

Direct reactions of weakly-bound nuclei within a one dimensional model

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Abstract. A line of research has been developed to describe structure and dynamics of weakly-bound systems with one or more valence particles. To simplify the problem we are assuming particles moving in one dimension and, despite the drastic assumption, the model encompasses many characteristics observed in experiments. Within this model we can describe, for example, one- and two-particle breakup and one- and two-particle transfer processes. We concentrate here in models involving weakly-bound nuclei with just one valence particle. Exact solutions obtained by directly solving the time-dependent Schroedinger equation can be compared with the results obtained with different approximation schemes (coupled-channels formalism, continuum discretization, etc). Our goal is to investigate the limitations of the models based on approximations, and in particular to understand the role of continuum in the reaction mechanism.

1. Introduction

In medium-light mass region of nuclear chart the drip-lines are characterized by the presence of the so-called halo nuclei. These systems can be described in terms of a core surrounded by one or more weakly-bound nucleons and present different features with respect to well-bound nuclei. While in a mean-field description bound systems do not fill all bound single-particle levels and their excitation basically promotes nucleons to empty bound levels, weakly-bound systems fully occupy all available bound states and so their excitation necessarily involve the promotion to the continuum with subsequent emission. For this reason the description of halo nuclei is more involved (in particular for more than two valence particles), even considering inert cores.

In this contribution we study direct reactions (i.e. elastic and inelastic scattering, transfer and breakup) involving nuclei with just one valence neutron. One could model the reaction following the time evolution of the wavefunction of this neutron: starting from an initial wavefunction $\Psi(\vec{r}, t_i)$ describing the system at $t_i = -\infty$, to obtain at $t_f = +\infty$ the final wavefunction $\Psi(\vec{r}, t_f)$. The projection of the final wavefunction on the different channels will lead to the probability for the different processes. The problem is traditionally solved in the so-called coupled-channels approach (or in its lower order approximations) in which the wavefunction of the system is expanded in a certain basis, and the solution of the problem moves to the study of the time evolution of the expansion coefficients. When this scheme is applied to weakly-bound systems, one has to deal with continuum: this involves a procedure of defining, discretizing and truncating



the continuum part of the basis. The different discretization methods involve approximations and subsequent limitations that should be tested. Our proposal here is to test these approximations reducing the problem to one dimension. In this simplified case we can easily follow the time evolution for the exact total wavefunction $\Psi(x, t)$ and for the expansion coefficients, thus checking the validity of the different approximations connected to the discretization methods.

2. The model

Let us start by specifying the model Hamiltonian and the initial conditions.

The system is described by the Hamiltonian

$$\mathcal{H}(x, t) = -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial x^2} + V_T(x) + V_P(x - x_P(t)), \quad (1)$$

where we include two potentials V_T and V_P , chosen with a Woods-Saxons shape and associated to target and to projectile respectively. In our “semiclassical” model the target is at rest and only the projectile potential moves according to a fixed trajectory¹

$$x_P(t) = x_0 + \sqrt{\rho^2 + (vt)^2} - \rho, \quad (2)$$

that accounts for the target-projectile relative motion with asymptotic energy $E_P = 1/2\mu v^2$ and a distance of closest approach x_0 . The variation of these two parameters can simulate different kinematical conditions due to different bombarding energies and impact parameters (partial waves), while the choice of the parameters of the two potentials accounts for the different masses of the colliding nuclei, the Q-values of the different final channels as well as the possibility of simulating weak-binding conditions.

By solving separately the time independent Schroedinger equation for each well we obtain two asymptotic sets of bound levels [1]; depending on the kind of reaction under study we will select one of these levels as the initial single-particle wavefunction $\Psi(x, t_i)$. For example, to describe a pick-up reaction we will choose as initial state a single-particle level in the target.

We can then proceed with the “exact” time evolution of the function $\Psi(x, t)$ by directly solving the time dependent Schroedinger equation [2, 3, 4]

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \mathcal{H}(x, t) \Psi(x, t). \quad (3)$$

At the end of time evolution we can evaluate the final probabilities for each reaction channel by projecting the final wavefunction onto the bound levels of each well (target states for elastic and inelastic scattering, projectile states for transfer channel)

$$\mathcal{P}_{elastic} = |\langle \Phi_{g.s.}^T(x) | \Psi(x, t_f) \rangle|^2, \quad (4)$$

$$\mathcal{P}_{inelastic} = |\langle \Phi_{i \neq g.s.}^T(x) | \Psi(x, t_f) \rangle|^2, \quad (5)$$

$$\mathcal{P}_{transfer} = |\langle \Phi_i^P(x) | \Psi(x, t_f) \rangle|^2; \quad (6)$$

we can also measure breakup probability either by direct subtraction

$$\mathcal{P}_{breakup} = 1 - \mathcal{P}_{elastic} - \mathcal{P}_{inelastic} - \mathcal{P}_{transfer}, \quad (7)$$

¹ According to the trajectory used, the projectile is changing its velocity v over the distance ρ , thus simulating the nuclear interaction with the target; in fact at $t = \pm\infty$ the trajectory tends to a uniform motion with constant velocity.

or by projection onto proper continuum states $\Phi(x, k)$ with momentum k

$$\mathcal{P}_{breakup} = \int |\langle \Psi(x, t_f) | \Phi(x, k) \rangle|^2 dk. \quad (8)$$

The same problem can then be solved along the popular framework used to describe a variety of reaction channels, namely the coupled-channels approach. To this end the system wavefunction is expressed as a combination of target and projectile states

$$\Psi(x, t) = \sum_{j=1}^{N_T} c_j^T(t) \Phi_j^T(x) e^{iE_j^T t/\hbar} + \sum_{j=1}^{N_P} c_j^P(t) \Phi_j^P(x) e^{iE_j^P t/\hbar}, \quad (9)$$

and, following reference [5], the solution of the problem is reduced to the time evolution of the coefficients $c_i(t)$ within the system of differential coupled equations

$$\begin{aligned} i\hbar \frac{\partial c_j^T}{\partial t} &= \sum c_k^T \langle \omega_j^T | V^P | \Phi_k^T \rangle + \sum c_k^P \langle \omega_j^T | V^T | \Phi_k^P \rangle, \\ i\hbar \frac{\partial c_j^P}{\partial t} &= \sum c_k^T \langle \omega_j^P | V^P | \Phi_k^T \rangle + \sum c_k^P \langle \omega_j^P | V^T | \Phi_k^P \rangle. \end{aligned} \quad (10)$$

Since the two bases are not orthonormal we introduce a dual basis $\omega_j^{T,P}$ associated to each well, which is time dependent and is defined in terms of overlaps between the two bases states. In this way states of a dual basis are orthonormal to states of the potential basis

$$\langle \Psi_m^I | \omega_n^J \rangle = \delta_{I,J} \delta_{n,m}. \quad (11)$$

In order to describe breakup processes we also include continuum states, following different discretization procedures. At the end, the probability to populate the different final channels is measured as

$$P_j^{(T,P)} = |c_j^{(T,P)}(t_f)|^2. \quad (12)$$

Note that this has to be calculated only for $t_f = +\infty$ since, due to non-orthonormality of the basis, the total probability is not conserved during the collision. Only at the end, when overlaps between target and projectile states vanish, the total probability gets unitary again.

3. Results

By comparing the “exact” and coupled-channels results, we can check the necessary approximations (such as bases truncation, continuum discretization, ...) which are particularly relevant in case of weakly-bound systems. For this reason we have compared results obtained for well-bound and weakly-bound systems in a variety of situations [6].

We report here two specific cases. For the well-bound case we choose a situation in which the initial state is bound by an energy of -7.39 MeV, as it is presented in figure 1. The projectile asymptotic energy is fixed at 1 MeV. In figure 2 we show the potentials (lower panel) and the final squared wavefunction (upper panel), obtained with different methods, at the end of the time evolution.

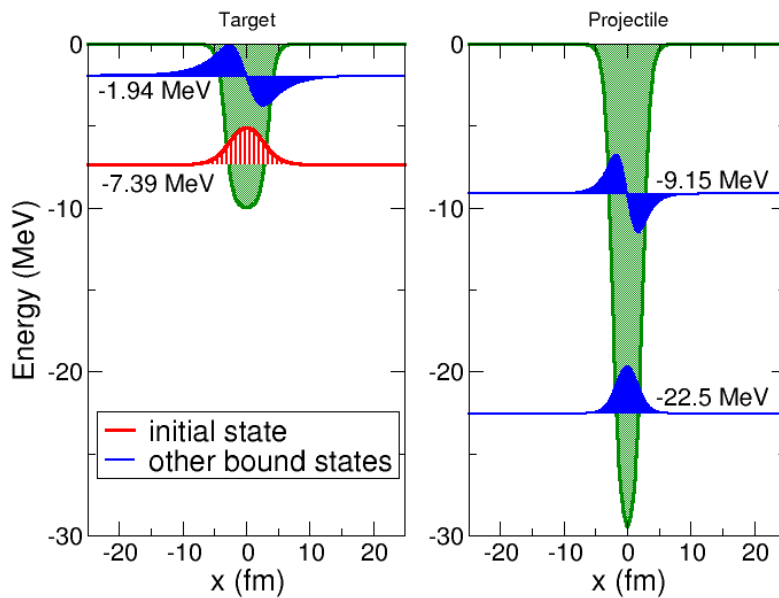


Figure 1: The target (left) and projectile (right) potentials with their bound states and respective binding energies. In red is highlighted the initial state of the reaction in the bound case.

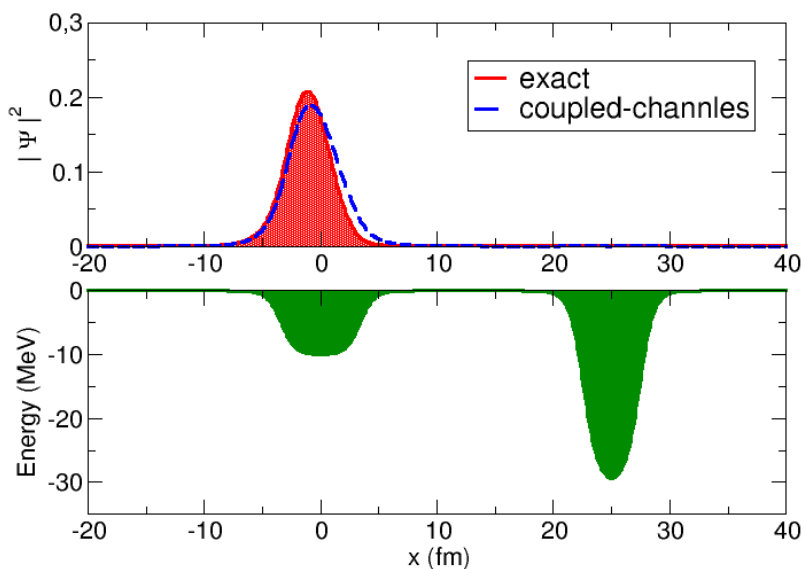


Figure 2: Lower panel: the target and projectile potentials at the end of the time evolution. Upper panel: the final squared “exact” wavefunction (red) and the final squared wavefunction obtained with the coupled-channel method (blue).

The results obtained by solving the time-dependent Schroedinger equation and those obtained in the coupled-channels formalism without including the continuum in the bases are listed in table 1. The two methods agree quite well, which means that in this bound case the continuum does not have a crucial role in the reaction mechanism.

Table 1: Results for the different reaction channels for the well-bound case obtained by solving the time dependent Schroedinger equation (*“Exact”*) and the coupled-channels formalism without including the continuum (*“Coupled-channels - no continuum”*).

	Exact	Coupled-channels - no continuum
Elastic	83%	84%
Inelastic	14%	16%
Transfer (g.s)	0.6%	0.002%
Transfer (1)	0.03%	0.2%
Breakup	2.4%	-

By following the time evolution of the probability calculated using equation (12), presented in figure 3, we can notice the non-conservation of total probability during collision, but we can also see the convergence to the final value after the collision, restoring the unitarity of the problem. This is due to non-orthonormality of the separated target and projectile bases, as it was mentioned in the previous Section 2.

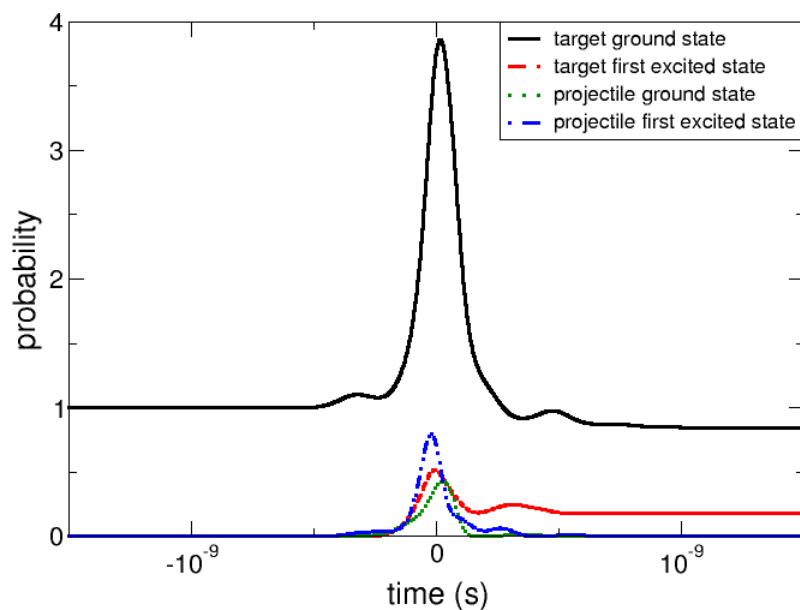


Figure 3: Time evolution of the probability for the coupled-channels method in the bound case.

To simulate the drip-line conditions we consider now a weakly-bound case as the one illustrated in figure 4. As initial state we choose a particle moving in the target weakly-bound ground state with energy -0.28 MeV. The projectile asymptotic energy is fixed at 5 MeV. In figure 5 we present the final “exact” wavefunction for this weakly-bound case.

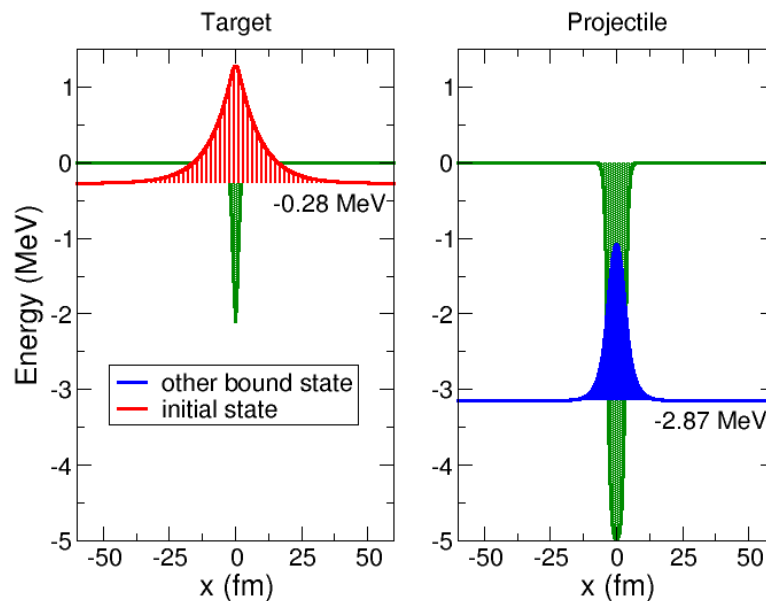


Figure 4: The target (left) and projectile (right) potentials with their bound states and respective binding energies. The squared initial wavefunction is highlighted in red.

In table 2 we show the final probabilities for each reaction channel, obtained applying different methods. First of all, by following the “exact” time evolution, we find that the breakup probability is 74%, and in fact looking at figure 5 the wavefunction is spread in the continuum. Therefore, the coupled-channels solution with only bound states does not give a satisfactory response: in this case the inclusion of the continuum is mandatory.

By adding the continuum part to the target basis we get close to the “exact” results. In this specific case we have defined the continuum states in a range of radius 40 fm centered on the target potential (note that target functions can not overlap with projectile states at the end of the time evolution because of model definition), and we have discretized the basis by imposing boundary conditions generated by an infinite square well.

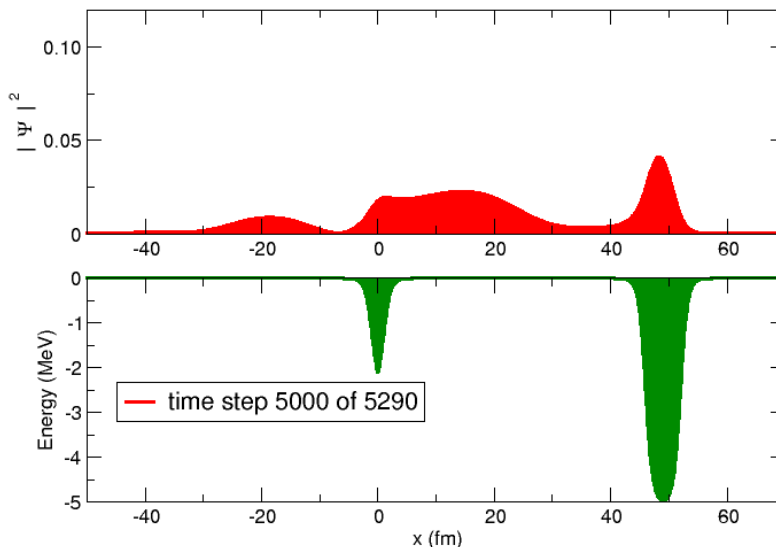


Figure 5: Lower panel: the position of the target and projectile potentials at the end of the time evolution. Upper panel: the final squared “exact” wavefunction.

Table 2: Results for the population of the different reaction channels in the weakly-bound case obtained by solving the time dependent Schroedinger equation (“*Exact*”), the coupled-channels formalism without including the continuum (“*Coupled-channels - no continuum*”), and the coupled-channels formalism including the lowest fifty continuum states (“*Coupled-channels - target continuum*”).

	Exact	Coupled-channels - no continuum	Coupled-channels - target continuum
Elastic	21%	95%	21%
Transfer	5%	5%	0.04%
Breakup	74%	-	79%

It is important to note that continuum waves can be obtained in other different ways: through the use of pseudostates constructed by diagonalizing the potentials in different bases (harmonic oscillator, transformed harmonic oscillator, ...), or grouping oscillating waves into bins in a CDCC-like method; the procedure choice affects the results.

The duration of the time evolution is also fundamental not to compromise the correct solution. If it is too short, the overlap between target and projectile bases is not zero when we calculate the final probabilities. On the contrary, if the system evolution is too long, the continuum component of the final wavefunction could leave the potentials area and go outside the range where we defined the bases.

4. Conclusions

We have developed a one dimensional model to describe direct reactions involving halo nuclei with one valence neutron. The approach consists in following the time evolution of the system total wavefunction by solving the time dependent Schroedinger equation, and the time evolution along a coupled-channels approach. By comparing the two results we can clarify the validity of the necessary approximations: basis truncation and continuum discretization. The latter point is particularly relevant in the case of weakly-bound systems where the inclusion of the continuum is practically unavoidable.

References

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