of the pumping rate that has been used is:

$$R(t) = \begin{cases} R_m\ \phi(t); & 0 < t < 2t_p \\ 0; & t < 0, t > 2t_p \end{cases} \tag{3}$$

where R_m is the maximum value of the pumping rate at the time t_p and

$$\phi(t) = \frac{1}{2}\ [1 - \cos(\frac{\pi t}{t_p})] \tag{4}$$

The value of $R(t)$ is represented in Figure 2 with the legend "Pumping".

The purpose of this work is the study of the CA approach to model the dynamics of pulsed lasers and the comparison with the results obtained by the corresponding laser rate equations.

2 CA Model

We have recently introduced a CA model for simulating laser dynamics, which is described in reference [3]. Further details are discused in references [4,5]. This CA laser model has a similar nature to other models which have been used to describe different reactive phenomena, such as the Belousov-Zhabotinsky reaction [6] or the population dynamics of prey-predator systems [7]. In order to simulate the behavior of pulsed pumped lasers, two main modifications from the original model have been introduced in this work:

i) The pumping probability here is time dependent, whereas in the original model it was constant.
ii) The spontaneous emission process is associated with the decaying of the population inversion. This is more realistic than in the original model, where this was introduced as a random level of noise photons.

The laser system is modeled by a 2-dimensional square lattice of 400 $\times$ 400 cells with periodic boundary conditions. The lattice represents a simplified model of the laser cavity.

2.1 States of the Cells

Two variables $a_i(t)$ and $c_i(t)$ are associated to each node of the lattice. The first one, $a_i(t)$, represents the state of the electron in node i at a given time t. An electron in the laser ground state takes the value $a_i(t) = 0$, while an electron in the upper laser state takes the value $a_i(t) = 1$. A temporal variable $\tilde{a}_i(t) \in \{0, 1, 2, ..., \tau_a\}$ is also introduced, in order to take into account the finite time, τ_a, that an electron can remain in the upper state. If the electron is in the base state $\tilde{a}_i(t) = 0$, otherwise $\tilde{a}_i(t+1) = \tilde{a}_i(t) + 1$ until the maximum value τ_a is reached and then $\tilde{a}_i(t+1) = 0$.

The second variable $c_i(t) \in \{0, 1, 2, ..., M\}$ represents the number of photons in node i at time t. A large enough upper value of M is taken to avoid saturation of the system. There is also another temporal variable, $\tilde{c}_i^j(t) \in \{0, 1, 2, ..., \tau_c\}$, which measures the amount of time since a photon $j \in \{1, 2, ..., M\}$ was created at node i. τ_c is the life time of each photon. For a given photon j, $\tilde{c}_i^j(t+1) = \tilde{c}_i^j(t) + 1$ until the life time τ_c is reached and then $\tilde{c}_i^j(t+1) = 0$.

2.2 Neighborhood

The *Moore neighborhood* has been used. Nine neighbors are taken into account to compute the transition rules of any given cell: the cell itself, its four nearest neighbors and the four next neighbors.

2.3 Transition Rules

The transition rules determine the state of any particular cell of the system at time $t + 1$ depending on the state of the cells included in its neighborhood at time t. The evolution of the temporal variables $\tilde{a}_i(t)$ and $\tilde{c}_i^j(t)$ was described beforehand. Here we describe only the evolution of $a_i(t)$ and $c_i(t)$.

- **R1**. If $a_i(t) = 0$ then

$$a_i(t+1) = 1 \qquad \textit{with probability } \lambda(t)$$

  where $\lambda(t) = \lambda_m \ \phi(t)$ and λ_m is the maximum value of the pumping probability $\lambda(t)$.

– **R2**. If $a_i(t) = 1$ and $\Gamma_i(t) = \sum_{neighbors} c_i(t) > \delta$ then

$$\begin{cases} c_i(t+1) = c_i(t) + 1 \\ a_i(t+1) = 0 \end{cases}$$

where Γ_i is the number of photons included in the neighborhood of the cell i, and δ is a threshold value, which has been taken to be 1 in our simulations.

– **R3**. If $c_i(t) > 0$ and there is one photon j for which $\tilde{c}_i^j(t) = \tau_c$ then

$$c_i(t+1) = c_i(t) - 1$$

– **R4**. If $a_i(t) = 1$ and $\tilde{a}_i(t) = \tau_a$ then

$$\begin{cases} a_i(t+1) = 0 \\ c_i(t+1) = c_i(t) + 1 \qquad \textit{with probability } \theta \end{cases}$$

These transition rules represent the different physical processes at the microscopical level in a laser system. Rule **R1** represents the probabilistic electron pumping process. Rule **R2** models the stimulated emission: if the electronic state of a cell has a value of $a_i(t) = 1$ and the number of laser photons in the neighborhood is greater than a certain threshold, then at the time $t+1$ a new photon will be created in that cell and the electron will decay to the ground level. Rule **R3** represents the photon decay. Rule **R4** represents the electron decay in a similar way to rule **R3**: after a time τ_a of the excited electrons, these electrons will decay to the ground level. This rule represents both radiative and non-radiative processes: a fraction θ of the electron decay processes involves creating a new photon in the same cell. This models the spontaneous emission with a probability θ. As in an ideal four level laser the population of level E_1 is negligible, stimulated absorption has not been considered.

3 Results

The expected behaviour of a pulsed pumped laser can be otained by integration of the differential equations (1) and (2). The initial conditions that have been used are: $n(0) = N(0) = 0$ and the values of the parameters are: $\tau_a = 45$, $\tau_c = 3$, $\varepsilon = 10^{-5}$, $K = 6 \cdot 10^{-6}$. The pumping pulse parameters are: $R_m = 8000$, $t_p = 2000$. Here τ_a, τ_c and t_p are measured in *time steps*, K and R_m in $(\textit{time steps})^{-1}$.

In this study, the values of τ_a, τ_c and t_p have been chosen so that τ_a and τ_c are much smaller than t_p. In this regime, usually referred to as *quasistationary pumped laser*, the solutions of the differential equations consist of a laser emission pulse which follows the pumping pulse, and essentially no relaxation oscillations appear. Figure 2 shows the results obtained by the integration of equations (1)

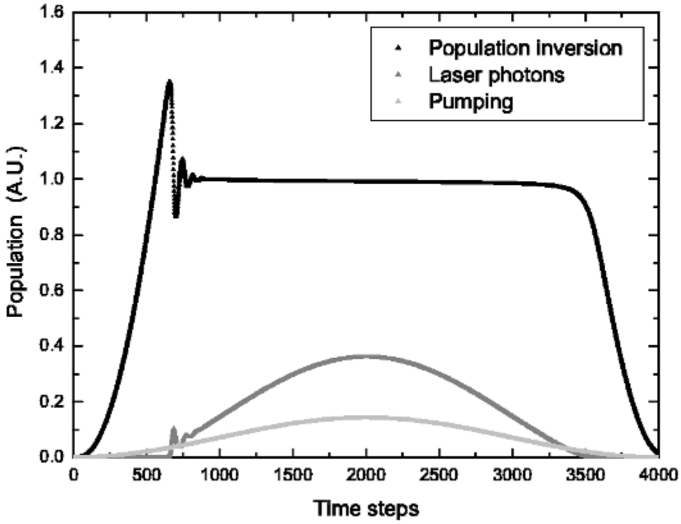

Fig. 2. Evolution of the system obtained by numerical integration of the differential equations, using the values of the parameters indicated in the text. The y-axis values have been normalized relative to the population inversion of the central plateau. The curve with the legend "Pumping" is the value of $R(t)$.

and (2). In the initial phase, the population inversion $N(t)$ increases in response to the pumping, but until a threshold pumping is reached, stimulated emission is not important and very few laser photons are created. When this threshold is reached, the laser process is activated and the number of laser photons $n(t)$ starts to increase. When the pumping is over the threshold value, stimulated emission reduces the population inversion, so $N(t)$ reaches a maximum. After that, in the differential equations solutions $N(t)$ reaches a stable value and $n(t)$ increases towards a maximum, to decrease again, following the pumping pulse. Finally, when the pumping decreases down the threshold value, stimulated emission is no longer important so $n(t)$ decays to zero. After that, $N(t)$ decreases following the tail in the pumping, to reach a zero value when the pumping falls to zero.

For pulsed pumped lasers the dynamics of $n(t)$ and $N(t)$ are strongly dependent on the pumping function $R(t)$. So in order to compare the output of the laser rate equations with the CA we must stress that the pumping process has a probabilistic nature following rule **R1** for the CA, whereas for the laser rate equations the pumping is a smooth function. Then a comparison between both outputs should be made only from a qualitative point of view, because the pumping is not exactly the same.

A typical result by the CA simulation is shown in Figure 3. For this simulation $\lambda_m = 0.1$ and $\theta = 0.01$. Initially there is no laser photon and all

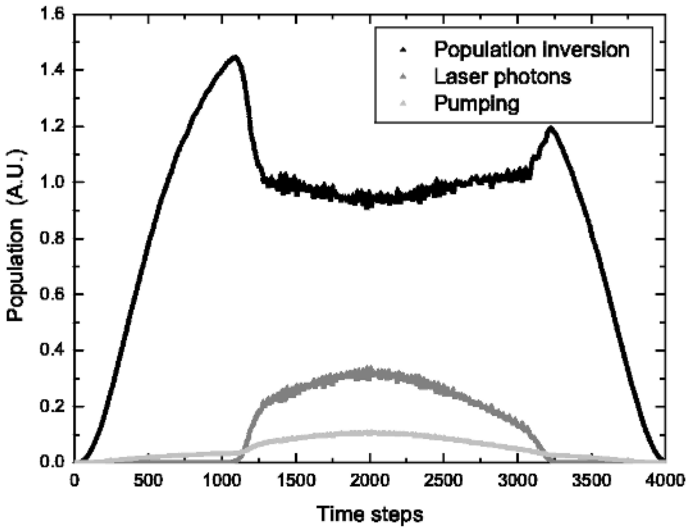

Fig. 3. Evolution of the system obtained by running a simulation using the cellular automata model, for values of the parameters equivalent to those of Figure 2. The y-axis values have been normalized relative to the population inversion of the central plateau. The curve with the legend "Pumping" is the number of electron cells which are pumped by rule **R1** in each time step.

the atoms are in the ground state $(a_i(0) = 0,\ c_i(0) = 0)$. The results of the CA simulation reproduce in a qualitative way the behavior exhibited by the solutions of the differential equations. It is remarkable that the CA model reproduces the plateau in the population inversion, which is one of the main features predicted by the equations: after the laser is above threshold, the population inversion remains approximately constant. Another feature which is reproduced by the CA model is the peak in the population inversion when the threshold pumping is reached. A similar -but smaller- peak appears in the CA simulation in the phase in which the pumping is decreasing, after the pumping threshold is surpassed. This peak does not appear in the equations solutions, but has a direct physical interpretation: after the stimulated emission decays to zero, it doesn't contribute to reduce the population inversion, so there is a net increase in $N(t)$, until it decreases again due to the fall in the pumping.

Finally we have found that the results from the integration of equations (1) and (2) are not very sensitive on the values of the parameter ε. By running different simulations changing the value of the equivalent parameter in the CA model (θ), it can be observed that the results are not very sensitive on this value either, in good agreement with the differential equations behavior.

4 Conclusions

Cellular automata models like the one used in this work have an interesting heuristic nature, which helps to understand the physical phenomena being modeled, in a more direct way than the differential equations [8,9]. In addition, this kind of models can be useful as an alternative modeling tool for laser dynamics when numerical difficulties arise, for example in lasers governed by stiff differential equations, with convergence problems. Due to their intrinsic parallel nature, they can be implemented in parallel computers and offer a great advantage in computing time [10,11].

In this work, we report a work in progress involving modifications in a previously introduced cellular automata model of laser dynamics, in order to simulate pulsed pumped lasers. We have compared the results obtained by the numerical integration of the laser rate equations for a quasistationary pulsed pumping, with those resulting from a simulation by a cellular automata model of this type of laser. The results of the CA model reproduce in a qualitative way the behavior exhibited by the numerical integration of the equations. After these preliminary results, more work will be done in order to compare both results in a more quantitative way.

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